Abstract

The Cambridge Banking Model is a stress-test framework to monitor systemic risk in financial systems. The framework simulates the propagation of losses throughout the system using different contagion channels. It integrates various network measures and combines them with conventional risk measures such as VaR. The current version uses data from Bankscope to reconstruct random interbank networks quarterly from 2003 - 2014 and applies predefined shocks to each of them. Observation of various losses gives insights in the state of the global banking system and it's evolution over time.

Network based Macroprudential stress testing Framework

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Centre for **Risk Studies**

UNIVERSITY OF CAMBRIDGE Judge Business School

Disclaimer

Results represented in this presentation use a methodology developed by Battiston, D'Errico et. al, applied to a dataset assembled by Cambridge Centre for Risk Studies:

Battiston, Stefano and Caldarelli, Guido and D'Errico, Marco and Gurciullo, Stefano, *Leveraging the Network: A Stress-Test Framework Based on DebtRank* (February 22, 2016). Available at SSRN: http://ssrn.com/abstract=2571218 or http://dx.doi.org/10.2139/ssrn.2571218

Introduction - Cambridge Banking Model

We apply the methodology developed by Battiston et al. (2015) to a new data set and stress-test the global financial system.

Existing Models

- empirical work to study systemic risk in a single country or region (EU) between banks (bank to bank)
- empirical work to study systemic risk between countries (country to country)

Why this Model is different?

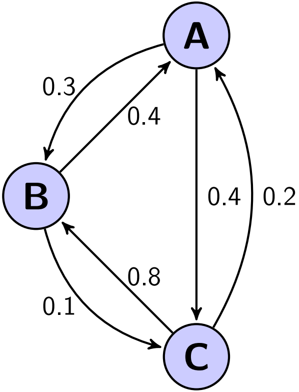
- this frameworks incorporates both: inter country and inter bank relationships on a bank to bank level (36 countries)
- bottom-up model: piecing together of systems to give rise to more complex systems
- realistic IB network: structure is defined by real (known) constraints, models are pretty good.

Overview

Financial Network Model

n institutions (banks)

Banks can invest in $n-1$ institutions



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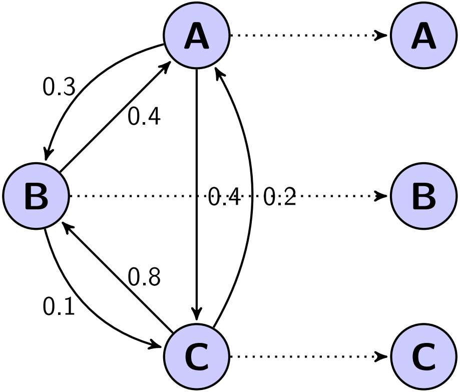
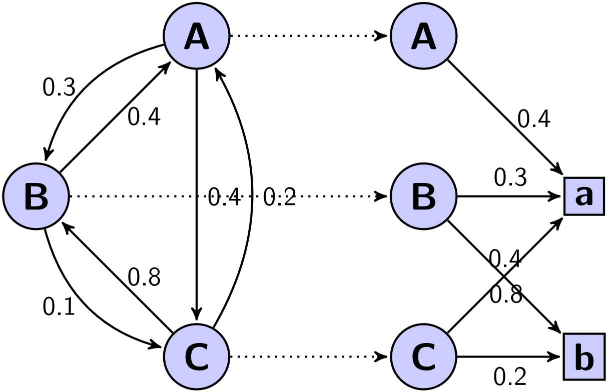


Financial Network Model

n institutions (banks)

m external assets

Banks can invest in $n-1$ institutions or m assets.



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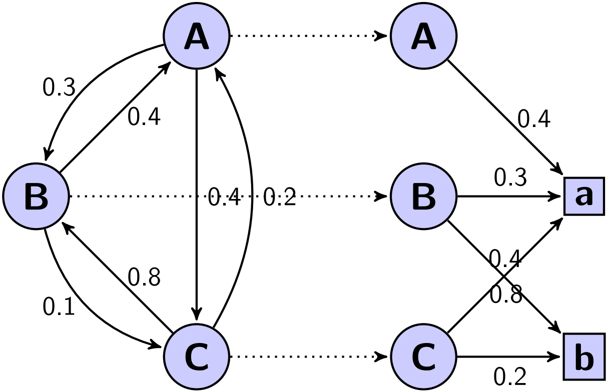

Assets (liabilities) can be external or inter-bank, with totals as

$$A_i^e = \sum_{k=1}^{m} A_{ik}^e \text{ and } A_i^b = \sum_{j=1}^{n} A_{ij}^b$$

Balance Sheets

State Variables

- $E_i(t)$ equity of institution i at time t
- $A_i(t)$ total assets of institution i at time t
- $D_i(t)$ total liabilities of institution i at time t
- A_{ij}^b amount institution i lends to institution j
- A_{ik}^e amount institution i invests in asset k
- $l_i(t)$ leverage of institution i at time t

Assets	**Liabilities**
$A^e = 0.4$	$D^b = 0.6$
$A^b = 0.7$	E

Table: Balance Sheet of Bank A

The balance sheet is defined as

$$A_i^e(t) + A_i^b(t) = A_i(t) = D_i(t) + E_i(t)$$

Leverage of a bank is the ratio of assets and equity

$$l_i(t) = \frac{A_i(t)}{E_i(t)}$$

.

Balance Sheets (cont.)

Financial System

$l_i(t)$ leverage of institution i at time t

$l_{ik}^e(t)$ external leverage of institution i with respect to asset k at time t

$l_{ij}^b(t)$ inter-bank leverage of institution i towards institution j at time t

$l_i^e(t)$ total external leverage of institution i at time t

$l_i^b(t)$ total inter-bank leverage of institution i at time t

Leverage (disaggregated) of a bank is the sum of it's external and inter-bank leverage.

$$l_i(t) = \frac{A_i^e(t)}{E_i(t)} + \frac{A_i^b(t)}{E_i(t)}$$
$$= l_{ik}^e(t) + l_{ij}^b(t)$$

l_{ik}^e can be seen as elements of the adjacency matrix of an bi-partite external leverage network and l_{ij}^b of a mono-partite interbank leverage network. The totals would be the sum along the columns:

$$l_i^e = \sum_{k=1}^{m} l_{ik}^e \text{ and } l_i^b = \sum_{j=1}^{n} l_{ij}^b$$

Loss in Equity Suffered

Distress or Vulnerability

$h_i(t)$ cumulative relative equity loss of institution i at time t

$H(t)$ cumulative relative equity loss of of the financial system at time t

losses of banks relative to it's equity and with respect to a baseline at $t = 0$:

$$h_i(t) = \min\{1, \frac{E_i(0) - E_i(t)}{E_i(0)}\}$$

with bank under distress for $h_i(t) \in (0, 1] \forall t$ and default for $h_i(t) = 1$.

losses of the financial system relative to total equity and with respect to a baseline at $t = 0$ is the weighted average cumulative relative equity loss of each bank:

$$\begin{aligned} H(t) &= \sum_{i=1}^{n} w_i h_i \\ &= \sum_{i=1}^{n} \frac{E_i(0)}{\sum_{j=1}^{n} E_j(0)} h_i \end{aligned}$$

Loss in Equity Induced to the System

Impact

DR_i global relative equity loss induced by the default of institution i

DebtRank DR_i is the impact induced by the default of each bank individually on the system:

$$DR_k(t) = \sum_{i=1}^{n} h_i(T) E_i(0)$$

.

This is the exact solution for systemic risk as defined in BCBS (2013)

Generalised Framework[1]

set of set of banks that have not defaulted up to time t

$$\mathcal{A}(t) = \{j : E_j(t) > 0\}$$

balance sheet identity for bank i at time t

$$E_i(t) = A_i^E(t) - L_i^E(t) + \sum_{j \in \mathcal{A}(t-1)} A_{ij}^B(t) - \sum_{j=1}^{N} L_{ij}^B(t)$$

where mark-to-market valuation for A_{ij}^B and face value for L_{ij}^B,
information about the default of other banks is received by bank i with a delay
mechanism for shock propagation from borrowers to lenders

$$A_{ij}(t+1) = \begin{cases} A_{ij} \frac{E_j(t)}{E_j(t-1)}, & \text{if } j \in \mathcal{A}(t-1) \\ 0, & \text{if } j \notin \mathcal{A}(t-1) \end{cases}$$

relative changes in the equity of borrowers are reflected in equal relative changes of interbank assets of lenders at the next time-step

[1] Bardoscia et al. (2015)

Generalised Framework (cont.)

relative loss of equity of bank i

$$h_i(t) = \frac{E_i(0) - E_i(t)}{E_i(0)}$$

contagion dynamics in terms of relative loss

$$h_i(t+1) = \min\left[1, h_i(t) + \sum_{j-1}^{N} \Lambda_{ij}(t)\left[h_j(t) - h_j(t-1)\right]\right]$$

with interbank leverage matrix Λ

$$\Lambda_{ij}(t) = \begin{cases} \frac{A_{ij}(0)}{E_j(0)}, & \text{if } j \in \mathcal{A}(t-1) \\ 0, & \text{if } j \notin \mathcal{A}(t-1) \end{cases}$$

Generalised DebtRank: as long as banks receive shocks, it will keep propagating them.

Generalised Framework (cont.)

properties of interbank leverage matrix Λ[2]:

$|\lambda_{max}| < 1$ fixed point at

$$\Delta h(t) = h(t) - h(t-1) = 0$$

$|\lambda_{max}| > 1$ shock amplification (at least one bank will default)

where $|\lambda_{max}|$ is the largest eigenvalue of $\Lambda(t)$

[2] although the final losses depend on the shock size, the stability of the system does not. It is a property of Λ.

Distress Propagation Circle

Asset Losses

negative shock on the value of assets causes losses in banks, which is absorbed by equity.

Inter-Bank Losses

Inter-Bank Losses: distress from asset losses puts inter bank obligations under pressure. Those losses are again absorbed by equity.

Fire Sale

banks need to adjust their leverage to meet regulatory requirements by selling assets. The price impact leads to further pressure on asset prices. This closes the virtuous circle.
[3]

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[3] Battiston et al. (2015)

Asset Losses

Price Shock

$p_k(t)$ unit price of asset k at time t

$r_k(t)$ relative price (shock) of asset k at time t

a shock

$$r_k(1) = \frac{p_k(0) - p_k(1)}{p_k(1)} < 0$$

on the value of asset k reduces the value of the investment in external assets in bank i by

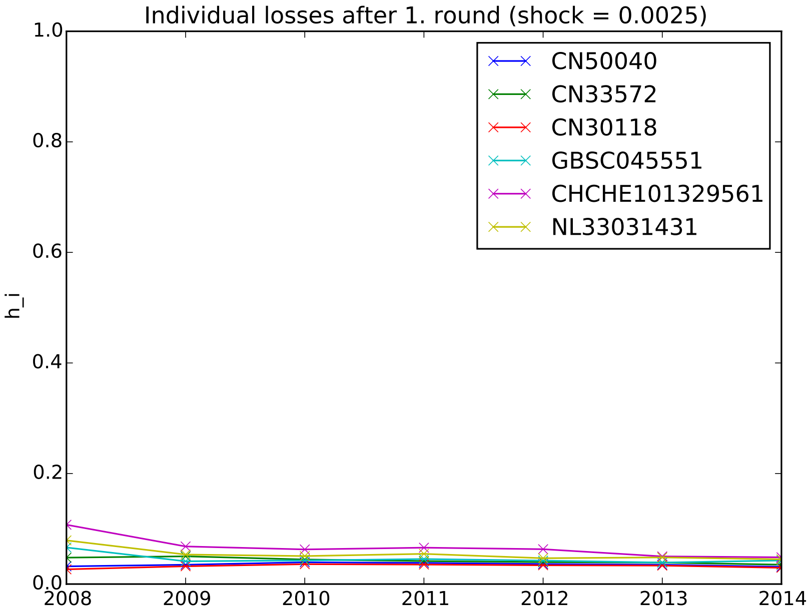
$$\sum_k r_k(1) A_{ik} = \sum_k r_k(1) l_{ik} E_i = E_i \sum_k r_k(1) l_{ik}$$

the loss needs to be compensated by reduction in equity

$$A^e_{ik}(0) - A^e_{ik}(1) = \sum_k r_k(1) A^e_{ik}(0) = E_i(0) - E_i(1)$$

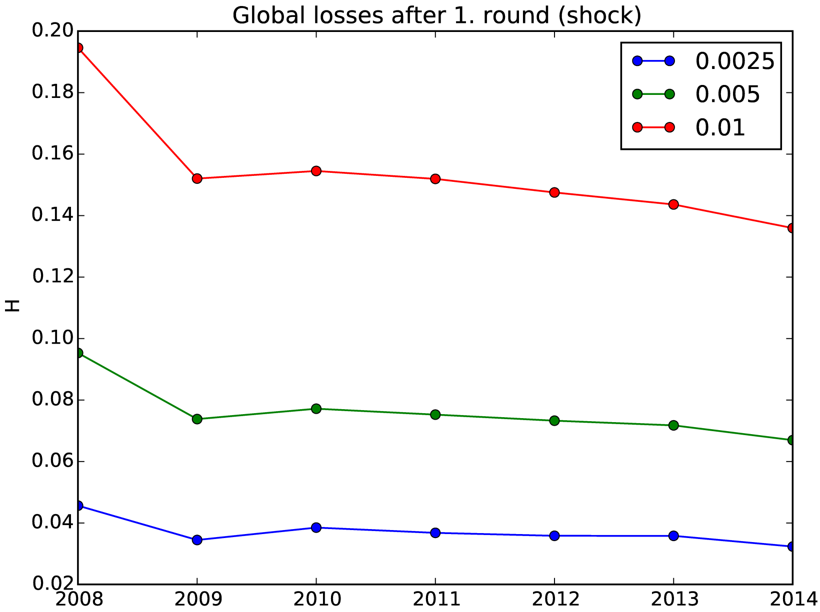
individual and global relative equity loss at time $t = 1$ are:

$$h_i(1) = \min\{1, \sum_k l_{ik} r_k(1)\} \text{ and } H(1) = \sum_{i=1}^{n} w_i h_i(1)$$



[5]

[5] compare Battiston et al. (2015)



[6]

[6] compare Battiston et al. (2015)

Inter-Bank Losses

Distress Propagation

$V_t(A_{ij})$ market to market value of A_{ij}

The distress that propagates from j into each of the lenders i is the relative loss with respect to the original face value

$$\frac{A_{ij} - V_t(A_{ij})}{A_{ij}} = f(h_j(t-1)).$$

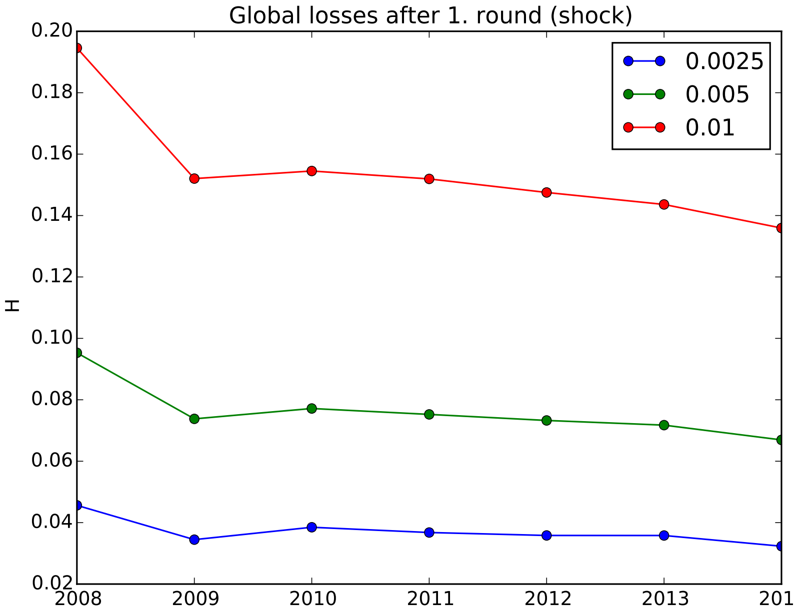
individual relative loss in equity:

$$h_i(t) = \frac{E_i(t) - E_i(0)}{E_i(0)} = \min\left\{1, \sum_{i \in S_A(t)} l_{ij} f(h_i(t-1))\right\}$$

$$= \left(l_i^e + \sum_j l_{ij}^b l_j^e\right) r(1)$$

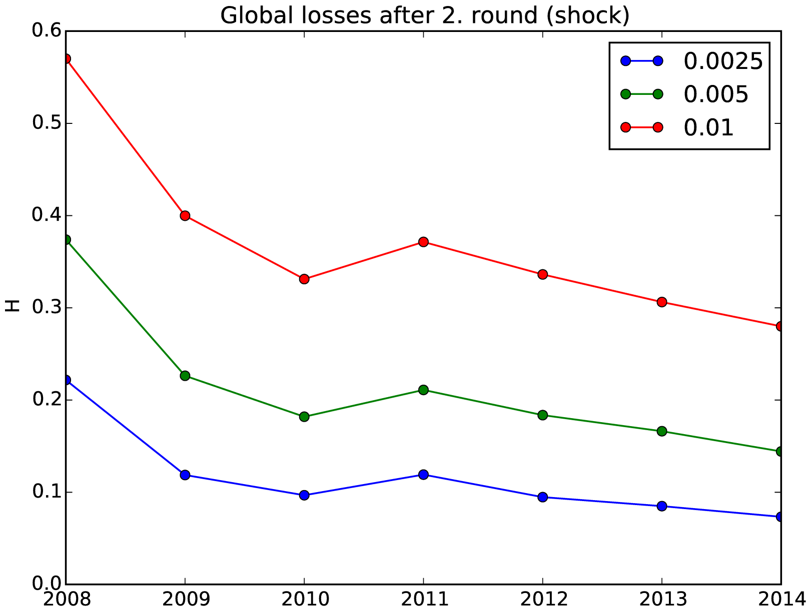
where $S_A(t)$ is the set of active[7] nodes.

[7] nodes that transmit distress at time t, as in Battiston et al. (2012)



[8]

[8]compare Battiston et al. (2015)



[9]

[9] compare Battiston et al. (2015)

Fire Sale

Price Impact

- Q_i quantity of assets of bank i
- $\hat{p}$ shock price
- η price impact factor

Banks try to sell external assets in order to repay obligations to move to the original leverage:

$$l_i(0) = l_i(t) = \frac{A_i^e(t) + A_i^b(t)}{E_i(t)} = \frac{(Q_i(0) + \Delta Q)\hat{p} + A_i^b(t)}{E_i(t)}$$

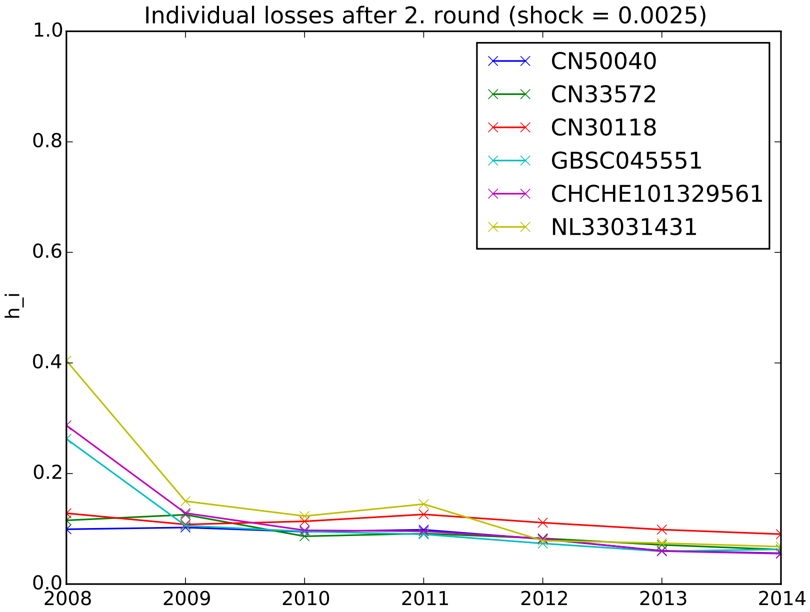
price impact[10] is linear (proportional to relative change in demand):

$$r(t) = \eta \frac{\Delta Q_i}{Q_i(0)} = \eta \frac{D_i(0)}{Q_i(0)\hat{p}} (l_i^e)^2 r(1)$$

relative loss in equity:

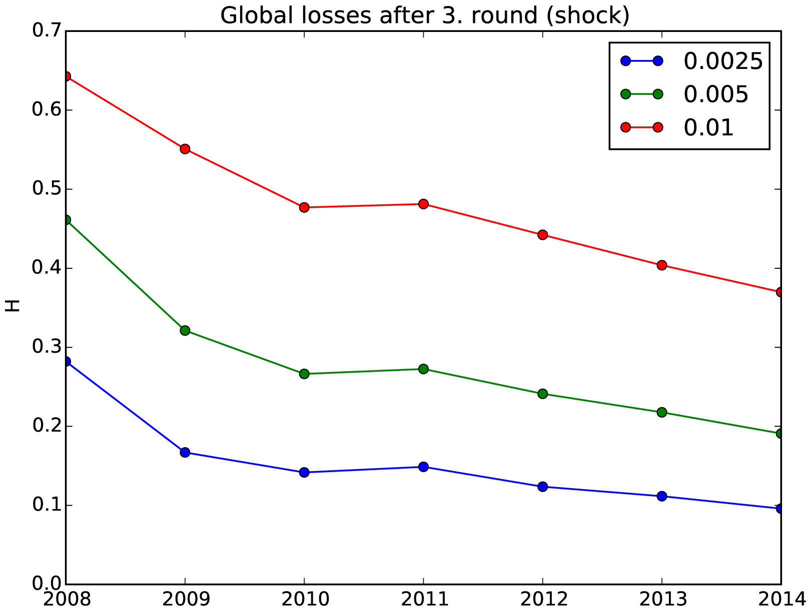
$$gh_i(t) = \frac{E_i(t) - E_i(0)}{E_i(0)} = \left(l_i^e + \sum_j l_{ij}^b l_j^e \right) r(1) + \eta \frac{D_i(0)}{Q_i(0)\hat{p}} (l_i^e)^2 r(1)$$

[10] Battiston et al. (2015)



[11]

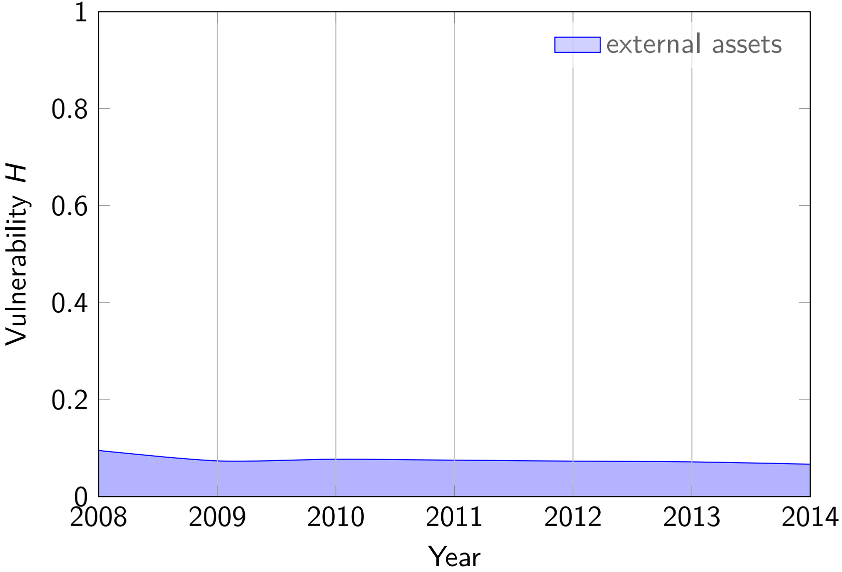
[11] compare Battiston et al. (2015)



[12]

[12] compare Battiston et al. (2015)

Stress Test Results (shock = 0.005)



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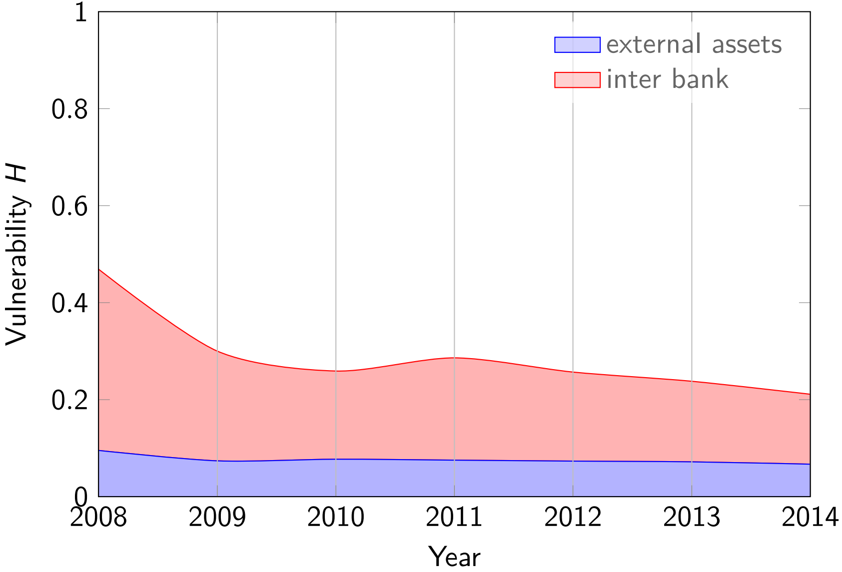

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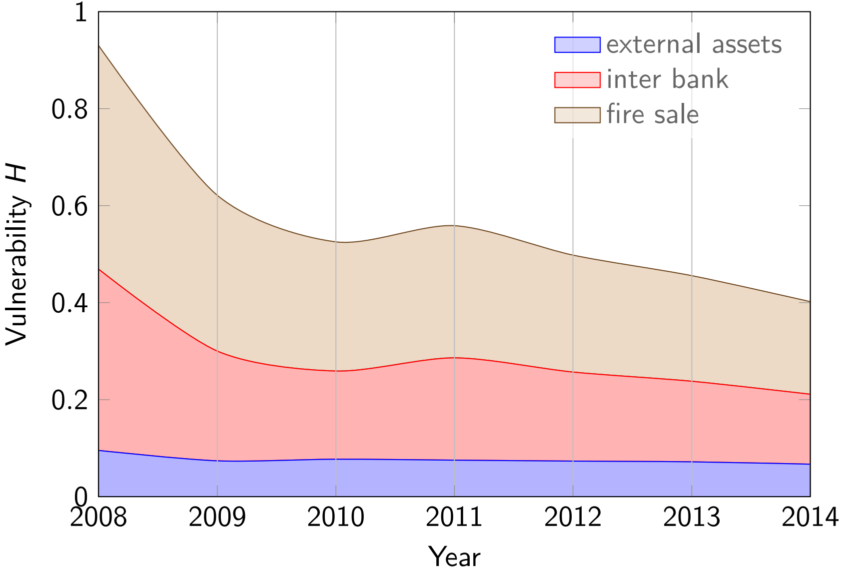
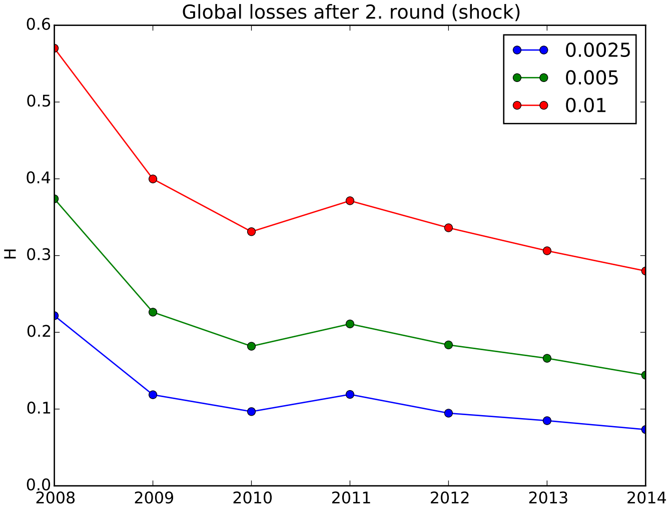


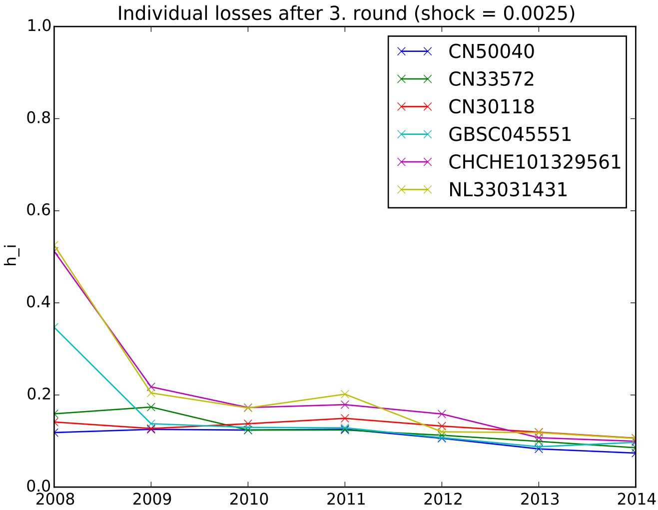
[13]

[13]compare Battiston et al. (2015)

Decrease in global leverage may explain decrease in equity losses

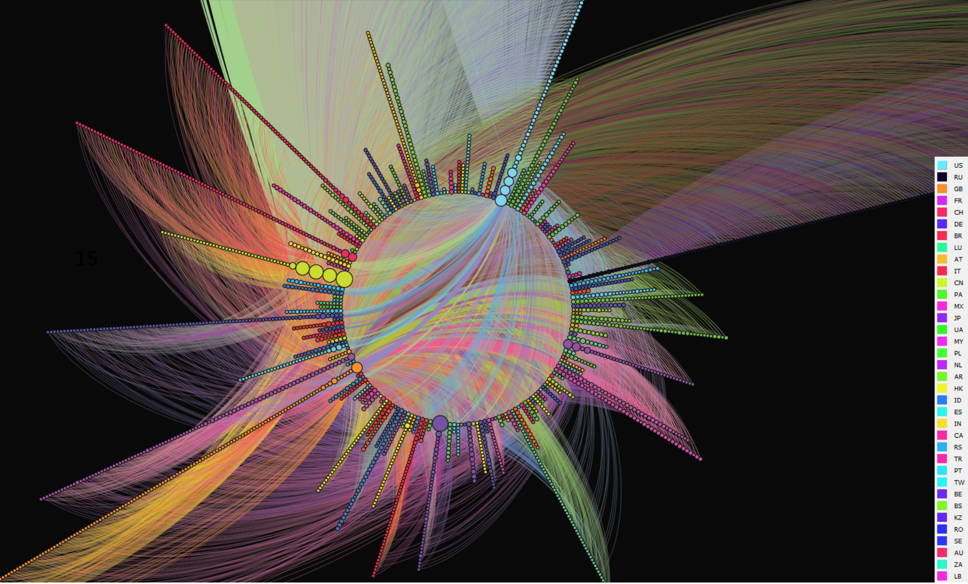


Balance sheet size did not shrink as consequence of deleveraging.



[14]

[14] x e3



[15] One instance of a reconstructed global banking system with 5.074 banks (vertices) and 31.587 inter-bank lending / borrowing relationships (edges). Visualisation created by Ali Shaghaghi with Gephi

Data

Bureau Van Dijk Bankscope Database for x Banks[16]

- total amount of interbank lending
- total amount of interbank borrowing
- equity
- total assets
- total liabilities

year	2008	2009	2010	2011	2012	2013	2014
banks	1526	2162	2166	1794	1821	1781	1435

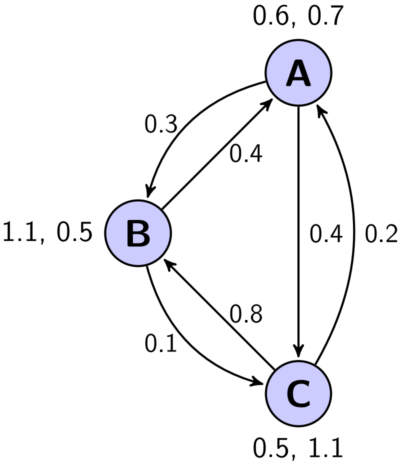
Table: Number of banks per year

banks from 36 countries

[16] publicly traded

Network reconstruction

Inter-Bank Network



$$\begin{pmatrix} 0.0 & 0.3 & 0.4 \\ 0.4 & 0.0 & 0.1 \\ 0.2 & 0.8 & 0.0 \end{pmatrix} \quad \begin{pmatrix} 0.7 \\ 0.5 \\ 1.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & 1.1 & 0.5 \end{pmatrix}$$

Network reconstruction

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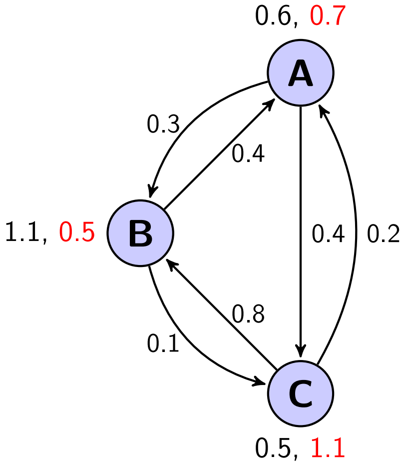

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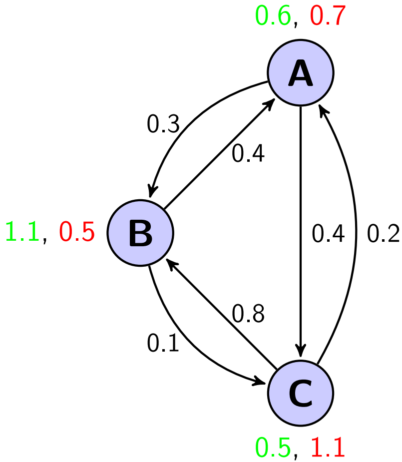

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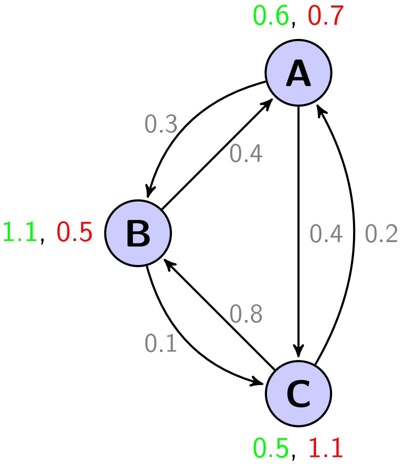

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Quiz

Why are this two matrices similar?

$$\begin{pmatrix} 0.0 & 0.2 & 0.5 \\ 0.5 & 0.0 & 0.0 \\ 0.1 & 0.9 & 0.0 \end{pmatrix} \qquad \begin{pmatrix} 0.0 & 0.3 & 0.4 \\ 0.4 & 0.0 & 0.1 \\ 0.2 & 0.8 & 0.0 \end{pmatrix}$$

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$$\begin{pmatrix} 0.6 & 1.1 & 0.5 \end{pmatrix} \qquad \begin{pmatrix} 0.6 & 1.1 & 0.5 \end{pmatrix}$$

Both matrices have the same sum over rows and columns

- no unique mapping between marginals and exposure
- possible networks range from maximum entropy to minimum density (e.g. diversification vs. costs for relationships)

Network reconstruction

Fitness Model

x_i^{in} lending propensity

x_i^{out} borrowing propensity

p_{ij} exposure probability

Lending and borrowing propensity is the relative exposure

$$x_i^{in} = \frac{A_i}{\sum_j A_j} \text{ and } x_i^{out} = \frac{L_i}{\sum_j L_j}$$

Fitness model applied to interbank network we assume x_i to be the fitness level.

The probability that bank i lends to bank j is :

$$p_{ij} = \frac{z x_i^{in} x_j^{out}}{1 + z x_i^{in} x_j^{out}},$$

where z is a free parameter. The total number of links is equal to the expected value $\frac{1}{2}\sum_i \sum_{j \neq i} p_{ij}$

(De Masi et al., 2006)

Network reconstruction (cont.)

Exposure Volume Allocation

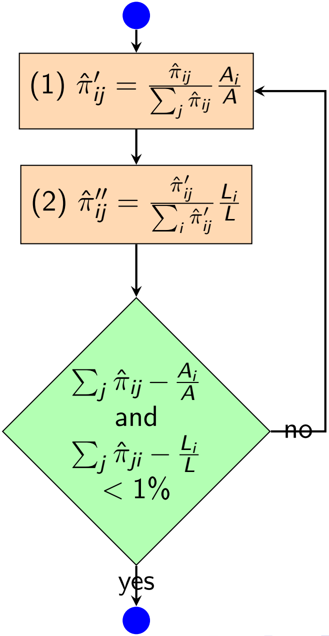
π_{ij} average relative exposure

$$\pi_{ij} = \frac{1}{2}\left(x_{ij}^{in} + x_{ij}^{out}\right)$$

Constraint: sum of exposures equal total assets of bank i

$$1 = \sum_j \pi_{ij}$$

Interactive prop. fitting algorithm: estimate the relative exposure π_{ij} iterating (1) and (2).



Conclusion

- We have a stress test model based on micro-foundation for the global banking system
  - exact algorithm for conter-party impact and vulnerability
  - measure to quantify dynamics/stability of the system
  - Block model to add more structure
  - network condensation to operate on aggregated level
- We have shown that stress test on condensed graph underestimates impact
- We just starting to understand for what it can be used
  - asset shock scenario
  - sovereign default scenario
  - ...
- It needs more work

calibration	network parameter, contagion parameter
integration	adding more scenarios, connect to macro / DSGE model

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De Masi, G., Iori, G., and Caldarelli, G. (2006). Fitness model for the italian interbank money market. *Phys Rev E Stat Nonlin Soft Matter Phys*, 74(6 Pt 2):066112.

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