Modelling Risk in Financial Markets

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Introduction

- Risk Management
  - Active (portfolio management)
  - Passive (risk monitoring and regulation)
- Multivariate Models of Conditional Volatility and Correlation
- Types of Risks
  - Measurement Uncertainty
  - Parameter Uncertainty
  - Model Uncertainty (structural breaks)
  - Policy Uncertainty
- Model Averaging as a Risk Diversification Option
- Global Trends in Volatilities and Correlations
- Concluding Remarks
Background Papers and Programs

- "Dynamic Conditional Correlation Models with multivariate t Distribution (TDCC)", Pesaran and Pesaran (2009, under preparation)
The Decision Problem: Active Risk Management

Change in value of the portfolio

\[
\Delta V_t = \sum_{j=1}^{N} n_{j,t-1} \left( \frac{P_{jt} - P_{j,t-1}}{E_{jt}} \right) = \sum_{j=1}^{N} \left( \frac{n_{j,t-1} P_{j,t-1}}{E_{j,t-1}} \right) \left( \frac{r_{jt}}{1 + r_{jt}^e} \right),
\]

where

- \( n_{jt} \) number of contracts, long (+) or short (–)
- \( r_{jt} = 100(P_{jt} - P_{j,t-1})/P_{j,t-1} \) return on \( jth \) asset
- \( r_{jt}^e = 100(E_{jt} - E_{j,t-1})/E_{j,t-1} \% \) change in FX rate against US $
Ignoring second-order effects, the portfolio return $\rho_t$ becomes

$$
\rho_t = \frac{\Delta V_t}{C_{t-1}} \approx \sum_{j=1}^{N} \frac{n_{j,t-1}P_{j,t-1}}{E_{j,t-1}C_{t-1}} r_{jt}
$$

(2)

$$
= \sum_{j=1}^{N} \omega_{j,t-1} r_{jt} = \omega'_{t-1} r_t,
$$

(3)

where

- $C_{t-1}$ notional capital
- $\omega_{j,t-1} = n_{j,t-1}P_{j,t-1}/(E_{j,t-1}C_{t-1})$ portfolio exposures
- $\omega_{t-1} = (\omega_{1,t-1}, \omega_{2,t-2}, \ldots, \omega_{N,t-1})'$, $r_t = (r_{1t}, r_{2t}, \ldots, r_{Nt})'$
Mean-Variance Problem Subject to the VaR Constraint

- Maximize

\[ Q(\omega_{t-1}|\mathcal{F}_{t-1}) = \omega'_{t-1} E(r_t|\mathcal{F}_{t-1}) - \frac{\delta_{t-1}}{2} \omega'_{t-1} V(r_t|\mathcal{F}_{t-1}) \omega_{t-1}, \]  

(4)

- Subject to

\[ \text{Pr}(\omega'_{t-1} r_t < -L_{t-1} | \mathcal{F}_{t-1}) \leq \alpha, \]  

(5)

where \( L_{t-1} > 0 \) is a pre-specified maximum (daily) loss (e.g. 1%).

- Unlike MV solutions, the MV subject to VaR constraint requires a \text{complete} knowledge of the \text{conditional joint probability distribution of returns}, \( f(r_t|\mathcal{F}_{t-1}) \).
Recent surveys are provided in Bauwens, Laurent, and Rombouts (2006, JAE) and McAleer (2005, ER).
The Riskmetrics specifications popularized by J.P.Morgan
The conditionally constant correlation (CCC) model of Bollerslev
The orthogonal GARCH model of Alexander
The dynamic conditional correlation (DCC) model advanced by Engle
Asymmetric DCC (ADCC) model of Cappiello, Engle and Sheppard.
The T-DCC model based on de-volatized returns of Pesaran and Pesaran (2009)
Modelling Conditional Correlation Matrix of Asset Returns

Following Bollerslev (1990) and Engle (2002) consider the decomposition

$$\sum_{t-1} = D_{t-1} R_{t-1} D_{t-1},$$

where

$$D_{t-1} = \begin{pmatrix} \sigma_{1,t-1} \\ \sigma_{2,t-1} & 0 \\ 0 & \ddots \\ \sigma_{m,t-1} \end{pmatrix}$$

$$R_{t-1} = \begin{pmatrix} \rho_{ij,t-1} \end{pmatrix}$$

$$\rho_{ij,t-1} = \frac{\text{Cov} \left( r_{it}, r_{jt} \mid \Omega_{t-1} \right)}{\sigma_{i,t-1} \sigma_{j,t-1}}.$$
Bollerslev (1990) considers (6) with a constant correlation matrix $R_{t-1} = R$.
Engle (2002) allows for $R_{t-1}$ to be time-varying and proposes a class of multi-variate GARCH models labeled as dynamic conditional correlation (DCC) models.

The decomposition of $\sum_{t-1}$ allows separate specification of the conditional volatilities and conditional cross-asset returns correlations.

For example, one can utilize the GARCH (1,1) model for $\sigma_{i,t-1}^2$, namely

$$V(r_{it} \mid \Omega_{t-1}) = \sigma_{i,t-1}^2 = \bar{\sigma}_i^2 (1 - \lambda_1 - \lambda_2) + \lambda_1 \sigma_{i,t-2}^2 + \lambda_2 r_{i,t-1}^2, \quad (7)$$

where $\bar{\sigma}_i^2$ is the unconditional variance of the $i$-th asset return.
For cross-asset correlations Engle propose the use of exponential smoother applied to the “standardized returns”

\[
\hat{\rho}_{ij,t-1}(\phi) = \frac{\sum_{s=1}^{\infty} \phi^{s-1}z_{i,t-s}z_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty} \phi^{s-1}z_{i,t-s}^2} \sqrt{\sum_{s=1}^{\infty} \phi^{s-1}z_{j,t-s}^2}},
\]

(8)

where the standardized returns are defined by

\[
z_{it} = \frac{r_{it}}{\sigma_{i,t-1}(\lambda_i)}.
\]

(9)
Pesaran and Pesaran (PP) consider an alternative formulation of $\rho_{ij,t-1}$ that makes use of realized volatilities. Andersen, Bollerslev and Diebold show that daily returns on foreign exchange and stock returns standardized by realized volatility are approximately Gaussian. The transformation of returns to Gaussianity is important since correlation as a measure of dependence can be misleading in the case of non-Gaussian returns. See Embrechts et al. (2003).
PP base the specification of the cross correlation of volatilities on devolatized returns. Let
\[ \tilde{r}_{it} = \frac{r_{it}}{\sigma_{it}^{\text{realized}}}, \quad (10) \]
and use
\[ \tilde{\rho}_{ij,t-1}(\phi) = \frac{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{i,t-s} \tilde{r}_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{i,t-s}^2} \sqrt{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{j,t-s}^2}}, \quad (11) \]
where \(-1 < \tilde{\rho}_{ij,t-1}(\phi) < 1\) for all values of \(|\phi| < 1\).
In the absence of intradaily observations the following simple estimate of $\sigma_{it}$ based on daily returns, inclusive of the contemporaneous value of $r_{it}$, seem to work well in practice

$$\tilde{\sigma}_{it}^2(p) = \frac{\sum_{s=0}^{p-1} r_{i,t-s}^2}{p}.$$  \hspace{1cm} (12)

PP find that for $p = 20$ the de-volatized returns, $\tilde{r}_{it} \approx r_{it}/\tilde{\sigma}_{it}(p)$, are nearly Gaussian, with approximately unit variances, for all asset classes foreign exchange, equities, bonds or commodities.
Individual Multivariate Volatility Models

- 53 different specifications of $\Sigma_{it}$ grouped into 8 different model types are considered.

1. Equal-Weighted Moving Average – EQMA
2. Exponential-Weighted Moving Average – EWMA
3. Mixed Moving Average – MMA
4. Generalized Exponential-Weighted Moving Average – GEWMA
5. Constant Correlation (Bollerslev 1990) – CCC
7. Asymmetric Dynamic Conditional Correlation (Cappiello et al. 2006) – ADCC
8. $t$-Dynamic Conditional Correlation (Pesaran and Pesaran, 2009) – TDCC
Average Volatility Models

Average predictive density based on a set of models $\{M_i\}$ and weights $\{\lambda_i\}$

$$f(r_t | \mathcal{F}_{t-1}, \mathcal{M}) = \sum_{i=1}^{m} \lambda_{i,t-1} f(r_t | \mathcal{F}_{t-1}, M_i), \quad (13)$$

Three main strategies (based on AIC or BIC criteria)

1. Best model
2. Akaike weights or Schwartz weights. E.g. for AIC:

$$\lambda_{i,t-1} = \exp(\Delta_{i,t-1}) / \sum_j \exp(\Delta_{j,t-1})$$

$$\Delta_{i,t-1} = AIC_{i,t-1} - \text{Max}_j(AIC_{j,t-1})$$

3. Thick model averaging: top 10%, top 25%, etc. with equal weights
VaR Based Diagnostic Tests: Individual Models

- Distribution of portfolio return $\rho_t$ based on model $M_i(\hat{\theta}_{iT_0})$

  \[
  \rho_t \mid \mathcal{F}_{t-1}, M_i(\hat{\theta}_{iT_0}) \sim (\mu_{\rho t}, \sigma_{\rho t}^2(M_i))
  \]

  \[
  \mu_{\rho t} = \omega'_{t-1}\mu_t, \text{ and } \sigma_{\rho t}^2(M_i) = \omega'_{t-1}\Sigma_i\omega_{t-1}
  \]

  VaR:

  \[
  Pr(\rho_t < -\bar{\rho}_{i,t-1}(\omega_{t-1}, \alpha; \hat{\theta}_{iT_0})|\mathcal{F}_{t-1}, M_i) \leq \alpha
  \]  \hspace{1cm} (14)

  \[
  \Rightarrow \quad \bar{\rho}_{i,t-1}(\omega_{t-1}, \alpha; \hat{\theta}_{iT_0}) = -\mu_{\rho t} + c_{it}(\alpha)\sigma_{\rho t}(M_i)
  \]

  where $c_{it}(\alpha)$ is the $\alpha\%$ critical value.

- Empirical VaR exceedance frequency

  \[
  \hat{\pi}_i = \frac{1}{T_1} \sum_{t \in T_1} d_{it}(\hat{\theta}_{iT_0})
  \]  \hspace{1cm} (15)

  \[
  d_{it}(\hat{\theta}_{iT_0}) = I[-\bar{\rho}_{i,t-1}(\omega_{t-1}, \alpha; \hat{\theta}_{iT_0}) - \rho_t]
  \]
VaR Based Diagnostic Tests: Average Models

- Set of $m$ models $M = \bigcup_{i=1}^{m} M_i$ with probability distributions $F_{it}(\cdot)$.
- VaR constraint: need to find the value $\bar{\rho}_{b,t-1}$ which satisfies

$$ Pr(\rho_t < -\bar{\rho}_{b,t-1}(\omega_{t-1}, \alpha) | F_{t-1}, M) = \sum_{i=1}^{m} \lambda_{i,t-1} F_{it} \left( \frac{-\bar{\rho}_{b,t-1}(\omega_{t-1}, \alpha) - \omega'_{t-1} \mu_{it}}{\sigma_{\rho t}(M_i)} \right) \leq \alpha \quad (16) $$

(typically this has to be done numerically)

- Compute the empirical VaR exceedance frequency as before

$$ \hat{\pi}_b = \frac{1}{T_1} \sum_{t \in T_1} \hat{d}_{bt} \quad (17) $$

$$ \hat{d}_{bt} = I \left[ -\rho_t - \bar{\rho}_{b,t-1}(\omega_{t-1}, \alpha) \right] . $$
Tail probabilities using a mixture model and a Gaussian model with the same average volatility are not the same, namely

$$
\sum_{i=1}^{m} \lambda_{it-1} \Phi \left( \frac{g}{\sigma_{\rho t}(M_i)} \right) \neq \Phi \left( \frac{g}{\sqrt{\sum_{i=1}^{m} \lambda_{it-1} \sigma_{\rho t}^2(M_i)}} \right), \tag{18}
$$

unless $\Sigma_{it} = \Sigma_t$ for all $i$.

The following theorem states that a combined model will be more fat-tailed than the associated Gaussian model with the same average volatility measure, as long as $g < -\sqrt{3} \sigma_{\rho t}(M_i), i = 1, \ldots, m$. 
Statistical Tests

\[ \hat{v}_{it} = \int_{-\infty}^{\rho_t} \hat{f}(x|\mathcal{F}_{t-1}, M_i) dx, \text{ for } t = \tau + 1, \ldots, \tau + T_1 \]

\( \hat{f}(x|\mathcal{F}_{t-1}, M_i) \) is the estimated pdf of \( \rho_t \) under model \( M_i \).

\( \hat{v}_{it}, t \in \mathcal{T}_1 \) are i.i.d. uniformly distributed on the interval \([0, 1]\), \textbf{if} \( \hat{f}(x|\mathcal{F}_{t-1}, M_i) \) coincides with the ‘true’ but unknown conditional predictive density of \( \rho_t \).
Kolmogorov-Smirnov test

\[ KS = \max_{1 \leq j \leq T_1} \left| \frac{j}{T_1} - \hat{v}_j^* \right| \]

Kuiper test

\[ Ku = \max_{1 \leq j \leq T_1} \left( \frac{j}{T_1} - \hat{v}_j^* \right) + \max_{1 \leq j \leq T_1} \left( \hat{v}_j^* - \frac{j}{T_1} \right) \]

where \( \hat{v}_1^* \leq \hat{v}_2^* \leq ... \leq \hat{v}_{T_1}^* \) are the ordered values of the \( \hat{v}_{i, \tau+1}, ..., \hat{v}_{i, \tau+T_1} \).
Weakly returns on 31 assets over the period January 7, 1994 to October 30, 2009 (826 weakly observations)

- 11 currencies (GBP, EUR, JPY, CAD, AUD, CHF, SE, NO, NZ, SG, TW)
- 13 equity indices (SP, RL FTSE, DAX, CAC, IBEX, SM, EO, QC, NK, HK, AUS, SA)
- 7 bonds (BU, BCA, BE, BCH, BG, JGB, BA)
Conditional Volatilities of Weakly Returns on FX

27-May-94 to 30-Oct-09

Vol(JY) Vol(EU) Vol(BP) Vol(CH)
Vol(CD) Vol(AD) Vol(SE) Vol(NC)
Vol(NZ) Vol(SG) Vol(SE) Vol(TW)
Conditional Correlations of Equity Futures (Weakly Returns)

Cor(AUS, SP)  Cor(FTSE, SP)  Cor(FTSE, AUS)
Cor(NK, AUS)  Cor(NK, FTSE)  Cor(HK, SP)
Cor(CAC, NK)  Cor(SA, SP)  Cor(HK, AUS)
Cor(DAX, SP)  Cor(DAX, AUS)  Cor(HK, FTSE)
Cor(IBEX, SP)  Cor(IBEX, AUS)  Cor(HK, NK)
Cor(SM, SP)  Cor(SM, AUS)  Cor(CAC, SP)
Cor(SM, DAX)  Cor(SM, IBEX)  Cor(CAC, AUS)
Cor(SM, SA)  Cor(SM, EO)  Cor(CAC, FTSE)
Cor(EO, NK)  Cor(EO, SA)  Cor(CAC, HK)
Cor(QC, AUS)  Cor(QC, EO)  Cor(CAC, NK)
Cor(QC, IBEX)  Cor(QC, HK)  Cor(SM, HK)
Cor(QC, SM)  Cor(QC, QC)  Cor(SM, FK)
Cor(RL, NK)  Cor(RL, EO)  Cor(SM, SA)
Cor(RL, CAC)  Cor(RL, SA)  Cor(SM, DAX)
Cor(RL, QC)  Cor(RL, EO)  Cor(SM, SA)
Equity Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = 0.071111
5% Critical value = 0.11705
FX Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = 0.10519
5% Critical value = 0.11705
Bond Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = 0.081481
5% Critical value = 0.11705
Combined Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = 0.061111

5% Critical value = 0.11705

Theoretical CDF
Empirical CDF
Value at Risk of the Equity Portfolio (Weakly Returns)
Main Features of the Results

- Variations in asset returns have become more volatile - although they have been declining recently.
- Asset return correlations have been rising - recent crisis led to further increases.
- The rise in asset return correlations is more reflective of underlying trends - globalization and integration of financial markets.
Main Features of the Results (continued)

- To deal with model uncertainty we advocate the use of ‘average’ models, and explore their use in optimal portfolio choice.
- Simple decision-based model evaluation tests in terms of VaR performance are proposed.
- The test is applicable to individual as well as to average models - the TDCC specification passes the VaR based tests.
- The main problem is how to predict sudden shifts in volatilities and conditional correlations.