Filtering for Risk Assessment of Interbank Network

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Background: Overnight Market

- By regulations, banks in the United States are required to maintain reserves either as cash or as fed funds.
- The day-to-day banking activities are unlikely to leave banks with the desired level of reserve.
- To meet shortfalls, regulated banks can trade fed funds in the interbank market.
- Other sources of overnight liquidity include Repos and discount window.

Overall, the overnight market serves as:
1. the most immediate source of liquidity
2. an important indicator of system functionality
3. a crucial role for implementation of monetary policy

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Motivation: Interbank Functionality

- Does the interbank market mitigate or amplify shocks to individual banks or the sector as a whole?
  - e.g. Afonso et al (2011)
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- Theory proposes two possible channels that lead to disruptions:
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- e.g. Afonso et al (2011)

Theory proposes two possible channels that lead to disruptions:

1. Liquidity Hoarding
   - Each bank’s uncertainty about its own funding needs skyrocketed, Brunnermeier (2009)

2. Counterparty Risk
Motivation: Interconnectedness

- Different line of research looks at the interconnectedness of financial institutions and its impact on systemic risk.
- Adrian and Brunnermeier (2011): policies that focus on bank’s individual-level fail to respond to systemic risk
- Uncertainty about network structure and fire sales, Caballero and Simsek (2013)
There is mixed evidence about the interconnectedness and network resilience.

- Allen and Gale (2000) interconnectedness is beneficial from risk-diversification.
- However, there are limits for diversification benefits,
Can we reverse engineer signals from the interbank market and identify its network structure?
If yes, how can we assess its resilience to mitigate liquidity shocks?
Moreover, what role does the network structure (especially interconnectedness) play in the interbank market functionality?
Research Question

Can we reverse engineer signals from the interbank market and identify its network structure?
If yes, how can we assess its resilience to mitigate liquidity shocks?
Moreover, what role does the network structure (especially interconnectedness) play in the interbank market functionality?

The interbank market allows to answer the above mainly due to
1. the overnight transactions
2. the pairwise lender-to-borrower relationship
Contribution

- We propose a framework that
  1. Deduces and evaluates information from the interbank market
  2. Identifies the interbank network structure along with its interconnectedness
  3. Serves as early warning system to detect distress
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1. Deduces and evaluates information from the interbank market
2. Identifies the interbank network structure along with its interconnectedness
3. Serves as early warning system to detect distress

Using simulation studies, we find that interconnectedness is risk mitigating to some degree, beyond which systemic risk increases exponentially.

- Contribute to the debate e.g. Stiglitz (2010)
Model: Assumptions

- There are $N$ banks interacting during $T$ periods via overnight borrowing and lending.
- Interest rate is zero and banks hold zero cash reserve.
- Net cash flows, $x_{i,t} = x_{i,t}^A - x_{i,t}^D$, is the only source of stochasticity, $\forall i = 1,..,N$ and $t = 0,1,...,T$. 

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- Banks with positive net cash flows always lend, if there is demand.
- If no lending takes place, positive net cash flows are invested in assets.
  - This, hence, induces non-stationarity in net cash flows over time.
Model: Assumptions II

- For all $i = 1, ..., N$
  \[ x_{i,t} \sim N(\mu_{i,t}, \sigma_{i,t}) , \]  
  \( (1) \)

- For all $i, j$ and $t$
  \[ \rho_{ij,t} = \rho_{ij} \]  
  \( (2) \)
Model: Assumptions II

- For all $i = 1, ..., N$
  \[ x_{i,t} \sim \mathcal{N}(\mu_{i,t}, \sigma_{i,t}) \]  
  \hspace{1cm} (1)

- For all $i, j$ and $t$
  \[ \rho_{ij,t} = \rho_{ij} \]  
  \hspace{1cm} (2)

- There are two states: $\mu_{i,t} \in \{ \mu_{i}^{(1)}, \mu_{i}^{(2)} \}$ and $\sigma_{i,t} \in \{ \sigma_{i}^{(1)}, \sigma_{i}^{(2)} \}$, such that
  \[ \frac{\mu_{i}^{(1)}}{\sigma_{i}^{(1)}} > \frac{\mu_{i}^{(2)}}{\sigma_{i}^{(2)}} \]  
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  \[ \frac{\mu_{i}^{(1)}}{\sigma_{i}^{(1)}} > \frac{\mu_{i}^{(2)}}{\sigma_{i}^{(2)}} \]  
  \hspace{1cm} (3)

- The transition between the two states follows a Markov chain process of order 1,

\[
\begin{array}{c|cc}
   t \backslash t + 1 & 1 & 2 \\
   \hline
   1 & p_i & 1 - p_i \\
   2 & 1 - q_i & q_i \\
\end{array}
\]
Model: Dynamics

- At \( t = 0 \), if \( x_{i,0} < 0 \), bank \( i \) borrows \( y_{i,0} = -x_{i,0} \) in the interbank market.
- At \( t = 1 \), bank \( i \) repays \( y_{i,0} \), such that its net cash flows become \( x_{i,1} - y_{i,0} \).
- If \( x_{i,1} - y_{i,0} < 0 \), then \( i \) borrows again at \( t = 1 \):
  \[
y_{i,1} = -(x_{i,1} - y_{i,0})
  \]  
(4)
- Otherwise, \( y_{i,1} = 0 \)
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  \[
  y_{i,1} = -(x_{i,1} - y_{i,0})
  \]  \( (4) \)
- Otherwise, \( y_{i,1} = 0 \)
- To generalize, we have
  \[
  y_{i,t} = \begin{cases} 
  -[x_{i,t} - (y_{i,t-1} - y_{i,t-1})] & B_{i,t} = 1 \\
  0 & B_{i,t} = 0 
  \end{cases}
  \]
  \( (5) \)
  where \( B_{i,t} \) returns 1 if bank \( i \) borrows at time \( t \) and 0 otherwise.
There are two main challenges: truncation and non-stationarity

- We deal with the former by using properties of truncated distributions
- While in the latter, we use Hamilton (1989) filter
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We deal with the former by using properties of truncated distributions

While in the latter, we use Hamilton (1989) filter

For a given state $s$, the conditional likelihood for bank $i$ at time $t$ is

$$
\ell_{i,t}^{(s)} = \begin{cases} 
\phi \left( \left[ y.i,t - (c_{i,t-1} - \mu_i^{(s)}) \right] / \sigma_i^{(s)} \right) \left( 1 / \sigma_i^{(s)} \right) & B_{i,t} = 1 \\
1 - \Phi \left( \left[ c_{i,t-1} - \mu_i^{(s)} \right] / \sigma_i^{(s)} \right) & B_{i,t} = 0
\end{cases}
$$

where $c_{i,t-1} = y.i,t-1 - y.i.,t-1$. 

(6)
Given two states, i.e. $s \in \{1, 2\}$, the conditional likelihood at time $t$ for bank $i$ is

$$
\ell_{i,t} =
\begin{align*}
&= \xi_{i,t-1}^{(1)} \left[ p_i \cdot \ell_{i,t}^{(1)} + (1 - p_i) \cdot \ell_{i,t}^{(2)} \right] \\
&+ \xi_{i,t-1}^{(2)} \left[ (1 - q_i) \cdot \ell_{i,t}^{(1)} + q_i \cdot \ell_{i,t}^{(2)} \right]
\end{align*}
$$

(7)
Filtering System: Estimation II

- Given two states, i.e. \( s \in \{1, 2\} \), the conditional likelihood at time \( t \) for bank \( i \) is

\[
\ell_{i,t} = \xi_{i,t-1}^{(1)} \left[ p_i \cdot \ell_{i,t}^{(1)} + (1 - p_i) \cdot \ell_{i,t}^{(2)} \right] + \xi_{i,t-1}^{(2)} \left[ (1 - q_i) \cdot \ell_{i,t}^{(1)} + q_i \cdot \ell_{i,t}^{(2)} \right]
\]

(7)

- Given information up till time \( t \), the probability of \( i \) being in either state is

\[
\xi_{i,t}^{(1)} = \frac{\left[ p_i \cdot \xi_{i,t-1}^{(1)} + (1 - q_i) \cdot \xi_{i,t-1}^{(2)} \right] \cdot \ell_{i,t}^{(1)}}{\ell_{i,t}},
\]

(8)

and

\[
\xi_{i,t}^{(2)} = 1 - \xi_{i,t}^{(1)}.
\]

(9)
Figure: Total Borrowings by Bank $i$ over Time
Filtering System: Illustration II

**Figure:** State 2 Filter for Bank $i$ over Time

![Graph showing the State 2 Filter for Bank $i$ over time with various time points and filter values.](image-url)
We proxy interconnectedness between two banks by the correlation coefficient between their net cash flows, i.e. $\rho_{ij}$.

To do so, we derive the conditional joint likelihood between banks $i$ and $j$, which is given by:

$$
\ell_{ij,t} = \left[ P_i \otimes P_j \bar{\ell}_{ij,t} \right]^T \bar{\xi}_{ij,t-1}, \quad (10)
$$
Filtering System: Interconnectedness I

- We proxy interconnectedness between two banks by the correlation coefficient between their net cash flows, i.e. $\rho_{ij}$.
- To do so, we derive the conditional joint likelihood between banks $i$ and $j$, which is given by:

$$\ell_{ij,t} = \left[ P_i \otimes P_j \bar{\ell}_{ij,t} \right] ^T \bar{\xi}_{ij,t-1},$$

(10)

where

$$\bar{\ell}_{ij,t} = \left[ \ell_{ij,t}^{(11)} \ell_{ij,t}^{(12)} \ell_{ij,t}^{(21)} \ell_{ij,t}^{(22)} \right] ^T,$$

(11)

and

$$\ell_{ij,t}^{(s_1s_2)} = \begin{cases} 
  f(y.i,t, y.j,t \mid \Omega_{t-1}) & B_{i,t} = 1, B_{j,t} = 1 \\
  f(y.i,t, y.j,t = 0 \mid \Omega_{t-1}) & B_{i,t} = 1, B_{j,t} = 0 \\
  f(y.i,t = 0, y.j,t \mid \Omega_{t-1}) & B_{i,t} = 0, B_{j,t} = 1 \\
  f(y.i,t = 0, y.j,t = 0 \mid \Omega_{t-1}) & B_{i,t} = 0, B_{j,t} = 0
\end{cases}.$$

(12)
Let $\Theta_i$ denote bank’s $i$ specific parameters.

What determines the interbank market interconnectedness is the correlation matrix among all agents’ net cash flows.

Specifically, let $R$ denote the correlation matrix of the network.

The $i$th row and $j$th column element of $R$ denotes the correlation in net cash flows between bank $i$ and $j$, $\rho_{ij}$ for all $i \neq j$. 
Let $\Theta_i$ denote bank’s $i$ specific parameters.
What determines the interbank market interconnectedness is the correlation matrix among all agents’ net cash flows.
Specifically, let $R$ denote the correlation matrix of the network.
The $i$th row and $j$th column element of $R$ denotes the correlation in net cash flows between bank $i$ and $j$, $\rho_{ij}$ for all $i \neq j$.
Hence, given real transaction, the proposed filtering system identifies the interbank network by estimating $R$ and $\Theta_i, \forall i = 1, ..., N$. 
We look at \( N = 20 \) banks that interact over \( T = 30 \) periods. For each bank \( i \) we identify \( \Theta_i \).

We impose that banks fall into two clusters:

\[
R = \begin{bmatrix}
R_1 & R_{12} \\
R'_{12} & R_2
\end{bmatrix},
\]

The \textbf{within} correlation in \( R_1 \) and \( R_2 \) is uniform and equals to \( \rho_1 \) and \( \rho_2 \), respectively.

The \textbf{between} correlation in \( R_{12} \) is uniform and equals to \( \rho_{12} \).
We look at $N = 20$ banks that interact over $T = 30$ periods.
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We impose that banks fall into two clusters:

$$R = \begin{bmatrix} R_1 & R_{12} \\ R'_{12} & R_2 \end{bmatrix},$$

(13)

- The **within** correlation in $R_1$ and $R_2$ is uniform and equals to $\rho_1$ and $\rho_2$, respectively.
- The **between** correlation in $R_{12}$ is uniform and equals to $\rho_{12}$

### Special Cases

1. **Quasi-Autarky**: $\rho_1 = \rho_2 = 0.5$ while $\rho_{12} = 0$
2. **Full Integration**: $\rho_1 = \rho_2 = \rho_{12} = 0.25$. 
Let $S$ denote the network’s structure.

For each structure $S$, we simulate the network $R = 1000$ times.

We assess systemic risk using the following two metrics:

Let $F_{r}^{S}$ be the number of failed banks in the network in run $r$ and network structure $S$.

1. The average number of failed banks is

$$\bar{F}_{S} = \frac{\sum_{r=1}^{R} F_{r}^{S}}{R}$$  \hspace{1cm} (14)$$

2. The 99th percentile of the distribution of failed banks is

$$Q_{S,0.99} = \inf \left\{ Q \mid \frac{1}{R} \sum_{r=1}^{R} I \{ F_{r}^{S} \leq Q \} = 99\% \right\}$$  \hspace{1cm} (15)$$

Where $F_{r}^{S}$ is the number of failed banks in the network in run $r$ and network structure $S$. 
We define a shock of magnitude $k > 1$ on bank $i$ at time $t$ in the following manner:

$$x_{i,t} \rightarrow x_{i,t} - k \cdot |x_{i,t}|.$$  \hspace{1cm} (16)

Specifically, we set $k = 10$ and $t = 10$, while looking at the largest four banks in the network.
Simulation Study: Quasi-Autarky
Simulation Study: Full Integration
## Simulation Study: Quasi-Autarky versus Full Integration

<table>
<thead>
<tr>
<th></th>
<th>$k = 0$</th>
<th>1 bank when $k = 10$</th>
<th>2 banks when $k = 10$</th>
<th>4 banks when $k = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel (a)</td>
<td>Quasi Autarky</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{F}_S$</td>
<td>0.18</td>
<td>0.64</td>
<td>1.47</td>
<td>3.47</td>
</tr>
<tr>
<td>$Q_{S,0.99}$</td>
<td>4.01</td>
<td>5.00</td>
<td>5.00</td>
<td>7.00</td>
</tr>
<tr>
<td></td>
<td>Panel (b)</td>
<td>Full Integration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{F}_S$</td>
<td>0.13</td>
<td>0.56</td>
<td>1.53</td>
<td>3.52</td>
</tr>
<tr>
<td>$Q_{S,0.99}$</td>
<td>4.00</td>
<td>4.00</td>
<td>8.00</td>
<td>8.00</td>
</tr>
</tbody>
</table>

Simaan et al 2016: Interbank Network
Simulation Study: Sensitivity to Correlation I

![Graph showing the relationship between \( \rho_1 \) and average failure.](image1)

![Graph showing the relationship between \( \rho_{12} \) and average failure.](image2)

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Interbank Network
Simulation Study: Sensitivity to Correlation II

Figure: Average Failure

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Simulation Study: Sensitivity to Correlation III

Figure: 99th Percentile Failure

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**Simulation Study: Regression I**

**Dependent variable: average failure**

<table>
<thead>
<tr>
<th></th>
<th>no shock (1)</th>
<th>1 bank with $k = 10$ (2)</th>
<th>2 banks with $k = 10$ (3)</th>
<th>4 banks with $k = 10$ (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1 \cdot \rho_{12}$</td>
<td>0.876***</td>
<td>1.293***</td>
<td>1.154***</td>
<td>1.126***</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.091)</td>
<td>(0.069)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>$(\rho_1 \cdot \rho_{12})^2$</td>
<td>2.001***</td>
<td>2.698***</td>
<td>1.629***</td>
<td>1.345***</td>
</tr>
<tr>
<td></td>
<td>(0.198)</td>
<td>(0.293)</td>
<td>(0.260)</td>
<td>(0.233)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.204***</td>
<td>0.773***</td>
<td>1.611***</td>
<td>3.574***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.021)</td>
<td>(0.016)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Observations</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Adjusted R$^2$</td>
<td>0.652</td>
<td>0.672</td>
<td>0.735</td>
<td>0.775</td>
</tr>
</tbody>
</table>

**Note:** *p<0.1; **p<0.05; ***p<0.01
### Simulation Study: Regression II

**Dependent variable: 99th percentile**

<table>
<thead>
<tr>
<th></th>
<th>no shock</th>
<th>1 bank with $k = 10$</th>
<th>2 banks with $k = 10$</th>
<th>4 banks with $k = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1 \cdot \rho_{12}$</td>
<td>$9.997^{***}$</td>
<td>$14.047^{***}$</td>
<td>$7.735^{***}$</td>
<td>$9.350^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.725)$</td>
<td>$(1.045)$</td>
<td>$(0.667)$</td>
<td>$(0.754)$</td>
</tr>
<tr>
<td>$(\rho_1 \cdot \rho_{12})^2$</td>
<td>$12.888^{***}$</td>
<td>$9.355^{**}$</td>
<td>$1.620$</td>
<td>$19.714^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(2.935)$</td>
<td>$(4.528)$</td>
<td>$(2.951)$</td>
<td>$(2.673)$</td>
</tr>
<tr>
<td>Constant</td>
<td>$3.132^{***}$</td>
<td>$5.498^{***}$</td>
<td>$5.226^{***}$</td>
<td>$7.108^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.165)$</td>
<td>$(0.237)$</td>
<td>$(0.152)$</td>
<td>$(0.171)$</td>
</tr>
<tr>
<td>Observations</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.656</td>
<td>0.645</td>
<td>0.574</td>
<td>0.607</td>
</tr>
</tbody>
</table>

**Note:** *

$p < 0.1$; **$p < 0.05$; ***$p < 0.01$
We identify the interbank network using a statistical learning procedure that reverse engineers transactions in the overnight interbank market.

Given the structure of the network, we conduct a series of simulation studies for risk assessment.

Integration appears to be optimal when systemic risk is absent.

However, this evidence is undermined when systemically important institutions suffer liquidity shocks.
Thanks!

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