THE INTERCONNECTED NATURE OF FINANCIAL SYSTEMS: DIRECT AND COMMON EXPOSURES

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Summary

Background: Systemic risk

Proposal: Correlation networks

Correlation networks on quantities

- Graphical Gaussian models
- Partial correlation networks
- Direct and common exposures
- Application: BIS CBS data
Systemic Risk: Definition

- De Bandt & P. Hartmann (2000):
  A systemic crisis can be defined as an event that affects a considerable number of financial institutions or markets in a strong sense. [...] At the heart of the concept is the notion of contagion, a particularly strong propagation of failures from one institution, market or system to another.

- Hendricks (2009):
  A systemic risk is the risk of a phase transition from one equilibrium to another, much less optimal equilibrium, characterized by multiple self-reinforcing feedback mechanisms making it difficult to reverse.

- Benoit et al. (2015):
  The risk that many market participants are simultaneously affected by severe losses, which then spread through the system.
**Systemic risk: measurement**

1. Conditional quantiles:
   - Acharya et al. (2010), Adrian & Brunnermeier (2011), Brownlees & Engle (2012), ..
   - Identify SIFIs → **Do not describe contagion transmission**;

2. Regression methods:
   - Koopman et al. (2012), Betz et al. (2014), Hale et al. (2016)
   - Provide predictive models → **Do not describe contagion transmission**;

3. Network models:
   - Battiston et al. (2012), Billio et al. (2012), Brunetti et al. (2015)
   - Describe contagion transmission → **Do not predict**;
Correlation network models

- Summarise systemic risk into a single measure to identify SIFIs
- Are capable to produce a clear predictive early warning signal
- Can describe contagion transmission

Data sources

- Not on financial market prices of institutions (returns, CDS), but:
- Between countries, using interbank flows: quantities.
Graphical Gaussian models

Definition

Given an undirected graph $G = (V, E)$, a graphical Gaussian model is the family of all $N$-variate normal distributions $\mathcal{N}_N(0, \Sigma)$ that satisfy the constraints induced by the graph on the partial correlations, as follows:

$$e_{ij} = 0 \iff \rho_{ij} = 0$$

for all $1 \leq i < j \leq N$.

In a graphical Gaussian model the non null entries of the adjacency matrix correspond to the significant partial correlations.
**Eigenvector centrality**

**Definition**

The eigenvector centrality of a node $i$ is:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{N} a_{i,j} x_j,$$

where $x_j$ is the score of a node $j$, $a_{i,j}$ is the $(i,j)$ element of the adjacency matrix of the network, $\lambda$ is a constant and $N$ is the number of nodes of the network.

Once a graphical Gaussian model is selected, we can estimate the eigenvector centrality measure conditionally on the chosen model. Alternatively, we can model average such estimates.
Data

Giudici, Spelta (JBES, 2016), extended Minou and Reyes (2013), building a model averaged graphical Gaussian model on the total financial exposure of countries banking sectors (from BIS locational statistics).

Doing so, they establish bilateral links between countries, based on exposure correlations rather than on bilateral ones.
PARTIAL CORRELATION NETWORKS

THE SELECTED NETWORK
Partial correlation networks

Model averaged eigenvector centralities
Summary Background: systemic risk  Proposal: correlation networks

Correlation networks on quantities

Partial correlation networks

Summary findings

Results

Most systemic countries are:

- International Financial hubs such as US and UK;
- Off-shore countries such as LU, HK and KY:
- Countries with large cross-border financial activities, such as CH, NL and DE.

Warning: the approach works also for non-reporting countries but it does not exploit bilateral data — Analysis of direct and common exposures
Objective

▶ Compare (for the first time) physical and correlation networks in terms of predictive performance;

▶ To provide an approach to aggregate the direct and indirect components of countries’ exposures for a more encompassing measure of interconnectedness

▶ To develop early warning predictive measures
Network Links

A link between two countries:

- in a physical (direct) network it represents a flow of funds between a lender country and a borrowing country.
- in a correlation (common exposure) network, it measures the proximity between the funding composition or between the credit allocations (portfolio composition) of two countries, depending on whether in-flows or out-flows are considered.
**Correlation network proximity**

\[ d_{ij}^{ln} = 2 - \sqrt{2 \left(1 - C_{ln^i, ln^j}\right)} \]  

where \( C_{ln^i, ln^j} \) is the correlation between the funding sets of country \( i \) and \( j \).

A high value of \( d_{ij}^{ln} \) means that the funding that \( i \) and \( j \) receive has a similar composition: they have similar funding risk.

\( ln^i \in \mathbb{R}^{1 \times N} \): set of flows that country \( i \) receives from all other countries

**In-flow proximity**
Correlation networks on quantities

Direct and common exposures

Correlation network proximity

\[ Out^i \in \mathbb{R}^{1 \times N}: \text{set of flows invested by country } i \text{ in all other countries.} \]

Out-flow proximity

\[ d_{ij}^{Out} = 2 - \sqrt{2 \left( 1 - C_{Out^i,Out^j} \right)} \]  \hspace{1cm} (2)

where \( C_{Out^i,Out^j} \) is the correlation between the investor sets of country \( i \) and \( j \).

A high value \( d_{i,j}^{Out} \) means that \( i \) and \( j \) invest similarly in other countries: they have similar credit risk.

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The interconnected nature of financial systems: direct and common exposures
Direct network: in and out strength

- Interconnectedness is related to the detection of the most central players in the network.
- Let $w_{i,j,t}$ be the quantity lent from $j$ to $i$ at time $t$.

### In-strength

The in-strength of country $i$ in a direct network at time $t$ is:

$$S^I_{i,t} = \sum_j w_{i,j,t}$$

and represents the total funding of a country in that period.

### Out-strength

The out-strength of country $i$ in a direct network at time $t$ is:

$$S^O_{j,t} = \sum_i w_{i,j,t}.$$
**COMMON EXPOSURE: IN AND OUT STRENGTH**

### In-strength

The in-strength of country $i$ in a common exposure network at time $t$ is:

$$S_{i,t}^{I,C} = \sum_{j} d_{i,j,t}^{in}$$

### Out-strength

The out-strength of country $i$ in a common exposure network at time $t$ is:

$$S_{i,t}^{O,C} = \sum_{i} d_{i,j,t}^{out}$$

- The higher $S_{i,t}^{I,C}$ ($S_{i,t}^{O,C}$) the higher the similarity of the composition of the funding (portfolio allocation) of that country with respect to all other countries.
- A lower $S_{i,t}^{I,C}$ ($S_{i,t}^{O,C}$) means that country has a set of investors (portfolio) that is specific to that country.

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The interconnected nature of financial systems: direct and common exposures.
Direct and common exposures

**Strength types**

- $S^I_{i,t}^R$: total funding a country receives from the others;
- $S^O_{i,t}^R$: total credit of a country in other countries;
- $S^I_{i,t}^C$: total proximity of the funding composition of a country with respect to the others,
- $S^O_{i,t}^C$: total proximity of the credit allocation of a country with respect to the others.
Mixed links

▶ Aggregate direct and common exposure networks, in a measure of systemic risk that takes into account both direct and indirect effect of countries exposures

**Definition**

A mixed link $i,j$ at time $t$ can be defined as follows:

$$m_{i,j,t} = \alpha_t \hat{w}_{i,j,t} + (1 - \alpha_t) \hat{d}_{i,j,t}$$

(7)

where $\hat{w}_{i,j,t}$ and $\hat{d}_{i,j,t}$ are the normalized links (z-scores) between country $i$ and country $j$ at time $t$ obtained from the direct and from the common exposure matrix.

The parameter $\alpha_t$ governs the relative strength of the two components (normalized singular value).
We consider the consolidated banking statistics on an ultimate risk: CBS data, from Q3–1998 to Q4–2013.

For the funding side, we restrict the analysis to the 33 largest economies (for which loans sum up to 100000 billion dollars).

For the credit side we use the 15 fully reporting countries.

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1Countries selected for in-flows analysis: AT = Austria, AU = Australia, BE = Belgium, BR = Brazil, CA = Canada, CH = Switzerland, CN = Cina, CZ = Czech Republic, DE = Germany, DK = Denmark, ES = Spain, FI = Finland, FR = France, GB = Great Britain, GR = Greece, HK = Hong Kong, IE = Ireland, IN = India, IT = Italy, JP = Japan, KR = South Korea, KY = Cayman Islands, LU = Luxemburg, MX = Mexico, NL = The Netherlands, NO = Norway, NZ = New Zealand, PL = Poland, PT = Portugal, RU = Russia, SE = Sweden, SG = Singapore, US = United State.

2Countries selected for out-flows analysis: AT = Austria, AU = Australia, BE = Belgium, CH = Switzerland, DE = Germany, DK = Denmark, ES = Spain, FR = France, GB = Great Britain, IE = Ireland, JP = Japan, NL = The Netherlands, SE = Sweden, TW = Taiwan, US = United States.
In-strengths of each country for the direct (blue) and for the funding composition similarity (green) networks. Near the name of each country we also report the correlation coefficients between the two measures. The three vertical bars emphasize the crisis periods (2005-07, 2007-09 and 2009-11).
RESULTS FROM IN-STRENGTHS

Results: in strength

▸ Regarding the total funding BE, ES, FR, GR, GB, IE, IT, NL, PT in Europe, and the US show an increase of the in-strength up to the 2007 financial crisis, followed by an abrupt fall

▸ Some countries do not follow this general trend: AU, CA, BR, KR, LU, RU and the Baltic countries do not decrease after 2007, whereas CN, HK, CH, JP, KY, MX, SG, IN show an ever increasing in-strength

▸ Regarding the funding composition, it is generally decreasing for most countries, except for DE, BR, IN and MX

▸ The funding composition of most countries has become more and more concentrated on a limited number of specific lenders (concentration risk factor)
Results: in strength

- Before the crisis, we observe an increase in total funding of countries, given by specific investments in those countries and not by a generalized increase in the overall system funding.

- After the crisis, we observe a decrease in total funding which, it does not directly correspond to a higher diversification but rather to a further concentration.

- Germany (DE) is the only country with a remarkable positive correlation (0.67) between the in-strengths calculated starting from the direct and from the common exposure networks.

- Before the crisis, funding of DE was increasing in a diversified way, differently from other countries, DE has attracted new investors that were previously investing in other countries: a flight to quality effect.

- After the crisis, instead, the investor set of DE has become more country-specific, like that of other countries.
Out-strengths of each country for the direct (blue) and for the portfolio similarity (green) networks, near the name of each country we also report the correlation coefficients between the two measures. The three vertical bars emphasize the crisis periods.

The crisis affects the direct out-strength of many core European countries: CH, FR, NL, GB but not that of non EU-countries such as US, JP, AU and TW.

The trend of the out-strength of the portfolio similarity networks is decreasing for most of the countries as for the funding composition.
**Mixed strengths**

**Figure:** Weights associated to the direct (green) and to the funding composition similarity (blue) component of the mixed network for the in-flow network (a). Weights associated to the direct (green) and to the portfolio composition similarity (blue) component of the mixed network for the out-flow network (b).
Mixed In-strengths

In-strength of the combined adjacency matrix, near the name of each country we also report the correlation coefficients between the combined and the direct in-strength (M-R) and the in-strength of the funding composition similarity network (M-C).

The three vertical bars emphasize the crisis periods (2007-08; 2008-09 and 2009-10).
Mixed Out-strengths

The interconnected nature of financial systems: direct and common exposures

Strength of the combined out-flows adjacency matrix, near the name of each country we also report the correlation coefficients between the combined (M) and the direct out-strength (R) and the strength of the portfolio composition similarity network (C). The three vertical bars emphasize the pre-crisis crisis periods (2007-08; 2008-09 and 2009-10)

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The interconnected nature of financial systems: direct and common exposures
Results from Mixed Strength

The higher the mixed strength, the lower the risk – both from a total funding (lending) and from a funding (lending) composition viewpoints

Results: mixed

- Many countries present a fall of the mixed in-strength measure during the financial crisis. Some of them: ES, GR, IT, PT, AT, BE, CZ, PL, UK and JP have not yet recovered.
- Others, such as IE, FR, NL, LU, FI, SE, BR together with US, instead have recovered.
- Another group of countries has not been affected by the crisis, and maintain a low risk profile throughout: it includes off-shore countries (HK, LU, KY) flight to quality countries (CH, DE, DK, SG) and emerging countries (CN, IN, KR, MX).
- The mixed out-strength of most of the EU countries to decrease near the crisis period.
- The mixed out-strength of most of the EU countries to decrease near the crisis period, except for DE, FR and NL.
- Most of the non-EU countries display an increasing strength over most of the sampling periods.
We compare direct and common exposures networks with the RiskRank measure (Mezei and Sarlin, 2015), based on individual risk and interconnectedness.

We employ an Early-warning model to separate vulnerable and tranquil period and to discriminate between them by estimating the probability of being in a vulnerable state.

The crisis events are based upon the IMF database by Laeven and Valencia (2008), while the individual risk indicators include 14 macro-financial indicators.

The forecast horizon is of 5–12 quarters prior to crisis events, as common in the literature.

\[
RR_c(x_1, \ldots, x_n, x_c) = \sum_{i=1}^{n} \left( v(e_i) - \frac{1}{2} \sum_{j \neq i} I(c_i, c_j) \right) x_i + \sum_{i \neq j}^{n} I(c_i, c_j) \prod_{x_i, x_j}\]

\[
\text{Indirect effect of } j \text{ via } i \text{ on } c
\]

<table>
<thead>
<tr>
<th>Predicted Class</th>
<th>Actual Class</th>
<th>Pre-crisis period</th>
<th>Tranquil period</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Signal</td>
<td>Signal</td>
<td>Correct call</td>
<td>False alarm</td>
</tr>
<tr>
<td>True Positive (TP)</td>
<td>Missed crisis</td>
<td>False Positive (FP)</td>
<td>Correct silence</td>
</tr>
<tr>
<td>False Negative (FN)</td>
<td>Correct call</td>
<td>True Negative (TN)</td>
<td></td>
</tr>
</tbody>
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Table 1: A contingency matrix.
Predictive performance comparison: results

- Direct linkages are clearly outperformed by the other ways of defining linkages.

- Linkages based upon common exposures and a combination of direct and common exposures perform equally well for the in-flow network.

- While the difference between the common and the combined exposures is minor, the combined exposures outperform in the case of $\mu = [0.1, 0.4]$ for the in-flow network.

- Linkages based upon common exposures outperform linkages based on the combination of direct and common exposures for the out-flow network.

- Out-flow networks generally outperform In-flows network in signaling crisis events.
FUTURE RESEARCH

**Country specific PD**
From interest rate spreads, cds spreads or EDF (Tintchev, 2016)

**Contagion network**
Interbank flows $\rightarrow$ Physical network between countries.
Interbank flows similarities $\rightarrow$ Correlation network between countries

**Early warning predictive measure**
- Country PD corrected by contagion impact