Mapping Change in Complex Financial Markets


The views expressed in this presentation are the authors’ and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System.
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U.S. Overnight Money Market

Overnight loans in the United States (Sept. 2008)
Interbank Claims (BIS Locational Data)
We need a way to see the important details of the network structure.
How do we make maps?
good maps simplify
Not helpful!
good maps simplify and highlight
good maps **simplify**

and **highlight**

**relevant** structures
What is important in our context?

- not trying to get from St John’s Wood to King’s Cross
- We want to find the structures within the financial network that are important with respect to the flow of funds
Consider the overnight money market in the U.S.
The flow of funds is a large weighted, directed network.
Individual banks are the nodes.
  - The value of loans from node $\alpha$ to node $\beta$ determines the weight on the directed link from $\alpha$ to $\beta$.
  - Normalized weights become transition probabilities
In order to understand lending relationships we can use an information-theoretic network clustering technique developed by Rosvall and Bergstrom (2008)
Dual Problem

Compressing data ← Finding patterns

“If we can find a good code for describing flow on a network, we will have solved the dual problem of finding the important structures with respect to that flow.” RB
Maps by compressing flow of information on networks

Source: *Maps of random walks on complex networks reveal community structure* by M. Rosvall and C. T. Bergstrom
Describing flow is a coding game
— objects deserve unique names
Huffman Code

1. At each step the characters you have seen do not yet correspond to any item, or they correspond to exactly one
2. Encoded message is shortest satisfying 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>10</td>
</tr>
<tr>
<td>r</td>
<td>110</td>
</tr>
<tr>
<td>t</td>
<td>111</td>
</tr>
</tbody>
</table>

1001110110010111 = cataract

Prefix free code: a receiver can identify each word without requiring a special marker between words.
Modular Structure

- RB apply Huffman coding in a “tiered” way, saving code by using two types of code books
  - module codebooks & index codebook
- Can reuse code words in different modules
- Transforms the problem of minimizing the description length of places traced by a path into the problem of how we should best partition the network with respect to this flow
- Trade-off costs and benefits measured in terms of bits
Simplify and highlight
— important objects deserve unique names
Simplify and highlight

— highlight important objects and filter away details
naming places
Shannon’s Source Code Theorem

- We do not need to actually produce code for each partition
- Rather, we can calculate the theoretical limit for all of the different partitions and pick the one that is best (gives shortest description length)
- If you want to describe the states of a random variable $X$, that occurs with frequency $p_i$, then the average length of a codeword can be no less than the entropy of $X$:

$$H(X) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$
The map equation tells us the minimum description length for a particular modular structure

\[ L(M) = q \bowtie H(Q) + \sum_{i=1}^{m} p_i \bowtie H(P^i) \]
The map equation

\[ L(M) = q \cdot H(Q) + \sum_{i=1}^{m} p_i \cdot H(P_i) \]

- frequency of inter-module movements
- frequency of movements within module \( i \)
- code length of module names
- code length of node names in module \( i \)
Any numerical search algorithm developed to find a network partition that optimizes an objective function can be modified to minimize the map equation.

Alluvial Fan
Mapping change of payment flows driven by interbank lending market activity from July 2008 to December 2008.
The orange cluster is dominated by a set of Federal Home Loan Banks and a number of small and medium sized banks.

"...a combination of financial consolidation, credit losses, and changes to risk management practices has led at least some GSEs to limit their number of counterparties in the money market and to tighten credit lines." (Bech and Klee, *JME*, 2011)
- The fairly stable green cluster which is subsumed into the large cluster after the implementation of interest on reserves, is dominated by a prominent government-sponsored enterprise and one large money center bank.

- The break down of the cluster may reflect the reduction (or even elimination) of the lending relationship to the particular bank by the GSE.
The blue cluster is comprised of a Federal Home Loan Bank and a number of banks that tend to be located the same geographic region as the Home Loan Bank.

We speculate that this cluster reflects the fact that the Home Loan bank may have started to intermediate funds between its members by borrowing funds from some and making overnight advances (i.e., collateralized loans) to others during this period.
What are the key differences?

- What is real change and what is mere noise?
- Need to know which structures are statistically significant and which are not.
Significance clustering

Source: Mapping change in large networks by M. Rosvall and C. T. Bergstrom
Significance clustering

Real world

Resampling

Bootstrapped world

Clustering
Significance clustering

Real world

Resampling

Clustering

Bootstrap world

Clustering
Significance clustering

Real world

Resampling

Clustering

Significance clustering

Bootstrap world

Clustering
Mapping change

Time 1

Real world 1

Bootstrap world 1

Resampling

Clustering

Significance clustering

Mapping change with alluvial diagram

Time 2

Real world 2

Bootstrap world 2

Resampling

Clustering

Significance clustering
Analysis of Micro-Scale Rates of Change

- While the alluvial diagrams are very nice for showing the general patterns of how lending takes place in each period, they do not necessarily reveal the onset of change in the system.

- Changes in clustering patterns reflect tipping points at which the cumulative effect of multiple small changes in flows constitute a significant change.
Transformative Event: Lehman?
Transformative Event: IOER?
Suppose that lending configurations \(a, b, c, d, e, f, g, h\) all generate a system with module structure of type 1, and configurations \(i, j, k, l, m, n, o, p\) all generate a system with module structure of type 2.

<table>
<thead>
<tr>
<th>(T)</th>
<th>config</th>
<th>cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>b</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>g</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>h</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>i</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>j</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>j</td>
<td>2</td>
</tr>
</tbody>
</table>

Most rapid change from period 2 to 3, not 4 to 5.
Micro-Scale Rates of Change

- We would like to be able to differentiate between the two hypotheses that (1) Lehman's failure is associated with a shakeup in lending patterns and (2) paying interest on reserves is associated with the shakeup.
  - An $n \times n$ lending matrix is specified by a unique vector of length $n^2$.
  - Each time period corresponds to a vector in $n^2$ space.
- To measure the amount of change in the system, we look at how much the angle between the vector at each time $t$ changes going to time $t+1$.

$$\text{similarity} = \cos(\theta) = \frac{A \cdot B}{\|A\|\|B\|} = \frac{\sum_{i=1}^{n} A_i \times B_i}{\sqrt{\sum_{i=1}^{n} (A_i)^2} \times \sqrt{\sum_{i=1}^{n} (B_i)^2}}$$

- This is a standard approach in network theory, known as cosine distance.
- Orange trace compares the distance between each time period and a fixed time period (Sep 10).
- Black trace shows us the velocity of change from one period to the next over time.
- Largest change occurs in the Oct 8 maintenance period, which covers the time period from September 25 to October 8, after collapse of Lehman and before IEOR.
While changes in borrowing and lending patterns do not fully reveal themselves in our maps until after the implementation of Interest on Reserves, examination of the micro-scale rates of change strongly suggests that the collapse of Lehman Brothers was the driving force.
Concluding Remarks

- Advanced network techniques can help stakeholders in the financial system to understand its structural features and to analyze the impact of transformative events.
- As illustrated here, the map equation appears to be a very useful tool for understanding funding flows.
  - More appropriate than “competing” techniques that impose community structure or ignore flows
- The lending flows in the interbank lending market changed in a fundamental way between September 10 and October 22, 2008, and the alluvial diagrams reveal this clearly.
- However, there is evidence that the underlying shifts in network flows that led to these structural changes may have been initiated well before the tipping point was reached.
Concluding Remarks

- Considerable caution is required when deriving causal inferences from alluvial diagrams.
  - We advocate the use of two tools for analyzing the structure of financial networks.
  - Each complements the other and neither is sufficient in isolation.
- The map equation reveals community structure and changes can be visualized via alluvial diagrams.
  - However, these maps do not reveal details of micro-scale rates of change that precipitated change.
- The cosine distance analysis is useful in this regard, but itself is not informative: it reveals change, but not the content of that change.
Thank You