How Interdependent are Systemic Risk Indicators? 
A Network Analysis

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Introduction
Measurement of **systemic risk** has become a prominent topic of research amongst academics, regulators and policymakers.

Large panels of risk indicators are now commonly available.

Coherent synthesis is not trivial:
Do systemic risks indicators comove? Do they comove in the same direction? Are there clusters of indicators that signal distress simultaneously? Are there more interdependent indicators?
The ESRB risk dashboard

- a common set of quantitative and qualitative indicators to identify and measure systemic risk. (ESRB Regulation)

- It includes more than 500 time series indicators covering 28 countries (all EU) about risks in banking, insurance and securities markets.

- Covers six risk categories:
  1. Interlinkage and contagion
  2. Macro risk
  3. Credit risk
  4. Funding and liquidity risk
  5. Profitability risk
  6. Market risk

- Different scope and nature of data.

Brownlees & Frison (2014)
In this work we propose a **Factor–Network** modelling approach to synthesize the cross–sectional dependence in the ESRB Dashboard.

We focus in particular on the analysis of the Network component.

Network estimation is carried out using robust methods that deal with the empirical characteristics of the data.

**Highlights of the methodology:**
- It allows to study large multivariate systems – hundreds of series
- It provides a synthetic dependence map among the indicators
- It allows to identify bellwethers of systemic risk

Brownlees & Frison (2014)
1. Factor component explains a relative small portion of the cross-sectional dependence in the panel.

2. On the other hand, Networks component is prominent.

3. Majority of interdependence is positive and network exhibits typical empirical characteristics of power law networks.

4. We find, that Macro, Credit and Funding risk indicators are the most central and highly interconnected categories.

5. In particular, we find that corporate debt-to-gdp and banking loans-to-deposit indicators are relevant bellwethers of systemic risk of the dashboard.
Methodology
We model cross-sectional dependence using a factor-network approach.

Let $\mathbf{y}_t$ denote a panel of $n$ risk indicators. We assume the indicators to be stationary. Also, for interpretation, we suggest to standardize indicators so that a positive realization of the indicator signals an increase in systemic risk.

We assume that the $i$th indicator is described by

$$y_{t,i} = \sum_{j}^{F} \beta_{ij} f_{t,j} + \epsilon_{t,i}$$

where

- $f_{t,j}$ systematic factors
- $\epsilon_{t,i}$ idiosyncratic shock with cross-sectional network dependence across $i$
What is Network Dependence?

- The network associated with the $\epsilon_t$ is an **undirected graph** where:

  1. the components of $\epsilon_t$ denote **vertices**
  2. the absence of an **edge** between $i$ and $j$ denotes that $i$ and $j$ are **conditionally independent**

$$\epsilon_i \perp \epsilon_j \mid \epsilon_k \quad \forall k \neq i, j$$

- We work under the assumption that the network is not known and we are interested in detecting which linkages are present from data.

Brownlees & Frison (2014)
Network Dependence for Normal Data

- If $\epsilon$ is multivariate normal, than conditionally independence is equivalent as the absence of (linear) partial correlation.

- We can equivalently characterize the network between idiosyncratic default shocks using the concentration matrix $K = \Sigma^{-1}$ with entries $k_{ij}$

- Then two indicators are conditionally independent iff $k_{ij} = 0$, in other words
  \[ \epsilon_i \perp \epsilon_j \mid \epsilon_k \iff k_{ij} = 0, \]

- Relevant for estimation. We can reformulate the problem of estimating the network as the problem of estimating a sparse concentration matrix!

Brownlees & Frison (2014)
What if data is not normal? We can assume $\epsilon$ to be a member of the nonparanormal family

- Let $f = (f_1, \ldots, f_n)$ be a set of monotonic univariate functions and let $\Sigma$ be positive correlation matrix.

- We say that $\epsilon_t$ is nonparanormal

$$\epsilon_t \sim NPN(f, \Sigma)$$

if

$$f(\epsilon_t) = (f_1(\epsilon_{1t}), \ldots, f_N(\epsilon_{Nt}))' \sim \mathcal{N}(0, \Sigma)$$
The nonparanormal family is equivalent to the Gaussian copula family.

The nonparanormal is a much wider class of distribution than the normal. In particular, it allows the marginal distribution of the data for skewness and kurtosis.

The advantage of this definition is that conditional independence is still encoded in the sparsity structure of $K = \Sigma^{-1}$.
We are interested in estimating the network implied by $\mathbf{K}$, however we do not want formulate assumptions on $f$.

Important result: If $\epsilon_t \sim NPN(f, \Sigma)$ then

$$\Sigma_{ij} = 2 \sin \left( \frac{\pi}{2} \rho_{ij}^S \right)$$

where $\rho_{ij}^S$ is Spearman’s rank correlation.

The network estimation can be carried out by the GLASSO

$$\hat{\mathbf{K}} = \arg \min_{\mathbf{K} \in S^n} \left\{ \text{tr}(\hat{\Sigma} \mathbf{K}) - \log \det(\mathbf{K}) + \lambda \sum_{i \neq j} |k_{ij}| \right\}$$

where $\hat{\Sigma}$ is the sample analog of $\Sigma$ matrix computed using Spearman’s
Factors: estimation by OLS

Network: estimated by LASSO

- LASSO (Tibshirani, R. (1996)) allows to simultaneously select nonzero edges and estimate the partial correlations.

Highlight of the procedure: it allows to estimate the network in a **sparse** high-dimensional setting. LASSO is consistent even when the number of partial correlation to estimate is higher than the number of observations to the extent that the network is sparse.
Empirical Results
We build an unbalanced panel of 156 time series covering the period from 1999 to 2013; indicators are divided into 6 risk categories following the ESRB classification.

Data are transformed to:

1. guarantee stationarity
2. same frequency (monthly) and
3. have univocal risk directionality (e.g. $\uparrow$ value of indicator implies more risk)
Empirical Results

Factor Component

- Factor component explains a relatively small proportion of covariation (loading from first two PCA is 21%)

- We analyse to which series the first factors are associated with:

  1. First principal component closely related to the economic cycle (Factor 1: EU quarterly GDP growth)

  2. Second principal component closely related to financial sector volatility (Factor 2: changes in the VSTOXX index).

Brownlees & Frison (2014)
First principal component and EU quart. GDP growth ($\rho = 62\%$)
Second principal component and VSTOXX index ($\rho = 58\%$)
Empirical Results

Rank Correlation Factor Residuals

Brownlees & Frison (2014)
1. Using simple correlation to measure cross-section interdependence means looking at nearly 12,000 possible combinations ⇒ no trivial interpretation.

2. Red/orange areas represent pockets of correlation, typically among same risk indicators/risk categories (i.e. interlinkage, macro and market).

3. Profitability seems to be poorly correlated with other risk categories and among themselves (perhaps because they are measured at country rather than bank level).

Brownlees & Frison (2014)
1. Around 400 non-zero partial correlation, or approximately 3% of total 12,000 possible edges; it is therefore a spare network, although 87% of indicators have at least one neighbour.

2. Indicators of Macro, credit and funding risk are the most central and highly interconnected in the network (represent the more dense part of the network). These three risk are also economically intertwined: typically a rise in credit risk $\Rightarrow$ rise in firms’ funding problems $\Rightarrow$ general worsening of macroeconomic outlook.

3. After controlling for stock market volatility, several credit and interlinkage risk indicators are fairly periferic in the network (e.g. profitability indicators).

Brownlees & Frison (2014)
## Empirical Results

**Network Component**

### Results - Network component

#### Edges vis–á–vis other risk categories (%)

<table>
<thead>
<tr>
<th></th>
<th>Credit</th>
<th>Funding</th>
<th>Macro</th>
<th>Interlink.</th>
<th>Market</th>
<th>Profit.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Credit</strong></td>
<td>35.03</td>
<td>15.82</td>
<td>46.89</td>
<td>1.69</td>
<td>0</td>
<td>0.56</td>
<td>100</td>
</tr>
<tr>
<td><strong>Funding</strong></td>
<td>25.23</td>
<td>23.42</td>
<td>44.14</td>
<td>4.51</td>
<td>0</td>
<td>2.71</td>
<td>100</td>
</tr>
<tr>
<td><strong>Macro</strong></td>
<td>36.09</td>
<td>21.3</td>
<td>40.87</td>
<td>1.74</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td><strong>Interlinkages</strong></td>
<td>10.34</td>
<td>17.24</td>
<td>13.79</td>
<td>34.48</td>
<td>6.91</td>
<td>17.24</td>
<td>100</td>
</tr>
<tr>
<td><strong>Market</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>28.57</td>
<td>71.43</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td><strong>Profitability</strong></td>
<td>5.26</td>
<td>15.79</td>
<td>0</td>
<td>26.32</td>
<td>0</td>
<td>52.63</td>
<td>100</td>
</tr>
</tbody>
</table>

Brownlees & Frison (2014)
1. Network relations are stronger among indicators within the same risk categories (elements in the diagonal). Note: this is in part due to the fact that some indicators are reported for 9 countries/regions.

2. Credit risk indicators are highly correlated with macro risks one (nearly 50% of edges stemming from credit risk’s indicators are linked to a macro risk variable).

3. Funding and liquidity risks also are highly correlated with macro. Possible interpretation: financial markets’ liquidity and funding of bank is key to reduce systemic risk at macro level.

Brownlees & Frison (2014)
Empirical Results

Network Trace Plot

Brownlees & Frison (2014)
3 Empirical Results

Network Trace Plot

1. Clear predominance of positive correlations.

2. Stable proportion of positive over negative correlation (aprox. 75% are positive)

3. BIC information criteria suggest using lambda equal to 0.31

Brownlees & Frison (2014)
Empirical Results

Degree and Partial Correlation Distribution

Partial Correlation

Degree

Brownlees & Frison (2014)
### Empirical Results

**Degree and Partial Correlation Distribution**

1. The distribution of partial correlations ranges from -22% to +29% with fat tails; 73% of are positive, in other words 3 out of 4 of any partial correlation signal a simultaneous increase in systemic risk.

2. Heterogeneous number of edges ⇒ Network exhibit “power law” behaviour (many indicators with few links + few indicators with many links) ⟺ small world effect

3. Top 5 edges account for almost 25% of total edges in the network
### Empirical Results

#### Goodness-of-Fit - Factor–Network regressions

**Regression’s $R^2$ by risk categories**

<table>
<thead>
<tr>
<th>Risk Category</th>
<th>Factor regression: average $R^2$</th>
<th>Network regression: average $\Delta R^2$</th>
<th>Average numb. of edges per vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit</td>
<td>11.04</td>
<td>55.29</td>
<td>5.82</td>
</tr>
<tr>
<td>Funding</td>
<td>4.77</td>
<td>58.06</td>
<td>7.21</td>
</tr>
<tr>
<td>Interlinkages</td>
<td>20.91</td>
<td>21.09</td>
<td>2.79</td>
</tr>
<tr>
<td>Macro</td>
<td>22.44</td>
<td>41.84</td>
<td>6.00</td>
</tr>
<tr>
<td>Market</td>
<td>32.90</td>
<td>40.00</td>
<td>1.50</td>
</tr>
<tr>
<td>Profitability</td>
<td>6.70</td>
<td>18.39</td>
<td>1.53</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>15.73</strong></td>
<td><strong>42.54</strong></td>
<td><strong>5.03</strong></td>
</tr>
</tbody>
</table>
## Empirical Results

**Top-20 most central vertexes**

<table>
<thead>
<tr>
<th>Page rank</th>
<th>Indicator</th>
<th># of edges</th>
<th>Factor-$R^2$</th>
<th>Network’ $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NFC debt-to-GDP - Spain</td>
<td>18</td>
<td>5.4</td>
<td>86.9</td>
</tr>
<tr>
<td>2</td>
<td>MFI loans-to-deposits ratio - France</td>
<td>18</td>
<td>5.4</td>
<td>91.8</td>
</tr>
<tr>
<td>3</td>
<td>Current account balance - Programme</td>
<td>17</td>
<td>19.9</td>
<td>69.6</td>
</tr>
<tr>
<td>4</td>
<td>MFI loans-to-deposits ratio - CEE</td>
<td>22</td>
<td>5.6</td>
<td>92.8</td>
</tr>
<tr>
<td>5</td>
<td>Residential property prices-Netherlands</td>
<td>16</td>
<td>38.8</td>
<td>58.6</td>
</tr>
<tr>
<td>6</td>
<td>Real GDP growth - Spain</td>
<td>15</td>
<td>68</td>
<td>29.2</td>
</tr>
<tr>
<td>7</td>
<td>Residential property prices - Italy</td>
<td>14</td>
<td>15.6</td>
<td>77.8</td>
</tr>
<tr>
<td>8</td>
<td>Current account balance - Spain</td>
<td>14</td>
<td>35.4</td>
<td>60.3</td>
</tr>
<tr>
<td>9</td>
<td>MFI loans-to-deposits ratio - Spain</td>
<td>14</td>
<td>9.2</td>
<td>69.6</td>
</tr>
<tr>
<td>10</td>
<td>Lending in FX - Latvia</td>
<td>11</td>
<td>5.8</td>
<td>59.3</td>
</tr>
<tr>
<td>11</td>
<td>MFI loans-to-deposits ratio-Programme</td>
<td>15</td>
<td>3.3</td>
<td>89.7</td>
</tr>
<tr>
<td>12</td>
<td>NFC debt-to-GDP - CEE</td>
<td>17</td>
<td>5.3</td>
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<tr>
<td>13</td>
<td>Residential property prices - Spain</td>
<td>14</td>
<td>33.8</td>
<td>59</td>
</tr>
<tr>
<td>14</td>
<td>Current account balance - Italy</td>
<td>17</td>
<td>2.4</td>
<td>93.5</td>
</tr>
<tr>
<td>15</td>
<td>NFC debt-to-GDP - Nordic countries</td>
<td>10</td>
<td>2.7</td>
<td>88.4</td>
</tr>
<tr>
<td>16</td>
<td>Real GDP growth - Programme</td>
<td>8</td>
<td>46.5</td>
<td>39.3</td>
</tr>
<tr>
<td>17</td>
<td>MFI loans-to-deposits ratio - Germany</td>
<td>12</td>
<td>5.3</td>
<td>86.4</td>
</tr>
<tr>
<td>18</td>
<td>Real GDP growth - Germany</td>
<td>9</td>
<td>79.4</td>
<td>15.4</td>
</tr>
<tr>
<td>19</td>
<td>Residential property prices - France</td>
<td>9</td>
<td>51.1</td>
<td>43.7</td>
</tr>
<tr>
<td>20</td>
<td>NFC debt-to-GDP - UK</td>
<td>9</td>
<td>9.7</td>
<td>63</td>
</tr>
</tbody>
</table>
1. Most central vertexes are linked to the credit cycle and the macro economy (corporate leverage, property prices, banks leverage, etc.).

2. Some of these indicators, e.g. banking sector leverage, property prices and credit-to-GDP gap have been identified as good predictor of systemic crisis by other studies (Behn et al., 2013).

3. We would consider these indicators (especially the top 5) as bellwether of systemic risk in the European economy, i.e. highly interdependent risk indicators that signal a trend in systemic risk.
Conclusions
We provide a factor-network methodology to synthesize the cross-sectional dependence structure of a large number of indicators. The methodology is robust to nonlinear type dependence across indicators.

The empirical application to the ESRB dashboard reveals a number of findings:

1. Relevance of network component
2. Power law network structure
3. High centrality of Macro, credit and funding indicators
Questions?

Thanks!