The Price of Complexity in Financial Networks

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**Financial Networks**

**Definition**

**Financial network**

\[ G = \{ V, E \} \], with \( V \) set of players (e.g. banks or financial institutions) and 
\[ E^Y = \{(i, j)^Y \mid i, j \in V\} \] a set of contracts of type \( Y \) between players \( i, j \).

**Exposure matrix**, weighted adjacency matrix \( A_{ij}^Y \in \mathcal{R}^+ \)

**Leverage matrix**\(^1\): exposure of \( i \) to \( j \) relative to \( i \)'s regulatory capital (ability to absorb losses from \( j \))

\[
\Lambda_{ij}^Y = \frac{A_{ij}^Y}{E_i} \in \mathcal{R}^+
\]

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General model set-up

- **Time 0**: banks allocate assets/liabilities (with any rule). Time 1: known shock.

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The Price of Complexity in Financial Networks
Time 2: unknown shocks hit banks’ external assets, some banks may default.
Time $T > 2$: debt contracts mature. Defaulted banks’s assets are liquidated, creditors get recovery rate $R$ (endogenous or exogenous).
Time $t \leq 2 \leq T$: players want to value counterparties’s debt, based on default probability computation.
Time 1: banks allocate assets and liabilities, including derivative contracts (dependent on other bank’s default)
External assets (investments outside the financial network)

\[ a_i^E(2) = a_i^E(1)(1 + \mu + \sigma u_i), \]

with \( u_i \) a r.v. with mean 0 and variance 1, \( \mu_i \) expected return and \( \sigma_i > 0 \) scaling factor. Shock joint probability distribution: \( p(u_1, \ldots, u_n) \): correlation is accounted.
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**Liabilities** (obligation of players to internal/external creditors)

- \( \ell_j \) constant for bank \( j \). Unitary value of j’s obligation for j’s counterparties: \( x_j^B(2) = 1 \) OR \( x_j^B(2) = R \) (if default) with \( R \) recovery rate (endogenous or exogenous)
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**Interbank assets** (investments in the debt of other players in the financial network)

- \( B_{ij} \): fraction of \( i \)'s interbank assets invested at time 1 in the liability of \( j \). \( x_j^B \): unitary value of \( j \)'s interbank liability, \( x_j^B(1) = 1 \ \forall j \).

- Interbank assets of bank \( i \), \( a_i^B(2) = a_i^B(1) \sum_j B_{ij} x_j^B(2) \).

*The Price of Complexity in Financial Networks*

Battiston, Caldarelli, Roukny, May, Stiglitz, 2016 PNAS
Default condition

Special case: $R$ exogenous

- Default condition: iff negative equity at time 2

$$e_i(2) = a_i^E(2) + a_i^B(2) - \ell_i =$$

$$a_i^E(1)(1 + \mu + \sigma u_i) + a_i^B(1) \sum_j B_{ij} x_j^B(2) - \ell_i < 0$$

- $e_i(2) < 0$ iff $\frac{e_i(2)}{e_i(1)} < 0$, thus we can rewrite

$$\varepsilon_i(1 + \mu + \sigma u_i) + \beta_i \sum_j B_{ij} x_j^B(2) - \lambda_i < 0$$

where $\varepsilon_i$ leverage over external assets, $\beta_i$ leverage over interbank assets, $\lambda_i = \varepsilon_i + \beta_i - 1$ debt leverage.

- Default indicator: $\chi_i = 1$ (i’s default) and $\chi_i = 0$ otherwise.

- $u_i$ stochastic: default condition, with $\theta_i$ default threshold:

$$u_i < \theta_i \equiv \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i \mu + \beta_i(1 - \sum_j B_{ij} x_j^B(\chi_j) - 1),$$

1. no bank defaults $\theta_i = \theta_i^- = -\frac{1}{\varepsilon_i \sigma}(\varepsilon_i \mu + 1)$

2. All banks default $\theta_i = \theta_i^+ = \frac{1}{\varepsilon_i \sigma}(-\varepsilon_i \mu + \beta_i(1 - R) - 1)$
Default condition

$\theta_1^-\quad \theta_1^+\quad \theta_2^-\quad \theta_2^+$
Remarks on Recovery Rate Mechanisms

- Endogenous recovery rate from recursion: (e.g. Eisenberg-Noe 2001; Elsinger ea. 2006; Rogers and Veraart 2013, NEVA Barucca ea. 2016)

\[
p_i^* = \min \left\{ \beta \sum_{j=1}^{n} \prod_{ij}^T p_j^* + \alpha A_i^e, \bar{p}_i \right\}
\]  

(1)

- Exogenous recovery rate (Furfine 2003; SYMBOL (EC-JRC Ispra); DebtRank (Battiston ea. 2012); “Leverage Networks” (Battiston ea. 2016); Price of complexity PNAS (Battiston ea. 2016); Uncertainty (Roukny ea. 2016).

  NOTE: to capture situations of systemic risk and great uncertainty on the value of external assets, exogenous R may be more appropriate. Legal procedure for liquidation may take months or years (see e.g. Lehman case)

- General results holding for both cases: NEVA (Network Asset Valuation Model, Barucca ea. 2016)
For any state of the default indicator vector $\chi = [\chi_1, \chi_n]$ of all banks, determine the set of threshold values $\theta_i(\chi)$.

Default probability of bank $i$, $P_i$ and the systemic default probability $P^{sys}$ is unique (for any given $\chi_0$):

$$\forall i \quad P_i = \int \chi_i(u, \chi_0) p(u) \, du, \quad (2)$$

$$P^{sys} = \int \chi^{sys}(u) p(u, \chi_0) \, du, \quad (3)$$

with $p(u)$ joint density function of shocks
Three basic architectures\(^1\) with three nodes: a star, a chain and a ring, uniform i.i.d. shocks in \([-1, 1]\); \(\theta_i^+(\theta_i^-)\) threshold with all (none) \(i\)'s counterparties defaulting.

Systemic default probability (area of shocks where all banks default):

\[
\begin{align*}
P_{\text{star}}^{\text{sys}} &= (1/2^3)(1 + \theta_1^+)(1 + \theta_2^-)(1 + \theta_3^-); \\
P_{\text{chain}}^{\text{sys}} &= (1/2^3)(1 + \theta_1^+)(1 + \theta_2^+)(1 + \theta_3^-); \\
P_{\text{ring}}^{\text{sys}} &= (1/2^3)(1 + \theta_1^+)(1 + \theta_2^+)(1 + \theta_3^+).
\end{align*}
\]

Note: \(1 + \theta^+ = (\epsilon \sigma - \epsilon \mu - 1 - \beta(1 - R))/(\epsilon \sigma) = (\text{constant} + \beta(1 - R))/(\epsilon \sigma).\)

Instead, \(\theta^- = (-\epsilon \mu - 1)/(\epsilon \sigma).\)

Case of homogenous banks:

\[
\begin{align*}
P_{\text{star}}^{\text{sys}} &= (1/2^3)(1 + \theta_1^-)^2(1 + \theta_1^+) = (1/2^3)(1 + \theta_1^-)^2 (\text{constant} + \beta(1 - R))/(\epsilon \sigma)); \\
P_{\text{chain}}^{\text{sys}} &= (1/2^3)(1 + \theta_1^-)(1 + \theta_1^+)^2 = (1/2^3)(1 + \theta_1^-) (\text{constant} + \beta(1 - R))/(\epsilon \sigma))^2; \\
P_{\text{ring}}^{\text{sys}} &= (1/2^3)(1 + \theta_1^+)^3 = (1/2^3) (\text{constant} + \beta(1 - R))/(\epsilon \sigma))^3
\end{align*}
\]

\(^1\)Battiston, Caldarelli, Roukny, May, Stiglitz, 2016, The Price of Complexity in Financial Networks, PNAS
Network architectures and systemic risk

Systemic default probability in the three architecture increases from star to chain to ring:

\[ P_{\text{sys,ring}} \geq P_{\text{sys,chain}} \geq P_{\text{sys,star}} \]

as long as \( \beta(1 - R))/\sigma > 1 \) (empirically relevant).

Network architectures and errors on systemic risk

Sensitivity of the default probability on the recovery rate R increases from star to chain to ring:

\[ \frac{\partial P_{\text{sys,ring}}}{\partial R} \propto \left( \frac{\beta}{\sigma \epsilon} \right)^3; \]
\[ \frac{\partial P_{\text{sys,chain}}}{\partial R} \propto \left( \frac{\beta}{\sigma \epsilon} \right)^2; \]
\[ \frac{\partial P_{\text{sys,star}}}{\partial R} \propto \frac{\beta}{\sigma \epsilon}. \]

as long as \( \beta/(\sigma \epsilon) > 1 \) (empirically relevant).
Small errors on contracts characteristics lead to large errors on systemic risk.
Errors on network structure lead to large errors on systemic risk
Findings: complexity and errors on systemic risk

- Misestimations of systemic risk (by market players with imperfect information) leads to social costs (inadequate buffers, moral hazard, regulatory capture).
- Amplification is intrinsic: small errors on 1) contracts characteristics or 2) network structure can lead to large errors on probability of systemic default.
- Mechanism: errors e.g. on recovery rate $R$ on individual contracts get compounded multiplicatively along chains of connected banks.
- Network complexity may increase not only systemic risk but also inaccuracy on the estimation of systemic risk.  

More research needed to tame complexity in financial ecosystems.

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$^a$ Battiston, Caldarelli, Roukny, May, Stiglitz, 2016, The Price of Complexity in Financial Networks, PNAS

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Asymmetric Information, Externalities and Networks

- Legacy of economics of information:
  1. recognition that information is typically costly, imperfect, and asymmetric
  2. this “deeply affects fundamental understanding of economics such as welfare theorem and characterization of a market economy, and provides explanations of economic and social phenomena that otherwise would be hard to understand.”

- With perfect information: externalities akin coordination problem.

- In contrast, with imperfect and asymmetric information: qualitatively different challenges, e.g. agents with different information sets on origin/magnitude of externalities can play strategically.

- Asymmetric information associated with important externalities affecting specific actors at specific times and along specific pathways (e.g. chains of actors and contracts) in a network

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\[^2\] adapted from [Stiglitz, 2000]
Relations btw Information Economics and Network Economics:

- Many (if not most) relevant externalities are associated with imperfect, asymmetric information.
- The impact of actors onto and from the system depends on their positions in a network of relations.
- While standard approaches are inadequate to capture these dependences, the network approach allows to characterize the microeconomic mechanics of how externalities emerge and how they lead to systemic effects.
- As a result, network economics succeeds in delivering a number of policy insights in various areas that could not be obtained otherwise.
Two specific areas in which financial networks matter.

1. **Financial stability.** Linkages can have ambiguous effects: reduce individual risk but propagate financial distress (assets or/and liability side). Issues remain open but much work done\(^3\)

2. **Macroprudential policy.**
   - Incentives to get too-connected-to-fail and too-correlated-to-fail\(^4\).
   - Empirically: tightly-knit structures\(^5\) and gain exposures to similar risks\(^6\).
   - Structure alters incentives inducing collective moral hazard\(^7\) whereby groups of institutions are altogether to-big-to-fail. This gives institutions greater market power and increases the risk of regulatory capture.

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\(^4\) [Acharya, 2009]


\(^6\) [Gai et al., 2011, Battiston et al., 2016c]

\(^7\) [Farhi and Tirole, 2012]
Financial network literature at a glance

Non-exhaustive list of streams and works:

- **Models of default contagion**, pioneering work 8, boosted by 2008 aftermath 9.
- **Models of distress contagion**: propagation even if not default 10, DebtRank applications11.
- **Models of contagion on liability side**: liquidity hoarding 12.
- **Models of common asset exposures**: common asset exposures trigger price-leverage spirals 13.

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8 [Allen and Gale, 2001, Eisenberg and Noe, 2001]
10 [Battiston et al., 2012a, Tasca and Battiston, 2016]
11 [Battiston et al., 2012c, Battiston et al., 2016a, Di Iasio et al., 2013, Tabak et al., 2013, Poledna and Thurner, 2014, Thurner and Poledna, 2013, Poledna et al., 2015, Fink et al., 2016, Puliga et al., 2014, Bardoscia et al., 2015a, Bardoscia et al., 2015, Bardoscia et al., 2015b, Battiston et al., 2016b, Barucca et al., 2016]
Financial network literature at a glance

Non-exhaustive list of streams and works:

- **Empirical analysis** of financial networks, e.g. equity holdings \(^{14}\) and claims on debt obligations, \(^{15}\).

- **Network reconstruction**: estimation from partial information on the contracts and robustness of the estimations of systemic risk \(^{16}\).

- **Correlation measures in market data** linkages estimated from time series \(^{17}\), Note: different networks may not be compared \(^{18}\).

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\(^{16}\) [Upper and Worms, 2004, Mistrulli, 2011, Musmeci et al., 2013, Anand et al., 2015, Cimini et al., 2014b, Cimini et al., 2014a, Cimini et al., 2014c, Squartini et al., 2013]

\(^{17}\) [Bonanno et al., 2003, Onnela et al., 2004, Billio et al., 2011, ?, Kaushik and Battiston, 2012]

\(^{18}\) [Puliga et al., 2014]
Two fundamental features of financial systems

- **Uncertainty**: traditional focus, valuation of corporate obligations building on Merton 1974: ex-ante valuation. Mostly disregards interdependence between claims’ values.


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19 as acknowledged also in original Eisenberg-Noe 2001
Uncertainty vs interdependence

Two fundamental features of financial systems

- **Uncertainty**: traditional focus, valuation of corporate obligations building on Merton 1974: ex-ante valuation. Mostly disregards interdependence between claims’ values.


When uncertainty and interdependence are both accounted, the valuation today of claims with maturity in the future is non-trivial.\(^\text{19}\)

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Models Landscape

Models of Interconnectedness and Contagion.

Models of Distress Contagion

Models of default Contagion, e.g. Eisenberg-Noe 2001, ...


Models of asset valuation in presence of uncertainty (Merton 1974, ...)
- Complete diversification not optimal in presence of amplification of distress
- Monetary value of losses, conditional to shock on one or more banks
- Impact vs vulnerability
EN, RV, DR can be written in terms of **leverage networks**

- Ordering losses $EN \leq RV \leq cDR$
- EN-based stress-tests: network is **irrelevant**: aggregate losses across banks and creditors equal initial losses to shocked bank.

Each model implies precise assumptions on recovery rate and uncertainty
- There exist pathways to instability:
  - Risk diversification
  - Market integration
- Max eigenvalue goes above 1
EN, RV, DR can be written in terms of leverage networks.

Ordering losses EN ≤ RV ≤ cDR

EN-based stress-tests: network is irrelevant: aggregate losses across banks and creditors equal initial losses to shocked bank.

Each model implies precise assumptions on recovery rate and uncertainty.
Models Landscape

Models of Distress Contagion

Liaisons Dangereuses, Battiston, JEDC 2012


The Price of Complexity, PNAS 2016

Uncertainty as source of systemic risk, Roukny ea. 2016

Clearing with Credit Default Swaps, Schuldenzucker ea. 2016

Leverage Networks, Battiston ea. SRM 2016, JAI 2016

Rethinking Financial Contagion, Visentin 2016

Pathways to instability, Bardoscia ea. 2016


Risk Compression in CDS D’Errico ea. 2016

Climate stress-test of financial system, Battiston ea. 2016

“Cyclic Debtrank, Bardoscia ea. 2015 PlosONE

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- **Liaisons Dangereuses**, Battiston, JEDC 2012
- **Leverage Networks**, Battiston ea., SRM 2016, JAI 2016
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DebtRank: Systemic Impact vs Vulnerability

DebtRank computes, conditional to initial shock (on one or more banks) and taking into account the obligation network.

1. Systemic vulnerability $h_i$ of bank $i$ (i.e. relative equity loss), as well as global vulnerability $H$

2. Systemic impact $DR_i$ of each bank (i.e. weighted sum of equity loss induced on others)

DebtRank (Battiston et al. SciRep 2012); Leverage Networks (Battiston et al. SRM 2016; JAI 2016)

Vulnerability depends on Leverage Network

$$h_i(t + 1) = \min \left\{ 1, h_i(t) + \sum_{j \in A(t)} \Lambda_{ij} h_j(t) \right\} \text{ with } \Lambda_{ij} = \frac{A_{ij}}{E_i(0)} \text{ interbank leverage of } i \text{ towards } j; \ R \text{ exogenous recovery rate.} \ [\text{Battiston et al. 2016 JAI; Battiston et al. 2016 Leveraging, SRM}]$$

Network effects as large as direct effects

$$h_i \approx \sum_k \epsilon_{ik} s_k + \sum_{j,k} \beta_{ij} \epsilon_{jk} s_k \approx \epsilon s + \beta \epsilon s, \text{ with } s_k \text{ relative shock on asset } k. \ [\text{Battiston et al. 2016 JAI; Battiston et al. 2016 Leveraging, SRM}]$$
Conservation of losses in Eisenberg-Noe based models

In EN-based stress-tests: network is irrelevant: aggregate losses across banks and creditors equal initial losses to shocked bank. Overestimation of soundness of financial systems, no matter what size and complexity.

Systematic comparison across contagion models

Leverage framework allows to compare losses
- across models: $EN \leq RV \leq cDR$
- asset types
- shock scenarios
- recovery rate

[Visentin ea. 2016 Rethinking]
Existence and convergence to a consistent valuation

- If each bank computes the expected value of its claims on other banks’s obligations at time $t$ as a function of other banks’ equity and its own external assets, based on local information,
- then market players can agree on consistent value for all obligations, taking into account both the uncertainty on external shocks and interdependence via the network.
- Various types of contracts are covered: loans, bonds, equity holdings. However, e.g. no naked-CDS.
- All previous contagion models are a special case.

Note on Imperfect Information

- Information can be imperfect and asymmetric (e.g. players could believe in different shock distributions on different portions of securities). For any given $j$, no need that $\mathcal{V}_{ij} = \mathcal{V}_{kj}$ for all $i, k$.
- More research needed to understand how to possibly incorporate players’ reactions.
NEVA encompasses all previous models, including DebtRank.

NEVA includes previous models
By assigning the valuation functions appropriately the $\Phi(E)$ maps is equivalent to the map in the following models: Eisenberg-Noe 2001, Furfine 2003, Rogers-Veraart 2013, DebtRank 2012.

The analytical meaning of DebtRank
DebtRank is the consistent network valuation of interbank securities in the case of ex-ante uncertainty with a given uniform distribution of shocks on external assets at time $T$ and external assets recovery rate $\alpha = 0$.

New: Endogenous DebtRank with generic shock distribution
DebtRank distress propagation can be combined with EN idea of endogenous recovery. By assigning the valuation functions appropriately the $\Phi(E)$ map is equivalent to a map of EN in the limit of $t \to T$ and for $t < T$ provides consistent valuation with endogenous recovery and ex-ante uncertainty with generic underlying shock distribution.
Valuation across models

Asset Devaluations ($\beta = 1$, $\sigma = 0.01$),

$A_e = \{10, 8, 6\} = I_e$ and $L_e = \{9, 7, 5\}$.
Valuation across models

- Asset Devaluations ($\beta = 1, \sigma = 0.1$),
- $A^e = \{10, 8, 6\} = I^e$ and $L^e = \{9, 7, 5\}$. 


