An Agent-based model of liquidity and solvency interactions

Grzegorz Hałaj

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Liquidity risk is a systemic and a system-wide concept, turbulently driven by behaviors of market participants.

Complexity: connectivity of balance sheets and markets + liquidity coupled with solvency.

Severity: liquidity problems may unwind rapidly and turbulently.

Expected takeaway: a method to measure, monitor, counteract.

Approach taken: agent-based modelling (ABM) of interacting agents responding to shocks.
• Mechanisms of systemic liquidity risk
  – Recent example(s)
  – Some theory on the drivers

• Components of the Agent-based Model (ABM)
  – Role of ABM in finance
  – **Six steps of a (liquidity+solvency) default chain**
  – For policy assessment: randomly generated systems vs real data on interbank

• Simulations
  – Macroprudential policy analysis: Liquidity Coverage Ratio (LCR)-type limits + Capita Adequacy Ratio (CAR) limits
  – Conclusion: a lot of nonlinearity and cliff effects
Liquidity risk – how it became a global problem (1)

- 2007-2008 crisis partly related to liquidity and contagion – largest globally active, highly interconnected market players caught in an illiquidity trap (Drudi et al. (2011), ECB WP 1467)
- 2010 sovereign phase of the crisis also implying for banks issues with liquidity and contagion
- Second stage of the crisis: bank – sovereign nexus
- Before that: liquidity a “forgotten risk”...

![European banks cost of funding and CDS](chart.png)

- Banks CDS (LHS), average across selected European banks
- Spread EURIBOR12-EONIA (RHS)
- Spreads between interbank rates and repo rates (RHS)
Liquidity crisis – insight into mechanism

- Cifuentes, Ferrucci & Shin (2006)
  [...]When the market's demand for illiquid assets is less than perfectly elastic, sales by distressed institutions depress the market prices of such assets. Marking to market of the asset book can induce a further round of endogenously generated sales of assets, depressing prices further and inducing further sales...]

Small shock can cause big troubles!

- Brunnermeier & Pedersen (2008)
  market liquidity:
  (i) can suddenly dry up
  (ii) has commonality across securities,
  (iii) is related to volatility,
  (iv) is subject to “flight-to-quality,”
  (v) co-moves with the market

Lots of non-linearity and reinforcement!
In financial context:

• Giansante et al. (2012): study of interactions between liquidity and solvency
  – Liquidity and solvency conditions for economic agents determine the bilateral flows based on the assessment of counterparty solvency and liquidity scoring index

• Klimek et al. (2015): deal with the efficiency of the bank resolution mechanisms
  – Confirming the intuition that a bail-in mechanism may perform better than other closer to the bail-out concept

• Bookstaber et al.: towards Agent-based modelling (ABM) approach
  – Interacting players: liquidity demanders, suppliers, market makers
  – Endogenising liquidity supply fluctuations (cyclical with periods of crises)
**LST TD tools – an ABM approach**

- **6-step approach**: a liquidity outflow triggers a chain of events in the banking system

- **Shocking the system:**
  - Outflow of deposits in a given segment (random / deterministic shocks, e.g. $\sigma, 2\sigma, \ldots$)

- **Shock transmission chain**: chain of events is activated

- **Shock impact:**
  - Illiquidity (default on liquidity if buffers insufficient) – number of banks, liquidity ratio reduction,
  - Solvency: P&L+CAR impact
Sensitivity analysis of key parameters

• Creating banks (randomly, 100) composed of assets / liabilities from a given set, with given parameters, s.t. liquidity and solvency is admissible
• Accounting for the heterogeneity of sizes (Gamma distribution)
• Applying a liquidity shock and going through the sequence of events
• Repeating many times and aggregating the (CAR) results (e.g. 1000 simulations of banks * 50 interbank markets * 100 scenarios of shocks → 5 million simulations)
**Targeting solvency – CAR threshold to mitigate contagion**

- **(A)** Liquidation above eligible securities
- **(B):** Fire-sale impact
- **(C):** Interbank losses due to cash hoarding
- **(D):** Funding cost shock following $\Delta$CAR
- **(E):** Peers funding cost impacted
- **(F):** Insolvency spread via cross-holding of debt

Note: Sequence of simulations: shock to corporate deposits and covered bonds in country A; x-axis – outflow (%); y-axis – average CAR; lines correspond to CAR threshold in (8%; 9%)
Targeting liquidity – LCR to mitigate contagion

(A) Liquidation above eligible securities
(B): Fire-sale impact
(C): Interbank losses due to cash hoarding

(D) Funding cost shock following $\Delta$CAR
(E) Peers funding cost impacted
(F) Insolvency spread via cross holding of debt

Note: Sequence of simulations: shock to corporate deposits and covered bonds in country A; x-axis – outflow (%); y-axis – average CAR; lines correspond to LCR in (1; 1.1)
Real system: structure of the BS input data

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20K datapoints for ≈120 banks to analyse
Rubric

• Assess the contagion risk: scope and magnitude of transmission
• Identify systemic institutions: these nodes that cause a cascade of problems
• Assess effectiveness of various policy measures aimed at mitigating these risks: LE limits, RWs and capital buffers on the systemically important institutions

Network models based on financial institutions’ (in particular banks’) exposures help to

<= Own debt issued (simulated) network based on 2014 EBA ST data

Unsecured interbank lending (simulated) network based on 2014 EBA ST data =>

Circle indicate a banks (size ~ log(total assets)); link = exposure
Sequence of simulations: outflow of corp. depo and covered bonds in country A

- (A) Liquidation above eligible securities
- (B): Fire-sale impact
- (C): Interbank losses due to cash hoarding
- (D) Funding cost shock following $\Delta$CAR
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Note: x-axis – outflow (%); y-axis – CAR
CAR impact on the 2.5% shock, percentiles across randomly selected banks affected by initial liquidity shock

Note: top pane – CAR; bottom pane – deviation from initial car (bps)
Heatmap of vulnerabilities

Design:
- Set funding outflow parameter = 20%
- Choose funding category $c$ and bank $b$ and run the 6-step simulation for a 20% outflow for category $c$ in bank $i$
- Aggregate the results per country XX (panel a) reports results for all pairs $(i,c))$ → total systemic risk effects
- Compute the average difference between the average CAR before and after 6-steps for all banks outside country XX (panel b) reports results for all pairs $(i,c))$ → cross-border effects
Conclusions

- Still work in progress (further parameterization work, robustness checks)
- Country instead of currency perspective: data available
- Correlated shocks on investment (asset) and funding (liability)
- Multi-period extension: a fully ABM implementation -> liquidity shock -> balance sheet reaction -> liquidity shock -> balance sheet reaction...
- Optimising banks: endogenising the behaviours
ANNEX

Model details
Model – input parameters (I)

- Balance sheet of bank $b$ is composed of some $N$ generic classes of assets $(a_1^b, \ldots, a_N^b)$, $M$ liabilities $(l_1^b, \ldots, l_M^b)$ and capital $e^b$ and the balance sheet sum identity holds ($\tilde{N}: = \{1, \ldots, N\}$, $\tilde{M}: = \{1, \ldots, M\}$):

$$\sum_{n} a_n^b = \sum_{m} l_m^b + e^b$$

- Outflow parameter $f_m^{0,b}$, $m \in \tilde{M}$: fraction of volume that flows out (or is not rolled over)

- Haircut $h_n$: a haircut on the book value of the securities in case of liquidation
Model – input parameters (II)

- Eligibility $\mathcal{E} \subset \tilde{N}$: a set of asset classes that are eligible in CB operations; noneligible: $\mathcal{N} \mathcal{E} := \tilde{N}/\mathcal{E}$
- Fire sales elasticity $\alpha_n$: implying revaluation of assets by a factor of $1 - \exp(-\sum_{n \in \mathcal{N} \mathcal{E}} \alpha_n \Delta a_n^b)$
- Solvency conditions: risk weights applied to the exposures $\Rightarrow$ RWA $\Rightarrow$ (divided by capital) capital ratio (CR)
- Interbank market: of direct lending (matrix $I$, $l_{k1k2}$: $k_2$ lends to $k_1$) and cross-holdings of debt securities issued by banks (matrix $B$)
- Maturity profile of funding $\mu_m$: deterioration of CR $\Rightarrow$ maturing wholesale funding repriced at a higher cost
- MtM-recognised assets impacted by the revaluation: $\mathcal{M} \subset \tilde{N}$
Model – liquidity shock

A liquidity shock is practically an outflow of deposits, which is a bank-specific vector \( s^b \), s.t. \( s^b_m = f^{O,b}_m l^b_m \in [0, l^b_m] \).

Assets are characterised by liquidity haircuts \( h_n \in [0, 1] \).

If

\[
\sum_{n \in \mathcal{E}} (1 - h_n) a^b_n > \sum_{m \in \mathcal{M}} s^b_m
\]

then deficiency observed: \( D^b : = \sum_{m \in \mathcal{M}} s^b_m - \sum_{n \in \mathcal{E}} (1 - h_n) a^b_n \) – needs to be supplemented by 'fire sales'
Model – impact of fire sales

MtM revaluation of securities

\[ e^{fs*, b} = e^b - \sum_{n \in \mathcal{M}} a_n^b \exp\left( - \sum_{(k,x) \in \mathcal{L}^k} \mathbb{I}_{\{k=n\}} \alpha_k x \right) \]

repricing of a given asset class by the aggregate volume of liquidated assets on the market depending on the depth of the market (\(\alpha\))

Spill over to the interbank market

Debtors need to search for other sources of funding which is a costly process (cost \(\equiv\) fraction of volume).

\[ e^{fs, b} = e^{fs*, b} - \sum_{k \in \mathcal{D}\mathcal{L}} c^l I_{bk} \]
Model – banks $\mathcal{ES}$ with significantly affected capital ratio

$$
\mathcal{T}_d : = [\tau_{d-1}, \tau_d] \quad \mathcal{ES} : = \{ b \left| \frac{e^b}{\Omega_b} - \frac{e^{fs,b}}{\Omega_b} > \Delta^T \right. \}
$$

**Direct effects**

$$
e^{ln,c,b} = e^{fs,b} + \sum_n r_n a_n^b - \sum_m c_m l_m^b
$$

Accounting for the relationship between funding cost and solvency leads to:

$$
e^{lncS,b} = e^{ln,c,b} - \sum_m \left( \sum_d \mathbb{I} \left\{ \frac{e^b - e^{fs,b}}{\sum_n \omega_n s_n^{fs,b} \in \mathcal{T}_d} \Delta c_m^d l_m^b \right\} \right)
$$

**Indirect effects**

The peers of $\mathcal{ES}$ are affected as well. A peer: any bank with a similar structure of the balance sheet (cosine of balance sheet vectors is $> 1 - \epsilon$)
Model – solvency defaults

• If for any reason the capital ratio falls below a regulatory threshold $\tau^b \rightarrow$ the bank defaults.

• We assume that it means default of payment of the interbank liabilities (with a given LGD, set uniformly for the interbank market) and defaults on the bonds issues that are held across the market.

• These two layers of interconnectedness transfer the shocks of the solvency defaults throughout the interbank market