CoRisk: measuring systemic risk through default probability contagion

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SUMMARY

INTRODUCTION
Definitions and Essential Bibliography
Paper’s overview

METHODOLOGY
Stochastic Processes
Networks & PD
TPD & CoRisk

EMPirical FINDINGS
Data & Networks
PD, CoRisk & TPD
Comparisons

CONCLUSIONS
**SYSTEMIC RISK**

» De Bandt & P. Hartmann (2000):  
A systemic crisis can be defined as a systemic event that affects a considerable number of financial institutions or markets in a strong sense. [...] At the heart of the concept is the notion of *contagion*, a particularly strong propagation of failures from one institution, market or system to another.

» Hendricks (2009):  
Systemic risk is the risk of a phase transition from one equilibrium to another, much less optimal equilibrium, characterized by multiple self-reinforcing feedback mechanisms making it difficult to reverse.

» Benoit et al. (2015):  
Systemic risk is the risk that many market participants are simultaneously affected by severe losses, which then spread through the system.
1. Measures of systemic risk for the banking sector:
   ▶ Conditional probabilities of default, based on that of the others/system → Bivariate approach, \textbf{No contagion effects};

2. Network models:
   ▶ Billio et al. (2012), Barigozzi & Brownlees (2013), Diebold & Yilmaz (2014), Ahelegbey et al. (2015),
   ▶ Channels of contagion → \textbf{No predictions};

3. Econometric causal methods:
   ▶ Duffie et al. (2000), Lando & Nielsen (2010), Koopman et al. (2012), Betz et al. (2014),
   ▶ Conditional probabilities of default, based on exogenous factors → \textbf{No contagion effects};
Motivation & Objectives (1/3)

- Predictive capability;
- Endogeneity and non-linearity;
- Systematic & Systemic risk;
- Channels of contagion & PD.

CROSS-SECTIONAL DIMENSION + TIME DIMENSION

Multivariate Stochastic Processes

1. Sovereign Risk
2. Corporate Risk
3. Bank Risk

- Linear combination of an idiosyncratic and a systematic factor;
- Spread measure $Z_t$. 
Motivation & Objectives (2/3)

Institution-specific PD

Spread measure $Z_t \rightarrow$ PD of each economic sector in each country.

+ 

Partial Correlation Networks

Spread measure $Z_t \rightarrow$ partial correlation networks for each economic sector.

= 

CoRisk

- Change in the PD of each economic sector in each country due to contagion;
- "Total" PD;
- Aggregate PD at the country level.
Motivation & Objectives (3/3)

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Motivation & Objectives (3/3)

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### Stochastic Processes

#### Sectorial Spreads

- \( N \) countries, 1 economic sector;
- For each country \( i = 1, \ldots, N \), we define:

\[
Z^i_t = y^i_t - S_t, \\
\begin{aligned}
    dS_t &= (a - \nu S_{t-1}) dt + b \sqrt{S_{t-1}} dB_t, \\
    dy^i_t &= (\theta^i_1 - \theta^i_2 y^i_{t-1}) dt + \theta^i_3 \sqrt{y^i_{t-1}} dW_t,
\end{aligned}
\]  

\[
\begin{align}
    \text{Corr}[y^i_t, y^j_t] &= \rho^{ij}, \\
    \text{Corr}[S_t, y^i_t] &= \gamma^i. 
\end{align}
\]
**CORRELATED SECTORIAL SPREADS**

- $N$ countries ($V = \{1, ..., N\}$), 3 economic sectors ($W = \{1, 2, 3\}$);
- For the three sectors (1=sovereign, 2=corporate, 3=banking), for $i \in V$ and $\{m, n\} \in V \times W$ we define:

\begin{align}
Z_{t,1}^i &= y_{t,1}^i - S_t, \\
Z_{t,2}^i &= y_{t,2}^i - S_t, \\
Z_{t,3}^i &= y_{t,3}^i - S_t,
\end{align}

(3)

**Correlation Structure between Countries and Sectors**

\begin{align}
\text{Corr}[y_t^m; y_t^n] &= \rho^{mn}, \\
\text{Corr}[y_t^m; S_t] &= \gamma^m,
\end{align}

(4)
Covariance Matrix

From the correlation structure between countries and sectors we derive the covariance matrix, which can be decomposed as

\[ A = \Phi \cdot \Theta^T; \]  

\[ A^m = [\beta^m b \sqrt{S_0}, \beta^m, \alpha^m \sqrt{S_0 y^m b \theta^m \Gamma^m}, \alpha^m \sqrt{y_0^m \theta^m} \sqrt{[P]^m}], \]

\[ \Theta^T = \begin{bmatrix} \beta^n b \sqrt{S_0} \\ \alpha^n \sqrt{S_0 y_0^n b \theta^n \Gamma^n} \\ \beta^n \\ \alpha^n \sqrt{y_0^n \theta^n} \sqrt{[P]^n} \end{bmatrix}. \]
From the inverse of the covariance matrix \( A^{-1} \), elements \( \sigma^{mn} \), partial correlations can be derived:

\[
\rho_{mn|S} = \frac{-\sigma^{mn}}{\sqrt{\sigma^{mm} \sigma^{nn}}}. \tag{6}
\]

Correlations between two elements, conditional on the remaining elements of the system \( S \).

Undirected graph \( G = (P, E) \) based on partial correlations:
- \( P = V \times W = \{1, \ldots, 3N\} \) vertex set,
- \( E = P \times P \) edge set,
- An edge between two nodes \( m, n \) is present if and only if the corresponding partial correlation \( \rho_{mn|S} \) is significantly different from zero.
Institution-specific PD

- Risk-free context;
- Dynamic of debt = dynamic of risk-free debt;

\[
\begin{align*}
D_{t+1}^m &= (1 - PD_t^m)e^{y_t^m} D_t^m, \\
D_{t+1}^m &= e^{s_t} D_t^m.
\end{align*}
\] (7)

- Default probability of each economic sector in each country, based on the spread measure \( Z_t \):

\[
PD_t^m = 1 - e^{-Z_t^m}.
\] (8)
TPD & CoRisk

FROM PARTIAL CORRELATIONS TO TPD

- \{m, n\} \in (V \times W), S = (V \times W) \setminus \{m, n\};
- It can be demonstrated that \(|\rho_{mn}|_S| = |\rho_{nm}|_S| = \sqrt{a_{mn}|S| \cdot a_{nm}|S|}, \) where

\[
\begin{align*}
Z^m &= a_m + \sum_{n \neq m} a_{mn}|S| Z^n; \\
Z^n &= a_n + \sum_{m \neq n} a_{nm}|S| Z^m.
\end{align*}
\]

Objective

- Is the default probability of node \(m\) affected by contagion with neighbours \(n\)?
- \(PD^m = f(Z^m, t) \rightarrow TPD^m = f(PD^{n\neq m}, \rho_{mn}|S, t)\)

It can be demonstrated that:

TPD

\[
TPD_t^m = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn}|S} \tag{9}
\]
**From TPD to CoRisk**

\[ CoRisk_{in}^m, t = 1 - \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn}|S} \] (10)

\[ CoRisk_{in}^m = \frac{(1 - PD^m) - (1 - TPD^m)}{1 - PD^m} \cdot \]

- **CoRisk\_in** = change in the survival probability of an agent \( m \) when contagion deriving from its first-order neighbours is included.

- To what extent agent \( m \) affects its neighbours:

\[ CoRisk_{out}^m, t = 1 - \prod_{n \neq m} (1 - PD_t^n)^{\rho_{nm}|S} = 1 - (1 - PD_t^m)^{\sum_{n \neq m} \rho_{nm}|S} \] (11)

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CoRisk\textsubscript{in} OR VULNERABILITY: ILLUSTRATIVE EXAMPLE

1. \(\text{CoRisk}_{\text{in}} > 0, \ TPD > PD;\)

2. \(\text{CoRisk}_{\text{in}} < 0, \ TPD < PD;\)

3. \(\text{CoRisk}_{\text{in}} > 0, \ TPD > PD\) but lower than in the first example.
**CoRisk** \textsubscript{out} **OR SYSTEMIC IMPORTANCE: ILLUSTRATIVE EXAMPLE**

1. \( \text{CoRisk}_{out} > 0; \)
2. \( \text{CoRisk}_{out} < 0; \)
3. \( \text{CoRisk}_{out} > 0, \) but lower than in the first example.
TPD & CoRisk

Aggregated TPD

- TPD of each economic sector $\rightarrow$ aggregate TPD of each country;

Assumption

A country defaults if **at least one** of its economic sectors defaults.

- $TPD^m = $ conditional probability;
- $A^i_1, A^i_2, A^i_3 = $ sets of defaults;
- $S^i = \{A^m; \forall m \in V \times W, m \in ne(i, j), m \neq (i, j)\}$
- We are looking for $P(\bigcup_{j \in W} A^i_j | S^i)$

Aggregate TPD

$$TPD_{country,t}^i = 1 - [1 - TPD^i_{1,t}] \cdot [1 - TPD^i_{2,t}] \cdot [1 - TPD^i_{3,t}], \quad (12)$$
Data & Networks

Bonds, Loans & Deposits

Interest Rates on Bonds - 11 European countries from 2003 until 2015

Interest Rates on Loans to Corporates - 11 European countries from 2003 until 2015

Interest Rates on Deposits - 11 European countries from 2003 until 2015

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Data & Networks

Partial Correlation Networks
Partial Correlation Networks

1. SOVEREIGNS:
   - Financial crisis: number of significant correlations starts decreasing,
   - Sovereign crisis: a clustering effect starts emerging,
   - Post-crisis: two distinct clusters (Core vs Peripheral countries);

2. CORPORATES:
   - Financial crisis: number of significant correlations starts decreasing,
   - Post-crisis: few significant correlations, at the zero-lower-bound the systematic component is not significant;

3. BANKS:
   - Sovereign crisis: sparse network,
   - Post-crisis: a clustering effect starts emerging.
Institution-specific PD, CoRisk\textsubscript{in} & TPD
Institution-specific PD, CoRisk$_{in}$ & TPD

1. SOVEREIGNS:
   - PDs reflect interest rates on bonds,
   - $CoRisk_{in}$: high values for countries positively connected to peripheral ones (France, Belgium),
   - Loop effects: clusters $\rightarrow$ peripheral countries strongly affected not only by high PD, but also by contagion to each other $\rightarrow$ TPD becomes higher and higher;

2. CORPORATES:
   - PD less volatile across both countries and time,
   - $CoRisk_{in} > 0$ in all countries during the financial crisis,
   - $CoRisk_{in}$ prevailing effect;

3. BANKS:
   - PDs suffered only the financial crisis,
   - $CoRisk_{in}$ as expected until 2012: positive for core economies (contagion with peripheral countries) and low or negative for peripheral economies,
   - $CoRisk_{in}$ increases during the post-crisis period because of clustering effects.

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PD, CoRisk & TPD

Aggregated TPD

- Financial crisis: more homogenous impact across countries;
- Peculiarities: France (positive correlations with Italy) and Ireland (deep crisis in 2011 and strong reforms afterwards);
- Pre- vs Post- crisis years: stable and homogenous situation → high volatilities in all countries + two clusters;
- Persisting effect of the sovereign crisis.
## Systemic Risk Contributions

<table>
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<tr>
<th></th>
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</tr>
</thead>
<tbody>
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<td>Austria</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Finland</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>France</td>
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<td>0.8</td>
<td>1.0</td>
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<td>Germany</td>
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<td>Greece</td>
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<tr>
<td>Ireland</td>
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<td>Italy</td>
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<td>0.4</td>
<td>0.6</td>
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<tr>
<td>Portugal</td>
<td>0.2</td>
<td>0.4</td>
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<td>0.8</td>
</tr>
<tr>
<td>Spain</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
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</tr>
</tbody>
</table>

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PD, CoRisk & TPD

**Systemic Risk Contributions**

- **Sovereign contribution**: larger in peripheral countries;
- **Corporate contribution**: larger during "normal" times, depending on
  - institution-specific PD (peripheral countries),
  - contagion (core countries);
- **Bank contribution**: high bank CoRisk in core economies during the sovereign crisis, because of exposition to peripheral banks;
- **Sovereign & Bank contributions**: increased during the financial and sovereign crisis;
- **Distribution** of the TPD in its six components: quite homogenous before the financial crisis, but not back to normality in recent years:
  - contagion risk in core economies,
  - high institution-specific PD + clustering effects (loops) in peripheral economies.

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**CoRisk\textsubscript{in} vs CoRisk\textsubscript{out}**

*CoRisk\textsubscript{in}: vulnerability* \hspace{1cm} vs \hspace{1cm} *CoRisk\textsubscript{out}: systemic importance*

- Pre-crisis and financial crisis: *CoRisk\textsubscript{in} ~ CoRisk\textsubscript{out}* because PDs were homogenous across countries;
- Post-crisis: peripheral countries have highest *CoRisk\textsubscript{out}*. 

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Comparisons

**CoRisk** vs Degree of Connectivity and Eigenvector Centrality

- Comparison of results and rankings;
- Non-parametric Spearman correlation coefficient/test on rankings:

<table>
<thead>
<tr>
<th>Period</th>
<th>DC</th>
<th>Eigen.</th>
<th>DC</th>
<th>Eigen.</th>
<th>DC</th>
<th>Eigen.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003-2006</td>
<td>0.436</td>
<td>0.136</td>
<td>0.936</td>
<td>0.373</td>
<td>0.764</td>
<td>0.245</td>
</tr>
<tr>
<td>2007-2009</td>
<td>0.582</td>
<td>0.064</td>
<td>0.809</td>
<td>0.811</td>
<td>0.936</td>
<td>0.518</td>
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<tr>
<td>2010-2012</td>
<td>0.136</td>
<td>-0.736</td>
<td>0.691</td>
<td>0.655</td>
<td>0.982</td>
<td>0.345</td>
</tr>
<tr>
<td>2013-2015</td>
<td>0.736</td>
<td>0.018</td>
<td>0.927</td>
<td>0.245</td>
<td>0.982</td>
<td>0.573</td>
</tr>
</tbody>
</table>

- **CoRisk vs DC**: CoRisk weights each link considering partial correlation + PD of neighbours;
- **CoRisk vs Eigen.**: Eigenvector centrality considers the importance of each node according to its links to important nodes, without considering PD of neighbours → amplification of the distance between the two measures (especially during crisis years).
Comparisons

$\text{CoRisk}_{\text{in}}$ AND $\text{CoRisk}_{\text{out}}$ VS $\Delta \text{CoVar}$

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CONCLUSIONS

1. SOVEREIGN RISK:
   ▶ Larger in peripheral than in core countries,
   ▶ Core countries: mainly due to contagion (CoRisk),
   ▶ Peripheral countries: due both to contagion (loop effects) and institution-specific PD;

2. CORPORATE RISK:
   ▶ Most important risk during "normal" times,
   ▶ Core countries: mainly due to contagion (CoRisk),
   ▶ Peripheral countries: mainly due to institution-specific PD;

3. BANK RISK:
   ▶ High in all countries during the financial crisis,
   ▶ Core countries: mainly due to contagion (CoRisk) during the financial crisis (exposition to peripheral banks),
   ▶ Peripheral countries: due both to contagion (loop effects) and institution-specific PD.
Conclusion 1

Sovereign crisis (clustering → loop effects) greater than Financial crisis (homogenous across countries)

Post-crisis period: distribution of risk not homogenous in its six components

WHY?
Different reactions to the financial crisis: peripheral countries (high public debt), no fiscal space, imbalances emerged.

Conclusion 2

\( CoRisk_{in} \) vs \( CoRisk_{out} \)

- Peripheral countries: systemically importance prevails
- Core countries: vulnerability prevails