Collectibles Tokenization & Optimal Security Design

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Abstract

I develop a model of collectibles tokenization to understand whether recent tokenization efforts create or destroy value. While issuing divisible security tokens against illiquid collectibles lowers transaction costs and facilitates greater portfolio diversification, security design complications arise because ownership rights and viewing rights are necessarily separated. Current efforts squander the viewing rights, likely making tokenization welfare-reducing, thereby explaining limited adoption. Instead, renting the viewing rights would make tokenization welfare-improving. While collectibles rental markets are immature, I show empirically that only modest rental yields are needed to make tokenization welfare-improving, which suggests this security design shortcoming can be resolved.

Keywords: Collectibles, Tokenization, Blockchain, Illiquidity, Convenience Yields, Art

JEL Codes: G11, G12, G23, Z11

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Investors have long expressed interest in collectibles, which have low correlations with other major asset classes. However, collectibles investment has remained elusive to most. Collectibles are expensive and indivisible, posing a problem for diversification, and round-trip transaction costs of 20-30% of the sales price erode already low financial returns. Financial innovation has struggled to overcome these frictions. For example, art investment funds have failed to improve liquidity or reduce transaction costs.[1] More recently, however, blockchain technology has given rise to a fundamentally different solution - tokenization.

Tokenization transfers ownership rights from a single physical asset to many digital security tokens, facilitating nearly infinite divisibility and low-cost trading on exchanges. While collectibles tokenization reduces transaction costs and facilitates improved diversification through fractionalization, security design complications arise because ownership rights and viewing rights are necessarily separated. Maecenas and TheArtToken, the first firms to successfully tokenize art, squander the viewing rights by keeping the artworks in secure storage. This outcome is puzzling because the viewing rights (ie: private-value dividends) make the collectibles valuable in the first place. Limited interest in their offerings suggests this may not be the optimal security design, although no fundamentally different alternative exists.[2]

I seek to resolve this security design problem because tokenization has the potential to improve efficiency in a large market. There are 198,342 ultra high net worth individuals (UHNWIs) with net worths exceeding $30 million and UHNWIs allocate, on average, 4% of their investment portfolios to collectibles (Knight Frank (2019)).

I begin by developing a model of collectibles tokenization. Agents can invest in two assets: collectibles and a risky stock. My model accounts for a variety of unique features of collectibles including large transaction costs, short-sales constraints, indivisibility, heterogeneous valuations of convenience yields, and the bundling of ownership rights and viewing rights. I show that current tokenization efforts are welfare-reducing if the value of the squandered

[1] The capital contribution period lasts three to five years and the fund life is an additional five to seven years with extensions at the discretion of the fund manager. In addition, transaction costs are not reduced because art funds buy and sell through third-party auction houses.

[2] Competitors including Masterworks, Otis, Monart, and CoArt do not deviate meaningfully with respect to security design.

convenience yields exceeds the savings on transaction costs and the incremental diversification benefits from fractionalization. In contrast, tokenization is unambiguously welfare-improving if the viewing rights are rented on a competitive market, which enables the token owners to receive financial dividends. This suggests a key friction impeding the spread of tokenization is the lack of well-developed collectible rental markets. Theoretically, it is clear that tokenization can be welfare-improving even if rental markets are less than perfectly competitive, although it is not clear what this sufficiency would look like in practice.

To bridge theory and practice, I empirically quantify how large rental yields would need to be for collectibles tokenization to be welfare-improving. I consider a representative agent portfolio allocation problem in which the investor allocates his wealth between the market portfolio and collectibles tokens to maximize the Sharpe ratio. The minimum rental yield needed to make tokenization welfare-improving is the increment to the observed average collectible return that makes the investor demand a dollar-value investment exceeding the total collectible float. Market clearing would then require the investor pays a premium for the collectibles tokens, reflecting welfare gains for some combination of buyers and sellers. To be conservative, I use the maximum rental yield from 48 specifications for each collectible category, assessing robustness across the version of the Sharpe ratio, the composition of the market portfolio, the data's time horizon, and the collectible category's estimated float. For the nine considered collectibles categories, the annual rental yields would only need to be 4.0-14.4% to ensure tokenization is welfare-improving. Most of these values are significantly less than the annual rental yields earned in existing art rental markets, meaning collectibles tokenization can be welfare-improving even if rental markets are far from well-functioning.

An interesting issue raised by the model is that the status quo of bundled ownership and viewing rights is inefficient when there are heterogeneous valuations of private-value dividends. For example, the world's foremost Picasso enthusiast would never purchase all of Picasso's works even if he could afford to do so - the concentrated price risk would wreak havoc on his investment portfolio. However, he would happily rent the viewing rights if these could be separated from the ownership rights (and the investment risk). This emphasizes that collectibles tokenization can improve over physical collectibles in three major ways: (1) by efficiently allocating viewing rights; (2) by reducing transaction costs; and (3) by augmenting diversification benefits through fractionalization.

1 Literature Review

The relevant literature can be divided into three categories: (1) the impact of blockchain technology on finance; (2) models of trade of real and private-value assets; and (3) mean-variance analyses of collectibles.

"Blockchain" has rapidly gained popularity in the financial press since 2015, propelled by its potential to transform the way in which finance is conducted. Asset tokenization is simply one such application of blockchain technology. The revolutionary role of blockchain technology has been more widely studied in applications including trading settlement (Chiu and Koeppl (2019)), record keeping (Abadi and Brunnermeier (2018)), contracting (Cong and He (2019)), platform finance (Cong et al. (2019a)), [crypto-]currencies (Cong et al. (2019b)), and financing entrepreneurial ventures (Chod and Lyandres (2018) and Howell et al. (2018)).

Models of trade of real and private-value assets are distinguished from other asset pricing models by explicitly accounting for unusual features of the asset. Important features of collectibles include heterogeneous agent valuations, short-sales constraints, private-value dividends, large transaction costs, and indivisibility. First, short-sales constraints can lead to overpricing when investors have heterogeneous beliefs about the asset's value (Miller (1977) and Scheinkman and Xiong (2003, 2004)). Second, private-value dividends from assets like art serve as insurance in downturns, resulting in lower equilibrium returns (Mandel (2009)). Heterogeneity in private-value dividends also leads the agents with the highest valuations to pay more on average and be more likely to sell in distress at a lower price (Penasse and Renneboog (2017) and Lovo and Spaenjers (2018)). Third, larger transaction costs lead to less frequent trade (Grossman and Laroque (1990)) and may actually increase prices by lengthening holding periods and thereby reducing required risk premia (Vayanos (1998)). Last, indivisibility prevents agents from optimizing and ensuring their first-order conditions hold with equality (Robison and Barry (1980)), which makes welfare increase in the degree of divisibility (Taber and Wallace (1999)). My model seeks to understand how all of these features interact, especially in the context of asset tokenization.

Empirically, a key feature of collectibles returns is positive autocorrelation stemming from infrequent valuations. Positive autocorrelation leads volatility to be underestimated and correlation to be misestimated. It is thus concerning that previous mean-variance analyses of collectibles have largely failed to account for autocorrelated returns in estimating variances and correlations.[3] Notably, Campbell (2008) and Dimson and Spaenjers (2011) use the Geltner (1993) procedure to unsmooth art returns and stamp returns, respectively. However, I find that the strong assumptions of the procedure are strongly violated for all collectibles returns in my sample. I develop a model-free estimator of autocorrelation-adjusted variances & covariances to more appropriately address the collectibles return autocorrelation, yielding results that should not necessarily be compared to the results in prior studies.

My paper contributes to the literature in two primary ways. First, it provides a tractable theoretical model to understand the strengths and weaknesses of the security design used in current tokenization efforts. In particular, my model identifies a major security design shortcoming and proposes a welfare-improving solution. Second, my paper provides an empirical assessment of the size of a key friction impeding collectibles tokenization efforts, showing it to be surmountable.

2 Theoretical Model

I motivate the model with a discussion of key frictions in the collectibles market. I then explain the security design used for the world's first successful case of collectibles tokenization; this is precisely the security design I will model. Last, I present the model and use it to understand the strengths and weaknesses of the security design employed in current collectibles tokenization efforts.

[3] Such papers focusing on art include Tucker et al. (1995), Worthington and Higgs (2003, 2004) , Mei and Moses (2005), Kräussl and Logher (2010), and Vecco et al. (2015). These problems also plague studies of wine by Masset and Weisskopf (2010) and Dimson et al. (2015).

2.1 Collectibles Market Frictions

The key frictions in the collectibles market are large transaction costs, indivisibility, and the bundling of ownership rights and viewing rights.

I focus on auction transaction costs because the secondary market for collectibles is predominantly intermediated by auction houses. Round-trip transaction costs, the sum of the buyer's premium and the seller's commission, are typically 20-30% of the sales price, which substantially hinders trade.[4]

Collectibles are indivisible and can be quite expensive, which creates challenges for obtaining diversified investment exposure to collectibles. For example, any investment portfolio containing Leonardo da Vinci's $450 million "Salvator Mundi" or the $71 million Pink Star 59.6 carat diamond cannot help but be undiversified. Nevertheless, Petterson and di Torcello (2017) document that investors are seeking investment exposure to collectibles and are pressuring wealth managers to incorporate collectibles into their wealth management offering.

Last, rental markets for collectibles are fairly immature, leading collectibles' ownership rights and viewing rights to be inseparable. Since collectibles are ultimately priced as the present value of expected future private-value dividends, this inseparability leads to inefficient outcomes if the owner of the collectible and the piece's biggest enthusiast (with the highest valuation of the private-value dividends) are not the same person. Even greater inefficiency arises if the owner squanders the viewing rights, which give the collectible its value in the first place. Surprisingly, this problem is actually quite commonplace. The largest museums display only about 5% of their collection at any one time, keeping the rest in secured storage (Groskopf (2016)). In addition, many individuals evade sales and use taxes by keeping their collectibles in secured storage facilities in freeports and custom-free zones.[5]

[4] For example, Penasse and Renneboog (2017) find that total transaction costs are minimally 20% of the sales price while Campbell (2008) notes that total transaction costs can be as much as 30%. Kräussl and NasserEddine (2018) shed light on this by reporting auction house-level fees, which are fairly standardized across major auction houses in key international cities. At the end of 2015, Christie's New York marginal buyer's premium was 25% of the sales price up to $100,000, 20% up to $2 million, and 12% above $2 million. While the seller's commission has generally been estimated at 10% of the sales price, it is negotiable and can sometimes be waived.

[5] For example, Switzerland's Federal Audit Office estimated goods worth $103 billion were being held in

2.2 Security Design for Tokenization

The tokenization in my model is based on the first successful instance of collectible tokenization. In July 2018, Maecenas tokenized Andy Warhol's (1980) "14 Small Electric Chairs," selling a 31.5% stake to 100 investors for $1.7 million. The 1 million security tokens, which can be fractionalized up to 18 decimal places, trade on an exchange (ATEX).

I illustrate the four main steps of tokenization in Figure 1. While this explanation is grounded in the experience of Maecenas, other companies pursuing collectibles tokenization differ only trivially in their security design.

- **Step 1**: Verify the authenticity of the collectible, document its condition, insure it, and then store it in a secure location. Maecenas used the Andy Warhol Art Authentication Board in New York for the certification and currently has the artwork in secure storage in Switzerland.[6]
- **Step 2**: Mint digital security tokens representing fractional ownership of the collectible and transfer the ownership rights to the security tokens. All of the information about the collectible (eg: provenance, location, insurance, independent valuation, condition, verification) is recorded on the blockchain in an immutable and transparent public record. The blockchain will also record ownership and transfers of ownership, facilitating the transfer of the authentication in a frictionless way.
- **Step 3**: The security tokens are sold on the primary market through an auction. Maecenas had over 800 bidders in its dutch auction, with 100 securing partial ownership.
- **Step 4**: The security tokens can be bought and sold on a secondary market (ie: exchange).

such Swiss facilities (Perman (2015)).

[6] Maecenas has indicated on its website that it intends for its artworks to be stored in a purpose-built storage facility with high-security access and optimal conditions for preserving art (humidity, lighting, temperature) to lower the risk of damage or theft. Maecenas has also indicated that it is possible that works could be stored at museums or certain galleries given they meet certain guidelines and are approved by the insurance provider.

2.3 Theoretical Model

I consider the market for two types of assets: collectibles and a risky stock. The model has one period with two dates: $t = 0, 1$. The value of the collectible at $t = 1$ is composed of three pieces:

$$\widetilde{C} = (\underbrace{\eta}_{\text{Intrinsic Value}} - \underbrace{v}_{\text{Verification Cost}})\mathbb{I}\{\text{Verify}\} + \underbrace{d_i}_{\text{Emotional Dividend}} + \underbrace{\epsilon}_{\text{Unobservable Value}} \quad \text{s.t. } \epsilon \sim \mathcal{N}(0, \sigma_C^2)$$

To reflect the fact that collectibles only have intrinsic value if their authenticity and rarity is verified, I specify that the intrinsic value can only be obtained by paying a verification cost. This verification cost is the model-equivalent of transaction costs, mimicking the role of auction houses (eg: Christie's, Sotheby's) in charging transaction fees to primarily guarantee authenticity. As the secondary market is primarily intermediated by auction houses, I assume that agents always choose to verify the authenticity of the collectible, meaning the verification cost v is always paid and the intrinsic value η is always received.

Agents differ only in their valuation of the emotional dividend d_i, so I index agents by their mean valuation $\mu_i^C \equiv \eta - v + d_i$. I assume that μ_i^C is uniformly distributed around μ in an interval $[\mu - \kappa, \mu + \kappa]$. This leads to a continuum of agent types ranging from enthusiasts (with high emotional dividends d_i) to investors (with low emotional dividends d_i). I also assume that all agents can borrow or lend at a risk-free interest rate of zero, short-sales of the collectible are prohibited, and the total supply of the collectible is Q.[7]

There is also a risky stock with liquidation value at $t = 1$ of:

$$\widetilde{S} \sim \mathcal{N}(\mu_S, \sigma_S^2)$$

The risky stock can be sold short and the total supply of the risky stock is Y.

At $t = 0$, each agents chooses his asset demand to maximize his expected utility of

[7] I assume there are many collectibles that are both distinct and identically valued by each agent. This allows me to avoid explicitly modeling the indivisibility of collectibles. I still account for the improved divisibility following tokenization by assuming agents take advantage of fractionalization to construct more diversified collectibles portfolios, which reduces the volatility of collectibles returns.

terminal wealth at $t = 1$:

$$\max \; \mathbb{E}\left[- e^{-\gamma W_1} \right] \tag{1}$$

where γ is the agent's risk aversion and W_1 is the agent's terminal wealth.

I use this model to study the market equilibrium in 3 different settings: (1) only the risky stock exists; (2) both the risky stock and physical collectibles exist; and (3) both the risky stock and tokenized collectibles exist.

2.3.1 Equilibrium 1: Only Risky Stock

When only the risky stock exists, equation (1) specializes to

$$\begin{aligned} &\max_{x_i^S} \; \mathbb{E}\left[- e^{-\gamma W_1} \right] \\ \text{s.t.} \quad & W_1 = W_0 + x_i^S(\widetilde{S} - p_0^S) \end{aligned}$$

where x_i^S is the agent's demand for the risky stock, W_0 is the agent's initial wealth, and p_0^S is the market price of the risky stock. The maximization problem is identical for all agents. It is immediate that

$$x_i^S = \frac{\mu_S - p_0^S}{\gamma \sigma_S^2}$$

The market clearing condition $\int_i x_i^S \frac{d\mu_i}{2\kappa} = Y$ then implies the equilibrium price and allocation.

Proposition 1. *When only the risky stock exists, the equilibrium price and allocation are given by*

$$\begin{aligned} p_0^S &= \mu_S - \gamma \sigma_S^2 Y \\ x_i^S &= Y \end{aligned}$$

2.3.2 Equilibrium 2: Risky Stock and Physical Collectibles

When both the risky stock and the physical collectibles exist, equation (1) specializes to

$$\begin{aligned} &\max_{x_i^C, x_i^S} \ \mathbb{E}\left[-e^{-\gamma W_1} \right] \\ \text{s.t.} \quad & W_1 = W_0 + x_i^C(\widetilde{C} - p_0^C) + x_i^S(\widetilde{S} - p_0^S) \\ & x_i^C \geq 0 \end{aligned} \tag{2}$$

where x_i^C is the agent's demand for the collectibles and p_0^C is the market price of the collectibles. Accounting for the short-sales constraint, the agent's demand for the physical collectibles is

$$x_i^C = \max\{\frac{\mu_i^C - p_0^C}{\gamma\sigma_C^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(\mu_S - p_0^S)}{\gamma\sigma_C\sigma_S(1-\rho_{CS}^2)}, 0\}$$

In the absence of the short-sales constraint and if collectibles were the only asset, the demand would be $\frac{\mu_i^C - p_0^C}{\gamma\sigma_C^2}$, analogous to the demand for the risky stock when only the risky stock exists. Aside from the effect of the short-sales constraint, the agent is now concerned with using the second asset to reduce the volatility of his portfolio.

The agent's demand for the risky stock is

$$x_i^S = \begin{cases} \frac{\mu_S - p_0^S}{\gamma\sigma_S^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(\mu_i^C - p_0^C)}{\gamma\sigma_C\sigma_S(1-\rho_{CS}^2)} & , \text{ if } x_i^C > 0 \\ \frac{\mu_S - p_0^S}{\gamma\sigma_S^2} & , \text{ if } x_i^C = 0 \end{cases}$$

For tractability, I assume the short-sales constraint never binds (ie: $x_i^C > 0 \ \forall i$). Equivalently, this is an assumption that all agents *choose* to participate in the collectibles market. This requires that the heterogeneity of agents' emotional dividend valuations is sufficiently small so that $\kappa < \gamma\sigma_C^2(1-\rho_{CS}^2)Q$ holds. The market clearing conditions $\int_{\mu-\kappa}^{\mu+\kappa} x_i^C \frac{d\mu_i}{2\kappa} = Q$ and $\int_{\mu-\kappa}^{\mu+\kappa} x_i^S \frac{d\mu_i}{2\kappa} = Y$ then imply the equilibrium prices and allocations.

Proposition 2. *When both the risky stock and physical collectibles exist, and assuming all agents demand positive amounts of collectibles, the equilibrium prices and allocations are given* *by*

$$
\begin{aligned}
p_0^C &= \mu - \gamma\rho_{CS}\sigma_C\sigma_S Y - \gamma\sigma_C^2 Q \\
p_0^S &= \mu_S - \gamma\rho_{CS}\sigma_C\sigma_S Q - \gamma\sigma_S^2 Y \\
x_i^C &= Q + \frac{\mu_i^C - \mu}{\gamma\sigma_C^2(1-\rho_{CS}^2)} \\
x_i^S &= Y - \frac{\rho_{CS}(\mu_i^C - \mu)}{\gamma\sigma_S\sigma_C(1-\rho_{CS}^2)}
\end{aligned}
$$

2.3.3 Equilibrium 3: Risky Stock and Tokenized Collectibles

As is standard with tokenization, I now assume that the ownership rights and viewing rights are separated when the collectibles are tokenized. I consider two cases in which: (1) the viewing rights are squandered, consistent with Maecenas' storage of the Warhol painting; and (2) the viewing rights are rented in a perfectly competitive market. In both cases, the verification cost is only paid when the assets are originally tokenized and never again when they are traded, so I assume the tokens already exist, meaning the intrinsic value η is obtained without paying verification cost v. When the viewing rights are squandered, $\mathbb{E}[\widetilde{C}] = \eta = \mu - \kappa + v$. When the viewing rights are rented in a perfectly competitive market, then since d_i is known, $\mathbb{E}[\widetilde{C}] = \eta + \max_i d_i = \mu + \kappa + v$. Since this expected value is a constant in either case, I will denote it z before specializing to one of the two considered cases.

As security tokens can be fractionalized up to 18 decimals, tokenization also facilitates more diversified holdings of collectibles. As this greater diversification would lower the volatility of collectibles returns, I denote the volatility of the tokenized collectibles σ_{CT}. This allows for the possibility that $\sigma_{CT} < \sigma_C$, that is, the volatility of the agent's portfolio of tokenized collectibles is less than the volatility of the agent's portfolio of physical collectibles.

The agent's maximization problem is identical to equation (2) with the new definition of $\mathbb{E}[\widetilde{C}] = z$. The key difference is that the heterogeneous agents now value the collectibles tokens identically. It is immediate that

$$
x_i^C = \max\{\frac{z - p_0^C}{\gamma\sigma_{CT}^2(1-\rho_{CS}^2)} - \frac{\rho(\mu_S - p_0^S)}{\gamma\sigma_S\sigma_{CT}(1-\rho_{CS}^2)}\ ,\ 0\}
$$

$$x_i^S = \begin{cases} \frac{\mu_S - p_0^S}{\gamma\sigma_S^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(z-p_0^C)}{\gamma\sigma_{CT}\sigma_S(1-\rho_{CS}^2)} & \text{, if } x_i^C > 0 \\ \frac{\mu_S - p_0^S}{\gamma\sigma_S^2} & \text{, if } x_i^C = 0 \end{cases}$$

Since agents' demand functions are identical and the supply of both types of assets is fixed, market clearing will ensure that prices adjust so that the short-sales constraint never binds (ie: $x_i^C > 0 \; \forall i$). The market clearing conditions then imply the equilibrium prices and allocations.

Proposition 3. *When both the risky stock and tokenized collectibles exist, the equilibrium prices and allocations are given by*

$$\begin{aligned} p_0^S &= \mu_S - \gamma\rho_{CS}\sigma_S\sigma_{CT}Q - \gamma\sigma_S^2 Y \\ p_0^C &= z - \gamma\rho_{CS}\sigma_S\sigma_{CT}Y - \gamma\sigma_{CT}^2 Q \\ x_i^S &= Y \\ x_i^C &= Q \end{aligned}$$

When the viewing rights are squandered, $z = \mu - \kappa + v$, and when the viewing rights are rented, $z = \mu + \kappa + v$.

2.3.4 Welfare Analysis

To understand the relative desirability of these four equilibria, I compare the agents' aggregate welfare

$$\int_i \left(x_i^C \, \mathbb{E}[\widetilde{C}] + x_i^S \mu_S - \frac{\gamma}{2} \left[(x_i^C)^2 \sigma_C^2 + (x_i^S)^2 \sigma_S^2 + 2 x_i^C x_i^S \rho_{CS}\sigma_C\sigma_S \right] \right) \frac{d\mu_i}{2\kappa}$$

where the omission of equilibrium prices reflects the implicit assumption that aggregate welfare is the sum of buyers' welfare and sellers' welfare. To ensure the various equilibria are comparable, I assume that the supply of the risky stock is equal to the supply of the collectibles (ie: $Q = Y$). In addition, for the first equilibrium with only the risky stock, I assume that the supply of the risky stock is equal to $2Y$; this ensures the total amount of assets is the same across all four equilibria.

I start by understanding whether any investment in physical collectibles is desirable.

Proposition 4. *Assume, as previously, that $\kappa < \gamma\sigma_C^2(1-\rho_{CS}^2)Y$. Suppose equilibrium 1 (only risky stocks) has total supply $2Y$ of the risky stock and equilibrium 2 (risky stocks and physical collectibles) has Y of the risky stock and Y of the physical collectibles. Equilibrium 2 improves aggregate welfare over equilibrium 1 if*

$$\underbrace{\frac{\gamma}{2}Y^2(3\sigma_S^2 - \sigma_C^2 - 2\rho_{CS}\sigma_C\sigma_S)}_{\textit{Diversification Benefits}} + \underbrace{\frac{\kappa^2}{6\gamma\sigma_C^2(1-\rho_{CS}^2)} + Y\kappa}_{\textit{Convenience Yields}} > \underbrace{Y\left[\mu_S - (\mu - \kappa)\right]}_{\textit{Lower Average Financial Return}}$$

The addition of collectibles (or, in this comparison, the replacement of half of the risky stock with physical collectibles) is welfare-improving if the collectibles' diversification benefits and convenience yields outweigh the cost of lower average financial returns. This provides intuition for the existence of investor-collectors - these individuals value the combination of diversification benefits and emotional dividends enough to compensate for the lower average financial returns of collectibles. Note that the higher transaction costs of physical collectibles are included in the average collectibles return μ.

I next seek to understand whether collectibles tokenization, as implemented by Macaenas and other start-ups, is an improvement over physical collectibles.

Proposition 5. *Assume, as previously, that $\kappa < \gamma\sigma_C^2(1-\rho_{CS}^2)Y$. Equilibrium 4 (risky stocks and tokenized collectibles, viewing rights squandered) improves aggregate welfare over Equilibrium 2 (risky stocks and physical collectibles) if*

$$\underbrace{-\kappa\big(1 + \frac{\kappa}{6\gamma\sigma_C^2(1-\rho_{CS})Y}\big)}_{\textit{Squandered Convenience Yields}} + \underbrace{v}_{\textit{Savings on Transaction Costs}} + \underbrace{\frac{\gamma}{2}Y\left[(\sigma_C^2 - \sigma_{CT}^2) + 2\rho_{CS}\sigma_S(\sigma_C - \sigma_{CT})\right]}_{\textit{Fractional Diversification Benefits}} > 0$$

Collectibles tokenization with squandered viewing rights is only an improvement over physical collectibles if the savings on transaction costs and the incremental diversification benefits from fractionalization exceed the value of the squandered convenience yields. Given that collectibles are principally valued for their convenience yields, it seems likely that such tokenization is actually value-destroying, which would explain limited interest in current security token offerings. For example, Maecenas could not sell a 49% stake of the Warhol painting, only 31.5%, and Maecenas quietly cancelled its plans to tokenize a Picasso painting in December 2018.

I consider whether the alternative version of collectibles tokenization, in which the viewing rights are not squandered but rather rented, is an improvement over physical collectibles.

Proposition 6. *Assume, as previously, that $\kappa < \gamma\sigma_C^2(1-\rho_{CS}^2)Y$. Then equilibrium 3 (risky stocks and tokenized collectibles, viewing rights rented) improves aggregate welfare over equilibrium 2 (risky stocks and physical collectibles) if*

$$\underbrace{\kappa\big[1-\frac{\kappa}{6\gamma\sigma_C^2(1-\rho_{CS}^2)Y}\big]}_{\text{Gains from Rentals}}+\underbrace{v}_{\text{Savings on Transaction Costs}}+\underbrace{\frac{\gamma}{2}Y\big[(\sigma_C^2-\sigma_{CT}^2)+2\rho_{CS}\sigma_S(\sigma_C-\sigma_{CT})\big]}_{\text{Fractional Diversification Benefits}}>0$$

which always holds.

Collectibles tokenization with rented viewing rights improves unambiguously over holding physical collectibles because steep verification costs are eliminated, the separation of ownership rights and viewing rights enables the viewing rights to be efficiently allocated, and additional diversification benefits can be gained from fractionalization. There is no tradeoff, only the need for well-functioning rental markets.

Last, while unsurprising, I emphasize that renting the viewing rights is much preferred to squandering them.

Proposition 7. *Equilibrium 3 (risky stocks and tokenized collectibles, viewing rights rented) improves aggregate welfare over equilibrium 4 (risky stocks and tokenized collectibles, viewing rights squandered) if*

$$\underbrace{2Y\kappa}_{\text{Gains from Renting Viewing Rights}}>0$$

which always holds.

This highlights that current tokenization efforts, which squander the viewing rights, would create much more value if the viewing rights were instead rented.

The key theoretical prediction is that collectibles tokenization with squandered viewing rights is likely welfare-reducing relative to holding physical collectibles, while collectibles tokenization with rented viewing rights is instead welfare-improving.

In practice, these two extremes amount to security tokens that pay no financial dividends (the security design status quo) and security tokens that pay financial dividends exactly equal to the agents' maximum convenience yields. Since security tokens without dividends are likely welfare-reducing while security tokens with full dividends are surely welfare-improving, a key friction to the success of collectibles tokenization is developing the infrastructure to rent the viewing rights. Theoretically, it is clear that collectible rental markets do not need to be perfectly competitive for collectibles tokenization to be welfare-improving. However, theory is not helpful for understanding how functional these rental markets need to be. To address this question, I turn to empirics.

3 Data

I construct two databases, one that contains collectibles price indices and another that contains price indices and returns for other major asset classes.

The collectibles price indices are presented in Table 1. Since collectibles are lauded for their portfolio diversification benefits, I want to ensure that covariances are accurately estimated. Given the relatively short histories of most collectibles price indices and my desire to use comparable time horizons, I consider only monthly and quarterly series.[8] My database is exhaustive in the sense that it includes all price indices that are relevant and either publicly available or available for purchase.[9]

While there are four collectibles categories that each have two price indices, I conclude that only one is usable for each category. First, the paintings and sculptures repeat-sales price indices both have higher-order return autocorrelation (eg: lags 11-13), and with short quarterly time series, autocorrelation-adjustment methods perform quite poorly. Second, the

[8] Regrettably, this means several well-constructed price indices cannot be used, including a semi-annual repeat-sales art price index by Mei and Moses (2002), an annual repeat-sales art price index by Goetzmann et al. (2011), an annual hedonic art price index by Renneboog and Spaenjers (2013), an annual repeat-sales stamp price index by Dimson and Spaenjers (2011), an annual repeat-sales wine price index by Dimson et al. (2015), and semi-annual diamond and gem repeat-sales price indices by Renneboog and Spaenjers (2015).

[9] There are several proprietary price indices that could not be acquired, namely the HAGI classic cars price indices, the PCGS 3000 coins price index, and the Wine Owners' wine price indices.

US CPI Jewelry price index contains only the basket of jewelry used in the construction of the US consumer price index (CPI), which is likely unrepresentative of the aggregate, global jewelry market. Last, the stamps repeat-sales price index is the only remaining series that is quarterly, not monthly, and it was discontinued at the end of 2012, making it only partially comparable to the other series. Thus, the price indices I will use to study the collectibles market consist of eight average-sales price indices and one repeat-sales price index (wine).

While both repeat-sales (RS) and average-sales (AS) price indices have a shared set of shortcomings,[10] RS price indices are preferred. This is because RS price indices control for quality while AS price indices do not. This issue is quite important because of substantial heterogeneity in the prices of collectibles sold at auction. In 2017, while da Vinci's "Salvator Mundi" fetched $450.3 million and Roy Lichtenstein's "Masterpiece" fetched $165 million, the median price for a Contemporary Art painting at auction houses around the world was only $1,300. To at least partially mitigate the effect of a long right tail in transaction prices, all AS price indices exclude the top and bottom 10% of transactions by sales price as in Worthington and Higgs (2004).

Table 2 presents the data used to represent the alternative investment opportunity set. I consider the perspective of a US-based investor who can invest across five major asset classes: (1) a "risk-free" asset (the 30-day US Treasury bill); (2) global fixed income; (3) global equity; (4) real estate; and (5) collectibles. I construct the value-weighted market portfolio (excluding collectibles) using Savills World Research (2018) estimates of 2017 market values for global equity ($83.3 trillion), securitized debt ($105.3 trillion), and global real estate ($280.6 trillion).[11]

[10] Both types of indices suffer from selection bias, as the underlying transactions come solely from auction markets. As argued by Goetzmann (1993), Korteweg et al. (2016), and Anderson et al. (2016), art with greater price appreciation is more likely to trade. In addition, both types of prices indices are biased upward because they do not include works that fail to sell at auction.

[11] As financial investments in commodities occur through futures, which are in zero net supply, I use a value weight of zero for commodities and thus exclude them.

3.1 Autocorrelation-Adjusted Variance & Covariance Estimation

Collectibles tend to be an understudied asset class for two primary reasons. First, as described above, price indices are known to imperfectly measure returns. Second, as explained by Andersen et al. (2017), positive autocorrelation, a prominent feature of collectibles returns, leads standard realized volatility measures to underestimate the true volatility. Dimson (1979) finds that standard realized covariance measures are also unreliable when returns are autocorrelated. As shown in Table 3, all of the quarterly collectibles returns have substantial first-, second- and higher-order autocorrelation.[12] Thus, the validity of any study of collectibles returns hinges on appropriately accounting for autocorrelation in the estimation of variances and covariances.

Much of the prior literature on collectibles returns has ignored return autocorrelation. The few papers that have addressed it have advocated "return unsmoothing."[13] The four most widely recognized unsmoothing procedures are those of Geltner (1993), Okunev and White (2003), Getmansky et al. (2004), and Amvella et al. (2010). These methods make assumptions about the relationship between the true (unsmoothed) returns and the observed (smoothed) returns in order to use the observed returns to recover the true returns. The assumptions underpinning these methods are strongly and consistently violated for all of the collectibles returns in my database. Consequently, I instead develop a model-free estimator of the unconditional return variance and covariance, extending French et al. (1987).

As daily market returns (c. 1987) tended to be autocorrelated from non-synchronous trading of securities, French et al. (1987) proposed estimating the monthly return variance as the sum of the squared daily returns plus twice the sum of the products of adjacent daily returns

$$\widehat{\sigma}_t^2 = \sum_{i=1}^{N_t} r_{ti}^2 + 2 \sum_{i=1}^{N_t - 1} r_{ti} r_{t,i+1}$$

[12] While the data I use is monthly, I present autocorrelation coefficients for all data series at a quarterly frequency to emphasize that autocorrelation is a common feature of any collectibles price index.

[13] See, for example, Campbell (2008) and Dimson and Spaenjers (2011).

where there are N_t daily returns r_{ti} in month t and i indexes the day. Implicitly, they assume the daily mean return is zero and daily returns are only autocorrelated up to one lag.

To extend their method, I estimate the monthly return variance for month t as

$$\widehat{\sigma}_t^2 = (r_t - \bar{r})^2 + 2\sum_{k=1}^{L}(r_t - \bar{r})(r_{t-k} - \bar{r})$$

where r_t is the monthly return for month t, $\bar{r}$ is the mean monthly return, and L is the number of statistically significant autocorrelation lags. The unconditional variance is then

$$\widehat{\sigma}^2 = \frac{1}{N - L - 1}\sum_{t=1}^{N-L-1}\widehat{\sigma}_t^2$$

where N is the number of monthly returns and dividing by $(N - L - 1)$ ensures the estimator is unbiased. Similarly, the monthly covariance between asset i and asset j for month t is

$$\widehat{\sigma}_{ijt} = (r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j) + \sum_{k=1}^{L}(r_{it} - \bar{r}_i)(r_{j,t-k} - \bar{r}_j) + \sum_{k=1}^{L}(r_{i,t-k} - \bar{r}_i)(r_{jt} - \bar{r}_j)$$

where r_{it} is the monthly return for asset i in month t, $\bar{r}_i$ is the mean monthly return for asset i, and L is the maximum number of statistically significant autocorrelation lags for the two return series. The unconditional covariance is then:

$$\widehat{\sigma}_{ij} = \frac{1}{N - L - 1}\sum_{t=1}^{N-L-1}\widehat{\sigma}_{ijt}$$

3.2 Comparison to Other Major Asset Classes

Table 4 presents summary statistics for the returns of collectibles, other major asset classes, and the value-weighted market portfolio. For return series with statistically significant autocorrelation, the compound total return is estimated using the autocorrelation-adjusted variance in a second-order Taylor series expansion.[14] With the exception of cars, coins, and wine,

[14] The estimator is constructed in Appendix B.

collectibles tend to have a low average return relative to other asset classes. Despite this, collectibles also tend to have quite large volatilities. Thus, many types of collectibles appear to be dominated single investments; for example, global bonds are strictly preferred to jewelry, stamps, furniture, and rugs.

Table 5 presents autocorrelation-adjusted correlations between collectibles returns and the returns of other major asset classes. Notably, collectibles tend to have fairly low, even negative, correlations with other major asset classes. This meshes well with findings by Goetzmann et al. (2011) and Pownall et al. (2019) that it takes time before wealth created in financial markets finds its way into collectibles markets. Thus, while collectibles are not attractive investments in isolation, they may be desirable from a risk-reducing portfolio diversification perspective.

4 Empirical Results

The key theoretical prediction I explore is that collectibles tokenization would be unambiguously welfare-improving if the viewing rights were rented on less than perfectly competitive rental markets. Specifically, I estimate the minimum annual rental yield that collectibles token owners would need to receive in order to be willing to pay a premium for the tokenized collectibles. The premium is surplus accruing to some combination of buyers and sellers, thus indicating that collectibles tokenization would be welfare-improving.

I am able to perform this test by exploiting a unique feature of collectibles price indices. A simple decomposition of collectibles returns is

$$\begin{aligned} \text{Collectibles Return} &= \text{Capital Gain} + \text{Convenience Yield} \\ &\quad - \text{Transaction Costs} - \text{Holding Costs} \end{aligned}$$

Collectibles price indices are based on sales prices (ie: hammer prices), which ignore both the transaction costs (ie: buyer's premium and seller's commission) and the private-value convenience yields received from owning the asset. Moreover, collectibles price indices are highly diversified. Since holding costs are, as an annual percentage of value, small enough to be inconsequential,[15] it follows that the collectibles price index actually measures the returns to a hypothetical well-diversified portfolio of collectibles tokens (without dividends), not the returns to investing in physical collectibles.

4.1 Empirical Design

I use a two-asset portfolio allocation problem to answer my empirical question. I assume all investors allocate their wealth between security tokens for a specific collectible category and the value-weighted market portfolio in order to maximize the Sharpe ratio. The key idea I leverage in this analysis is the following: if the optimal collectibles weighting ω_C^* exceeds the prevailing weighting at current market prices ω_C^{EQM}, then the investors would be willing to pay a premium to tokenize the entire supply of the collectible.

Instead of working in the space of weightings, I assess statistical significance in the space of means. I assume that all uncertainty is in the mean collectible return and focus on three important values:

1. μ_C^{EQM}: The mean collectibles return that would induce investors to hold the tokenized collectibles at their current market value weight ω_C^{EQM}.
2. μ_C^{TOK}: The mean collectibles return that exceeds μ_C^{EQM} at exactly the 5% significance level.
3. d^{IMP}: The difference between μ_C^{TOK} and the observed collectibles mean return μ^{OBS}.

Since statistical significance in the space of means ($\mu_C^{TOK} > \mu_C^{EQM}$) is equivalent to statistical significance in the space of weightings ($\omega_C^* > \omega_C^{EQM}$), d^{IMP} is thus the minimum annual rental yield that collectible token owners would need to receive in order to be willing to pay a premium for the tokenized collectibles. The remainder of this analysis will focus on estimating d^{IMP} under various assumptions to understand how large the minimum annual rental yield may need to be to ensure that collectibles tokenization is welfare improving.

[15] Holding costs are dominated by insurance but also include storage and maintenance. Roffino (2017) notes annual insurance costs are often only 0.05-0.2% of the insured value, while specialized fine art storage in Manhattan is roughly $5.00-12.50 per square foot per month.

4.2 Specifications

I test the results' robustness along four dimensions: the Sharpe ratio that is maximized, real estate's share of the market portfolio, the time horizon selected for returns, and the assumptions used to estimate the prevailing collectibles floats.

4.2.1 Sharpe Ratio

As highlighted by Sharpe (1994), the "Sharpe ratio," while clear theoretically, is practically ambiguous, with the ex-post and ex-ante Sharpe ratios being the two versions most commonly considered. Mechanically, the key difference is whether the ratio should be backward-looking (viewing the risk-free rate as part of historical excess returns) or forward-looking (in which case the prevailing risk-free rate is known):

$$SR_T^{\text{Ex-Post}} = \frac{\overline{R_p - R_f}}{\widehat{\sigma}(R_p - R_f)}$$
$$SR_T^{\text{Ex-Ante}} = \frac{\overline{R_p} - R_{fT}}{\widehat{\sigma}(R_p)}$$

As this portfolio allocation problem is intended to be forward-looking from the perspective of end-2017, and as future Treasury bill yields are expected to be much lower than in the past, the ex-ante Sharpe ratio seems most appropriate for this analysis. Nevertheless, for robustness, I consider both versions. Note that all variances and covariances are autocorrelation-adjusted.

4.2.2 Real Estate in the Market Portfolio

With the 2017 market value of all real estate at $280.6 trillion, real estate accounts for 59.8% of the value-weighted market portfolio. Particularly since the majority of real estate is not for sale, it seems important to consider including only investable real estate in the market portfolio, which has a 2017 market value of $103.6 trillion.[16] This reduces real estate's share of the market portfolio from 59.8% to 35.5%.

[16] Savills World Research (2016) finds that only about one-third of global real estate is readily investable at scale; I apply their 2015 ratio of investable-to-total real estate to the 2017 real estate market value to estimate the market value of investable global real estate.

4.2.3 Time Horizon

While I treat the 30-day Treasury bill rate as the risk-free rate, as shown in Figure 2, it appears to have undergone a regime shift from an era of higher rates in the 1990s to an era of much lower rates in the late 2000s and 2010s. Given the sensitivity of my analysis to the average level of the risk-free rate, I consider multiple time horizons, namely 1990-2017, 1994-2017, 1998-2017, and 2002-2017. This addresses the likelihood, as suggested by Carvalho et al. (2017) and Holston et al. (2017), that 30-day treasury bill rates will be much lower in the future than the 1990-2017 average would suggest.

4.2.4 Collectibles Floats

The Barclays (2012) report covers more than 2,000 high net worth individuals (HNWIs) and provides their self-reported ownership (as a percentage of wealth) for various types of collectibles, shown in Table 6.[17] To estimate the floats of these nine collectibles categories, I make simplifying assumptions to apply the results of the Barclays (2012) survey to the Credit Suisse Research (2017) global wealth distribution. For the sake of brevity, I present the results in Figure 3 and relegate the calculation details to Appendix C.

The lower, baseline, and upper estimates of the total floats of all 9 collectibles categories are \$17.9 trillion, \$21.1 trillion, and \$24.8 trillion. An immediate concern is that these values, based on survey results, may suffer from behavioral biases. For example, Tversky and Kahneman (1973) would suggest that availability bias leads people to overestimate the share of their wealth constituted by physical collectibles. Kahneman et al. (1990) would suggest that the endowment effect leads the survey respondents to overestimate the value of their collectibles, likely exacerbated by the fact that the owner of a unique, indivisible collectible tends to have the highest valuation of that collectible.

As these float estimates are only used to compute equilibrium weightings, all that matters is the size of these float estimates relative to the market portfolio, which is valued at either

[17] These HNWIs all have more than \$1.5 million of investable assets and 200 have more than \$15 million. Notably, the survey of UHNWIs in Knight Frank (2019) lumps various types of collectibles together as "luxury investments (art, wine, cars, etc)" and features what seems to be a lower percentage of clients that collect an investment of passion (28%). Accordingly, it is not clear that these two surveys are comparable.

\$469.2 trillion (all real estate) or \$292.2 trillion (only investable real estate). As will become evident through the sensitivity analysis, whether the equilibrium collectibles weight is as big as 7.8% or as small as 3.7% does not have much of an impact on the results. Moreover, if the collectibles floats are overestimated, then μ^{EQM} will be larger (to induce the investors to hold more collectibles), leading μ^{TOK} and d^{IMP} to also be larger. As the goal is to show that d^{IMP} is actually quite small, it follows that any bias in the estimation of collectibles floats works against the result I will show.

4.3 Empirical Results

Figure 4 summarizes the empirical results for the baseline collectibles float estimates. For each collectible category, two different Sharpe ratios are maximized for two different market portfolios for four different time horizons. As a result, there are 16 different estimates of the minimum annual rental yield needed to ensure that collectibles tokenization is unambiguously welfare improving. To be conservative, I take the *maximum* of these 16 estimates as the minimum annual rental yield.

These results reveal three broad groupings of collectibles categories. Sculptures, jewelry, and cars have a relatively low threshold, requiring at most a 5.3% rental yield for collectibles tokenization to be unambiguously welfare improving. Art, stamps, and coins have a relatively moderate threshold, requiring at most a 7.9% rental yield. Last, furniture, rugs, wine, and a float-weighted collectibles portfolio ("basket") have a relatively high threshold, requiring at most a 14.4% rental yield. Tables 7, 8, 9, & 10 present the precise numerical values resulting from these tests.[18]

Figures 5 & 6 summarize the empirical results for the lower and upper collectibles float estimates. These results are approximately indistinguishable from the results in Figure 4 because the estimates of the minimum annual rental yield typically differ by no more than 0.1 percentage point when only the value of the collectibles float is adjusted. The three thresholds, now based on 48 estimates for each category, only slightly increase from 5.3%,

[18] The thresholds by individual collectible category are as follows: 3.9% for sculptures, 4.0% for jewelry, 5.3% for cars, 7.2% for stamps, 7.4% for coins, 7.9% for art, 13.1% for the float-weighted collectibles portfolio ("basket"), 13.4% for furniture, 14.3% for wine, and 14.4% for rugs.

7.9%, and 14.4% to 5.3%, 8.0%, and 14.4%.[19] The precise numerical values resulting from these additional tests (32 for each collectible category) are relegated to the online appendix because of their similarity to the ones presented here.

4.4 Discussion

The empirical results establish annual rental yield thresholds of 5.3%, 8.0%, & 14.4% to make collectibles tokenization unambiguously welfare-improving. The welfare improvement stems from the fact that the representative agent investor would be willing to pay a premium for the tokenized collectibles.

While collectibles rental markets are generally quite immature, the rental yields earned in art rental markets provide appropriate context for judging the size of the rental yield thresholds. Table 11 lists a variety of annualized rental yields charged by firms renting heterogeneous qualities of art with listed sales prices ranging from under $300 to millions of dollars. Typical firm-level fee schedules have rental yields that decline as the listed sales price of the artwork increases. The lowest possible annualized rental yield for these firms is 6%, while most firms charge well in excess of 15% even for their most expensive pieces. Recalling that the median sales price of contemporary art was $1,300, which is fairly low on most of these rental fee schedules, the rental yield thresholds of 5.3%, 8.0%, & 14.4% seem fairly modest. This suggests that collectibles tokenization would be unambiguously welfare-improving in the presence of only modestly developed rental markets.

It is important to emphasize that this is a full-equilibrium analysis that considers tokenizing the entire supply of each collectible. The empirical results suggest it would be welfare-improving to transform the entire collectible market (for most, if not all, categories of collectibles) through tokenization. This would separate ownership rights from viewing rights and enable the owners of the collectibles tokens to receive financial dividends from renting the viewing rights.

[19] The overall thresholds by individual collectible category are as follows: 4.0% for sculptures, 4.1% for jewelry, 5.3% for cars, 7.2% for stamps, 7.4% for coins, 8.0% for art, 13.5% for the float-weighted collectibles portfolio ("basket"), 13.5% for furniture, 14.3% for wine, and 14.4% for rugs.

5 Concluding Remarks

I develop a tractable model of collectibles tokenization to understand the strengths and weaknesses of current tokenization efforts. My model accounts for a variety of unique features of collectibles including large transaction costs, short-sales constraints, indivisibility, heterogeneous valuations of convenience yields, and the bundling of ownership rights and viewing rights. I first show that collectibles investment is desirable if the diversification benefits and convenience yields compensate for the lower average financial returns. I then show that current tokenization efforts are welfare-reducing if the value of the squandered convenience yields exceeds the savings on transaction costs and the incremental diversification benefits from fractionalization. Last, I show that tokenization is unambiguously welfare-improving if the viewing rights are rented on a competitive market, in which case tokenization improves over physical collectibles in three ways: (1) by efficiently allocating viewing rights; (2) by reducing transaction costs; and (3) by creating further diversification benefits through fractionalization.

Theoretically, the key security design shortcoming hindering the success of collectibles tokenization is that current efforts squander the viewing rights. An obvious practical challenge is that collectibles rental markets are immature. Empirically, I use a representative agent portfolio allocation problem to show that rental markets do not need to be fully developed to provide rental yields that would make collectibles tokenization unambiguously welfare-improving. For example, annual rental yields of 4.1% are sufficient to make tokenizing sculptures and jewelry welfare-improving, while annual rental yields of more than 15% are common in existing art rental markets.

An important issue my model cannot satisfactorily address is the welfare gain for small investors. The indivisibility of expensive collectibles has generally barred small investors from gaining investment exposure to collectibles. As small investors are disadvantaged more generally (eg: higher mutual fund fees for retail share classes, prohibitive investment minimums for alternative investments), they may derive larger benefits from the substantial improvement to their investment opportunity set, suddenly being able to hold a well-diversified portfolio of financial dividend-paying collectible tokens.

Table 1: Collectibles Data

This table provides an overview of the collectibles price indices in my sample. For each asset class listed in the first column, the second column reports the name of the price index. The third column denotes the type of price index, either average-sales (AS) or repeat-sales (RS). The fourth column reports the frequency of the data, the fifth column reports the time horizon of the data, and the last column reports the source for the price index.

Asset Class	Name of Data Series	Type	Frequency	Time Horizon	Source
Paintings	Art 100 Index	AS	Monthly	1976-2017	Art Market Research
Paintings	Painting Index	RS	Quarterly	1998-2017	Artprice
Sculptures	European & North American Sculptures 100 Index	AS	Monthly	1985-2017	Art Market Research
Sculptures	Sculptures Index	RS	Quarterly	1998-2017	Artprice
Jewelry	General Jewelry Index	AS	Monthly	1986-2017	Art Market Research
Jewelry	US CPI Jewelry Non-Seasonally Adjusted	RS	Monthly	1987-2017	Bloomberg
Stamps	Stamps World Index	AS	Monthly	1976-2017	Art Market Research
Stamps	Stanley Gibbons 100 Stamp Price Index	RS	Quarterly	1999-2012	Bloomberg
Antique Furniture	English 18th Century Furniture	AS	Monthly	1976-2017	Art Market Research
Classic Cars	Classic Cars Index	AS	Monthly	1981-2017	Art Market Research
Rugs & Carpets	European and Eastern Rugs & Carpets	AS	Monthly	1985-2017	Art Market Research
Coins	English Coins Index	AS	Monthly	1976-2017	Art Market Research
Wine	Liv-ex 100 Benchmark Fine Wine Investables Index	RS	Monthly	1988-2017	Bloomberg

Table 2: Other Major Asset Classes Data

This table provides an overview of the non-collectibles price indices and return series in my sample. I take these to constitute the investor's alternative opportunity set. For each asset class listed in the first column, the second column reports the name of the price index or return series. The third column reports the time horizon of the data and the last column reports the source for the price index. Note that the global debt, global equity, and real estate price indices are total return indices that include dividends and coupons.

Asset Class	Name of Data Series	Time Horizon	Source
Risk-Free Rate	30-Day US Treasury Bill Return	1963-2017	Ken French
Global Debt	Bloomberg Barclays Global Aggregate Total Return Index (Unhedged USD)	1990-2017	Bloomberg
Global Equity	MSCI World Net Total Return USD Index	1987-2017	Bloomberg
Real Estate	FTSE NAREIT Equity REITs Total Return Index USD	1987-2017	Bloomberg

Table 3: Autocorrelation of Quarterly Returns: 1990-2017 (When Available)

This table presents estimates of first- through eighth-order autocorrelation coefficients for 1990-2017 quarterly returns (when available). The specific time horizons can be inferred from Tables 1 & 2. Note that "AS" refers to the average-sales price index and "RS" refers to the repeat-sales price index. I choose to present autocorrelation coefficients for quarterly returns so that all returns can be compared (both monthly and quarterly) and to more easily illustrate the higher-order autocorrelation.

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8
Global Bonds	0.05	-0.06	0.02	-0.18	0.11	-0.01	-0.07	0.02
Global Equity	0.02	0.05	-0.02	-0.04	-0.04	0.08	-0.13	0.01
Commodities	0.06	0.10	-0.05	-0.17	-0.13	-0.08	0.02	-0.08
Real Estate	0.11	-0.15	-0.09	-0.02	0.09	-0.01	-0.14	-0.09
Paintings (AS)	0.67^{***}	0.51^{***}	0.24^{**}	-0.02	-0.17	-0.22^{*}	-0.22^{*}	-0.21^{*}
Paintings (RS)	0.64^{***}	0.15	0.08	0.14	0.03	-0.14	-0.15	-0.07
Sculptures (AS)	0.41^{***}	0.28^{**}	0.15	0.03	0.22^{*}	0.37^{***}	0.30^{***}	0.15
Sculptures (RS)	0.48^{***}	-0.16	-0.10	0.09	-0.01	-0.17	-0.05	0.09
Jewelry (AS)	0.80^{***}	0.51^{***}	0.25^{**}	0.08	0.05	0.05	-0.00	-0.09
Jewelry (RS)	-0.45^{***}	0.14	-0.33^{**}	0.65^{***}	-0.42^{***}	0.20	-0.37^{***}	0.63^{***}
Stamps (AS)	0.87^{***}	0.59^{***}	0.30^{***}	0.07	-0.04	-0.10	-0.16	-0.24^{**}
Stamps (RS)	0.20	-0.20	0.01	0.08	0.13	0.05	-0.12	-0.12
Furniture (AS)	0.89^{***}	0.69^{***}	0.50^{***}	0.36^{***}	0.29^{**}	0.22^{*}	0.13	0.05
Cars (AS)	0.84^{***}	0.56^{***}	0.26^{**}	0.00	-0.07	-0.06	-0.02	-0.01
Rugs (AS)	0.42^{***}	0.35^{***}	0.29^{**}	0.17	0.26^{**}	0.05	-0.00	-0.16
Coins (AS)	0.88^{***}	0.65^{***}	0.44^{***}	0.31^{***}	0.27^{**}	0.24^{**}	0.15	0.05
Wine (RS)	0.20^{*}	-0.01	0.06	0.24^{**}	0.05	-0.07	-0.07	0.26^{**}

Note: $^{*}p<0.05$; $^{**}p<0.01$; $^{***}p<0.001$

Table 4: Collectibles Price Index Summary Statistics and Comparisons

This table provides summary statistics for the returns of collectibles, other major asset classes, and the value-weighted market portfolio (including either all real estate or only investable real estate). Standard deviations are autocorrelation-adjusted following the methodology in section 3.1 and the compound total return is also autocorrelation-adjusted following the procedure in Appendix B.

	1990-2017			2002-2017		
	Mean (Annualized %)	Std. Dev. (Annualized %)	Compound Total Return (%)	Mean (Annualized %)	Std. Dev. (Annualized %)	Compound Total Return (%)
Paintings	2.31	25.95	-25.07	7.26	23.83	103.69
Sculptures	3.69	6.03	165.64	5.90	4.31	152.77
Jewelry	3.47	9.10	134.60	4.67	9.50	96.36
Stamps	2.71	9.77	86.55	3.56	10.63	61.54
Furniture	-2.29	13.17	-58.57	-5.08	10.84	-59.71
Cars	4.97	15.13	191.17	9.42	8.21	325.40
Rugs	0.62	21.03	-35.73	-6.87	13.77	-71.48
Coins	5.84	12.72	306.49	8.35	12.78	233.29
Wine	10.82	26.68	663.64	8.04	22.31	143.78
Global Equity	7.76	14.69	540.44	7.71	14.83	186.53
Global Bonds	5.86	5.39	391.34	5.00	5.71	116.58
Commodities	5.03	20.74	122.52	8.64	22.64	161.55
Real Estate	12.08	19.40	1615.54	12.46	18.65	453.67
30-day T-Bill	2.72	0.67	113.55	1.18	0.45	20.80
VW Market (Inv RE)	8.61	9.65	864.62	8.42	11.25	246.49
VW Market (All RE)	9.92	12.97	1151.62	9.94	15.44	304.45

Table 5: Autocorrelation-Adjusted Correlations for 1990-2017 Returns

This table presents the correlation coefficients for the returns of the nine categories of collectibles with the returns of global equity, global bonds, real estate, and the value-weighted market portfolio (including either all real estate or only investable real estate). The covariance matrix used to compute the correlation matrix is autocorrelation-adjusted following the methodology in section 3.1.

	Global Equity	Global Bonds	Real Estate	VW Market (Inv RE)	VW Market (All RE)
Paintings	0.017	-0.239	-0.407	-0.281	-0.356
Sculptures	-0.214	-0.640	-0.236	-0.416	-0.331
Jewelry	0.331	0.180	0.103	0.254	0.176
Stamps	0.427	0.374	0.192	0.398	0.293
Furniture	0.053	0.239	0.048	0.097	0.074
Cars	0.159	-0.221	-0.061	-0.019	-0.044
Rugs	0.078	0.315	-0.009	0.091	0.037
Coins	0.139	0.019	0.534	0.427	0.500
Wine	0.403	0.171	0.349	0.458	0.409

Table 6: Reported HNWI Collectibles Holdings (2012)

This table presents select results from the Barclays (2012) "Wealth Insights" survey of high net worth individuals (HNWIs). The survey covered more than 2,000 HNWIs, all of which have at least \$1.5 million of investable assets and 200 of which have more than \$15 million of investable assets. These results concern the respondents' self-reported ownership of different types of collectibles. The first column is the collectibles category under consideration. The second column reports the percentage of individuals that reported owning that category of collectible. The third column reports the average percentage of wealth invested in that collectible category for those reporting ownership. The fourth column is the overall average percentage of wealth HNWIs hold in these collectibles, obtained by multiplying the second and third columns together. The last column is the share of collectibles wealth represented by each category, obtained by dividing each entry in column four by the sum of all entries in column four.

Asset Class	Percent that Own	Average Percentage of Wealth (Given Ownership)	Average Percentage of Wealth	Share of Collectibles Wealth
Precious Jewelry	70%	5%	3.50%	30.54%
Fine Art Paintings	49%	4%	1.96%	17.10%
Antique Furniture	37%	3%	1.11%	9.69%
Wine	28%	2%	0.56%	4.89%
Fine Art Tapestries & Rugs	26%	3%	0.78%	6.81%
Fine Art Sculptures	24%	4%	0.96%	8.38%
Classic Cars	19%	7%	1.33%	11.61%
Coin Collections	23%	4%	0.92%	8.03%
Stamp Collections	17%	2%	0.34%	2.97%

Table 7: Portfolio Problem: Ex-Ante Sharpe Ratio, All Real Estate, Baseline Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-ante Sharpe ratio. The market portfolio includes global equity, global bonds, and all real estate. The collectibles floats are the baseline values from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible return (μ^{EQM}), the observed mean collectible return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.059	-0.006	0.023	0.032	0.023	0.012	0.015	0.057	0.086	0.051
	μ^{OBS}	0.023	0.037	0.035	0.027	-0.023	0.050	0.006	0.058	0.108	0.050
	$t(\mu^{OBS} - \mu^{EQM})$	1.675*	3.274***	0.703	-0.267	-1.793*	1.325	-0.230	0.060	0.436	-0.026
	μ^{TOK}	0.037	0.020	0.055	0.068	0.073	0.068	0.094	0.106	0.186	0.138
	d^{IMP}	0.014	-0.017	0.021	0.041	0.096	0.018	0.087	0.047	0.077	0.088
1994-2017	μ^{EQM}	-0.046	-0.002	0.026	0.034	0.027	0.017	-0.003	0.064	0.095	0.058
	μ^{OBS}	0.058	0.041	0.040	0.028	-0.019	0.058	-0.025	0.071	0.116	0.064
	$t(\mu^{OBS} - \mu^{EQM})$	2.439**	3.106***	0.853	-0.288	-1.631	1.867*	-0.603	0.302	0.365	0.134
	μ^{TOK}	0.037	0.025	0.059	0.073	0.083	0.060	0.069	0.111	0.209	0.157
	d^{IMP}	-0.020	-0.016	0.019	0.044	0.102	0.002	0.094	0.040	0.093	0.092
1998-2017	μ^{EQM}	-0.039	-0.001	0.026	0.033	0.020	0.026	-0.000	0.071	0.068	0.048
	μ^{OBS}	0.069	0.049	0.039	0.024	-0.033	0.068	-0.027	0.077	0.062	0.072
	$t(\mu^{OBS} - \mu^{EQM})$	2.176**	3.606***	0.650	-0.378	-1.960*	1.688*	-0.735	0.216	-0.106	0.428
	μ^{TOK}	0.059	0.027	0.066	0.079	0.074	0.075	0.072	0.127	0.168	0.157
	d^{IMP}	-0.010	-0.023	0.027	0.055	0.107	0.007	0.100	0.050	0.105	0.085
2002-2017	μ^{EQM}	-0.051	-0.006	0.028	0.036	0.028	0.017	-0.001	0.069	0.078	0.067
	μ^{OBS}	0.073	0.059	0.047	0.036	-0.051	0.094	-0.069	0.084	0.080	0.086
	$t(\mu^{OBS} - \mu^{EQM})$	2.051**	5.430***	0.776	-0.033	-2.905***	3.738***	-2.077**	0.439	0.043	0.314
	μ^{TOK}	0.068	0.017	0.075	0.089	0.081	0.058	0.064	0.133	0.203	0.190
	d^{IMP}	-0.005	-0.042	0.029	0.053	0.132	-0.037	0.133	0.049	0.122	0.104

Note: *p<0.1; **p<0.05; ***p<0.01

Table 8: Portfolio Problem: Ex-Post Sharpe Ratio, All Real Estate, Baseline Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-post Sharpe ratio. The market portfolio includes global equity, global bonds, and all real estate. The collectibles floats are the baseline values from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible excess return (μ^{EQM}), the observed mean collectible excess return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible excess return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible excess return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible excess return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.056	0.010	0.011	0.018	0.000	0.009	-0.007	0.052	0.050	0.024
	μ^{OBS}	-0.004	0.010	0.007	-0.000	-0.050	0.022	-0.021	0.031	0.081	0.023
	t($\mu^{OBS} - \mu^{EQM}$)	1.048	-0.023	-0.196	-0.953	-2.346**	0.437	-0.405	-0.756	0.642	-0.027
	μ^{TOK}	0.042	0.049	0.043	0.055	0.043	0.069	0.061	0.105	0.146	0.107
	d^{IMP}	0.046	0.039	0.035	0.055	0.093	0.046	0.083	0.074	0.065	0.084
1994-2017	μ^{EQM}	-0.044	0.011	0.013	0.021	0.002	0.014	-0.013	0.056	0.056	0.028
	μ^{OBS}	0.034	0.017	0.017	0.004	-0.043	0.034	-0.048	0.047	0.092	0.040
	t($\mu^{OBS} - \mu^{EQM}$)	1.860*	0.287	0.211	-0.800	-2.014**	0.749	-1.111	-0.345	0.654	0.263
	μ^{TOK}	0.038	0.052	0.046	0.061	0.046	0.067	0.050	0.105	0.165	0.121
	d^{IMP}	0.005	0.035	0.030	0.057	0.089	0.033	0.099	0.058	0.073	0.081
1998-2017	μ^{EQM}	-0.049	0.007	0.014	0.022	0.002	0.018	-0.011	0.059	0.043	0.018
	μ^{OBS}	0.050	0.030	0.020	0.005	-0.052	0.049	-0.046	0.058	0.043	0.053
	t($\mu^{OBS} - \mu^{EQM}$)	2.066**	1.196	0.307	-0.671	-2.361**	1.038	-1.060	-0.017	0.010	0.710
	μ^{TOK}	0.046	0.046	0.054	0.070	0.047	0.077	0.053	0.115	0.139	0.114
	d^{IMP}	-0.005	0.015	0.033	0.065	0.098	0.028	0.099	0.057	0.096	0.061
2002-2017	μ^{EQM}	-0.067	-0.006	0.014	0.024	0.010	0.006	-0.006	0.064	0.053	0.034
	μ^{OBS}	0.061	0.047	0.035	0.024	-0.063	0.082	-0.081	0.072	0.069	0.075
	t($\mu^{OBS} - \mu^{EQM}$)	2.221**	3.164***	0.996	0.010	-2.956***	3.633***	-2.327**	0.254	0.265	0.737
	μ^{TOK}	0.047	0.027	0.056	0.074	0.059	0.048	0.058	0.123	0.171	0.143
	d^{IMP}	-0.014	-0.020	0.021	0.050	0.121	-0.035	0.138	0.051	0.103	0.068

Note: *p<0.1; **p<0.05; ***p<0.01

Table 9: Portfolio Problem: Ex-Ante Sharpe Ratio, Investable Real Estate, Baseline Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-ante Sharpe ratio. The market portfolio includes global equity, global bonds, and investable real estate. The collectibles floats are the baseline values from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible return (μ^{EQM}), the observed mean collectible return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.042	-0.013	0.030	0.042	0.025	0.015	0.024	0.054	0.108	0.081
	μ^{OBS}	0.023	0.037	0.035	0.027	-0.023	0.050	0.006	0.058	0.108	0.050
	$t(\mu^{OBS} - \mu^{EQM})$	1.331	3.815***	0.260	-0.799	-1.887*	1.195	-0.459	0.161	0.013	-0.687
	μ^{TOK}	0.054	0.013	0.062	0.078	0.076	0.072	0.103	0.103	0.207	0.167
	d^{IMP}	0.031	-0.024	0.028	0.051	0.098	0.022	0.097	0.045	0.099	0.117
1994-2017	μ^{EQM}	-0.026	-0.006	0.034	0.045	0.033	0.023	0.004	0.064	0.119	0.089
	μ^{OBS}	0.058	0.041	0.040	0.028	-0.019	0.058	-0.025	0.071	0.116	0.064
	$t(\mu^{OBS} - \mu^{EQM})$	1.969*	3.383***	0.371	-0.860	-1.826*	1.621	-0.781	0.272	-0.060	-0.491
	μ^{TOK}	0.058	0.021	0.068	0.084	0.088	0.066	0.076	0.112	0.233	0.188
	d^{IMP}	0.000	-0.020	0.027	0.056	0.108	0.008	0.100	0.041	0.118	0.124
1998-2017	μ^{EQM}	-0.020	0.002	0.033	0.043	0.018	0.030	-0.001	0.072	0.084	0.083
	μ^{OBS}	0.069	0.049	0.039	0.024	-0.033	0.068	-0.027	0.077	0.062	0.072
	$t(\mu^{OBS} - \mu^{EQM})$	1.796*	3.423***	0.312	-0.798	-1.858*	1.501	-0.705	0.182	-0.420	-0.202
	μ^{TOK}	0.078	0.029	0.073	0.089	0.071	0.080	0.071	0.128	0.184	0.191
	d^{IMP}	0.009	-0.020	0.034	0.065	0.104	0.012	0.098	0.051	0.121	0.120
2002-2017	μ^{EQM}	-0.041	-0.013	0.034	0.043	0.030	0.017	0.001	0.077	0.098	0.094
	μ^{OBS}	0.073	0.059	0.047	0.036	-0.051	0.094	-0.069	0.084	0.080	0.086
	$t(\mu^{OBS} - \mu^{EQM})$	1.886*	5.948***	0.536	-0.277	-3.007***	3.715***	-2.135**	0.199	-0.275	-0.119
	μ^{TOK}	0.078	0.011	0.081	0.095	0.084	0.058	0.066	0.140	0.223	0.218
	d^{IMP}	0.005	-0.048	0.035	0.060	0.134	-0.036	0.135	0.057	0.143	0.131

Note: *p<0.1; **p<0.05; ***p<0.01

Table 10: Portfolio Problem: Ex-Post Sharpe Ratio, Investable Real Estate, Baseline Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-post Sharpe ratio. The market portfolio includes global equity, global bonds, and investable real estate. The collectibles floats are the baseline values from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible excess return (μ^{EQM}), the observed mean collectible excess return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible excess return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible excess return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible excess return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.023	0.011	0.016	0.025	-0.001	0.016	-0.012	0.050	0.058	0.039
	μ^{OBS}	-0.004	0.010	0.007	-0.000	-0.050	0.022	-0.021	0.031	0.081	0.023
	$t(\mu^{OBS} - \mu^{EQM})$	0.379	-0.048	-0.503	-1.353	-2.257**	0.211	-0.274	-0.677	0.472	-0.384
	μ^{TOK}	0.075	0.049	0.048	0.063	0.041	0.076	0.057	0.103	0.154	0.122
	d^{IMP}	0.079	0.039	0.040	0.063	0.091	0.053	0.078	0.072	0.073	0.099
1994-2017	μ^{EQM}	-0.034	0.013	0.019	0.029	0.002	0.023	-0.010	0.055	0.066	0.044
	μ^{OBS}	0.034	0.017	0.017	0.004	-0.043	0.034	-0.048	0.047	0.092	0.040
	$t(\mu^{OBS} - \mu^{EQM})$	1.629	0.201	-0.132	-1.217	-2.009**	0.438	-1.190	-0.326	0.478	-0.073
	μ^{TOK}	0.048	0.054	0.052	0.069	0.046	0.075	0.053	0.105	0.174	0.137
	d^{IMP}	0.014	0.037	0.035	0.065	0.089	0.041	0.101	0.058	0.082	0.096
1998-2017	μ^{EQM}	-0.028	0.012	0.020	0.029	0.000	0.026	-0.012	0.056	0.050	0.035
	μ^{OBS}	0.050	0.030	0.020	0.005	-0.052	0.049	-0.046	0.058	0.043	0.053
	$t(\mu^{OBS} - \mu^{EQM})$	1.622	0.916	0.039	-0.980	-2.271**	0.765	-1.056	0.080	-0.143	0.364
	μ^{TOK}	0.067	0.051	0.059	0.078	0.045	0.085	0.053	0.112	0.147	0.131
	d^{IMP}	0.017	0.021	0.039	0.072	0.096	0.036	0.099	0.054	0.103	0.078
2002-2017	μ^{EQM}	-0.061	-0.003	0.017	0.029	0.012	0.008	-0.000	0.073	0.065	0.051
	μ^{OBS}	0.061	0.047	0.035	0.024	-0.063	0.082	-0.081	0.072	0.069	0.075
	$t(\mu^{OBS} - \mu^{EQM})$	2.121**	3.006***	0.845	-0.201	-3.019***	3.553***	-2.503**	-0.042	0.061	0.435
	μ^{TOK}	0.052	0.030	0.059	0.079	0.060	0.049	0.063	0.131	0.183	0.159
	d^{IMP}	-0.008	-0.017	0.024	0.056	0.123	-0.033	0.144	0.060	0.115	0.085

Note: *p<0.1; **p<0.05; ***p<0.01

Table 11: Art Rental Yields

This table presents the range of annualized rental yields charged by an assortment of firms that offer art rentals. The first column lists the names of the companies. The second column presents the range of annualized rental yields charged by these firms. The third column reports the range of quoted rental yields available on each company's website. When quoted in dollar values, I obtain rental yields by dividing the rental fee by the listed sales price. Note that, while most of the rental yields are monthly, some firms require a minimum number of months for the rental. The last column is the maximum sales price for any artwork that is listed for rent. Since the rental yields tend to decrease as the listed sales price increases, the maximum sales price is associated with the lowest rental yield.

Company	Annualized Rental Yield	Quoted Rental Yield	Maximum Price for Rented Art
Artemus	-	-	$100 M (luxury art in New York)
Art-Lease	6-8%	6-8% annually	Luxury art in Hong Kong
Ryan James Fine Arts	6-42%	0.5-3.5% per month	$7,000
Get Art Up	12-300%	1-25% per month	$4,000
Northwest Museum	16-72%	4-18% for 3 months	$3,800
Artolease	18-54%	1.5-4.5% per month	-
Teichert Gallery	24-120%	2-10% per month	$10,000
Agora Gallery	36%	3% per month for 3+ months	Minimum value of $20,000
Artforte	36-42%	3-3.5% per month	-
Artspay	48%	4% per month for 4-6 months	-
Hang Art	52-120%	13-30% for 3 months	$50,000
Riverfront Art Gallery	72-144%	6-12% per month	-
Rise Art	84-240%	7-20% per month	$16,000

Table 12: Distribution of Global Wealth (2017)

This table presents select results from the Credit Suisse Research (2017) "Global Wealth Report" on the distribution of global wealth. The first column is the range of individual net worth considered, the second column is the number of adults (percent of total adults) that fall in that category, and the last column is the total value of wealth (percent of total wealth) that the individual net worth category constitutes. Note that the definitions of "wealth" used in the Barclays (2012) and Credit Suisse Research (2017) reports are the same.

Individual Net Worth	Number of Adults	Total Value of Wealth
>$1 M	36 M (0.7%)	$128.7 T (45.9%)
$100,000 - $1 M	391 M (7.9%)	$111.4 T (39.7%)
$10,000 - $100,000	1,054 M (21.3%)	$32.5 T (11.6%)
<$10,000	3,474 M (70.1%)	$7.6 T (2.7%)

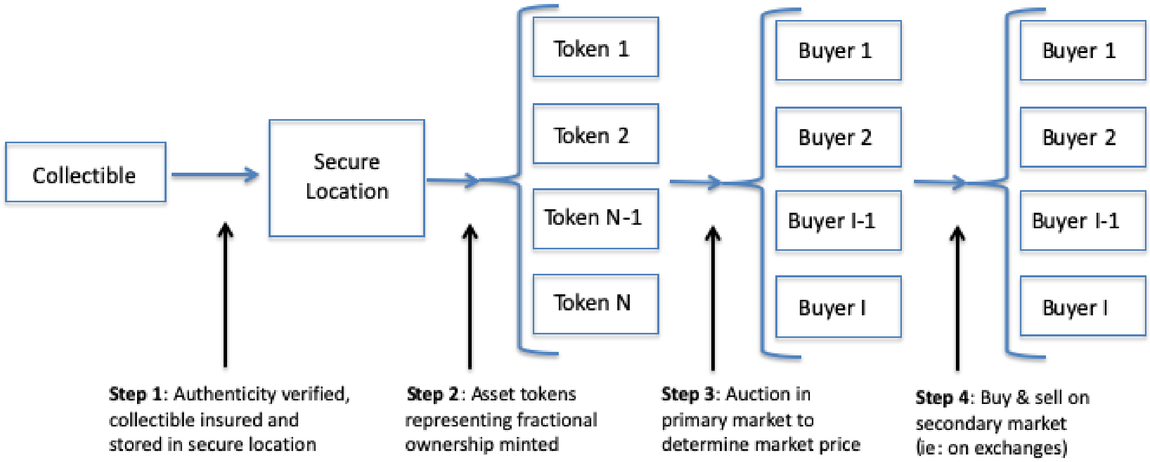

Figure 1: Tokenization Process

This figure illustrates the steps involved in tokenizing a physical collectible, using Maecenas' successful tokenization of Andy Warhol's (1980) "14 Small Electric Chairs" for guidance on the details of the process. The input is a single physical collectible and the output is many collectible tokens that can be traded on an exchange.

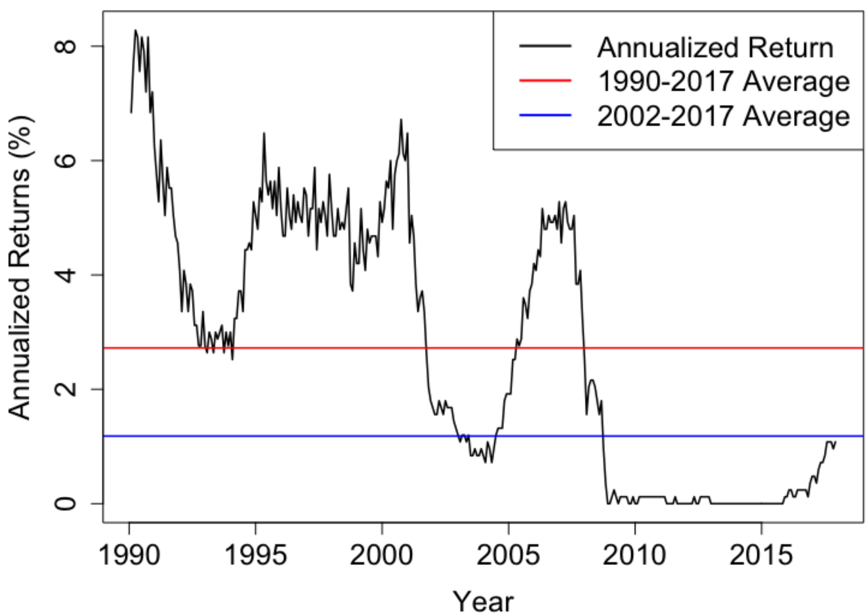

Figure 2: 30-Day Treasury Bill Returns

This graph presents the annualized 30-day Treasury bill rate for the 1990-2017 time period of interest. The 1990-2017 average of 2.72% is shown in red and the 2002-2017 average of 1.18% is shown in blue.

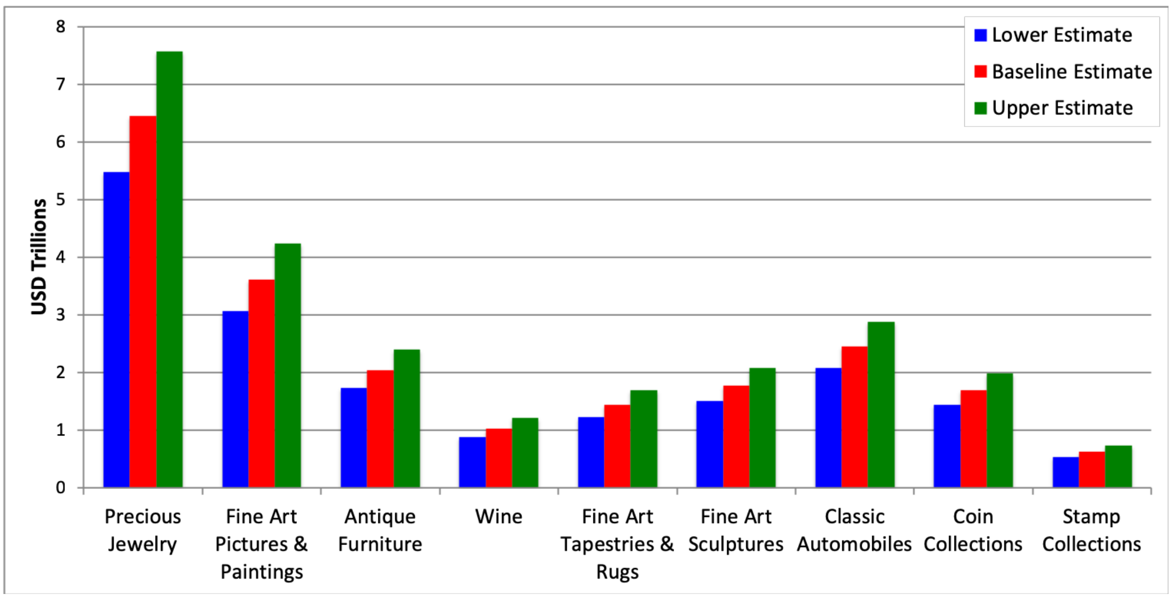

Figure 3: Total Float of the Collectibles Categories (2017)

This chart presents the lower, baseline, and upper estimates for the total float of each of the nine collectibles categories. These values are obtained by making assumptions about the applicability of the Barclays (2012) survey of high net worth individuals' collectibles ownership as a percentage of wealth and applying the survey results to the Credit Suisse Research (2017) global wealth distribution. This process is described in detail in Appendix C.

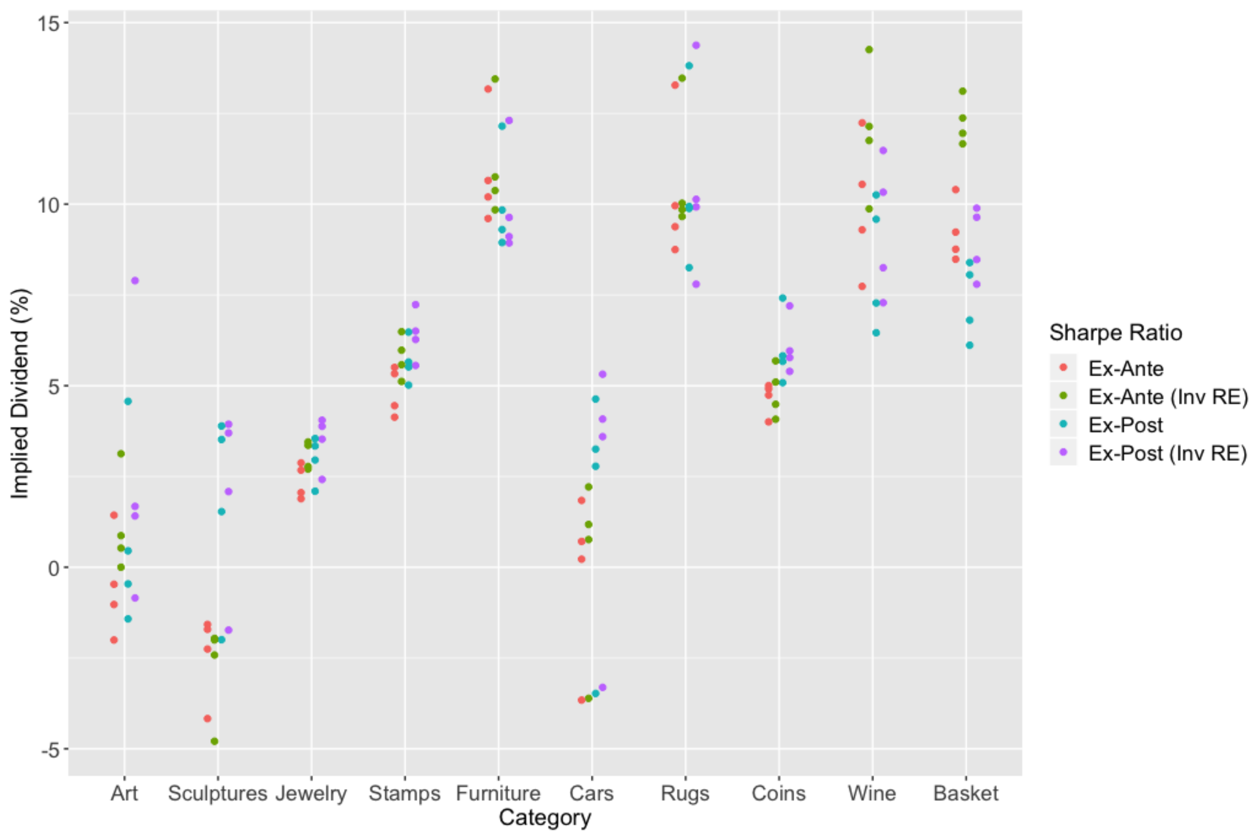

Figure 4: Financial Dividend Needed for Welfare-Improving Tokenization: Baseline Floats

This chart presents the annual dividend yield that each type of collectible token would need to pay in order for tokenization to be unambiguously welfare-improving. Recall that the dividend yield is supported by renting the viewing rights of the collectibles. There are 16 specifications considered for each collectible category, with each value being the result of a 2-asset portfolio problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the Sharpe ratio. I consider two versions of the Sharpe ratio (ex-ante and ex-post), two versions of the market portfolio (with all real estate and with only investable real estate), and four time horizons (1990-2017, 1994-2017, 1998-2017, and 2002-2017). Notably, the results presented here are all for the baseline estimates of the collectibles floats from Figure 3. The precise numerical values of the implied dividend yields are presented in Tables 7, 8, 9, & 10.

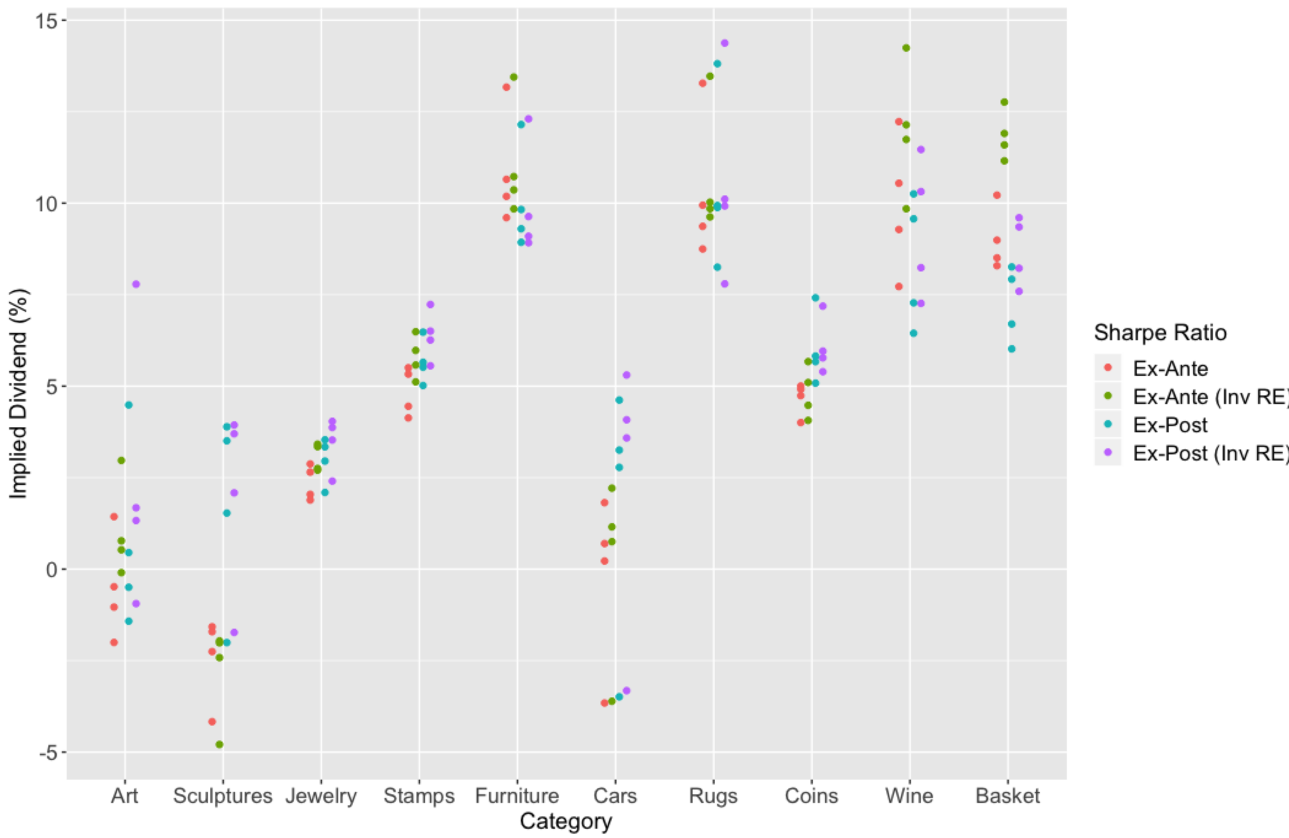

Figure 5: Financial Dividend Needed for Welfare-Improving Tokenization: Lower Floats

This chart presents the annual dividend yield that each type of collectible token would need to pay in order for tokenization to be unambiguously welfare-improving. Recall that the dividend yield is supported by renting the viewing rights of the collectibles. There are 16 specifications considered for each collectible category, with each value being the result of a 2-asset portfolio problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the Sharpe ratio. I consider two versions of the Sharpe ratio (ex-ante and ex-post), two versions of the market portfolio (with all real estate and with only investable real estate), and four time horizons (1990-2017, 1994-2017, 1998-2017, and 2002-2017). Notably, the results presented here are all for the lower estimates of the collectibles floats from Figure 3. The precise numerical values of the implied dividend yields are presented in the online appendix.

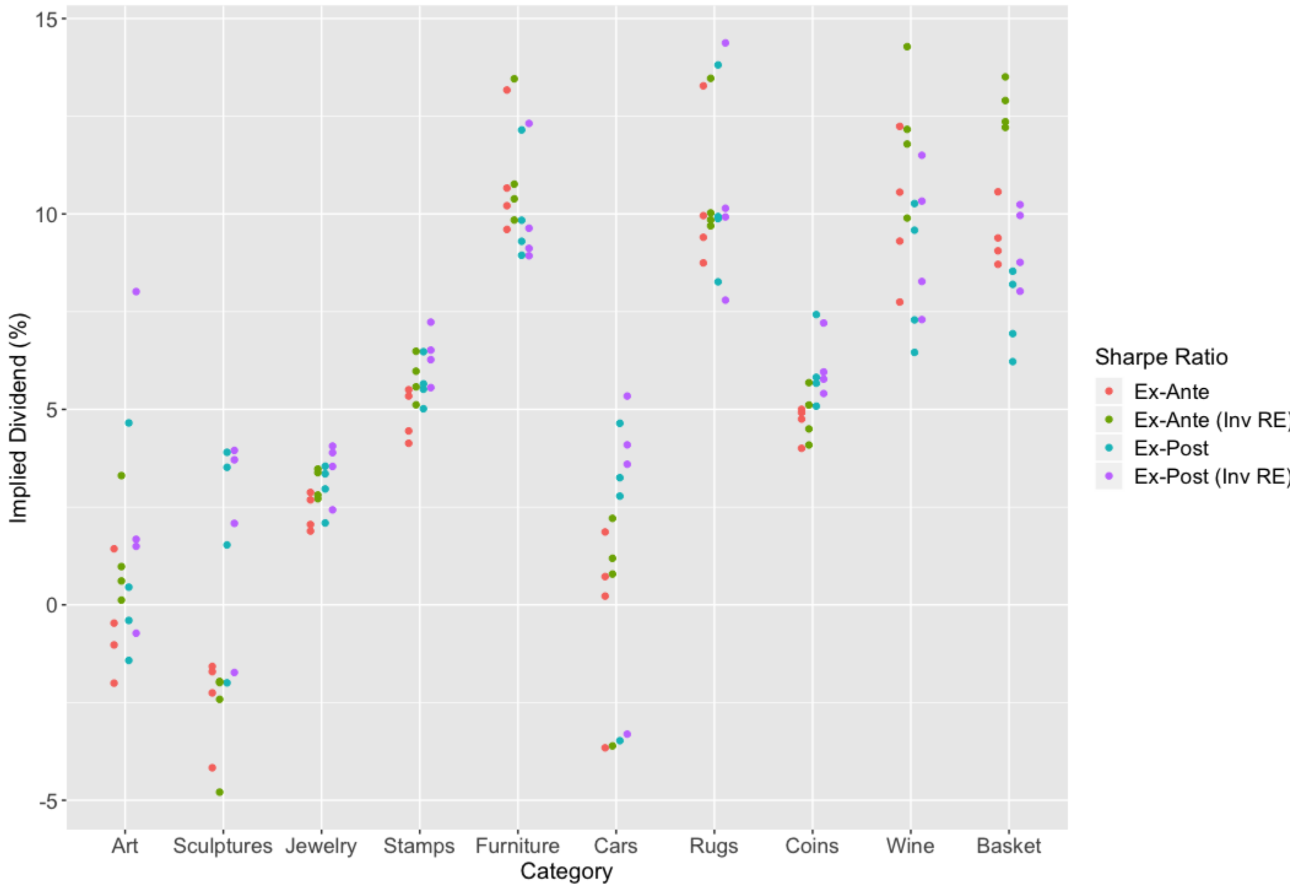

Figure 6: Financial Dividend Needed for Welfare-Improving Tokenization: Upper Floats

This chart presents the annual dividend yield that each type of collectible token would need to pay in order for tokenization to be unambiguously welfare-improving. Recall that the dividend yield is supported by renting the viewing rights of the collectibles. There are 16 specifications considered for each collectible category, with each value being the result of a 2-asset portfolio problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the Sharpe ratio. I consider two versions of the Sharpe ratio (ex-ante and ex-post), two versions of the market portfolio (with all real estate and with only investable real estate), and four time horizons (1990-2017, 1994-2017, 1998-2017, and 2002-2017). Notably, the results presented here are all for the upper estimates of the collectibles floats from Figure 3. The precise numerical values of the implied dividend yields are presented in the online appendix.

Appendix

In section A, I provide proofs of the model's propositions. In section B, I present the approximation to the compound total return that accounts for return autocorrelation. In section C, I provide the details of the method I use to estimate the total floats of the nine collectibles categories.

A Model Proofs

A.1 Proof of Proposition 1

The agent's problem is

$$\begin{aligned}
&\max_{x_i^S} \mathbb{E}\left[-e^{-\gamma[W_0+x_i^S(\widetilde{S}-p_0^S)]}\right] \\
&= \max_{x_i^S} \; -e^{-\gamma W_0+\gamma x_i^S p_0^S-\gamma x_i^S \mu_S+\frac{\gamma^2}{2}(x_i^S)^2\sigma_S^2}
\end{aligned}$$

The first order condition with respect to x_i^S simplifies to

$$\begin{aligned}
\gamma p_0^S - \gamma\mu_S + \gamma^2 x_i^S \sigma_S^2 = 0 \\
\implies x_i^S = \frac{\mu_S - p_0^S}{\gamma\sigma_S^2} \qquad \text{(A.1)}
\end{aligned}$$

Plugging equation (A.1) into the market clearing equation $\int_i x_i^S \frac{d\mu_i^C}{2\kappa} = Y$ allows me to solve for the equilibrium price

$$\begin{aligned}
\frac{\mu_S - p_0^S}{\gamma\sigma_S^2} = Y \\
\implies p_0^S = \mu_S - \gamma\sigma_S^2 Y \qquad \text{(A.2)}
\end{aligned}$$

Then plugging equation (A.2) into equation (A.1) gives the equilibrium demand $x_i^S = Y$.

A.2 Proof of Proposition 2

The agent's problem is

$$
\begin{aligned}
&\max_{x_i^C, x_i^S} \ \mathbb{E}\left[-e^{-\gamma[W_0 + x_i^C(\widetilde{C} - p_0^C) + x_i^S(\widetilde{S} - p_0^S)]}\right] \\
\text{s.t.} \quad & x_i^C \geq 0 \\
\Longrightarrow &\max_{x_i^C, x_i^S} \ -e^{-\gamma\left[W_0 - x_i^C p_0^C - x_i^S p_0^S + x_i^C \mu_i^C + x_i^S \mu_S - \frac{\gamma}{2}\left((x_i^C)^2\sigma_C^2 + (x_i^S)^2\sigma_S^2 + 2x_i^C x_i^S \sigma_{CS}\right)\right]} \\
\text{s.t.} \quad & x_i^C \geq 0
\end{aligned}
$$

Taking the first order conditions with respect to x_i^S and x_i^C yield

$$x_i^S = \frac{\mu_S - p_0^S - \gamma x_i^C \sigma_{CS}}{\gamma \sigma_S^2} \tag{A.3}$$

$$x_i^C = \max\{\frac{\mu_i^C - p_0^C - \gamma x_i^S \sigma_{CS}}{\gamma \sigma_C^2}, \ 0\} \tag{A.4}$$

Plugging equation (A.3) into equation (A.4) and vice-versa yields

$$x_i^C = \max\{\frac{\mu_i^C - p_0^C}{\gamma\sigma_C^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(\mu_S - p_0^S)}{\gamma\sigma_C\sigma_S(1-\rho_{CS}^2)}, 0\} \tag{A.5}$$

$$x_i^S = \begin{cases} \frac{\mu_S - p_0^S}{\gamma\sigma_S^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(\mu_i^C - p_0^C)}{\gamma\sigma_C\sigma_S(1-\rho_{CS}^2)} & , \text{ if } x_i^C > 0 \\ \frac{\mu_S - p_0^S}{\gamma\sigma_S^2} & , \text{ if } x_i^C = 0 \end{cases} \tag{A.6}$$

For tractability, I focus only on the case in which the short-sales constraint doesn't bind. The market clearing conditions $\int_i x_i^C \frac{d\mu_i^C}{2\kappa} = Q$ and $\int_i x_i^S \frac{d\mu_i^C}{2\kappa} = Y$ yield

$$p_0^C = \mu - \frac{\rho_{CS}\sigma_C(\mu_S - p_0^S)}{\sigma_S} - \gamma\sigma_C^2(1-\rho_{CS}^2)Q \tag{A.7}$$

$$p_0^S = \mu_S - \frac{\rho_{CS}\sigma_S(\mu - p_0^C)}{\sigma_C} - \gamma\sigma_S^2(1-\rho_{CS}^2)Y \tag{A.8}$$

Plugging equation (A.7) into equation (A.8) and vice-versa yields the equilibrium prices

$$p_0^C = \mu - \gamma\rho_{CS}\sigma_C\sigma_S Y - \gamma\sigma_C^2 Q \tag{A.9}$$

$$p_0^S = \mu_S - \gamma\rho_{CS}\sigma_C\sigma_S Q - \gamma\sigma_S^2 Y \tag{A.10}$$

Plugging equations (A.9) & (A.10) into equations (A.5) & (A.6) yield the equilibrium demands

$$x_i^C = Q + \frac{\mu_i^C - \mu}{\gamma\sigma_C^2(1-\rho_{CS}^2)} \tag{A.11}$$

$$x_i^S = Y - \frac{\rho_{CS}(\mu_i^C - \mu)}{\gamma\sigma_S\sigma_C(1-\rho_{CS}^2)} \tag{A.12}$$

To ensure the short-sales constraint never binds, it must be that $x_i^C > 0 \; \forall i$. It suffices to show that this holds for the agent with the lowest value of μ_i^C, which is $\mu_i^C = \mu - \kappa$. Then

$$x_i^C|_{\mu_i^C = \mu - \kappa} > 0 \iff \kappa < \gamma\sigma_C^2(1-\rho_{CS}^2)Q$$

A.3 Proof of Proposition 3

The agent's problem is

$$\begin{aligned}
&\max_{x_i^C, x_i^S} \; \mathbb{E}\left[-e^{-\gamma[W_0 + x_i^C(\widetilde{C} - p_0^C) + x_i^S(\widetilde{S} - p_0^S)]}\right] \\
&\text{s.t.} \quad x_i^C \geq 0 \\
\implies &\max_{x_i^C, x_i^S} \; -e^{-\gamma\left[W_0 - x_i^C p_0^C - x_i^S p_0^S + x_i^C z + x_i^S \mu_S - \frac{\gamma}{2}\left((x_i^C)^2\sigma_C^2 + (x_i^S)^2\sigma_S^2 + 2x_i^C x_i^S \sigma_{CS}\right)\right]} \\
&\text{s.t.} \quad x_i^C \geq 0
\end{aligned}$$

Taking the first order conditions with respect to x_i^S and x_i^C yield

$$x_i^S = \frac{\mu_S - p_0^S - \gamma x_i^C \sigma_{CS}}{\gamma\sigma_S^2} \tag{A.13}$$

$$x_i^C = \max\{\frac{z - p_0^C - \gamma x_i^S \sigma_{CS}}{\gamma\sigma_C^2} \; , \; 0\} \tag{A.14}$$

Plugging equation (A.13) into equation (A.14) and vice-versa yields

$$x_i^C = \max\{\frac{z - p_0^C}{\gamma\sigma_C^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(\mu_S - p_0^S)}{\gamma\sigma_C\sigma_S(1-\rho_{CS}^2)}, 0\} \tag{A.15}$$

$$x_i^S = \begin{cases} \frac{\mu_S - p_0^S}{\gamma\sigma_S^2(1-\rho_{CS}^2)} - \frac{\rho_{CS}(z-p_0^C)}{\gamma\sigma_C\sigma_S(1-\rho_{CS}^2)} & \text{, if } x_i^C > 0 \\ \frac{\mu_S - p_0^S}{\gamma\sigma_S^2} & \text{, if } x_i^C = 0 \end{cases} \tag{A.16}$$

The short-sales constraint never binds because agents have identical demand functions and the supply is positive, so prices will adjust to achieve an equilibrium. The market clearing conditions $\int_i x_i^C \frac{d\mu_i^C}{2\kappa} = Q$ and $\int_i x_i^S \frac{d\mu_i^C}{2\kappa} = Y$ yield

$$p_0^C = z - \frac{\rho_{CS}\sigma_C}{\sigma_S}(\mu_S - p_0^S) - \gamma\sigma_C^2(1-\rho_{CS}^2)Q \tag{A.17}$$

$$p_0^S = \mu_S - \frac{\rho_{CS}\sigma_S}{\sigma_C}(z - p_0^C) - \gamma\sigma_S^2(1-\rho_{CS}^2)Y \tag{A.18}$$

Then plugging equation (A.17) into equation (A.18) and vice-versa yields the equilibrium prices

$$p_0^C = z - \gamma\rho_{CS}\sigma_C\sigma_S Y - \gamma\sigma_C^2 Q \tag{A.19}$$

$$p_0^S = \mu_S - \gamma\rho_{CS}\sigma_C\sigma_S Q - \gamma\sigma_S^2 Y \tag{A.20}$$

Last, plugging equations (A.19) & (A.20) into equations (A.15) & (A.16) yields the equilibrium demands

$$x_i^C = Q$$

$$x_i^S = Y$$

A.4 Proof of Proposition 4

Aggregate welfare is measured as

$$\int_i \left(x_i^C\,\mathbb{E}[\widetilde{C}] + x_i^S \mu_S - \frac{\gamma}{2}\left[(x_i^C)^2 \sigma_C^2 + (x_i^S)^2 \sigma_S^2 + 2 x_i^C x_i^S \rho_{CS} \sigma_C \sigma_S \right] \right) \frac{d\mu_i}{2\kappa} \tag{A.21}$$

I plug the equilibrium values of $\{x_i^C, x_i^S\}$ derived in Propositions 1-3 into equation (A.21) to compute aggregate welfare for the four possible equilibria:

- Equilibrium 1 (Risky stock only): Aggregate welfare is $Y\mu_S - \frac{\gamma}{2} Y^2 \sigma_S^2$
- Equilibrium 2 (Risky stock & physical collectibles): Aggregate welfare is $Y\mu_S + Q\mu - \frac{\gamma}{2}(Q^2\sigma_C^2 + Y^2\sigma_S^2 + 2\rho_{CS}\sigma_C\sigma_S QY) + \frac{\kappa^2}{6\gamma\sigma_C^2(1-\rho_{CS}^2)}$
- Equilibrium 3 (Risky stock & tokenized collectibles, viewing rights rented): Aggregate welfare is $Y\mu_S + Q(\mu + \kappa + v) - \frac{\gamma}{2}(Q^2\sigma_{CT}^2 + Y^2\sigma_S^2 + 2\rho_{CS}\sigma_{CT}\sigma_S QY)$
- Equilibrium 4 (Risky stock & tokenized collectibles, viewing rights squandered): Aggregate welfare is $Y\mu_S + Q(\mu - \kappa + v) - \frac{\gamma}{2}(Q^2\sigma_{CT}^2 + Y^2\sigma_S^2 + 2\rho_{CS}\sigma_{CT}\sigma_S QY)$

I then standardize the comparisons by setting $Q = Y$ and considering a total supply of $2Y$ of the risky stock in equilibrium 1. This leads to the following aggregate welfare values for the four equilibria:

$$\text{Equilibrium 1:}\quad 2Y\mu_S - 2\gamma Y^2 \sigma_S^2 \tag{A.22}$$

$$\text{Equilibrium 2:}\quad Y(\mu_S + \mu) - \frac{\gamma}{2} Y^2 (\sigma_C^2 + \sigma_S^2 + 2\rho_{CS}\sigma_C\sigma_S) + \frac{\kappa^2}{6\gamma\sigma_C^2(1-\rho_{CS}^2)} \tag{A.23}$$

$$\text{Equilibrium 3:}\quad Y(\mu_S + \mu + v + \kappa) - \frac{\gamma}{2} Y^2 (\sigma_{CT}^2 + \sigma_S^2 + 2\rho_{CS}\sigma_{CT}\sigma_S) \tag{A.24}$$

$$\text{Equilibrium 4:}\quad Y(\mu_S + \mu + v - \kappa) - \frac{\gamma}{2} Y^2 (\sigma_{CT}^2 + \sigma_S^2 + 2\rho_{CS}\sigma_{CT}\sigma_S) \tag{A.25}$$

Proposition 4 follows as the condition for which equation (A.23) is greater than equation (A.22).

A.5 Proof of Proposition 5

Proposition 5 follows as the condition for which equation (A.25) is greater than equation (A.23).

A.6 Proof of Proposition 6

Proposition 6 follows as the condition for which equation (A.24) is greater than equation (A.23).

A.7 Proof of Proposition 7

Proposition 7 follows as the condition for which equation (A.24) is greater than equation (A.25).

B Approximation to Compound Total Return

Let P_0 denote the value of the price index at time 0, which is inclusive of any dividends or coupon payments. The compound total return from time 0 to T is thus:

$$\frac{P_T}{P_0} - 1 \tag{B.1}$$

Define the gross return from time $t-1$ to t as $R_t = \frac{P_t}{P_{t-1}}$. Note that

$$\begin{aligned} \frac{P_T}{P_0} &= \prod_{t=1}^{T} R_t \\ \implies \log\left(\frac{P_T}{P_0}\right) &= \sum_{t=1}^{T} \log(R_t) \end{aligned} \tag{B.2}$$

Since the second-order Taylor series expansion of $\log(R_t)$ centered at $R_t = \bar{R}$ is:[20]

$$\log(R_t) = \log(\bar{R}) + \frac{1}{\bar{R}}(R_t - \bar{R}) - \frac{1}{2\bar{R}^2}(R_t - \bar{R})^2$$

equation (B.2) can be rewritten as:

$$\begin{aligned}
\log\left(\frac{P_T}{P_0}\right) &= \sum_{t=1}^{T} \log(\bar{R}) + \frac{1}{\bar{R}} \sum_{t=1}^{T} (R_t - \bar{R}) - \frac{1}{2\bar{R}^2} \sum_{t=1}^{T} (R_t - \bar{R})^2 \\
\implies \frac{1}{T} \log\left(\frac{P_T}{P_0}\right) &= \log(\bar{R}) - \frac{1}{2\bar{R}^2} \frac{1}{T} \sum_{t=1}^{T} (R_t - \bar{R})^2 \\
\implies \frac{1}{T} \log\left(\frac{P_T}{P_0}\right) &= \log(\bar{R}) - \frac{1}{2\bar{R}^2} \frac{T-1}{T} \widehat{\sigma}^2 \qquad \text{(B.3)}
\end{aligned}$$

where $\widehat{\sigma}^2 = \frac{1}{T-1} \sum_{t=1}^{T} (R_t - \bar{R})^2$ is the unbiased estimate of the sample variance. Solving equation (B.3) for the compound total return in equation (B.1) yields:

$$\frac{P_T}{P_0} - 1 = \exp\left(T \log(\bar{R}) - \frac{T-1}{2\bar{R}^2} \widehat{\sigma}^2\right) - 1 \qquad \text{(B.4)}$$

Since the collectibles price indices are smoothed, equation (B.1) provides an unreliable estimate of the compound total return. A better estimate of the compound total return comes from substituting the autocorrelation-adjusted variance estimate into equation (B.4).

C Estimating the Total Float of Collectibles

I construct estimates of the total float of the nine collectibles sub-asset classes and of the collectibles market as a whole. My methodology seeks to apply a survey of high net worth individuals' (HNWIs) reported collectibles ownership as a percentage of wealth to the global wealth distribution. Note that the reported HNWI collectibles holdings are summarized in Table 6. Thus, I focus here on how additional information about the global wealth distribution

[20] Note that the Taylor series expansion of a real function $f(x)$ about a point $x = a$ is given by:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots$$

can be used to estimate the collectibles floats.

I rely on Credit Suisse Research (2017) for a characterization of the global wealth distribution in 2017, reproduced in Table 12. The Barclays (2012) survey can be viewed as approximately applying to individuals with more than \$1 million of wealth, but such surveys do not exist for the less wealthy categories. However, as US households with at least \$1.14 million of wealth own 84% of all stocks (Wolff (2017)), and as wealthier individuals find it less costly to own illiquid assets like collectibles, it seems reasonable to assume that adults with more than \$100,000 of wealth own virtually all collectibles. This simplifying assumption is useful for restricting attention to individuals in two wealth buckets: the \$100,000 - \$1 million wealth bucket and the \$1+ million wealth bucket.

To arrive at my baseline estimates for the total floats of the nine collectibles sub-asset classes, I make two additional assumptions:

1. The survey-based HNWI collectibles holdings are representative of the holdings of individuals in the \$1+ million wealth bucket. As collectibles are a luxury good, the HNWI collectibles holding percentage may be an overestimate for the less wealthy and an underestimate for the wealthier. For my baseline estimates, I assume these under- and over-estimations approximately cancel out.

2. Adults in the \$100,000 - \$1 million wealth bucket own, as a percentage of wealth, half the value of collectibles as adults in the HNWI survey. Since these adults have greater collectibles ownership than adults in the <\$100,000 wealth bucket (0%) but less collectibles ownership than adults in the HNWI survey, the midpoint seems apt for my baseline estimates.

To construct lower estimates, I assume adults in the \$100,000 - \$1 million wealth bucket own, as a percentage of wealth, 25% of the value of collectibles as adults in the HNWI survey. To construct upper estimates, I assume adults in the \$1+ million wealth bucket own, as a percentage of wealth, 125% of the value of collectibles as adults in the HNWI survey.

My baseline estimate for the total float of collectibles in 2017 is \$21.1 trillion, with lower and upper estimates of \$17.9 trillion and \$24.8 trillion, respectively.[21] Figure 3 presents the estimates for the collectibles category floats.

A key advantage of using a holdings-based estimation approach, as opposed to a sales-based estimation approach, is that I need not be concerned with the resale tendencies of the buyers (ie: private individuals vs. museums) or with the division of the supply-side market into the auction market (highly fragmented & transparent) and the dealer market (highly fragmented & opaque).

[21] Technically, these are estimates of the total value of private collectibles holdings. However, as individuals face no selling restrictions like museums and institutions, it seems reasonable to take these as estimates of the total float.

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Online Data Appendix for
Collectibles Tokenization & Optimal Security Design

February, 2020

Table of Contents

A Portfolio Problem Robustness: Collectibles Floats

A.1 Lower Float Estimates

Table A.1: Portfolio Problem: Ex-Ante Sharpe Ratio, All Real Estate, Lower Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-ante Sharpe ratio. The market portfolio includes global equity, global bonds, and all real estate. The collectibles floats are the lower estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible return (μ^{EQM}), the observed mean collectible return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.059	-0.006	0.023	0.032	0.023	0.012	0.015	0.057	0.086	0.049
	μ^{OBS}	0.023	0.037	0.035	0.027	-0.023	0.050	0.006	0.058	0.108	0.050
	t($\mu^{OBS} - \mu^{EQM}$)	1.675*	3.274***	0.710	-0.267	-1.793*	1.333	-0.230	0.060	0.438	0.031
	μ^{TOK}	0.037	0.020	0.055	0.068	0.073	0.068	0.094	0.106	0.185	0.135
	d^{IMP}	0.014	-0.017	0.020	0.041	0.096	0.018	0.087	0.047	0.077	0.085
1994-2017	μ^{EQM}	-0.046	-0.002	0.026	0.034	0.027	0.017	-0.003	0.064	0.095	0.055
	μ^{OBS}	0.058	0.041	0.040	0.028	-0.019	0.058	-0.025	0.071	0.116	0.064
	t($\mu^{OBS} - \mu^{EQM}$)	2.439**	3.106***	0.853	-0.288	-1.627	1.867*	-0.600	0.302	0.367	0.182
	μ^{TOK}	0.037	0.025	0.059	0.073	0.083	0.060	0.069	0.111	0.209	0.154
	d^{IMP}	-0.020	-0.016	0.019	0.044	0.102	0.002	0.094	0.040	0.093	0.090
1998-2017	μ^{EQM}	-0.039	-0.001	0.026	0.033	0.020	0.026	-0.000	0.071	0.068	0.046
	μ^{OBS}	0.069	0.049	0.039	0.024	-0.033	0.068	-0.027	0.077	0.062	0.072
	t($\mu^{OBS} - \mu^{EQM}$)	2.178**	3.606***	0.662	-0.378	-1.960*	1.692*	-0.731	0.216	-0.106	0.463
	μ^{TOK}	0.059	0.027	0.066	0.079	0.074	0.075	0.072	0.127	0.168	0.155
	d^{IMP}	-0.010	-0.023	0.026	0.055	0.107	0.007	0.099	0.050	0.105	0.083
2002-2017	μ^{EQM}	-0.051	-0.006	0.028	0.036	0.028	0.017	-0.001	0.069	0.078	0.065
	μ^{OBS}	0.073	0.059	0.047	0.036	-0.051	0.094	-0.069	0.084	0.080	0.086
	t($\mu^{OBS} - \mu^{EQM}$)	2.053**	5.430***	0.776	-0.033	-2.905***	3.738***	-2.077**	0.439	0.045	0.343
	μ^{TOK}	0.068	0.017	0.075	0.089	0.081	0.058	0.064	0.133	0.203	0.189
	d^{IMP}	-0.005	-0.042	0.029	0.053	0.132	-0.037	0.133	0.049	0.122	0.102

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.2: Portfolio Problem: Ex-Post Sharpe Ratio, All Real Estate, Lower Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-post Sharpe ratio. The market portfolio includes global equity, global bonds, and all real estate. The collectibles floats are the lower estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible excess return (μ^{EQM}), the observed mean collectible excess return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible excess return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible excess return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible excess return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.057	0.010	0.011	0.018	0.000	0.009	-0.007	0.052	0.050	0.023
	μ^{OBS}	-0.004	0.010	0.007	-0.000	-0.050	0.022	-0.021	0.031	0.081	0.023
	t($\mu^{OBS} - \mu^{EQM}$)	1.064	-0.023	-0.188	-0.953	-2.346**	0.441	-0.405	-0.756	0.644	0.004
	μ^{TOK}	0.041	0.049	0.043	0.055	0.043	0.069	0.061	0.105	0.145	0.106
	d^{IMP}	0.045	0.039	0.035	0.055	0.093	0.046	0.083	0.074	0.064	0.083
1994-2017	μ^{EQM}	-0.044	0.011	0.013	0.021	0.002	0.014	-0.013	0.056	0.056	0.027
	μ^{OBS}	0.034	0.017	0.017	0.004	-0.043	0.034	-0.048	0.047	0.092	0.040
	t($\mu^{OBS} - \mu^{EQM}$)	1.860*	0.293	0.211	-0.800	-2.009**	0.749	-1.111	-0.345	0.654	0.291
	μ^{TOK}	0.038	0.052	0.046	0.061	0.046	0.067	0.050	0.105	0.165	0.120
	d^{IMP}	0.005	0.035	0.030	0.057	0.089	0.033	0.099	0.058	0.073	0.079
1998-2017	μ^{EQM}	-0.049	0.007	0.014	0.022	0.002	0.018	-0.011	0.059	0.043	0.018
	μ^{OBS}	0.050	0.030	0.020	0.005	-0.052	0.049	-0.046	0.058	0.043	0.053
	t($\mu^{OBS} - \mu^{EQM}$)	2.073**	1.196	0.307	-0.671	-2.356**	1.038	-1.060	-0.017	0.012	0.730
	μ^{TOK}	0.045	0.046	0.054	0.070	0.047	0.077	0.053	0.115	0.139	0.113
	d^{IMP}	-0.005	0.015	0.033	0.065	0.098	0.028	0.099	0.057	0.096	0.060
2002-2017	μ^{EQM}	-0.067	-0.006	0.014	0.024	0.010	0.006	-0.006	0.064	0.053	0.033
	μ^{OBS}	0.061	0.047	0.035	0.024	-0.063	0.082	-0.081	0.072	0.069	0.075
	t($\mu^{OBS} - \mu^{EQM}$)	2.221**	3.171***	0.996	0.010	-2.956***	3.639***	-2.327**	0.254	0.265	0.757
	μ^{TOK}	0.047	0.027	0.056	0.074	0.059	0.047	0.058	0.123	0.171	0.142
	d^{IMP}	-0.014	-0.020	0.021	0.050	0.121	-0.035	0.138	0.051	0.103	0.067

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.3: Portfolio Problem: Ex-Ante Sharpe Ratio, Investable Real Estate, Lower Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-ante Sharpe ratio. The market portfolio includes global equity, global bonds, and investable real estate. The collectibles floats are the lower estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible return (μ^{EQM}), the observed mean collectible return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.044	-0.013	0.030	0.042	0.025	0.015	0.024	0.054	0.107	0.075
	μ^{OBS}	0.023	0.037	0.035	0.027	-0.023	0.050	0.006	0.058	0.108	0.050
	$t(\mu^{OBS} - \mu^{EQM})$	1.363	3.815***	0.275	-0.799	-1.887*	1.195	-0.450	0.166	0.018	-0.573
	μ^{TOK}	0.053	0.013	0.062	0.078	0.076	0.072	0.102	0.103	0.207	0.162
	d^{IMP}	0.030	-0.024	0.028	0.051	0.098	0.022	0.096	0.045	0.098	0.112
1994- 2017	μ^{EQM}	-0.027	-0.006	0.034	0.045	0.032	0.023	0.004	0.064	0.119	0.084
	μ^{OBS}	0.058	0.041	0.040	0.028	-0.019	0.058	-0.025	0.071	0.116	0.064
	$t(\mu^{OBS} - \mu^{EQM})$	1.991**	3.383***	0.371	-0.860	-1.817*	1.626	-0.781	0.277	-0.058	-0.398
	μ^{TOK}	0.057	0.021	0.068	0.084	0.088	0.066	0.076	0.111	0.233	0.183
	d^{IMP}	-0.001	-0.020	0.027	0.056	0.107	0.008	0.100	0.041	0.117	0.119
1998-2017	μ^{EQM}	-0.021	0.002	0.033	0.043	0.018	0.030	-0.001	0.072	0.084	0.079
	μ^{OBS}	0.069	0.049	0.039	0.024	-0.033	0.068	-0.027	0.077	0.062	0.072
	$t(\mu^{OBS} - \mu^{EQM})$	1.815*	3.432***	0.318	-0.798	-1.854*	1.511	-0.705	0.182	-0.420	-0.137
	μ^{TOK}	0.077	0.029	0.073	0.089	0.071	0.080	0.071	0.128	0.184	0.188
	d^{IMP}	0.008	-0.020	0.033	0.065	0.104	0.012	0.098	0.051	0.121	0.116
2002-2017	μ^{EQM}	-0.041	-0.013	0.033	0.043	0.030	0.017	0.001	0.077	0.098	0.090
	μ^{OBS}	0.073	0.059	0.047	0.036	-0.051	0.094	-0.069	0.084	0.080	0.086
	$t(\mu^{OBS} - \mu^{EQM})$	1.886*	5.948***	0.551	-0.277	-3.007***	3.715***	-2.135*	0.203	-0.273	-0.063
	μ^{TOK}	0.078	0.011	0.081	0.095	0.084	0.058	0.066	0.140	0.223	0.214
	d^{IMP}	0.005	-0.048	0.034	0.060	0.134	-0.036	0.135	0.057	0.142	0.128

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.4: Portfolio Problem: Ex-Post Sharpe Ratio, Investable Real Estate, Lower Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-post Sharpe ratio. The market portfolio includes global equity, global bonds, and investable real estate. The collectibles floats are the lower estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible excess return (μ^{EQM}), the observed mean collectible excess return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible excess return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible excess return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible excess return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.024	0.011	0.016	0.025	-0.002	0.016	-0.012	0.049	0.058	0.036
	μ^{OBS}	-0.004	0.010	0.007	-0.000	-0.050	0.022	-0.021	0.031	0.081	0.023
	$t(\mu^{OBS} - \mu^{EQM})$	0.400	-0.048	-0.496	-1.347	-2.251**	0.215	-0.274	-0.672	0.477	-0.315
	μ^{TOK}	0.074	0.049	0.048	0.062	0.041	0.076	0.057	0.103	0.154	0.119
	d^{IMP}	0.078	0.039	0.040	0.063	0.091	0.053	0.078	0.072	0.073	0.096
1994-2017	μ^{EQM}	-0.035	0.013	0.019	0.029	0.002	0.023	-0.011	0.055	0.065	0.041
	μ^{OBS}	0.034	0.017	0.017	0.004	-0.043	0.034	-0.048	0.047	0.092	0.040
	$t(\mu^{OBS} - \mu^{EQM})$	1.649	0.201	-0.132	-1.217	-2.003**	0.438	-1.182	-0.326	0.480	-0.012
	μ^{TOK}	0.047	0.054	0.052	0.069	0.046	0.075	0.053	0.105	0.174	0.134
	d^{IMP}	0.013	0.037	0.035	0.065	0.089	0.041	0.101	0.058	0.082	0.094
1998-2017	μ^{EQM}	-0.028	0.012	0.019	0.029	0.000	0.026	-0.012	0.056	0.050	0.033
	μ^{OBS}	0.050	0.030	0.020	0.005	-0.052	0.049	-0.046	0.058	0.043	0.053
	$t(\mu^{OBS} - \mu^{EQM})$	1.622	0.916	0.045	-0.980	-2.271**	0.769	-1.056	0.080	-0.140	0.406
	μ^{TOK}	0.067	0.051	0.059	0.078	0.045	0.085	0.053	0.112	0.147	0.129
	d^{IMP}	0.017	0.021	0.039	0.072	0.096	0.036	0.099	0.054	0.103	0.076
2002-2017	μ^{EQM}	-0.062	-0.003	0.017	0.029	0.012	0.008	-0.000	0.073	0.065	0.048
	μ^{OBS}	0.061	0.047	0.035	0.024	-0.063	0.082	-0.081	0.072	0.069	0.075
	$t(\mu^{OBS} - \mu^{EQM})$	2.137**	3.006***	0.850	-0.201	-3.019***	3.559***	-2.503**	-0.042	0.063	0.480
	μ^{TOK}	0.051	0.030	0.059	0.079	0.060	0.049	0.063	0.131	0.183	0.157
	d^{IMP}	-0.009	-0.017	0.024	0.056	0.123	-0.033	0.144	0.060	0.115	0.082

Note: *p<0.1; **p<0.05; ***p<0.01

A.2 Upper Float Estimates

Table A.5: Portfolio Problem: Ex-Ante Sharpe Ratio, All Real Estate, Upper Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-ante Sharpe ratio. The market portfolio includes global equity, global bonds, and all real estate. The collectibles floats are the upper estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible return (μ^{EQM}), the observed mean collectible return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.059	-0.006	0.023	0.032	0.023	0.012	0.015	0.057	0.086	0.054
	μ^{OBS}	0.023	0.037	0.035	0.027	-0.023	0.050	0.006	0.058	0.108	0.050
	$t(\mu^{OBS} - \mu^{EQM})$	1.675*	3.274***	0.703	-0.267	-1.793*	1.316	-0.230	0.055	0.433	-0.095
	μ^{TOK}	0.037	0.020	0.055	0.068	0.073	0.068	0.094	0.106	0.186	0.141
	d^{IMP}	0.014	-0.017	0.021	0.041	0.096	0.019	0.087	0.048	0.077	0.091
1994-2017	μ^{EQM}	-0.046	-0.002	0.026	0.034	0.027	0.017	-0.002	0.064	0.095	0.059
	μ^{OBS}	0.058	0.041	0.040	0.028	-0.019	0.058	-0.025	0.071	0.116	0.064
	$t(\mu^{OBS} - \mu^{EQM})$	2.439**	3.106***	0.853	-0.288	-1.635	1.867*	-0.610	0.302	0.363	0.103
	μ^{TOK}	0.037	0.025	0.059	0.073	0.083	0.060	0.069	0.111	0.209	0.158
	d^{IMP}	-0.020	-0.016	0.019	0.044	0.102	0.002	0.094	0.040	0.093	0.094
1998-2017	μ^{EQM}	-0.039	-0.001	0.026	0.033	0.021	0.026	-0.000	0.071	0.068	0.051
	μ^{OBS}	0.069	0.049	0.039	0.024	-0.033	0.068	-0.027	0.077	0.062	0.072
	$t(\mu^{OBS} - \mu^{EQM})$	2.176**	3.606***	0.644	-0.378	-1.965*	1.683*	-0.735	0.216	-0.108	0.387
	μ^{TOK}	0.059	0.027	0.066	0.079	0.074	0.075	0.072	0.127	0.168	0.159
	d^{IMP}	-0.010	-0.023	0.027	0.055	0.107	0.007	0.100	0.050	0.106	0.087
2002-2017	μ^{EQM}	-0.051	-0.006	0.028	0.037	0.028	0.017	-0.001	0.069	0.078	0.068
	μ^{OBS}	0.073	0.059	0.047	0.036	-0.051	0.094	-0.069	0.084	0.080	0.086
	$t(\mu^{OBS} - \mu^{EQM})$	2.051**	5.430***	0.776	-0.037	-2.905***	3.738***	-2.077**	0.439	0.043	0.287
	μ^{TOK}	0.068	0.017	0.075	0.089	0.081	0.058	0.064	0.133	0.203	0.192
	d^{IMP}	-0.005	-0.042	0.029	0.053	0.132	-0.037	0.133	0.049	0.122	0.106

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.6: Portfolio Problem: Ex-Post Sharpe Ratio, All Real Estate, Upper Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-post Sharpe ratio. The market portfolio includes global equity, global bonds, and all real estate. The collectibles floats are the upper estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible excess return (μ^{EQM}), the observed mean collectible excess return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible excess return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible excess return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible excess return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.055	0.010	0.011	0.018	0.000	0.009	-0.007	0.052	0.050	0.026
	μ^{OBS}	-0.004	0.010	0.007	-0.000	-0.050	0.022	-0.021	0.031	0.081	0.023
	t($\mu^{OBS} - \mu^{EQM}$)	1.031	-0.030	-0.196	-0.953	-2.346**	0.433	-0.408	-0.761	0.642	-0.061
	μ^{TOK}	0.042	0.049	0.043	0.055	0.043	0.069	0.062	0.105	0.146	0.108
	d^{IMP}	0.047	0.039	0.035	0.055	0.093	0.046	0.083	0.074	0.065	0.085
1994-2017	μ^{EQM}	-0.044	0.011	0.013	0.021	0.002	0.014	-0.013	0.056	0.056	0.030
	μ^{OBS}	0.034	0.017	0.017	0.004	-0.043	0.034	-0.048	0.047	0.092	0.040
	t($\mu^{OBS} - \mu^{EQM}$)	1.860*	0.287	0.204	-0.800	-2.014**	0.749	-1.111	-0.345	0.651	0.232
	μ^{TOK}	0.038	0.052	0.046	0.061	0.046	0.067	0.050	0.105	0.165	0.122
	d^{IMP}	0.005	0.035	0.030	0.057	0.089	0.033	0.099	0.058	0.073	0.082
1998-2017	μ^{EQM}	-0.048	0.007	0.014	0.022	0.002	0.018	-0.011	0.059	0.043	0.020
	μ^{OBS}	0.050	0.030	0.020	0.005	-0.052	0.049	-0.046	0.058	0.043	0.053
	t($\mu^{OBS} - \mu^{EQM}$)	2.053**	1.196	0.301	-0.671	-2.361**	1.038	-1.060	-0.017	0.010	0.688
	μ^{TOK}	0.046	0.046	0.054	0.070	0.047	0.077	0.053	0.115	0.139	0.115
	d^{IMP}	-0.004	0.015	0.034	0.065	0.098	0.028	0.099	0.057	0.096	0.062
2002-2017	μ^{EQM}	-0.067	-0.006	0.014	0.024	0.010	0.006	-0.006	0.064	0.053	0.035
	μ^{OBS}	0.061	0.047	0.035	0.024	-0.063	0.082	-0.081	0.072	0.069	0.075
	t($\mu^{OBS} - \mu^{EQM}$)	2.221**	3.164***	0.996	0.010	-2.956***	3.633***	-2.327**	0.254	0.263	0.713
	μ^{TOK}	0.047	0.027	0.056	0.074	0.059	0.048	0.058	0.123	0.171	0.144
	d^{IMP}	-0.014	-0.020	0.021	0.050	0.121	-0.035	0.138	0.051	0.103	0.069

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.7: Portfolio Problem: Ex-Ante Sharpe Ratio, Investable Real Estate, Upper Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-ante Sharpe ratio. The market portfolio includes global equity, global bonds, and investable real estate. The collectibles floats are the upper estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible return (μ^{EQM}), the observed mean collectible return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.040	-0.013	0.031	0.042	0.025	0.015	0.025	0.054	0.108	0.086
	μ^{OBS}	0.023	0.037	0.035	0.027	-0.023	0.050	0.006	0.058	0.108	0.050
	$t(\mu^{OBS} - \mu^{EQM})$	1.294	3.815***	0.238	-0.799	-1.887*	1.195	-0.468	0.156	0.008	-0.813
	μ^{TOK}	0.056	0.013	0.063	0.078	0.076	0.072	0.103	0.103	0.207	0.172
	d^{IMP}	0.033	-0.024	0.028	0.051	0.098	0.022	0.097	0.045	0.099	0.122
1994-2017	μ^{EQM}	-0.025	-0.006	0.034	0.045	0.033	0.023	0.004	0.064	0.120	0.094
	μ^{OBS}	0.058	0.041	0.040	0.028	-0.019	0.058	-0.025	0.071	0.116	0.064
	$t(\mu^{OBS} - \mu^{EQM})$	1.940*	3.383***	0.364	-0.860	-1.830*	1.610	-0.781	0.267	-0.066	-0.596
	μ^{TOK}	0.059	0.021	0.068	0.084	0.088	0.066	0.076	0.112	0.234	0.193
	d^{IMP}	0.001	-0.020	0.027	0.056	0.108	0.008	0.100	0.041	0.118	0.129
1998-2017	μ^{EQM}	-0.019	0.002	0.033	0.043	0.018	0.030	-0.001	0.072	0.084	0.087
	μ^{OBS}	0.069	0.049	0.039	0.024	-0.033	0.068	-0.027	0.077	0.062	0.072
	$t(\mu^{OBS} - \mu^{EQM})$	1.774*	3.415***	0.301	-0.798	-1.863*	1.497	-0.705	0.178	-0.425	-0.276
	μ^{TOK}	0.079	0.029	0.073	0.089	0.071	0.080	0.071	0.128	0.184	0.195
	d^{IMP}	0.010	-0.020	0.034	0.065	0.104	0.012	0.098	0.051	0.122	0.124
2002-2017	μ^{EQM}	-0.040	-0.013	0.034	0.043	0.030	0.017	0.001	0.077	0.098	0.098
	μ^{OBS}	0.073	0.059	0.047	0.036	-0.051	0.094	-0.069	0.084	0.080	0.086
	$t(\mu^{OBS} - \mu^{EQM})$	1.872*	5.948***	0.526	-0.277	-3.012***	3.715***	-2.135*	0.199	-0.278	-0.182
	μ^{TOK}	0.079	0.011	0.081	0.095	0.084	0.058	0.066	0.140	0.223	0.221
	d^{IMP}	0.006	-0.048	0.035	0.060	0.135	-0.036	0.135	0.057	0.143	0.135

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.8: Portfolio Problem: Ex-Post Sharpe Ratio, Investable Real Estate, Upper Floats

This table presents the results of a portfolio allocation problem in which the investor allocates his wealth between the tokenized collectible and the value-weighted market portfolio to maximize the ex-post Sharpe ratio. The market portfolio includes global equity, global bonds, and investable real estate. The collectibles floats are the upper estimates from Figure 3. Each test is repeated using data for four different time horizons: 1990-2017, 1994-2017, 1998-2017, and 2002-2017. The reported results include the equilibrium mean collectible excess return (μ^{EQM}), the observed mean collectible excess return (μ^{OBS}), the number of standard errors between these two means ($t(\mu^{OBS} - \mu^{EQM})$), the mean collectible excess return that is larger than μ^{EQM} at exactly the 5% significance level (μ^{TOK}), and the implied rental yield that is the difference between μ^{TOK} and μ^{OBS} (d^{IMP}). The equilibrium mean is the mean collectible excess return that induces the investors to hold the tokenized collectible at its current market value weight. μ^{TOK} is the mean collectible excess return that would induce the investors to demand a dollar-value of the tokenized collectible that exceeds its total float. d^{IMP} is thus the financial dividend yield the collectible tokens would need to pay (via rentals of the viewing rights) to make the investors willing to pay a premium for the tokenized collectibles. The mean standard error is estimated following my method in section 3.1.

		Art	Sculptures	Jewelry	Stamps	Furniture	Cars	Rugs	Coins	Wine	Basket
1990-2017	μ^{EQM}	-0.022	0.011	0.016	0.025	-0.001	0.016	-0.012	0.050	0.058	0.043
	μ^{OBS}	-0.004	0.010	0.007	-0.000	-0.050	0.022	-0.021	0.031	0.081	0.023
	t($\mu^{OBS} - \mu^{EQM}$)	0.354	-0.054	-0.511	-1.353	-2.262**	0.203	-0.274	-0.681	0.469	-0.466
	μ^{TOK}	0.076	0.049	0.048	0.063	0.041	0.076	0.057	0.103	0.154	0.125
	d^{IMP}	0.080	0.040	0.041	0.063	0.091	0.053	0.078	0.072	0.073	0.102
1994-2017	μ^{EQM}	-0.033	0.013	0.019	0.029	0.002	0.023	-0.010	0.055	0.066	0.047
	μ^{OBS}	0.034	0.017	0.017	0.004	-0.043	0.034	-0.048	0.047	0.092	0.040
	t($\mu^{OBS} - \mu^{EQM}$)	1.609	0.195	-0.139	-1.223	-2.009**	0.434	-1.194	-0.326	0.474	-0.141
	μ^{TOK}	0.049	0.054	0.052	0.069	0.046	0.075	0.053	0.105	0.175	0.140
	d^{IMP}	0.015	0.037	0.035	0.065	0.089	0.041	0.101	0.058	0.083	0.100
1998-2017	μ^{EQM}	-0.028	0.012	0.020	0.029	0.000	0.026	-0.012	0.056	0.050	0.038
	μ^{OBS}	0.050	0.030	0.020	0.005	-0.052	0.049	-0.046	0.058	0.043	0.053
	t($\mu^{OBS} - \mu^{EQM}$)	1.622	0.916	0.033	-0.980	-2.271**	0.765	-1.056	0.076	-0.143	0.317
	μ^{TOK}	0.067	0.051	0.059	0.078	0.045	0.085	0.053	0.112	0.147	0.133
	d^{IMP}	0.017	0.021	0.039	0.072	0.096	0.036	0.099	0.054	0.103	0.080
2002-2017	μ^{EQM}	-0.060	-0.003	0.017	0.029	0.012	0.008	-0.000	0.073	0.065	0.054
	μ^{OBS}	0.061	0.047	0.035	0.024	-0.063	0.082	-0.081	0.072	0.069	0.075
	t($\mu^{OBS} - \mu^{EQM}$)	2.100**	3.006***	0.839	-0.201	-3.024***	3.553***	-2.503**	-0.042	0.057	0.382
	μ^{TOK}	0.054	0.030	0.059	0.079	0.061	0.049	0.063	0.131	0.184	0.162
	d^{IMP}	-0.007	-0.017	0.024	0.056	0.123	-0.033	0.144	0.060	0.115	0.088

Note: *p<0.1; **p<0.05; ***p<0.01

B Comparing Average-Sales & Repeat-Sales Price Indices

To better understand the consequences of using AS instead of RS price indices, I compare the statistical properties of AS and RS returns for collectibles sub-asset classes for which both exist. For comparability, I use the maximum time horizon for which both return series exist.

Table B.1 presents the annualized mean returns, autocorrelation-adjusted volatilities, and autocorrelation-adjusted correlations between the matched AS-RS series and with global bonds, global equity, and real estate. Notably, there is no consistent unidirectional relationship between the AS and RS means, standard deviations, or correlations. The differences stem not only from the different price index estimation methods, but also from trimming the top and bottom 10% of transactions by price in constructing the AS price indices and from imperfect matching of the indices' underlying assets. For example, the RS jewelry index contains only the basket of jewelry considered for the US consumer price index (CPI), while the AS jewelry index aims to be globally representative and contains 1945-1975 jewelry, antique jewelry, Belle Epogue & Art Deco jewelry, and pearl jewelry.

Table B.1: Comparing Average-Sales and Repeat-Sales Returns

This table compares summary statistics for average-sales (AS) and repeat-sales (RS) price indices covering the same collectibles category, focusing on the annualized mean, the annualized standard deviation, the correlation between the AS & RS returns for the same collectible category, and the correlation with global bonds, global equities, and real estate. All standard deviations and correlations have been autocorrelation-adjusted following the methodology in section 3.1.

	Time Period	Mean (Annualized %)	Std. Dev. (Annualized %)	Cor(AS,RS)	Cor(Bonds)	Cor(Equities)	Cor(RE)
Paintings (AS)	Q1 1998 - Q4 2017	1.26%	12.36%	0.54	0.20	0.43	0.19
Paintings (RS)	Q1 1998 - Q4 2017	2.02%	5.88%	0.54	-0.06	-0.14	-0.29
Sculptures (AS)	Q1 1998 - Q4 2017	1.68%	1.65%	-0.23	-0.25	-0.24	-0.31
Sculptures (RS)	Q1 1998 - Q4 2017	1.55%	12.95%	-0.23	-0.22	0.68	0.52
Jewelry (AS)	Q1 1990 - Q4 2017	1.17%	2.83%	-0.07	-0.16	0.26	0.10
Jewelry (RS)	Q1 1990 - Q4 2017	1.48%	9.29%	-0.07	-0.21	-0.27	-0.04
Stamps (AS)	Q1 1999 - Q4 2012	1.32%	3.64%	0.09	0.08	0.68	0.24
Stamps (RS)	Q1 1999 - Q4 2012	5.72%	10.13%	0.09	0.06	0.46	0.41

The stark differences between the AS & RS price indices, stemming from a number of commingled reasons, make it challenging to draw any meaningful conclusions about their comparison. Certainly the two indices are quite different. As this comparison cannot be used to inform the construction of a synthetic RS collectibles price index, I can only suggest that, by removing the top and bottom 10% of transactions by sales price in forming the AS price indices, the resulting AS price indices provide a better picture of true returns than using the original AS price indices.[22]

[22] Note that this is an option when purchasing the AS price indices from Art Market Research, which is also selected by Worthington & Higgs (2004) for the AMR art price index they use. The underlying transaction data is not available through AMR, which prevents the construction of actual RS price indices.

C Unsmoothing Theory

I briefly cover the theory & assumptions underlying the four most common methods of unsmoothing serially correlated returns. The four methods are those of Geltner (1993), Okunev & White (OW) (2003), Getmansky, Lo, & Makarov (GLM) (2004), and Amvella, Meier, & Papageorgiou (AMP) (2010).

Throughout this section, I let r_t denote the observed (ie: smoothed) returns and r_t^* denote the unobserved (ie: unsmoothed) returns. The raw data coincides with the smoothed $\{r_t\}$ series, which these methods seek to transform into the unsmoothed $\{r_t^*\}$ series.

C.1 Geltner (1993) Unsmoothing

Geltner (1993) assumes that the observed (ie: smoothed) returns $\{r_t\}$ follow an AR(1) process:

$$\begin{aligned}\textbf{Assumption G.1:}\ \ & r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \\ \text{s.t.}\ \ & \epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2) \\ & |\alpha_1| < 1\end{aligned}$$

In addition, Geltner (1993) assumes a particular form of the smoothing:

$$\textbf{Assumption G.2:}\ \ r_t = (1-\alpha_1) r_t^* + \alpha_1 r_{t-1}$$

Assumptions G.1 & G.2 imply that the true unsmoothed returns can be calculated as:

$$r_t^* = \frac{1}{(1-\alpha_1)} \Big[r_t - \alpha_1 r_{t-1} \Big] \tag{C.1}$$

where the $\{r_t^*\}$ series has zero first-order autocorrelation by construction.

C.2 Okunev & White (2003) Unsmoothing

OW (2003) serve to extend Geltner (1993). For example, suppose the observed returns $\{r_t\}$ follow an AR(2) process. Letting $\{r_t^i\}$ denote returns adjusted i times, they rewrite equation

(C.1) as:

$$r_t^1 = \frac{1}{(1-\alpha_1)}\Big[r_t^0 - \alpha_1 r_{t-1}^0\Big]$$

Then the second-order autocorrelation can be removed as follows:

$$r_t^2 = \frac{1}{(1-\alpha_2)}\Big[r_t^1 - \alpha_2 r_{t-2}^1\Big]$$

However, the first-order autocorrelation of $\{r_t^2\}$ is no longer zero, so the first-order autocorrelation must be removed again:

$$r_t^3 = \frac{1}{(1-\alpha_3)}\Big[r_t^2 - \alpha_3 r_{t-1}^2\Big]$$

Depending on the magnitude of the remaining first- and second-order autocorrelation, this iterative process can be continued if needed. This process can be extended to correct for any order of autocorrelation.

C.3 GLM (2004) Unsmoothing

GLM (2004) assume that the observed demeaned returns follow a MA(k) process:

$$\begin{aligned}
\textbf{Assumption GLM.1:}\quad r_t - \mu &= \theta_0(r_t^* - \mu) + \theta_1(r_{t-1}^* - \mu) + \cdots + \theta_k(r_{t-k}^* - \mu) \\
&= \theta_0\nu_t + \theta_1\nu_{t-1} + \cdots + \theta_k\nu_{t-k} \\
\text{s.t}\ \ 1 &= \theta_0 + \theta_1 + \cdots + \theta_k \\
\nu_t &\overset{iid}{\sim} \mathcal{N}(0, \sigma_\nu^2)
\end{aligned}$$

Under assumption GLM.1, we can estimate the $\{\theta_j\}$ by maximum likelihood. Then we can solve for the unsmoothed return series as:

$$r_t^* = \frac{1}{\hat{\theta}_0}\Big[r_t - \hat{\theta}_1 r_{t-1}^* - \hat{\theta}_2 r_{t-2}^* - \cdots - \hat{\theta}_k r_{t-k}^*\Big] \tag{C.2}$$

C.4 AMP (2010) Unsmoothing

AMP (2010) find that the residual normality imposed by GLM (2004) often does not hold in practice. They formulate a more general method of solving for the $\{\theta_j\}$ which does not assume error normality. In particular, they assume that:

$$\begin{aligned} \textbf{Assumption AMP.1:}\ \ r_t - \mu &= \theta_0(r_t^* - \mu) + \theta_1(r_{t-1}^* - \mu) + \cdots + \theta_k(r_{t-k}^* - \mu) \\ &= \theta_0\nu_t + \theta_1\nu_{t-1} + \cdots + \theta_k\nu_{t-k} \\ \text{s.t.}\ \ 1 &= \theta_0 + \theta_1 + \cdots + \theta_k \\ \nu_t &\overset{iid}{\sim} \mathcal{D}(0, \sigma_\nu^2) \end{aligned}$$

where D is any distribution. Letting $X_t = r_t - \mu$, they formulate the method of moments conditions:

$$\begin{aligned} \mathbb{E}[X_t^2] &= (\theta_0^2 + \theta_1^2 + \cdots + \theta_k^2)\sigma^2\nu \\ \mathbb{E}[X_t X_{t-1}] &= (\theta_0\theta_1 + \theta_1\theta_2 + \cdots + \theta_{k-1}\theta_k)\sigma^2\nu \\ \mathbb{E}[X_t X_{t-2}] &= (\theta_0\theta_2 + \theta_1\theta_3 + \cdots + \theta_{k-2}\theta_k)\sigma^2\nu \\ \mathbb{E}[X_t X_{t-k}] &= \theta_0\theta_k\sigma^2\nu \end{aligned}$$

These (k+1) moment conditions and the restriction $\sum_{j=1}^{k}\theta_j = 1$ give us (k+2) equations with which to estimate the (k+2) parameters. With the $\{\hat{\theta}_j\}$ in hand, we can then use equation (C.2) to solve for the unsmoothed return series $\{r_t^*\}$.

C.5 Testing the Assumptions of the Unsmoothing Procedures

I summarize the assumptions imposed by the four unsmoothing procedures in Table C.1.

Table C.1: Comparing the Assumptions of the Four Unsmoothing Procedures

This table compares the unsmoothing assumptions of Geltner (1993), Okunev & White (OW) (2003), Getmansky, Lo, & Makarov (GLM) (2004), and Amvella, Meier, & Papageorgiou (AMP) (2010).

	Geltner (1993)	**OW (2003)**	**GLM (2004)**	**AMP (2010)**
Corrects for:	1st Order Autocorrelation	Any Order Autocorrelation	Any Order Autocorrelation	Any Order Autocorrelation
Underlying Process	AR(1)	AR(k)	MA(k)	MA(k)
Residuals	$\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$	$\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$	$\nu_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\nu^2)$	$\nu_t \overset{iid}{\sim} \mathcal{D}(0, \sigma_\nu^2)$

To check these assumptions against the data, I employ the following tests:

- **Ljung-Box Test for Autocorrelation of Original Series:** I use the Ljung-Box Test to test for overall first- and second-order autocorrelation of the original series. In other words, this test determines if unsmoothing is needed.

- **Komogorov-Smirnov Test for Same Data Generating Process:** I use the Komogorov-Smirnov Test to ensure that the unsmoothing procedure does not alter the returns to an extent that the smoothed & unsmoothed series likely come from two different data generating processes.

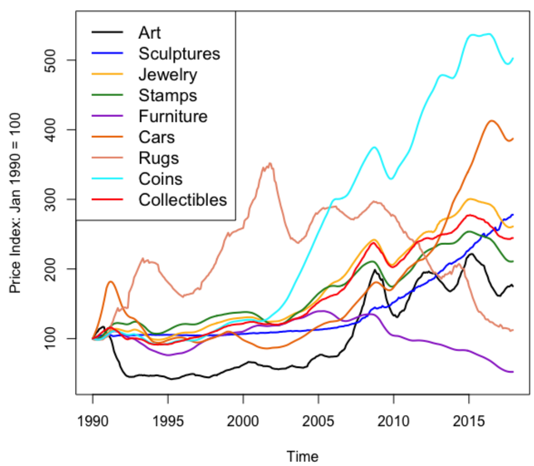
- **Ljung-Box Test for Autocorrelation of Residuals:** I use the Ljung-Box Test to test for overall first- and second-order autocorrelation of the residuals. The iid assumption on the residuals implies that they should be serially uncorrelated.

- **Ljung-Box Test for Autocorrelation of Squared Residuals:** I use the Ljung-Box Test to test for overall first- and second-order autocorrelation of the squared residuals. The constant variance assumption on the residuals implies that the squared residuals should be serially uncorrelated.

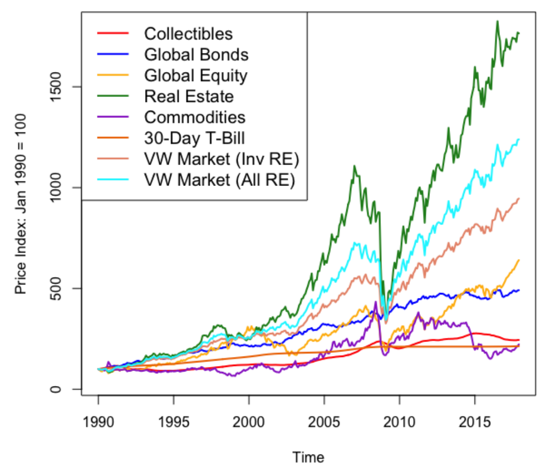
- **Jarque-Bera Test for Normality of the Residuals:** I use the Jarque-Bera Test to test that the residuals are normally distributed. This is assumed for the Geltner (1993), OW (2003), and GLM (2004) procedures, but *not* for the AMP (2010) procedure.

D Plotting the Price Indices

Figure D.1a compares the float-weighted collectibles price index to the price indices of the constituent components (ie: the nine categories). Figure D.1b compares the float-weighted collectibles price index to price indices for other major asset classes. Note that this graphical representation is only an approximation because the price indices are not comparable. This is because the price indices have different degrees of return autocorrelation and positive return autocorrelation leads the price indices to overstate compound total returns.



(a) Collectibles and Constituent Components



(b) Collectibles and Other Major Asset Classes

Figure D.1: Price Indices: Collectibles, Constituent Components, and Other Assets

Figure D.1a plots the price indices for the float-weighted collectibles index and eight of the nine categories. Wine is excluded because the wine price index has a maximum value of 1969, which substantially impairs the figure's legibility. Figure D.1b plots the float-weighted collectibles price index against the price indices for the other major asset classes of interest, namely global equity, global bonds, real estate, commodities, the 30-day Treasury bill, and the value-weighted market portfolio (with investable real estate and with all real estate).