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Building a Risk Measurement Framework for Hedge Funds and Funds-of-Funds[*]

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In the first of two papers, we present a factor-decomposition based framework that facilitates non-parametric risk analysis for complex hedge fund portfolios in the absence of portfolio level transparency. Our approach has been designed specifically for use within the hedge funds-of-funds environment, but is equally relevant to those who seek to construct risk-managed portfolios of hedge funds under less than perfect underlying portfolio transparency. Using dynamic multivariate regression analysis coupled with a top-down qualitative understanding of hedge fund return drivers, we are able to perform a robust factor decomposition to attribute risk within any hedge fund portfolio with an identifiable strategy. Furthermore, through use of Bayesian-adjusted correlated Monte Carlo simulation techniques, these factors can be employed to generate implied risk profiles at either the constituent fund or aggregate funds-of-funds level. As well as being pertinent to risk forecasting and monitoring, such methods also have application to style analysis, profit attribution, portfolio stress testing and diversification studies. In this first paper we present the technical foundations of such a framework. The follow-up paper (Part II) will present detailed application of the concepts discussed in Part I to a broad base of hedge fund strategies and funds-of-funds.

Section I. Introduction

Hedge funds are a class of investment vehicle that aim to generate market independent returns by utilising a range of 'non-traditional' investment techniques and investing across a range of markets. Institutional and individual investors generally access hedge funds through hedge funds-of-funds, since they offer diversified access to a range of external, underlying hedge funds, which are selected and then monitored by the hedge funds-of-funds manager. For such a hedge funds-of-funds manager, risk monitoring of hedge funds can be difficult not least for the following reasons:

- Limited transparency – many hedge funds will only disclose partial information on their underlying portfolio.
- Liquidity and pricing risk – even when an investor gets full transparency, the portfolio risk is often difficult to measure as a result of the instruments within; for example the portfolio may contain OTC instruments with only indicative prices or difficult-to-value derivatives.

[*] A reduced version of this paper - Factor-Based, Non-Parametric Risk Measurement Framework for Hedge Funds and Funds-of-Funds – was presented at the Forecasting Financial Markets 2004 conference.

- Model risk – traditional VaR-style risk measures will often mis-estimate the risk in a hedge fund portfolio. Many such portfolios will be constructed by their managers to have carefully matched long and short positions in pairs or baskets as a hedge, and so will have low VaR. However, this may not reflect the risk in the future since often such positions are event-driven with a severe covariance structure altering event yet to come. Similarly, a manager may invest in such a position straight after the event, giving 'higher VaR, lower risk'.
- Complexity – hedge fund portfolios can be vast and complex meaning there is just not enough time to accurately measure the risk of many hedge fund portfolios over a month.

As we can see from the above, and more generally in Lo (2001), using traditional techniques to measure the risks associated with a hedge fund portfolio can be difficult and inaccurate. In this paper we aim to develop a framework that can be used independently or in conjunction with traditional techniques to generate an accurate measure of risk for hedge fund portfolios despite the problems outlined above.

Fung and Hsieh (1999) have shown that normal mean-variance risk measurement techniques do not work for hedge funds due to the dominance of non-normal returns. To avoid distribution-based pitfalls such as underestimation of tail risk, we have attempted to make the framework non-parametric and forward-looking wherever possible. Furthermore, to avoid the problem of portfolio transparency, we have assumed zero transparency and instead built a factor-based model, which has been enhanced by our qualitative choice of factors based on knowledge of the drivers of hedge fund returns for each strategy. Such a factor-based approach is also useful for hedge funds-of-funds risk measurement and portfolio construction.

Using factors to understand hedge fund returns and risk is not a new approach. Based on Sharpe's (1992) style analysis, Fung and Hsieh (1997) and more recently Agarwal and Naik (2000a, 2000b) have used a factor based approach to help understand how hedge funds generate returns, with Brealey and Kaplanis (2001) applying this approach specifically to understand hedge fund risk. Fung and Hsieh (2002) extended the factors used in such analysis to 'Asset Based Style' factors, which are better descriptors of what hedge fund managers do but, given the active nature of such hypothetical factors, are inappropriate for risk measurement in this context. As a result, a recent working paper by Fung and Hsieh (2004) looking at hedge fund risk using asset based style factors is of interest but not extendable for this style of risk measurement. Finally, the most pragmatic and active use to date of style factor analysis in hedge funds is by Amenc *et al* (2003), where this analysis is applied to tactical asset allocation for hedge funds.

This paper, therefore, will be the first to extend factor-based analysis of hedge fund returns to form a pragmatic hedge fund risk evaluation framework that, given the non-parametric forward looking nature, should better estimate tail risk. The paper continues with a discussion of methodology in Section II, followed by extended analysis techniques in Section III, which build on the central risk model framework.

Section II. Methodology

In this section we outline a prescriptive methodology necessary to analyse a hedge fund, or hedge funds-of-funds portfolio within a non-parametric factor-based framework. We begin by considering the criteria that a factor-set should satisfy. This is followed by a discussion of the regression and simulation techniques employed and the results they generate.

2.1 Selecting a factor set

Within a non-transparent reporting framework, a set of fundamental standardised factors can be adopted as a proxy for asset-specific risk analysis at the portfolio level. Although an inherent generalisation over the use of asset-specific data, a comprehensive factor set operating at an aggregate level still has the ability to preserve top-down thematic trends and biases in the portfolio, thus permitting conventional style analysis in the vein of off-the-peg applications such as Barra or APT. Furthermore, the use of a top-down factor basis set provides an essential mechanism for cross-strategy fund analysis at the hedge funds-of-funds level, simplifying a very complex task when viewed from a bottom-up perspective.

Factors are introduced to the set with no outlier screening in a standardised form (1), where r is the raw factor return series and σ_r is the standard deviation of the set such that r' has a unit standard deviation and a mean of zero.

$$r' = \frac{r - \bar{r}}{\sigma_r} \tag{1}$$

In constructing the factor set (universe), there should be enough breadth to robustly describe all potential underlying funds within the chosen strategy bucket, whilst minimising factor redundancy and overlap. For example, a market-independent funds-of-funds universe, constructed according to instrument, region and duration, would typically have to contain 50 factors to reach a suitable level of descriptivity. In most instances it is convenient to employ major indices due to the timeliness, scope and integrity of data, although in some instances proprietary data must be employed, in which case the ramifications of interrupted data supply must also be considered. By adopting a parent-child relationship, the subset of factors that describe any given fund will comprise a core strategy-specific component and additional, fund-specific, factors that add 'flavour' to the strategy. In particular, this strategy-level factor inheritance approach allows new (short track record) funds to be analysed with increased confidence. It is important to stress that at this stage allocating factor sets at a strategy/manager level is more art than process, and is the result of on-going research and discussion with portfolio managers. However, a benefit of the process is that a well-constructed factor basis set, although temporally dynamic, should vary adiabatically, thus yielding a convergent set that does not change significantly over time[†].

[†] Assuming no discontinuous changes to the investment mandate of the underlying fund.

Having identified the universe, the first quantitative screen assesses each factor return series $R_i = [\ r_i^1, r_i^2, \ldots , r_i^T]$ for the impact of highly correlated factors. Unlike principal component analysis, multivariate linear regression (used to determine factor loadings to the underlying fund) does not dictate that factors form a mutually orthogonal basis, in which case these factors cannot be treated independently. However, potentially more concerning is the ability for factors to pair-up, forming independent microsets with high temporal coherence, effectively becoming unconstrained during regression, leading to irrelevant factor loadings. Factor pairs affected in this way can be removed from fund-specific subsets by screening the absolute confidence-weighted correlation matrix with a ceiling tolerance ρ_{max} of about 0.8. Confidence weighting of correlation coefficients provides a robust method of accommodating varying sample sizes within the factor universe, thereby not penalising the shorter factor sets. Typically we find a fund can be robustly described using 5-8 factors, selected using a detailed qualitative understanding of the fund's mandate. Over-description of a fund will inherently improve the initial regression analysis, but will subsequently reduce the information content of the resulting factor decomposition as factors become less significant. Furthermore a large factor subset is more likely to encounter the orthogonality constraints discussed above during construction.

2.2 Establishing factor significance

Having screened out unconstrained microsets, the process of establishing a fund-specific factor subset is three-fold. Initially the factor set is linearly regressed against the fund with an exclusion threshold of $R^2 > 0.05$ set on the regression significance. Secondly, given that both correlation ρ and R^2 are highly sensitive to the length of the time series N, a statistical significance check is performed to calculate the likelihood of such a correlation occurring randomly. This test comprises an *f*-statistic coordinate transform of the form $|\rho| / \sqrt{(1-\rho^2)/\nu}$ where $\nu = N$-2 are the system degrees of freedom[‡], which is then evaluated under a cumulative *t*-distribution to obtain the probability p of a statistically insignificant factor correlation, Barlow (1996). Using this test the factor subset is screened with an acceptance threshold of $p < 0.05$. For computational convenience, this test can be carried out in a single step using an incomplete beta function of the form (2), where Γ is the associated gamma function, and t represents the above ρ-transform, Koepf (1998).

$$p(\rho) = \frac{\Gamma(1-\rho^2 + \frac{1}{2})}{\Gamma(1-\rho^2)\Gamma(\frac{1}{2})} \int_0^{1-\rho^2} \frac{t^{\frac{N}{2}-1}}{\sqrt{1-t}}\, dt \tag{2}$$

Finally, following linear regression, a significance check of the exposure-weighted aggregate factor subset level is undertaken before accepting the subset as the fund basis set. If the factors employed for regression accurately represent the drivers of the fund, then all systematic information will have been accounted for, necessarily implying that any residual component will result purely from random stochastic influences and will therefore be normally distributed. This residual content from the regression $\varepsilon(t)$ is evaluated using a Kolmogorov-Smirnov methodology for goodness of fit to a standardised normal distribution, Conover (1980). The null hypothesis is

[‡] Ignores presence of autocorrelation structure, which would act to reduce ν.

rejected above the 90% confidence interval. Having screened individual factors prior to regression, the rejection frequency of a well-constructed basis set is minimal. When factor basis sets are rejected it is invariably due to the lack of a significant period of manager returns, in which case recourse to the default core strategy factors can be made.

2.3 Calculation of factor loadings through multivariate linear regression

The loading (exposures) of the multidimensional factor subset is determined through conventional linear regression according to (3), where β represent the factor loading matrix comprising D factors. Note the use of two linear terms (regressed α and residual ε) permitting separation of systematic translation of the mean (manager skill or pure alpha) and the zero-sum stochastic fluctuations. The avoidance of non-linear terms in establishing the time-dependent factor loading matrix is an intentional step to minimise the degrees of freedom in the system, thus maximising the significance of the loadings with respect to shorter time series. The regression uses an optimum-period exponentially weighted moving average (EWMA) rolling window, described by (4), where the decay constant λ = 0.97 corresponds to the 1-month weighting parameter identified by Fleming *et al* (2001).

$$r(t) = \sum_{d=1}^{D} \beta_d(t) r_d(t) + \alpha_t + \varepsilon_t \tag{3}$$

$$[r_i^{\,t-T}, r_i^{\,t-T+1} \ldots r_i^{\,t}] \xrightarrow{EWMA} \sqrt{\frac{1-\lambda}{1-\lambda^{T+1}}} \begin{pmatrix} r_{i,t} \\ \sqrt{\lambda}\, r_{i,t-1} \\ \vdots \\ \sqrt{\lambda^T}\, r_{i,t-T} \end{pmatrix}' \tag{4}$$

As the factor-loading matrix and alpha are both time-dependent, the choice of regression window length is a critical degree of freedom; too long and short-term reverting trends are washed out (temporal dampening), too short and statistical significance is forsaken. By forming the empirical function $R^2(T)$; regression quality as a function of window length T, the optimum period will manifest itself as an inflection (shoulder) in the curve $R^2(T)$, identifiable as a local minimum in the first derivative, known as a Scree test, Cattell (1966). In the present study, a rolling regression period of 12 – 18 months was identified as optimal. In addition, the optimum window length was also observed to be strategy dependent, in agreement with the findings of Mina and Xiao (2001), who identified dependencies at regional and asset class levels for long-only data. Finally, care should be taken to ensure use of sample sets where the investment mandate remains contiguous to avoid the introduction of style-based factor discontinuities.

2.4 Non-Parametric Factor-Correlated Monte Carlo Simulation

Having established the matrix of dynamic factor exposures $\beta(d,t)$ and the systematic linear offset vector $\alpha(t)$, these parameters are subsequently employed in conjunction with conventional multivariate Monte Carlo techniques to determine an associated time-dependent risk profile. Given the factor basis is non-orthogonal, a covariance structure exists, preventing independent variate sampling. To accommodate this inherent covariance structure, simulated return distributions for the factor subset are generated using Cholesky factorisation, Higham and Cheng (1998). This technique decomposes the time-dependent historic factor covariance matrix Σ_t into a triangular matrix A such that (5) is satisfied, allowing co-dependent factor return vectors $R_D(t)$ to be generated under conventional Monte Carlo simulation conditions according to (6), where φ_D is a vector of standardised, normally distributed random returns. Cholesky factorisation necessitates that Σ_t be Hermitian.

$$A^T A = \Sigma_t \tag{5}$$

$$R_D(t) = \varphi_D(t) A \tag{6}$$

Generation of co-dependent simulated return vectors (typically 10^5 samples) using such a technique permits faithful replication of the historic factor covariance matrix whilst allowing for the existence of extreme aggregate factor returns that have yet to be observed historically, i.e. the incorporation of extra-historic rare events. Cross-factor frequency analysis of these extra-historic tails suggests that these events typically account for around 2% of simulated returns.

In order to escape the undesirable assumption of normality implicit in the Monte Carlo simulated factor return matrix generated using Cholesky factorisation, a two-step probability reflection transformation is employed. This process allows probability intervals, corresponding to the generated returns for each of the D factors, to be mapped into a common, time-dependent, probability density space $P(r,t)$ from which they are subsequently transposed back through a perturbed form of the set of historic factor distributions which include the extra-historic factor events identified under the Cholesky decomposition back into return-space. Given that φ_D, and therefore the simulated returns R_D are normally distributed, the simulated return matrix $R_D(t)$ can be associated with the probability interval matrix $P_D(r,t)$ through a CDF (cumulative density function) transformation using a continuous standardised normal distribution $N(r)$ as shown in (7).

$$P_D(r,\tau) = \int_{-\infty}^{r} N(r,t)dr\Big|_{t=\tau} \tag{7}$$

The reverse phase of this transformation requires $P_D(r)$ for each time t to be mapped back into return space, thus yielding a set of distribution-aware (non-normal) correlated returns. Were the correlated factor probability interval matrix $P_D(t)$ reflected directly back via the historic factor cumulative density function, additional tail width introduced by the Monte Carlo simulation into the factor returns distribution would be lost and the simulated factor returns would just represent a subset of the historic distribution of the period t-T to t. However, by resampling the simulated factor return matrix with a sampling interval N/T, historic and simulated factor return vectors become equivalent, permitting simulated events outside the scope of historic returns to be concatenated into the historic series forming a perturbed historic factor return matrix R'_D. The perturbed historic returns distribution for each of the D factors are converted into a cumulative density function using an empirical

Kaplan-Meier approach, Cox and Oakes (1984), thus preserving the vital characteristic functionality and moments of the historic factor distributions whilst accounting for co-dependent events outside historic scope. Cumulative density functions $C(r)$ are related to probability density functions $P(r)$ through the first derivative dC(r)/dr.

An additional complication of the reverse transformation phase is that, given that the empirical cumulative density functions are discrete, reflection about $C_D(r)$ need not necessarily yield a unique solution. In order to force a closed form, a clamped local cubic spline interpolant has been employed. Clamping refers to the prescription of known boundary conditions for the first derivative of the spline (specifically the termination values, which will be zero in this instance), thus avoiding the erratic flailing-rope scenario associated with open-ended splines. The final consideration of this non-parametric, factor-driven Monte Carlo simulation is convergence. Under a normal assumption, convergence increases with N as the standard error drop like $1/\sqrt{N}$, therefore creating a trade-off between convergence tolerance and calculation time. In this instance $N = 10^5$ provides convergence to better than 99.5% in a suitable time frame.

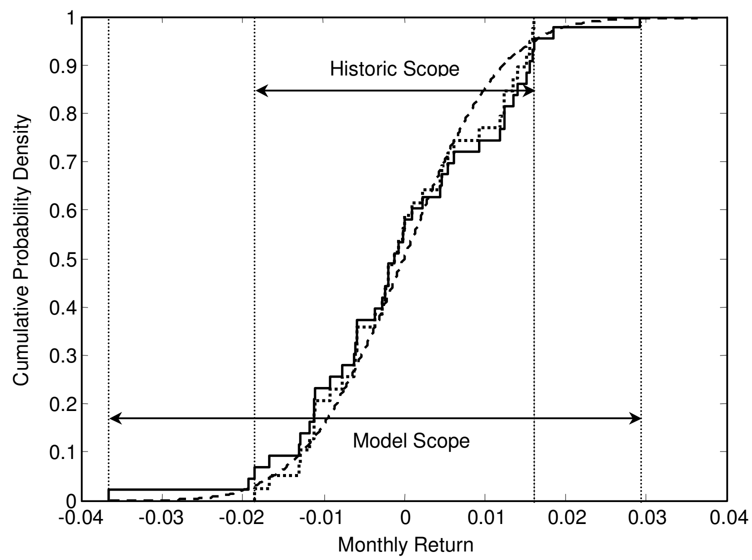


Figure 1: Typical monthly cumulative distribution functions constructed using; factor-driven non-parametric approach (solid), historic distribution (dotted) and a best-fit normal distribution (dashed), highlighting the impact of using historic data or placing an assumption of normality on the system.

Figure 1 illustrates the benefits of the present method by comparing typical cumulative distribution functions for the factor-driven non-parametric approach (solid curve), the equivalent historic distribution (dotted curve), and a best-fit normal distribution (dashed curve). The first derivative of the factor-driven non-parametric cdf yields the monthly implied risk distribution for the simulated system. As shown in figure 2, this implied risk distribution has significant contribution from non-static higher moments that importantly, rather being forced onto the problem as initial conditions, actually reveal themselves as solutions of the system. In some instances multi-model structure are also clearly evident.

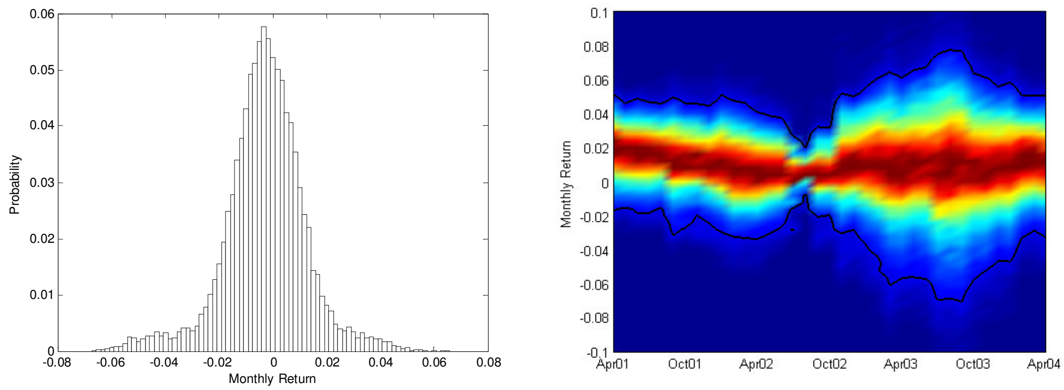


Figure 2: (Left) Typical return profile generated by the factor-driven non-parametric model using a six factor basis set and a Monte Carlo sample size of 5×10^5 events. (Right) Colour-map illustrating the temporal evolution of the implied risk profile, with red indicating regions of high probability density.

In summary, this factor-based Monte Carlo technique can be considered to provide a Bayesian adjustment to a historic return distribution in light of information elucidated through the use of the factor basis set to proxy the return time series. This allows for presence of additional tail risk beyond historic limits, and therefore the Bayesian-adjusted implied risk distribution is seen to provide a more appropriate gauge of potential volatility when compared against historic factor analysis. Furthermore, construction within an entirely non-parametric framework places absolutely no assumption on the normality of the underlying factors - again a significant source of tail-risk underestimation, Sortino and Satchell (2001). This technique also has distinct advantages over traditional single-series bootstrap methods, given that use of factor basis to describe a track record provides an extended-data set permitting out-of-sample simulation, Sharma (1996).

Section III. Extended Analysis Techniques

Having discussed the foundations for obtaining a non-parametric, factor-based implied risk distribution in Section, this section will examine, in some detail, a number of techniques employed to further extract pertinent information from the implied risk distribution. In addition to these methods, the availability of factor-based implied risk distributions for each underlying hedge fund allows any statistical test or performance measure to be empirically calculated, e.g. value-at-risk (VaR), extreme tail loss (ETL), Sortino ratio and Omega to name but a few. For a more comprehensive review of such indicators see, for example, Sortino and Satchell (2001).

3.1 Funds-of-Funds Aggregate Risk

Although factor-based risk profiles for funds-of-funds can be extracted in an identical fashion to the constituent hedge funds outlined above, it is significantly more informative to consider the funds-of-funds in its true form; as a composite structure of F underlying funds. In this case, the fund-of-fund aggregate factor exposure vector β_{FOF}, and factor-independent returns α_{FOF}, rather than being directly extracted through regression, will be given by the portfolio weighted sum of the component fund regression parameters, as shown in (8), where $w_i(t)$ is the weight of the i'th fund at time t. In this way, the composite funds-of-funds factor exposure matrix becomes unconstrained (in the absence of regression), permitting the entire universe of factors to have a presence, regardless of previous orthogonality constraints.

$$\begin{aligned} \beta_{FOF}(t) &= \sum_{i=1}^{F} w_i(t)\beta_D^i(t) \\ \alpha_{FOF}(t) &= \sum_{i=1}^{F} w_i(t)\alpha_i(t) \end{aligned} \tag{8}$$

3.2 Common Factor Risk Attribution

In this analysis, we assume that the origin of risk is two-fold; namely quantifiable common risk originating from factor exposure (factor risk) and specific risk, unique to each constituent fund. In this instance, we take specific risk as the standard deviation of the residual component of the multivariate regression ε, which by definition is normally distributed, yielding σ_ε. Given that specific risk is independent, the specific risk for set of n funds that together comprise the funds-of-funds is given by the diagonal specific volatility matrix (9).

$$\Lambda = \begin{pmatrix} \left(\sigma_\varepsilon^1\right)^2 & & 0 \\ & \ddots & \\ 0 & & \left(\sigma_\varepsilon^n\right)^2 \end{pmatrix} \tag{9}$$

Given a vector of constituent fund weights ω such that $\sum_i \omega \le 1$ (§), the weighted vector product $\omega^T\Lambda\omega$ yields the total specific risk in the funds-of-funds. Similarly, the factor risk matrix $\mathrm{B}^T\Sigma\mathrm{B}$ can be constructed from the rolling covariance matrix Σ_t , and B, the D by n matrix of constituent fund factor exposures (10). The aggregate funds-of-funds volatility is therefore given by (11), with the fraction of risk explained through factor exposure alone Ω_F, i.e. common block risk as oppose to specific risk, given by (12).

$$\mathrm{B} = \begin{pmatrix} \beta_1^1 & \cdots & \beta_1^n \\ \vdots & \ddots & \vdots \\ \beta_D^1 & \cdots & \beta_D^n \end{pmatrix} \tag{10}$$

$$\sigma_F{}^2 = \omega^T \mathrm{B}^T \Sigma \mathrm{B}\,\omega + \omega^T \Lambda \omega \tag{11}$$

§ If the funds-of-funds is not fully invested we necessarily exclude the risk-free cash component from the marginal contribution analysis.

$$\Omega_F = \frac{\omega^T B^T \Sigma B \omega}{\left(\omega^T B^T \Sigma B \omega + \omega^T \Lambda \omega\right)} \tag{12}$$

Observations of time-dependent common block risk fractions suggest that factor risk typically accounts for around 90% of total implied risk within a funds-of-funds. Additionally, increased Ω fractions have been observed for trading-style fund composites, highlighting the greater dependence these strategies place in pure factor trends to generate trading ideas in comparison to, say, a more research driven equity hedge manager.

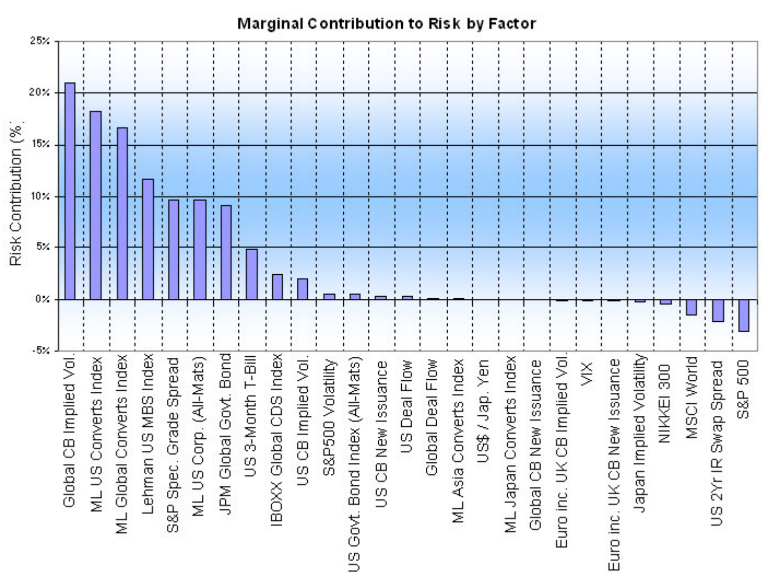
3.3 Factor Marginal Contribution to Total Risk

Component factor contributions to the funds-of-funds risk (see figure 3) can be recovered by partially differentiating the square root of (11) with respect to the weighted factor exposure matrix Bω, as shown in (13)[**], and multiplying in the weighted factor exposure vector $B\omega$, to give the vector of factor marginal contributions to risk σ_n, such that the sum of the marginal contributions equates to the common factor risk component $\Omega_F \sigma_F$.

$$\frac{\partial \sigma_F}{\partial B\omega} = \frac{\Sigma B \omega}{\sqrt{\omega^T B^T \Sigma B \omega}} \tag{13}$$

The fractional contribution of n^{th} factor can be found simply by renormalizing σ_n according to (14).

$$n^{\text{th}} \text{ Factor Risk Fraction } = \frac{1}{\sqrt{\omega^T B^T \Sigma B \omega}} \left(B_n \omega_n \left. \frac{\partial \sigma_F}{\partial B\omega} \right|_n \right) \tag{14}$$



[**] Note that we exclude the constant specific risk term in this instance as we only wish to consider common factor risk attribution, not total risk attribution.

Figure 3: Factor marginal contributions to risk, expressed as a fraction of total common factor risk. Negative contributions indicate diversifying factors.

3.4 Fund Marginal Contribution to Total Risk

Having considered factor attribution to risk, we now consider the other significant decomposition axis; risk attribution by constituent fund. Following an identical approach to the factor risk decomposition, the vector of fund marginal contributions to risk is given by the product of the partial derivative of the square root of (11) with respect to the portfolio weights vector ω (15), and ω itself. As with factor marginal contributions, fractional contribution of each constituent hedge fund can be evaluated by normalising with respect to total common factor risk, as shown in (16).

$$\frac{\partial \sigma_F}{\partial \omega} = \frac{\mathrm{B}\Sigma\mathrm{B}\,\omega}{\sqrt{\omega^T \mathrm{B}^T \Sigma\mathrm{B}\,\omega}} \tag{15}$$

$$f^{\text{th}} \text{ Fund Fractional Contribution } = \frac{1}{\sqrt{\omega^T \mathrm{B}^T \Sigma\mathrm{B}\,\omega}} \left(\mathrm{B}^f \omega_f \left.\frac{\partial \sigma_F}{\partial \omega}\right|_f \right) \tag{16}$$

As with the factor marginal contributions, diversification is also evident within common factor risk at a constituent fund level [††], although the degree of diversification is substantially reduced as a result of common investment themes permeating through different strategies and therefore the underlying hedge funds. Figure 4 illustrates this fund level risk attribution of a portfolio of 19 hedge funds H1 through H19.

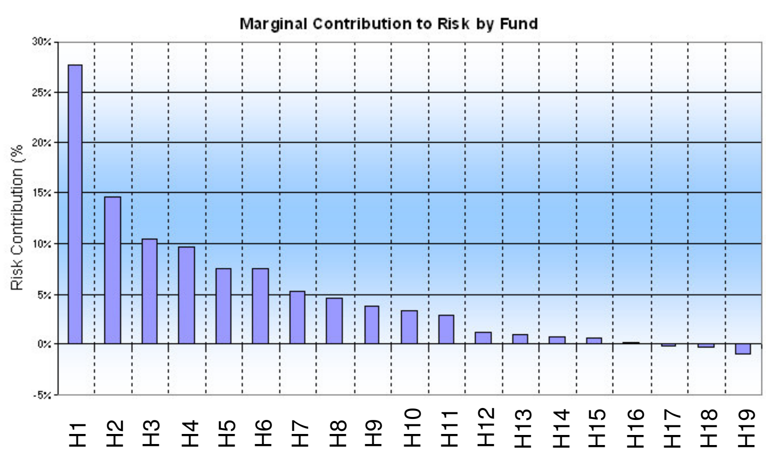


Figure 4: Funds-of-funds implied risk attribution at the constituent fund level. Note the reduction of diversification compared to factor level attribution, resulting from the reduced mutual orthogonality of this more complex set.

[††] Note specific risk will always positively contribute towards total risk for all constituent funds, even if diversification is observed in common factor risk.

3.5 Factor Timing

In addition absolute or realised performance measures, another measure of manager skill is return sourced from market timing, or how factor exposures have capitalised on underlying the factor dynamics.

Factor timing performance attribution $r_{MT}(t)$ is calculated from the first derivative of the set of n factor exposures vectors with respect to time d$\beta(t)$/dt and $R_D(t,n)$, the underlying set of factor returns at time t, according to (17).

$$r_{MT}(t) = X(t) \cdot \frac{d\beta(t)}{dt} \tag{17}$$

In essence this is simply the difference in performance obtained by a discrete change in factor exposure at time t, rather than propagating an unaltered factor exposure vector from time t-1. Importantly, this calculation only accounts for the relative change in exposure, and not the absolute level of exposure. In this analysis, the level of absolute exposure is regarded as strategic in origin, and therefore should not contribute to exposure timing returns. Once again, observation shows, as one would expect, market timing sourced returns are more prevalent in short-term equity trading-style and fixed income funds than the majority of other strategies.

Summary

In summary, we have shown how combining conventional multivariate linear regression with non-parametric Monte Carlo simulation within a factor basis framework provides access to both time-dependent factor exposures and implied risk profiles, facilitating active style analysis in addition to a wealth of risk analysis beyond the conventional, and often misleading, standard deviation multiple based value-at-risk measures. Furthermore, it has been shown that this framework can been employed as a foundation for advanced portfolio risk decomposition and attribution in a funds-of-funds scenario. Further work is in progress and should yield prescriptive information on building diversified hedge fund portfolios and managing their risk.

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