Modelling Risk in Financial Markets

Hashem Pesaran

Cambridge University and USC

Cambridge Centre for Risk Studies, November 19, 2009

Introduction

- Risk Management
  - Active (portfolio management)
  - Passive (risk monitoring and requlation)
- Multivariate Models of Conditional Volatility and Correlation
- Types of Risks
  - Measurement Uncertainty
  - Parameter Uncertainty
  - Model Uncertainty (structural breaks)
  - Policy Uncertainty
- Model Averaging as a Risk Diversification Option
- Global Trends in Volatilities and Correlations
- Concluding Remarks

Background Papers and Programs

- "Model Averaging in Risk Management with an Application to Futures Markets", by Pesaran, Schleicher and Zaffaroni, *Journal of Empirical Finance*, 2009.
- "Dynamic Conditional Correlation Mdoels with multivariate t Distribution (TDCC)", Pesaran and Pesaran (2009, under prepration)
- *Time Series Econometrics using Microfit 5*, Oxford University Press, Oxford and New York, 2009

The Decision Problem: Active Risk Management

Change in value of the portfolio

$$\Delta V_t = \sum_{j=1}^{N} n_{j,t-1} \left(\frac{P_{jt} - P_{j,t-1}}{E_{jt}} \right) = \sum_{j=1}^{N} \left(\frac{n_{j,t-1} P_{j,t-1}}{E_{j,t-1}} \right) \left(\frac{r_{jt}}{1 + r_{jt}^{e}} \right), \quad (1)$$

where

- n_{jt} number of contracts, long $(+)$ or short $(-)$
- $r_{jt} = 100(P_{jt} - P_{j,t-1})/P_{j,t-1}$ return on jth asset
- $r_{jt}^{e} = 100(E_{jt} - E_{j,t-1})/E_{j,t-1}$ % change in FX rate against US $

The Decision Problem: Active Risk Management

Ignoring second-order effects, the portfolio return ρ_t becomes

$$\rho_t = \frac{\Delta V_t}{C_{t-1}} \approx \sum_{j=1}^{N} \frac{n_{j,t-1}P_{j,t-1}}{E_{j,t-1}C_{t-1}} r_{jt} \tag{2}$$

$$= \sum_{j=1}^{N} \omega_{j,t-1} r_{jt} = \boldsymbol{\omega}'_{t-1}\mathbf{r}_t, \tag{3}$$

where

- C_{t-1} notional capital
- $\omega_{j,t-1} = n_{j,t-1}P_{j,t-1}/(E_{j,t-1}C_{t-1})$ portfolio exposures
- $\boldsymbol{\omega}_{t-1} = (\omega_{1,t-1}, \omega_{2,t-2}, ..., \omega_{N,t-1})'$, $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{Nt})'$

Mean-Variance Problem Subject to the VaR Constraint

- Maximize

$$Q(\omega_{t-1}|\mathcal{F}_{t-1}) = \boldsymbol{\omega}'_{t-1}E(\mathbf{r}_t|\mathcal{F}_{t-1}) - \frac{\delta_{t-1}}{2}\boldsymbol{\omega}'_{t-1}V(\mathbf{r}_t|\mathcal{F}_{t-1})\boldsymbol{\omega}_{t-1}, \quad (4)$$

- Subject to

$$\Pr\left(\boldsymbol{\omega}'_{t-1}\mathbf{r}_t < -L_{t-1}\,|\mathcal{F}_{t-1}\right) \leq \alpha, \quad (5)$$

  where $L_{t-1} > 0$ is a pre-specified maximum (daily) loss (e.g. 1%).
- Unlike MV solutions, the MV subject to VaR constraint requires a **complete** knoweldge of the **conditional joint probability distribution of returns**, $f(\mathbf{r}_t|\mathcal{F}_{t-1})$.

Literature on Multivariate Volatility Modelling

- Recent surveys are provided in Bauwens, Laurent, and Rombouts (2006, JAE) and McAleer (2005, ER).
- The Riskmetrics specifications popularized by J.P.Morgan
- The conditionally constant correlation (CCC) model of Bollerslev
- The orthogonal GARCH model of Alexander
- The dynamic conditional correlation (DCC) model advanced by Engle
- Asymmetric DCC (ADCC) model of Cappiello, Engle and Sheppard.
- T-DCC model based on de-volatized returns of Pesaran and Pesaran (2009)

Modelling Conditional Correlation Matrix of Asset Returns

Following Bollerslev (1990) and Engle (2002) consider the decomposition

$$\sum_{t-1} = \mathbf{D}_{t-1}\mathbf{R}_{t-1}\mathbf{D}_{t-1}, \quad (6)$$

where

$$\mathbf{D}_{t-1} = \begin{pmatrix} \sigma_{1,t-1} & & & \\ & \sigma_{2,t-1} & 0 & \\ & 0 & \ddots & \\ & & & \sigma_{m,t-1} \end{pmatrix}, \sigma^2_{i,t-1} = V\left(r_{it} \mid \Omega_{t-1}\right),$$

$$\mathbf{R}_{t-1} = \left(\rho_{ij,t-1}\right),$$

$$\rho_{ij,t-1} = \frac{Cov\left(r_{it}, r_{jt} \mid \Omega_{t-1}\right)}{\sigma_{i,t-1}\sigma_{j,t-1}}.$$

Bollerslev (1990) considers (6) with a constant correlation matrix $\mathbf{R}_{t-1} = \mathbf{R}$.
Engle (2002) allows for $\mathbf{R}_{t-1}$ to be time-varying and proposes a class of multi-variate GARCH models labeled as dynamic conditional correlation (DCC) models.
Sentana (2000) uses an unobserved common factor specification to model pair-wise correlations.
The decomposition of $\sum_{t-1}$ allows separate specification of the conditional volatilities and conditional cross-asset returns correlations.
For example, one can utilize the GARCH (1,1) model for $\sigma^2_{i,t-1}$, namely

$$V\left(r_{it} \mid \Omega_{t-1}\right) = \sigma^2_{i,t-1} = \bar{\sigma}^2_i \left(1 - \lambda_{1i} - \lambda_{2i}\right) + \lambda_{1i}\sigma^2_{i,t-2} + \lambda_{2i} r^2_{i,t-1}, \quad (7)$$

where $\bar{\sigma}^2_i$ is the unconditional variance of the i-th asset return.

For cross-asset correlations Engle proposes the use of exponential smoother applied to the "standardized returns"

$$\hat{\rho}_{ij,t-1}\left(\phi\right) = \frac{\sum_{s=1}^{\infty} \phi^{s-1} z_{i,t-s} z_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty} \phi^{s-1} z_{i,t-s}^{2}}\sqrt{\sum_{s=1}^{\infty} \phi^{s-1} z_{j,t-s}^{2}}}, \tag{8}$$

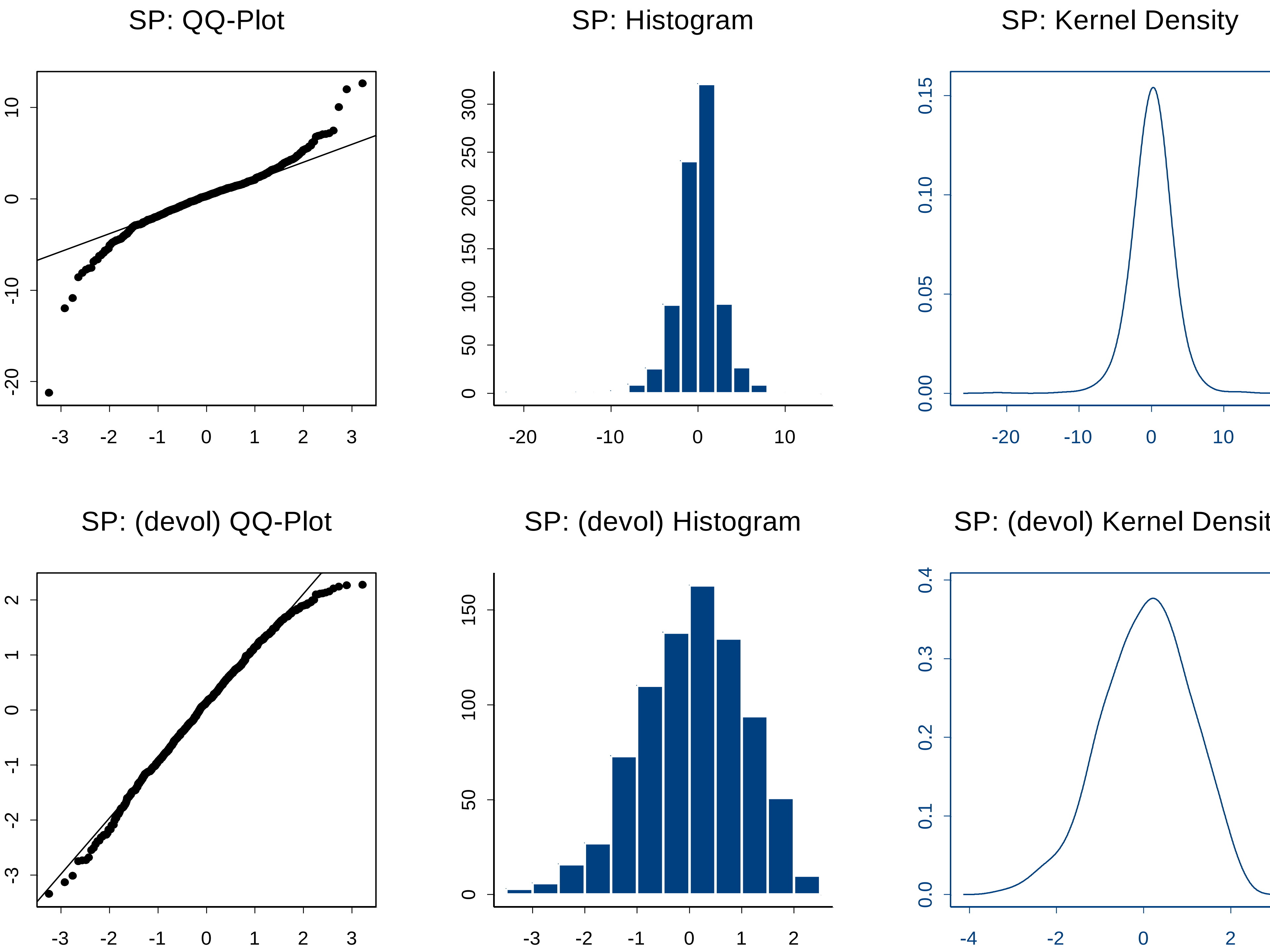
where the standardized returns are defined by

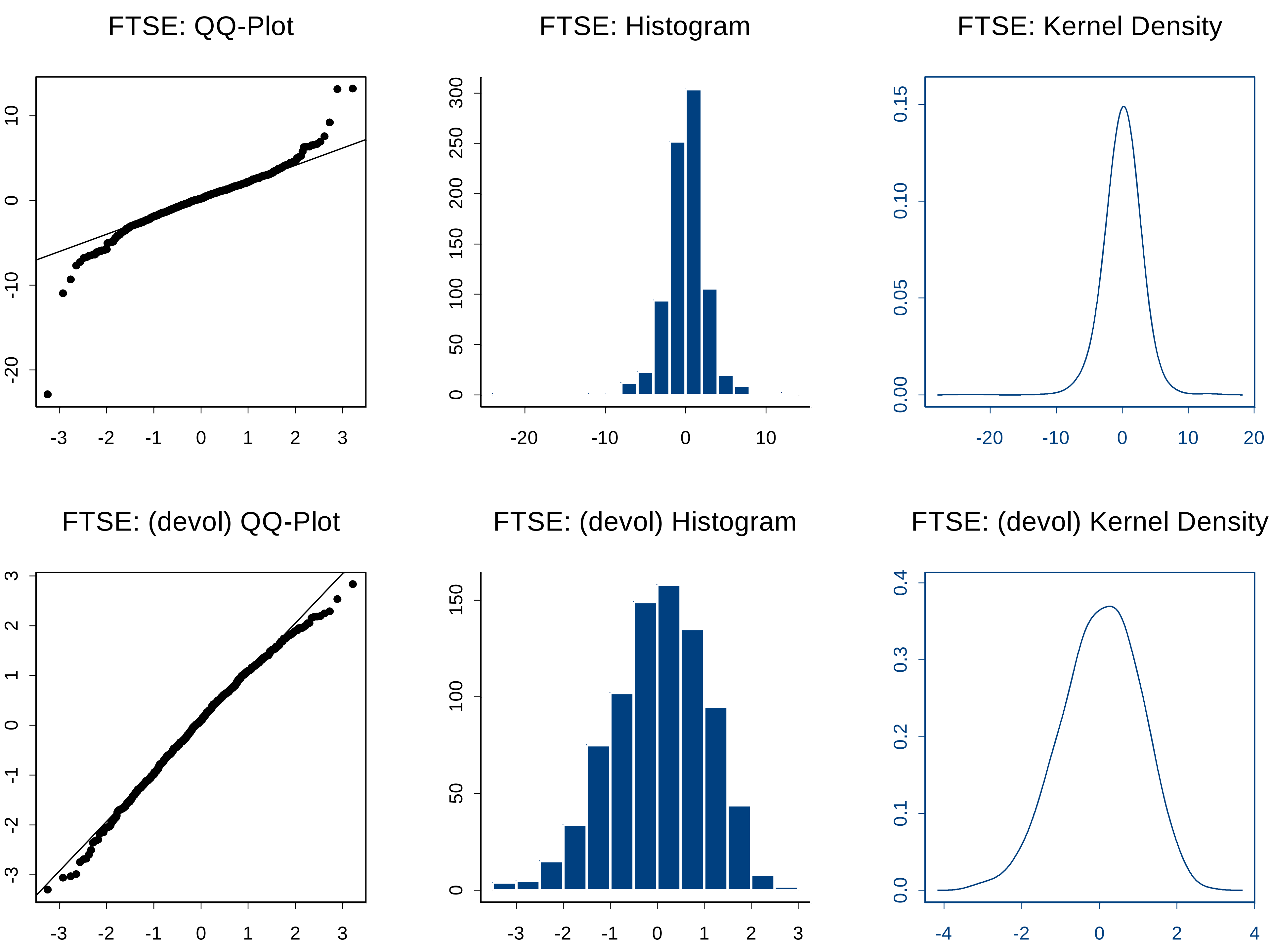
$$z_{it} = \frac{r_{it}}{\sigma_{i,t-1}\left(\lambda_i\right)}. \tag{9}$$

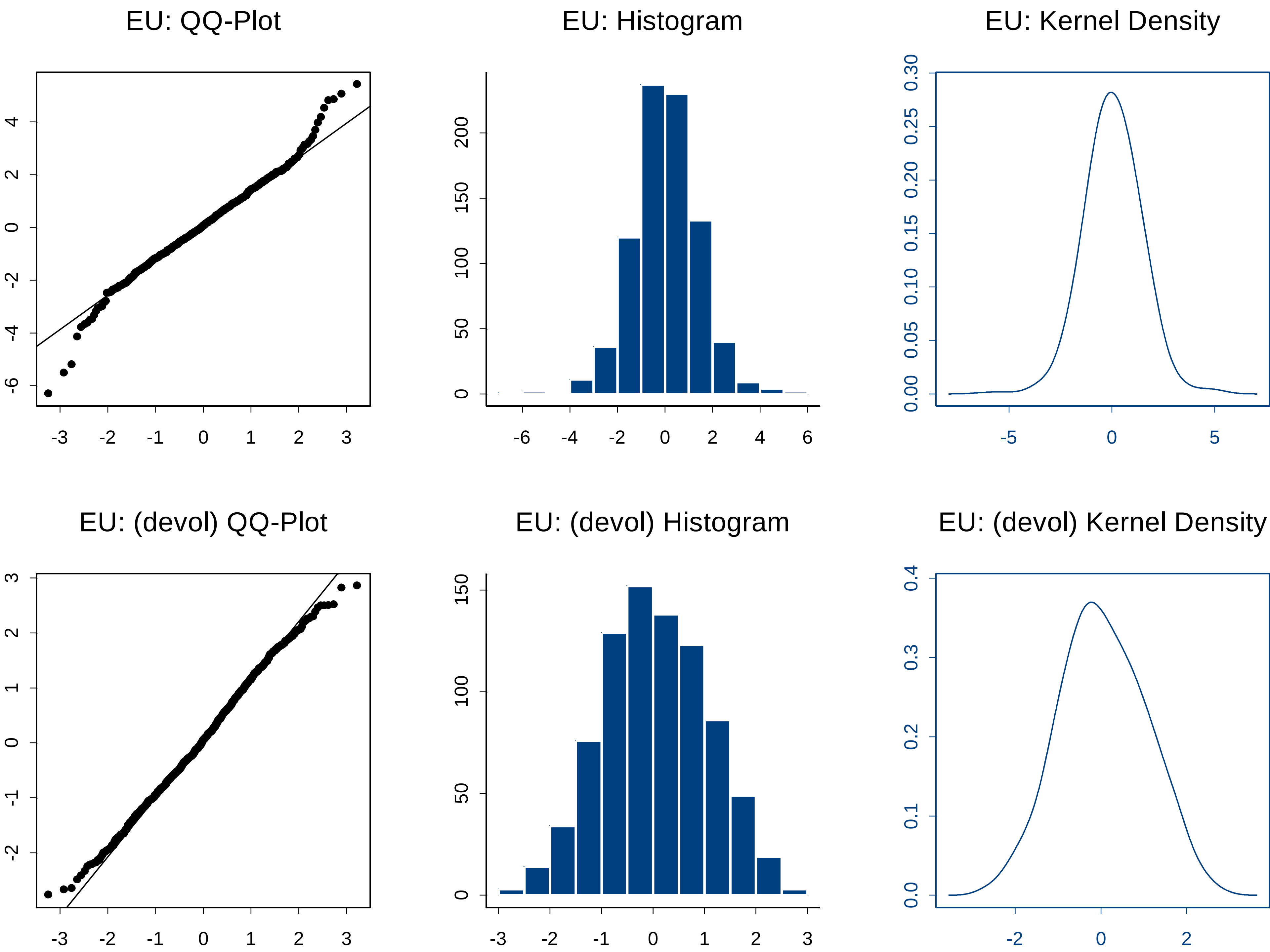
Pesaran and Pesaran (PP) consider an alternative formulation of $\rho_{ij,t-1}$ that makes use of realized volatilities.

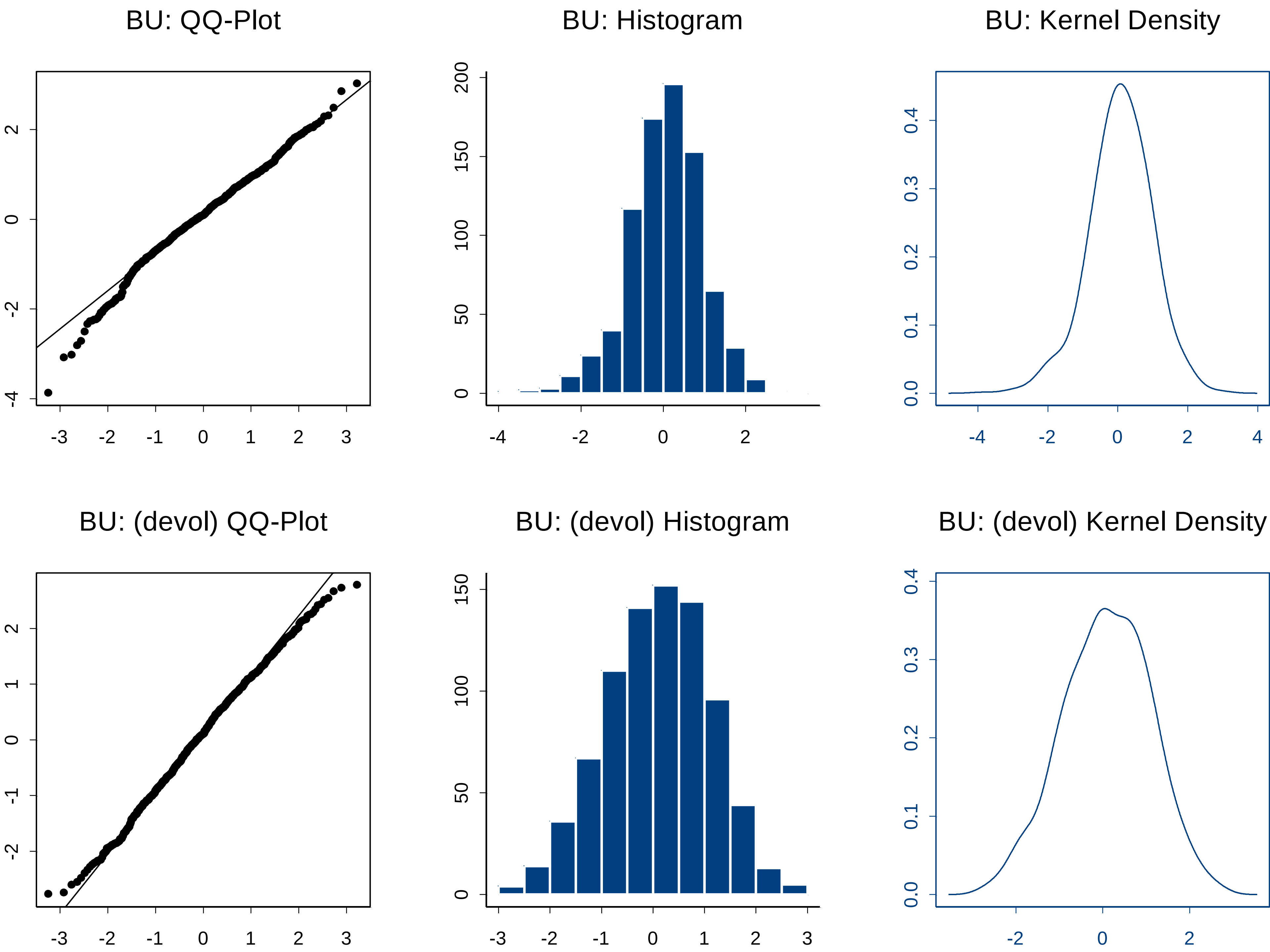
Andersen, Bollerslev and Diebold show that daily returns on foreign exchange and stock returns standardized by realized volatility are approximately Gaussian.

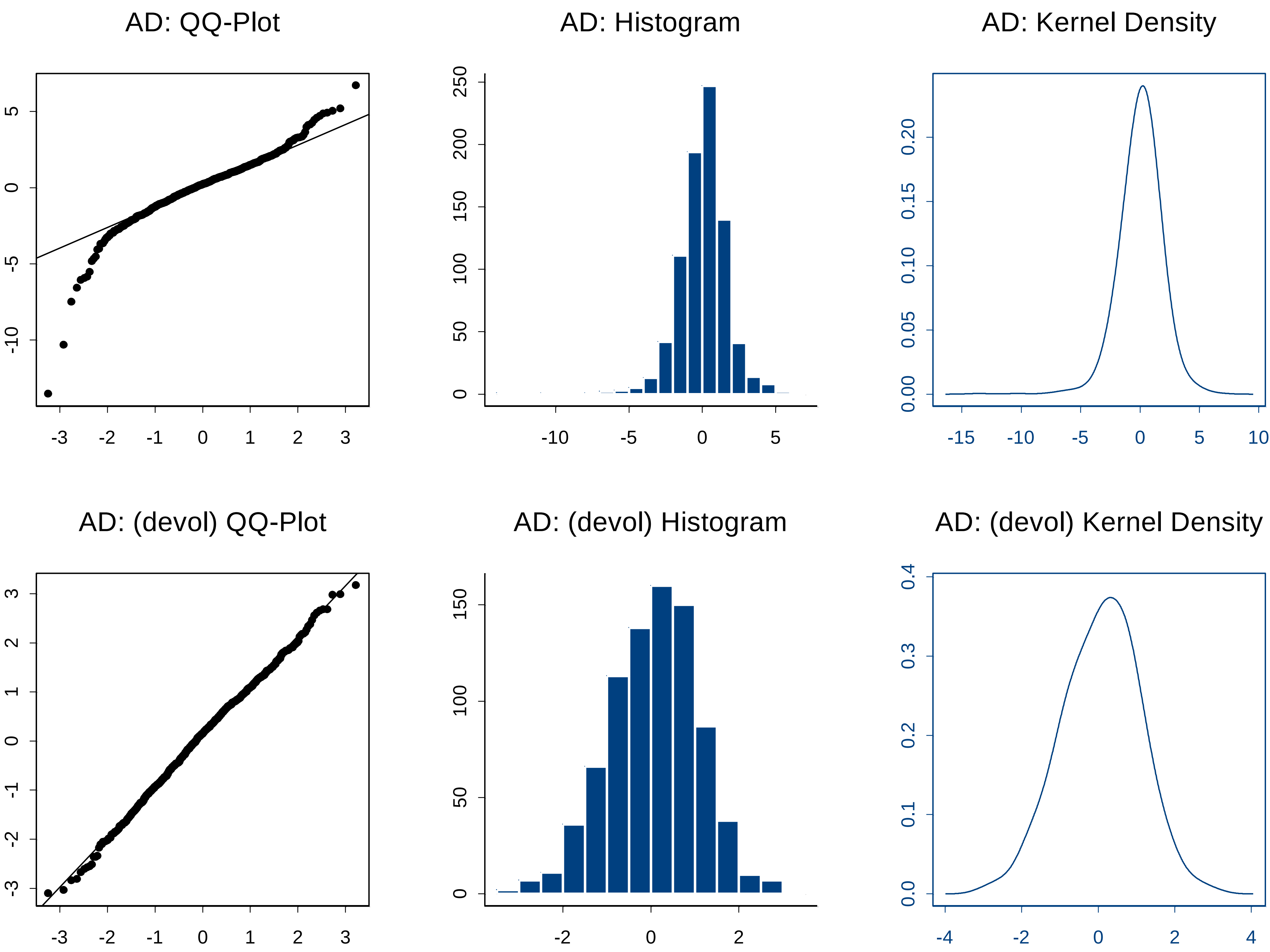
The transformation of returns to Gaussianity is important since correlation as a measure of dependence can be misdealing in the case of non-Gaussian returns. See Embrechts et al. (2003).


SP: QQ-Plot
10
0
-10
-20
-3 -2 -1 0 1 2 3
SP: Histogram
300
250
200
150
100
50
0
-20 -10 0 10
SP: Kernel Density
0.15
0.10
0.05
0.00
-20 -10 0 10
SP: (devol) QQ-Plot
2
1
0
-1
-2
-3
-3 -2 -1 0 1 2 3
SP: (devol) Histogram
150
100
50
0
-3 -2 -1 0 1 2
SP: (devol) Kernel Densit
0.4
0.3
0.2
0.1
0.0
-4 -2 0 2


FTSE: QQ-Plot
FTSE: Histogram
FTSE: Kernel Density
FTSE: (devol) QQ-Plot
FTSE: (devol) Histogram
FTSE: (devol) Kernel Density
-20 -10 0 10
-3 -2 -1 0 1 2 3
0 50 100 150 200 250 300
0.00 0.05 0.10 0.15
-20 -10 0 10 20
0 50 100 150
0.0 0.1 0.2 0.3 0.4
-4 -2 0 2 4


EU: QQ-Plot
EU: Histogram
EU: Kernel Density
EU: (devol) QQ-Plot
EU: (devol) Histogram
EU: (devol) Kernel Density


BU: QQ-Plot
BU: Histogram
BU: Kernel Density
BU: (devol) QQ-Plot
BU: (devol) Histogram
BU: (devol) Kernel Density


AD: QQ-Plot
5
0
-5
-10
-3 -2 -1 0 1 2 3
AD: Histogram
250
200
150
100
50
0
-10 -5 0 5
AD: Kernel Density
0.20
0.15
0.10
0.05
0.00
-15 -10 -5 0 5 10
AD: (devol) QQ-Plot
3
2
1
0
-1
-2
-3
-3 -2 -1 0 1 2 3
AD: (devol) Histogram
150
100
50
0
-2 0 2
AD: (devol) Kernel Density
0.4
0.3
0.2
0.1
0.0
-4 -2 0 2 4

PP base the specification of the cross correlation of volatilities on devolatized returns. Let

$$\tilde{r}_{it} = \frac{r_{it}}{\sigma_{it}^{realized}}, \tag{10}$$

and use

$$\tilde{\rho}_{ij,t-1}(\phi) = \frac{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{i,t-s} \tilde{r}_{j,t-s}}{\sqrt{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{i,t-s}^{2}} \sqrt{\sum_{s=1}^{\infty} \phi^{s-1} \tilde{r}_{j,t-s}^{2}}}, \tag{11}$$

where $-1 < \tilde{\rho}_{ij,t-1}(\phi) < 1$ for all values of $|\phi| < 1$.

In the absence of intradaily observations the following simple estimate of σ_{it} based on daily returns, *inclusive* of the contemporaneous value of r_{it}, seem to work well in practice

$$\tilde{\sigma}_{it}^{2}(p) = \frac{\sum_{s=0}^{p-1} r_{i,t-s}^{2}}{p}. \tag{12}$$

PP find that for $p = 20$ the de-volatized returns, $\tilde{r}_{it} \approx r_{it}/\tilde{\sigma}_{it}(p)$, are nearly Gaussian, with approximately unit variances, for all asset classes foreign exchange, equities, bonds or commodities.

Individual Multivariate Volatility Models

- 53 different specifications of Σ_{it} grouped into 8 different model types are considered.

1. Equal-Weighted Moving Average – EQMA
2. Exponential-Weighted Moving Average – EWMA
3. Mixed Moving Average – MMA
4. Generalized Exponential-Weighted Moving Average – GEWMA
5. Constant Correlation (Bollerslev 1990) – CCC
6. Dynamic Conditional Correlation (Engle 2002) – DCC
7. Asymmetric Dynamic Conditional Correlation (Cappiello et al. 2006) – ADCC
8. t-Dynamic Conditional Correlation (Pesaran and Pesaran, 2009) – TDCC

Average Volatility Models

Average predictive density based on a set of models $\{M_i\}$ and weights $\{\lambda_i\}$

$$f(\mathbf{r}_t \,|\mathcal{F}_{t-1},\, \mathcal{M}\,) = \sum_{i=1}^{m} \lambda_{i,t-1} f(\mathbf{r}_t|\mathcal{F}_{t-1}, M_i), \tag{13}$$

Three main strategies (based on AIC or BIC criteria)

1. Best model
2. Akaike weights or Schwartz weights. E.g. for AIC:

$$\lambda_{i,t-1} = exp(\Delta_{i,t-1}) / \sum_j exp(\Delta_{j,t-1})$$
$$\Delta_{i,t-1} = AIC_{i,t-1} - Max_j(AIC_{j,t-1})$$

3. Thick model averaging: top 10%, top 25%, etc. with equal weights

VaR Based Diagnostic Tests: Individual Models

- Distribution of portfolio return ρ_t based on model $M_i(\hat{\theta}_{iT_0})$

$$\rho_t|\mathcal{F}_{t-1}, M_i(\hat{\theta}_{iT_0}) \sim (\mu_{\rho t}, \sigma^2_{\rho t}(M_i))$$

$$\mu_{\rho t} = \boldsymbol{\omega}'_{t-1}\boldsymbol{\mu}_t, \text{ and } \sigma^2_{\rho t}(M_i) = \boldsymbol{\omega}'_{t-1}\Sigma_{it}\boldsymbol{\omega}_{t-1}$$

VaR:

$$Pr(\rho_t < -\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1}, \alpha; \hat{\theta}_{iT_0})|\mathcal{F}_{t-1}, M_i) \leq \alpha \quad (14)$$

- $\Rightarrow$

$$\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1}, \alpha; \hat{\theta}_{iT_0}) = -\mu_{\rho t} + c_{it}(\alpha)\sigma_{\rho t}(M_i)$$

where $c_{it}(\alpha)$ is the α% critical value.

- Empirical VaR exceedance frequency

$$\hat{\pi}_i = \frac{1}{T_1}\sum_{t \in \mathcal{T}_1} d_{it}(\hat{\theta}_{iT_0}) \quad (15)$$

$$d_{it}(\hat{\theta}_{iT_0}) = I[-\bar{\rho}_{i,t-1}(\boldsymbol{\omega}_{t-1}, \alpha; \hat{\theta}_{iT_0}) - \rho_t]$$

VaR Based Diagnostic Tests: Average Models

- Set of m models $M = \cup_{i=1}^{m} M_i$ with probability distributions $F_{it}(\cdot)$.
- VaR constraint: need to find the value $\bar{\rho}_{b,t-1}$ which satisfies

$$Pr(\rho_t < -\bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1}, \alpha) | \mathcal{F}_{t-1}, M) = \sum_{i=1}^{m} \lambda_{i,t-1} F_{it} \left(\frac{-\bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1}, \alpha) - \boldsymbol{\omega}'_{t-1} \boldsymbol{\mu}_{it}}{\sigma_{\rho t}(M_i)} \right) \leq \alpha \quad (16)$$

(typically this has to be done numerically)

- Compute the empirical VaR exceedance frequency as before

$$\hat{\pi}_b = \frac{1}{T_1} \sum_{t \in \mathcal{T}_1} \hat{d}_{bt} \quad (17)$$

$$\hat{d}_{bt} = I\left[-\rho_t - \bar{\rho}_{b,t-1}(\boldsymbol{\omega}_{t-1}, \alpha) \right].$$

Tail Behavior of Average Volatility Models

- Tail probabilities using a mixture model and a Gaussian model with the same average volatility are not the same, namely

$$\sum_{i=1}^{m} \lambda_{it-1} \Phi\left(\frac{g}{\sigma_{\rho t}(M_i)}\right) \neq \Phi\left(\frac{g}{\sqrt{\sum_{i=1}^{m} \lambda_{it-1} \sigma_{\rho t}^{2}(M_i)}}\right), \tag{18}$$

  unless $\Sigma_{it} = \Sigma_t$ for all i.
- The following theorem states that a combined model will be more fat-tailed than the associated Gaussian model with the same average volatility measure, as long as $g < -\sqrt{3}\sigma_{\rho t}(M_i), i = 1, ..., m$.

Statistical Tests

$$\hat{v}_{it} = \int_{-\infty}^{\rho_t} \hat{f}(x|\mathcal{F}_{t-1}, M_i)dx, \text{ for } t = \tau + 1, ..., \tau + T_1$$

$\hat{f}(x|\mathcal{F}_{t-1}, M_i)$ is the estimated pdf of ρ_t under model M_i.
$\hat{v}_{it}$, $t \in \mathcal{T}_1$ are *i.i.d.* uniformly distributed on the interval $[0, 1]$, **if** $\hat{f}(x\,|\mathcal{F}_{t-1}, M_i\,)$ coincides with the 'true' but unknown conditional predictive density of ρ_t.

Statistical Tests

Kolmogorov-Smirnov test

$$KS = \max_{1 \leq j \leq T_1} \left| \frac{j}{T_1} - \hat{v}_j^* \right|$$

Kuiper test

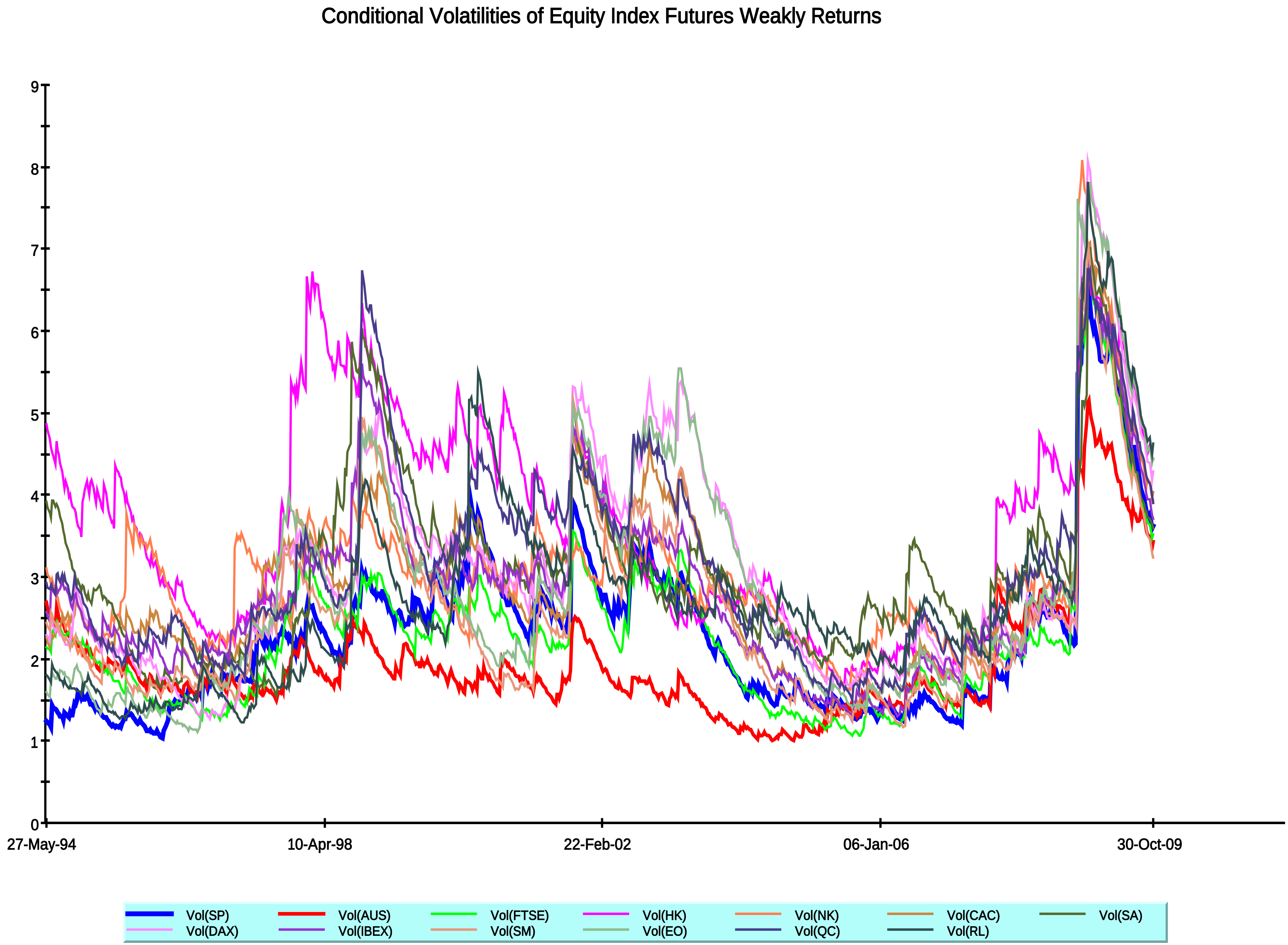
$$Ku = \max_{1 \leq j \leq T_1} \left(\frac{j}{T_1} - \hat{v}_j^* \right) + \max_{1 \leq j \leq T_1} \left(\hat{v}_j^* - \frac{j}{T_1} \right)$$

where $\hat{v}_1^* \leq \hat{v}_2^* \leq ... \leq \hat{v}_{T_1}^*$ are the ordered values of the $\hat{v}_{i,\tau+1}, ..., \hat{v}_{i,\tau+T_1}$.

FX, Equities and Bonds

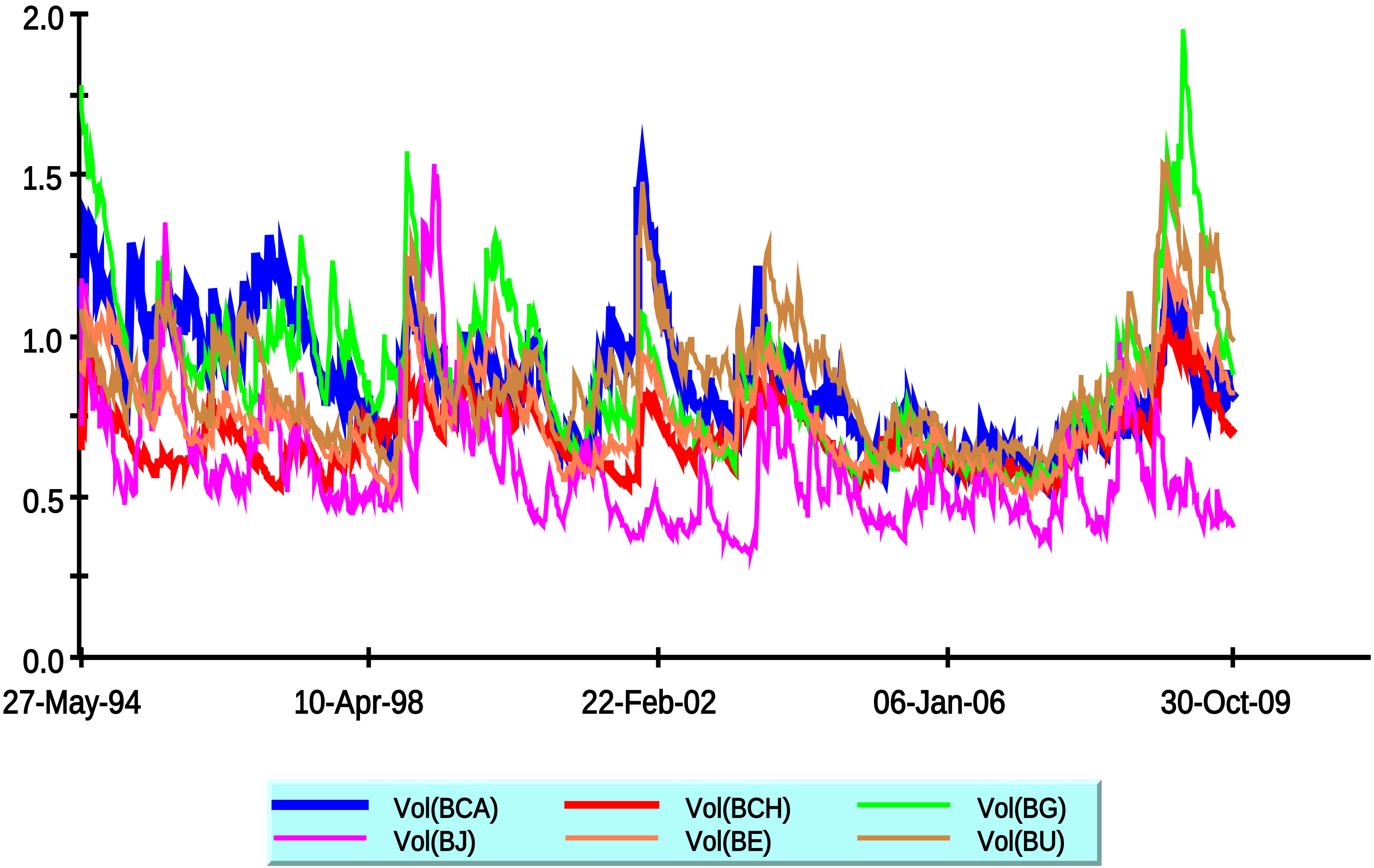
Weakly returns on 31 assets over the period January 7, 1994 to October 30, 2009 (826 weakly observations)

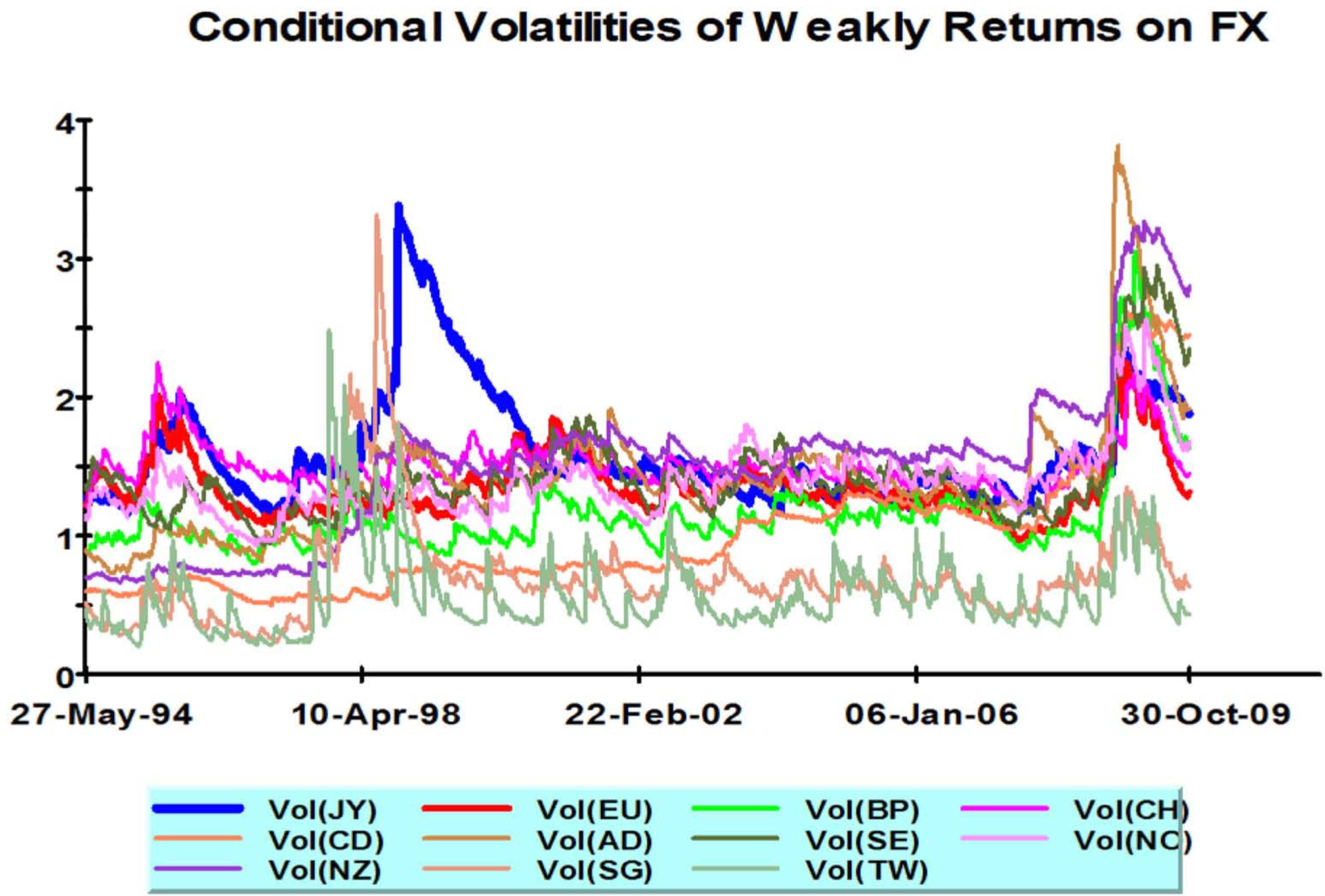
- 11 currencies (GBP, EUR, JPY, CAD, AUD, CHF, SE, NO, NZ,SG,TW)
- 13 equity indices (SP, RL FTSE, DAX,CAC,IBEX,SM,EO,QC, NK,HK,AUS, SA)
- 7 bonds (BU, BCA, BE, BCH,BG, JGB, BA)



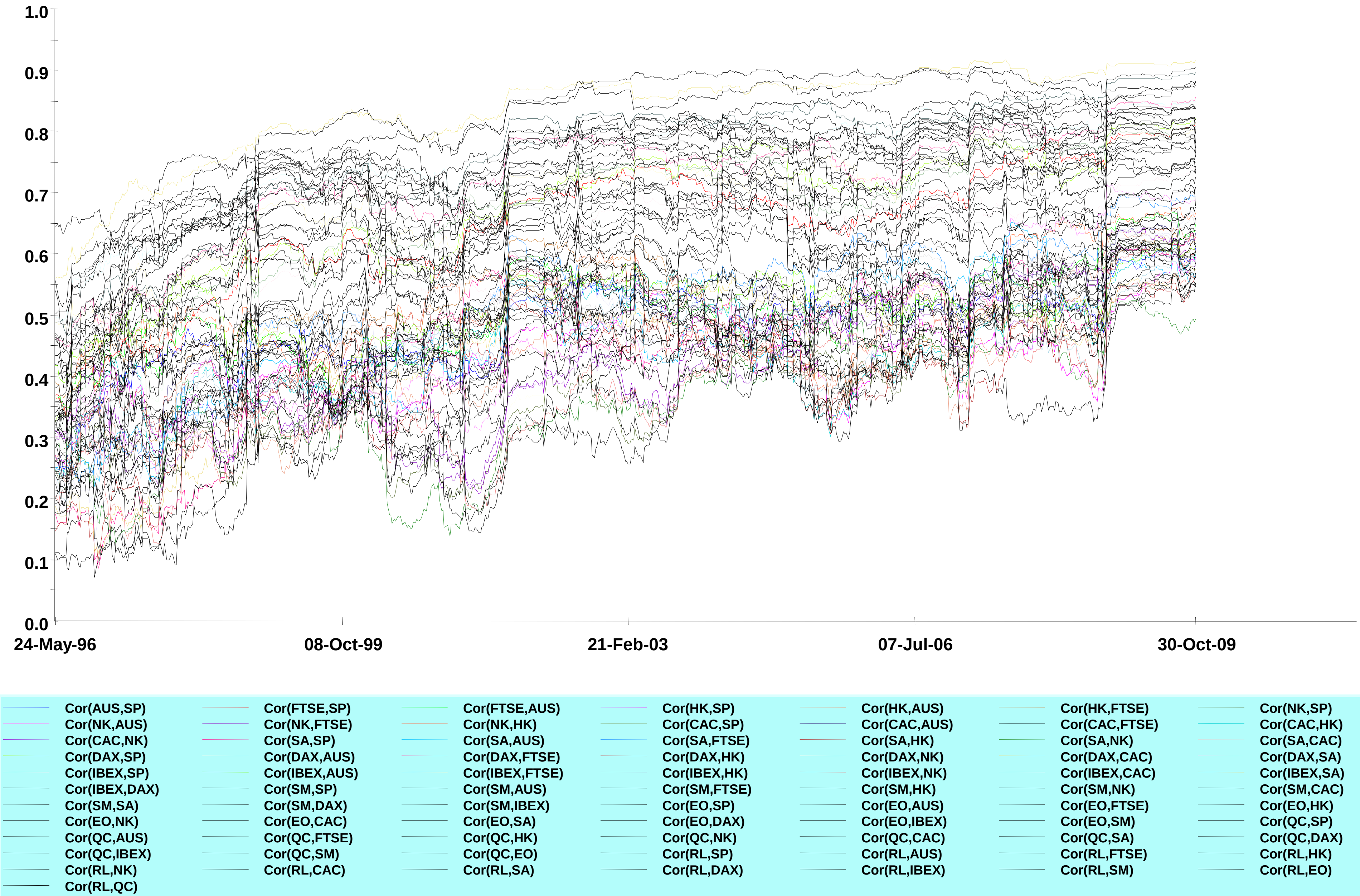
Conditional Volatilities of Equity Index Futures Weakly Returns
0
1
2
3
4
5
6
7
8
9
27-May-94
10-Apr-98
22-Feb-02
06-Jan-06
30-Oct-09
Vol(SP)
Vol(AUS)
Vol(FTSE)
Vol(HK)
Vol(NK)
Vol(CAC)
Vol(SA)
Vol(DAX)
Vol(IBEX)
Vol(SM)
Vol(EO)
Vol(QC)
Vol(RL)

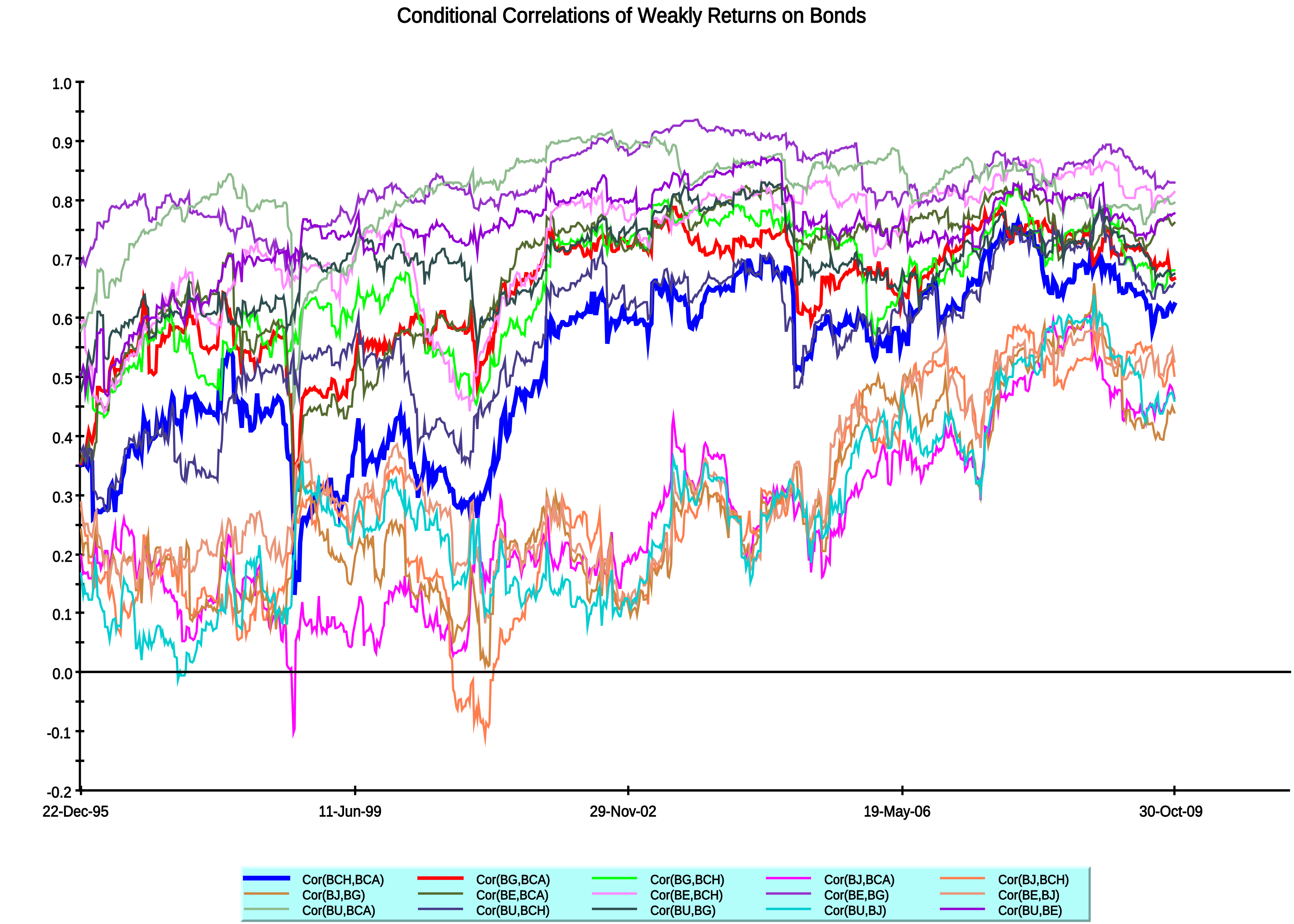
Conditional Volatilities of Weakly Returns on Bond Futures




Conditional Volatilities of Weakly Retums on FX
0
1
2
3
4
27-May-94
10-Apr-98
22-Feb-02
06-Jan-06
30-Oct-09
Vol(JY)
Vol(EU)
Vol(BP)
Vol(CH)
Vol(CD)
Vol(AD)
Vol(SE)
Vol(NC)
Vol(NZ)
Vol(SG)
Vol(TW)

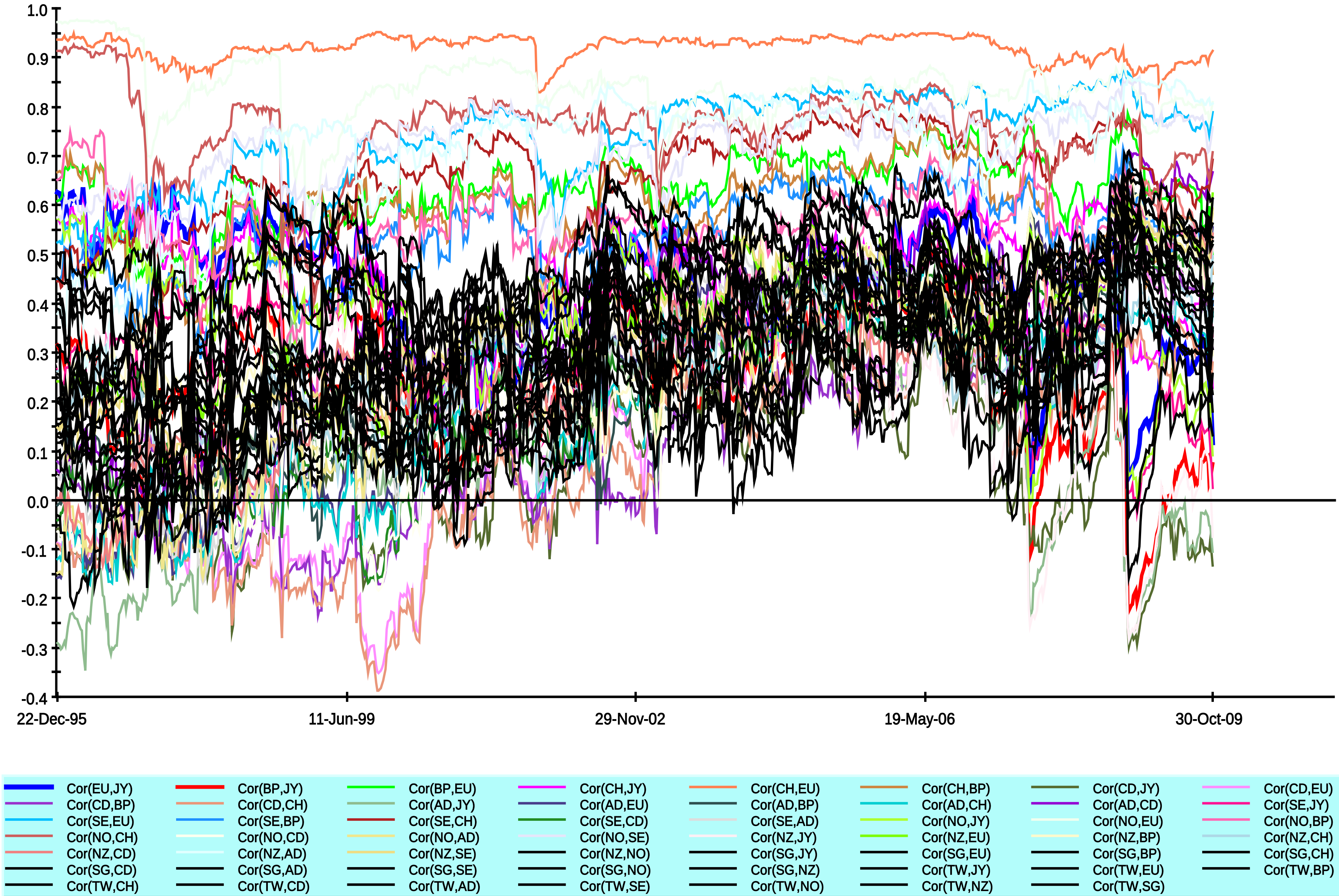
Conditional Correlations of Equity Futures (Weakly Returns)





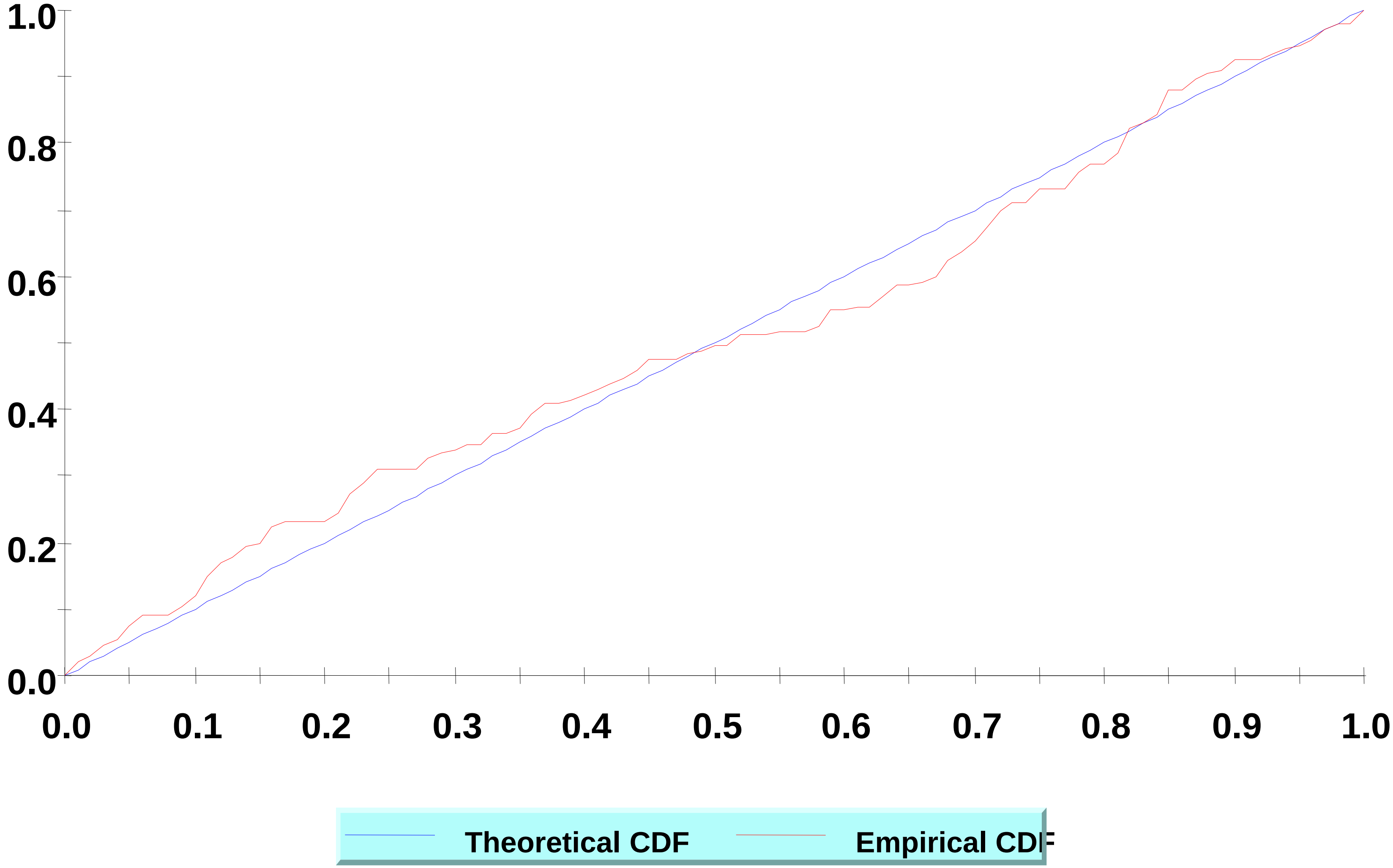
Conditional Correlations of Weakly Returns on Bonds
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
22-Dec-95
11-Jun-99
29-Nov-02
19-May-06
30-Oct-09
Cor(BCH,BCA)
Cor(BG,BCA)
Cor(BG,BCH)
Cor(BJ,BCA)
Cor(BJ,BCH)
Cor(BJ,BG)
Cor(BE,BCA)
Cor(BE,BCH)
Cor(BE,BG)
Cor(BE,BJ)
Cor(BU,BCA)
Cor(BU,BCH)
Cor(BU,BG)
Cor(BU,BJ)
Cor(BU,BE)

Conditional Correlations of Weakly Returns on FX

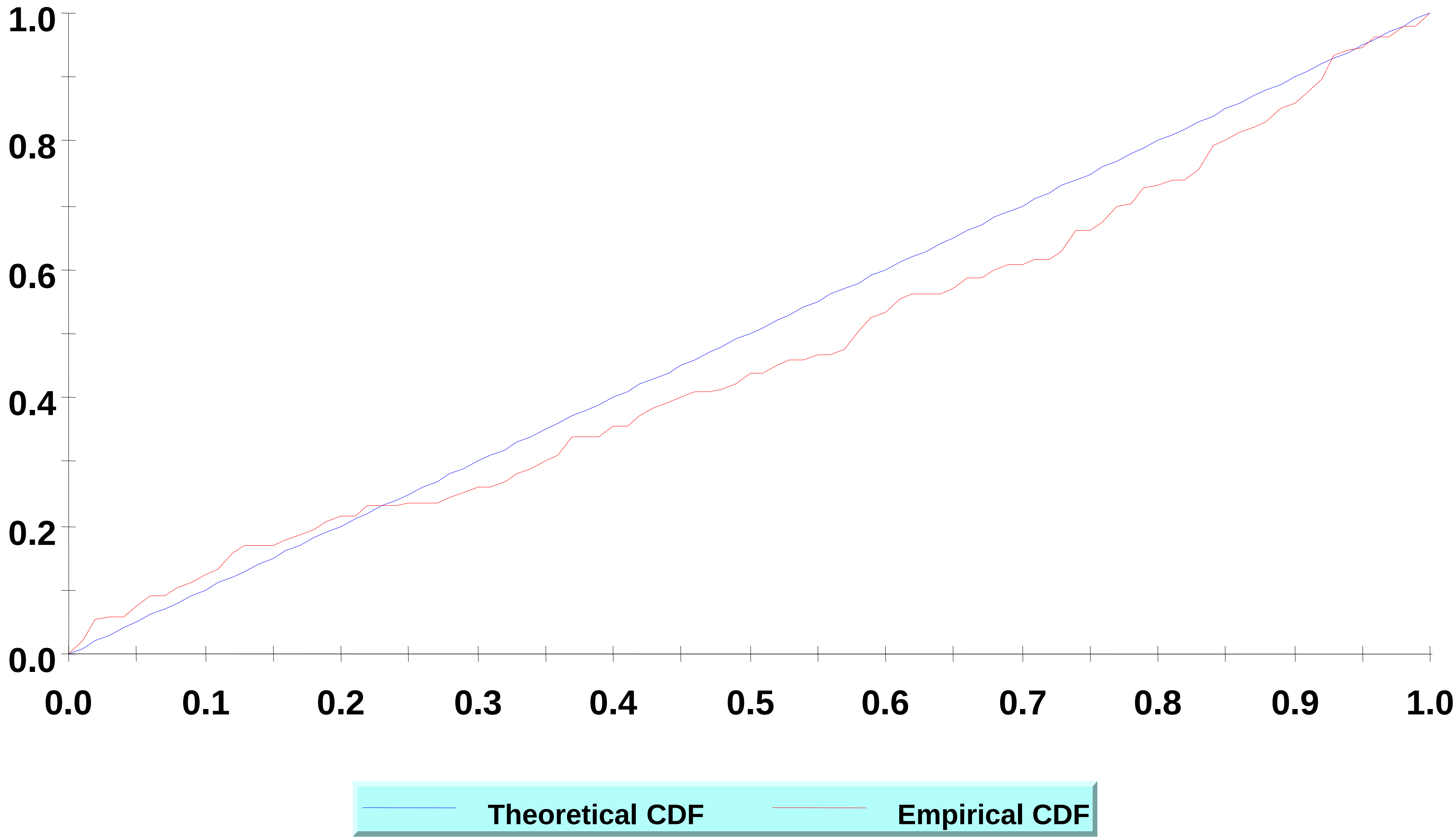


Equity Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = .071111

5% Critical value = .11705

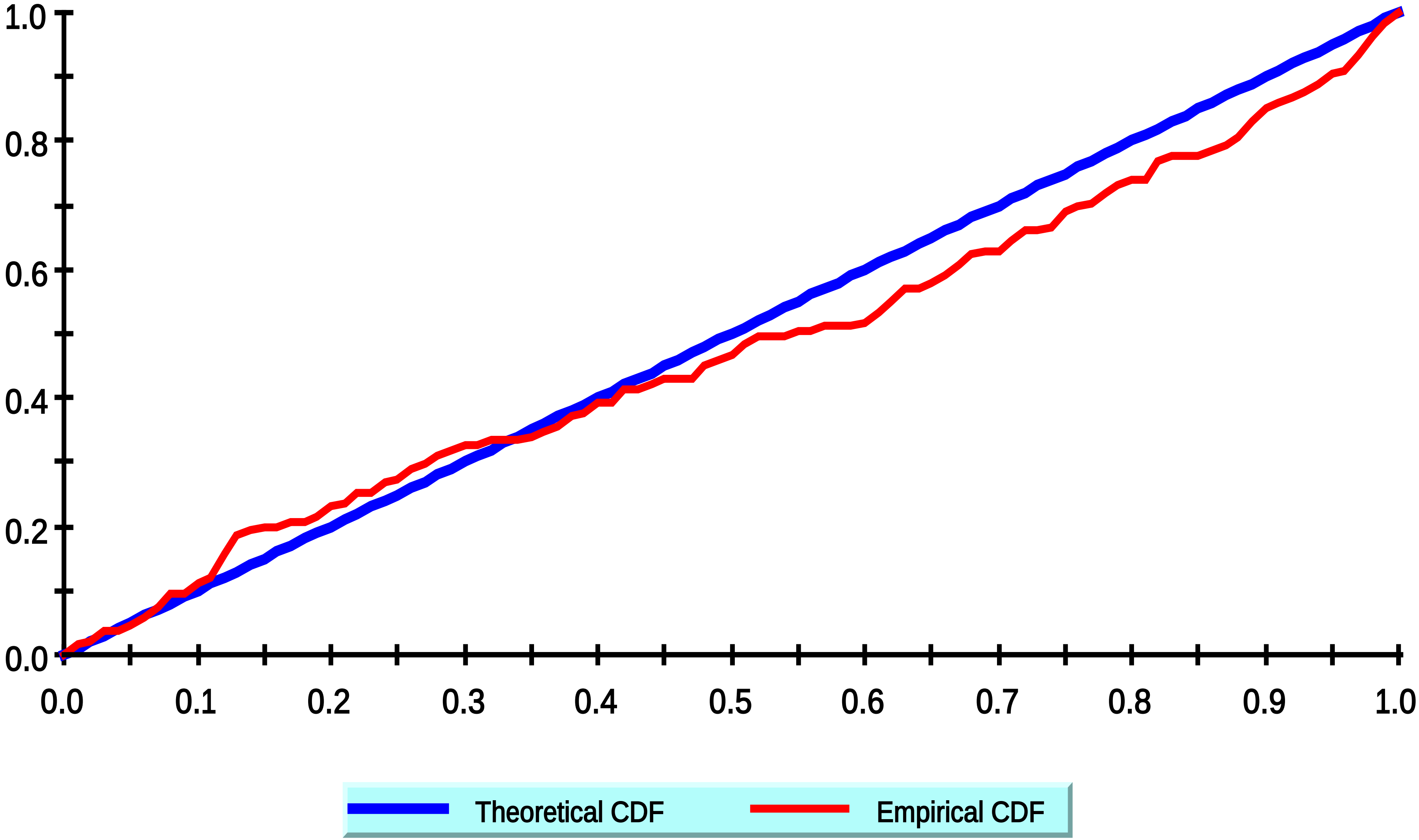


FX Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = .10519
5% Critical value = .11705



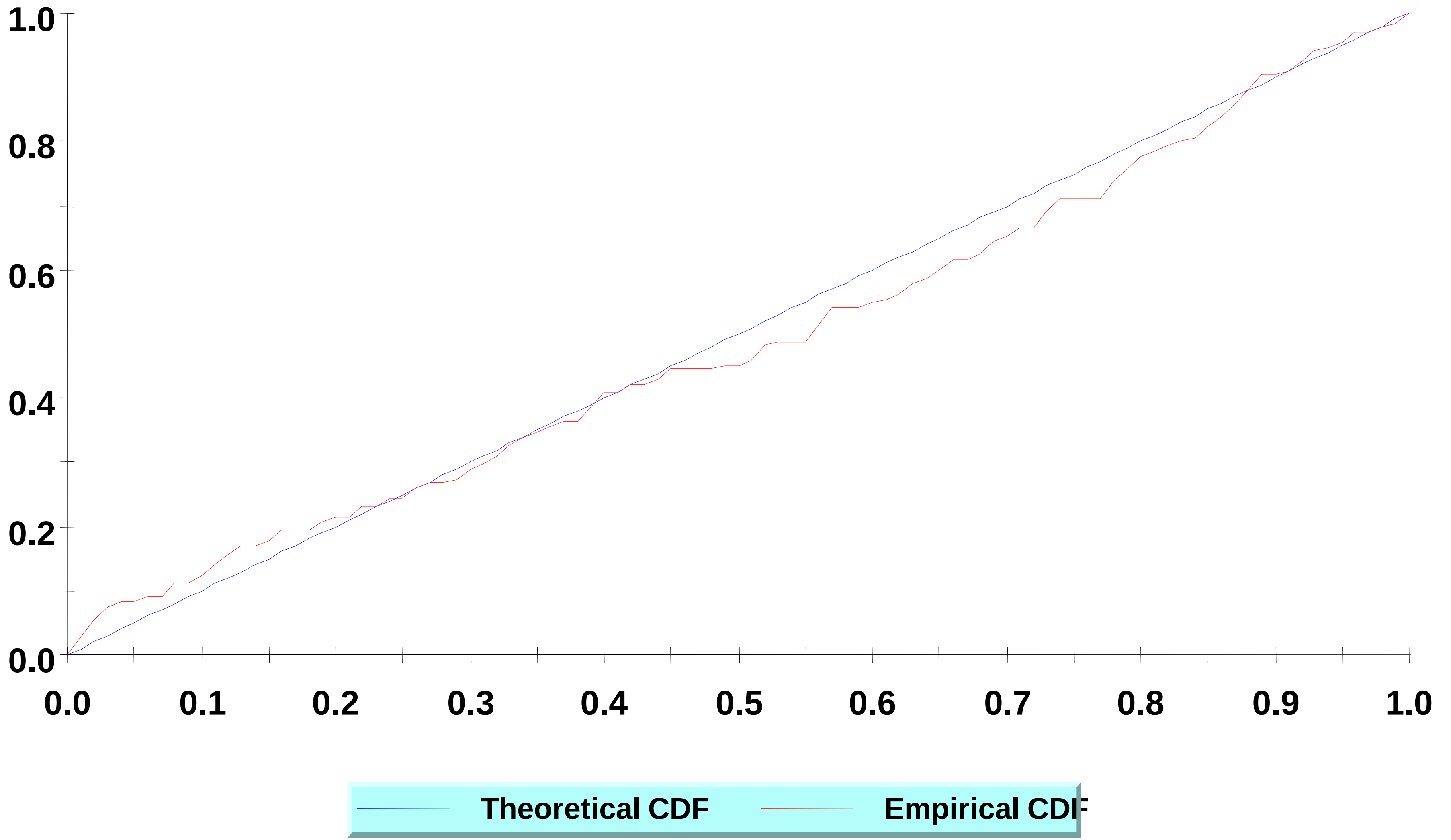
Bond Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = .081481

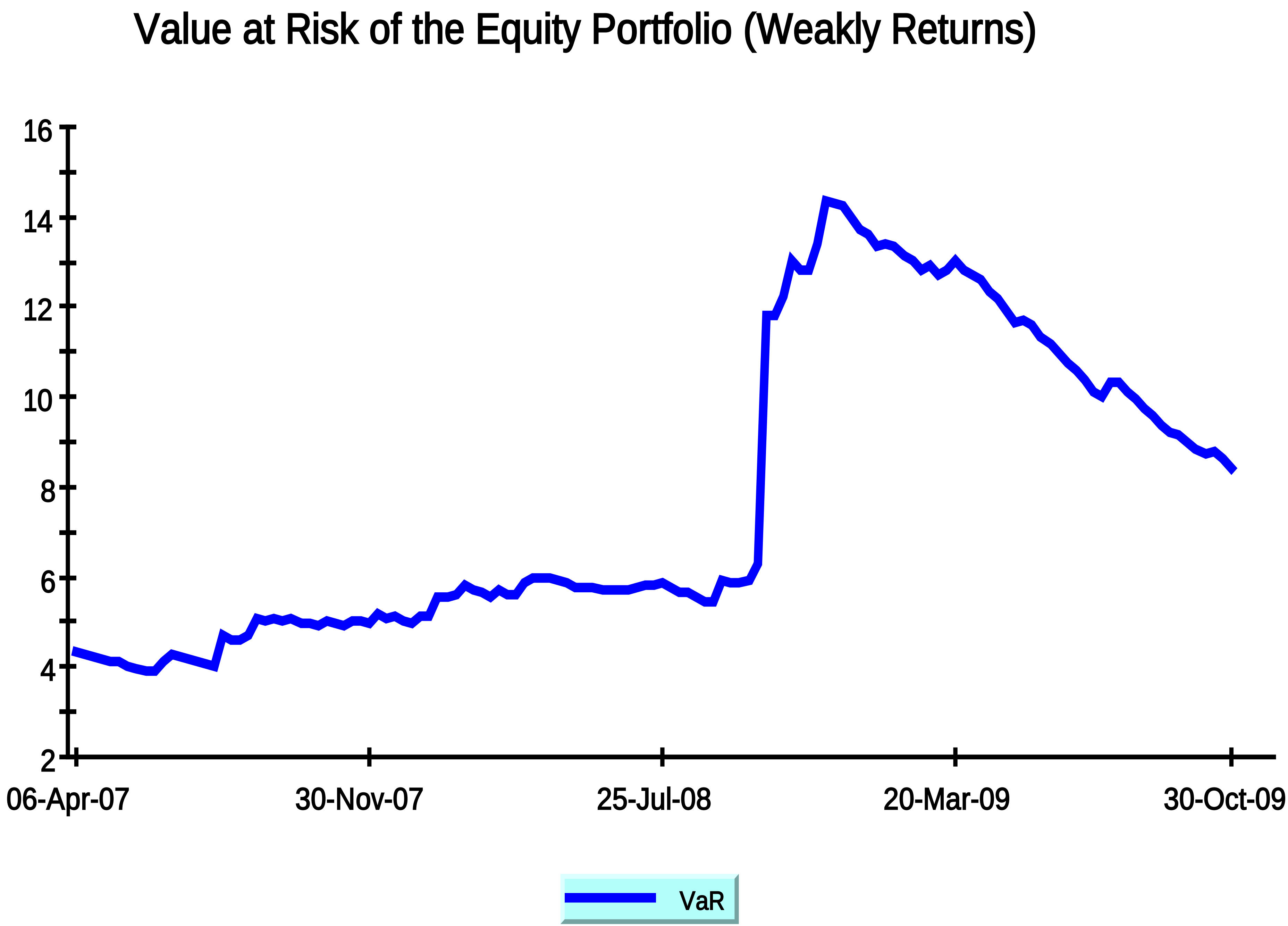
5% Critical value = .11705

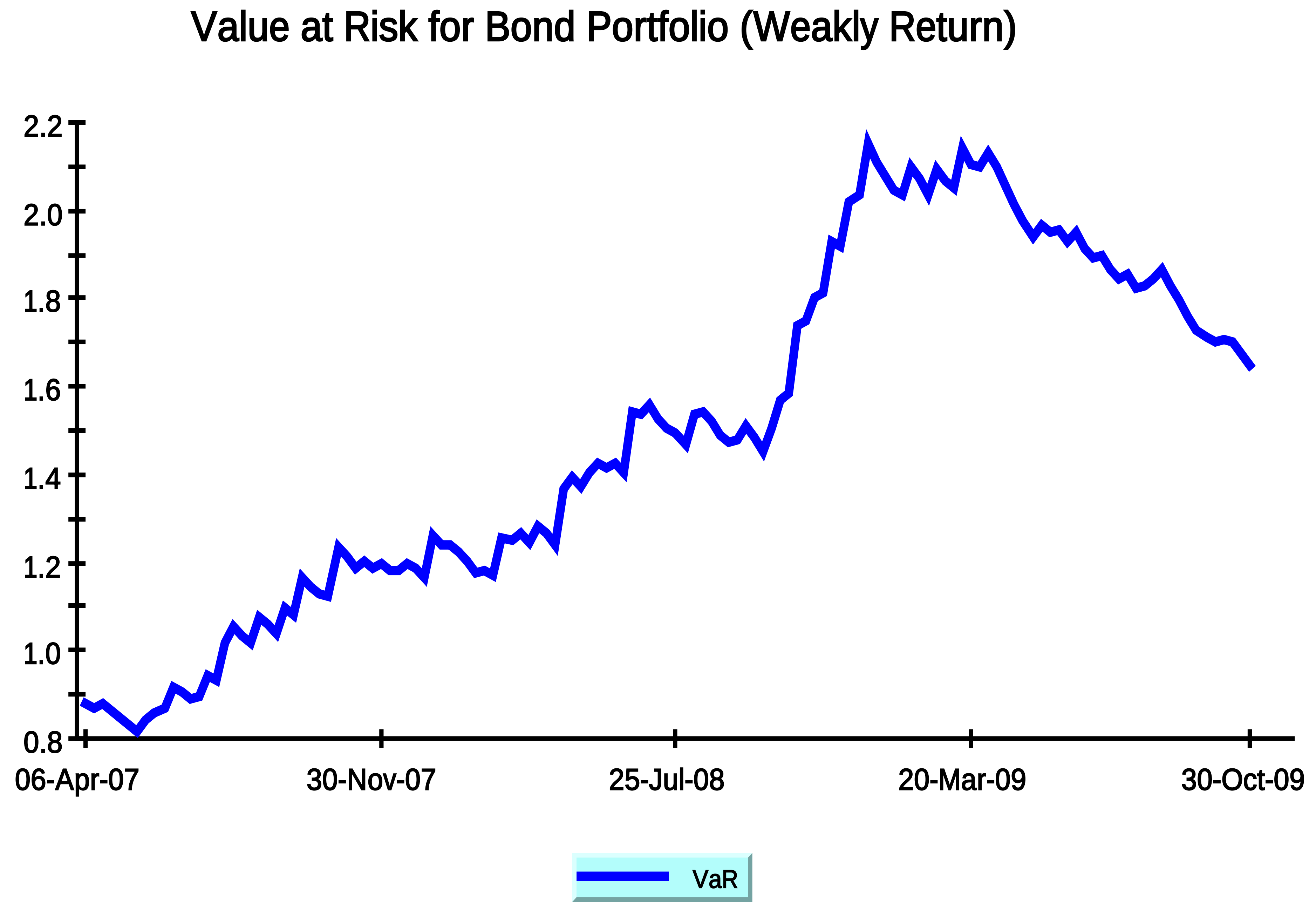


Combined Portfolio - Kolmogorov-Smirnov Goodness-of-Fit Test = .061111

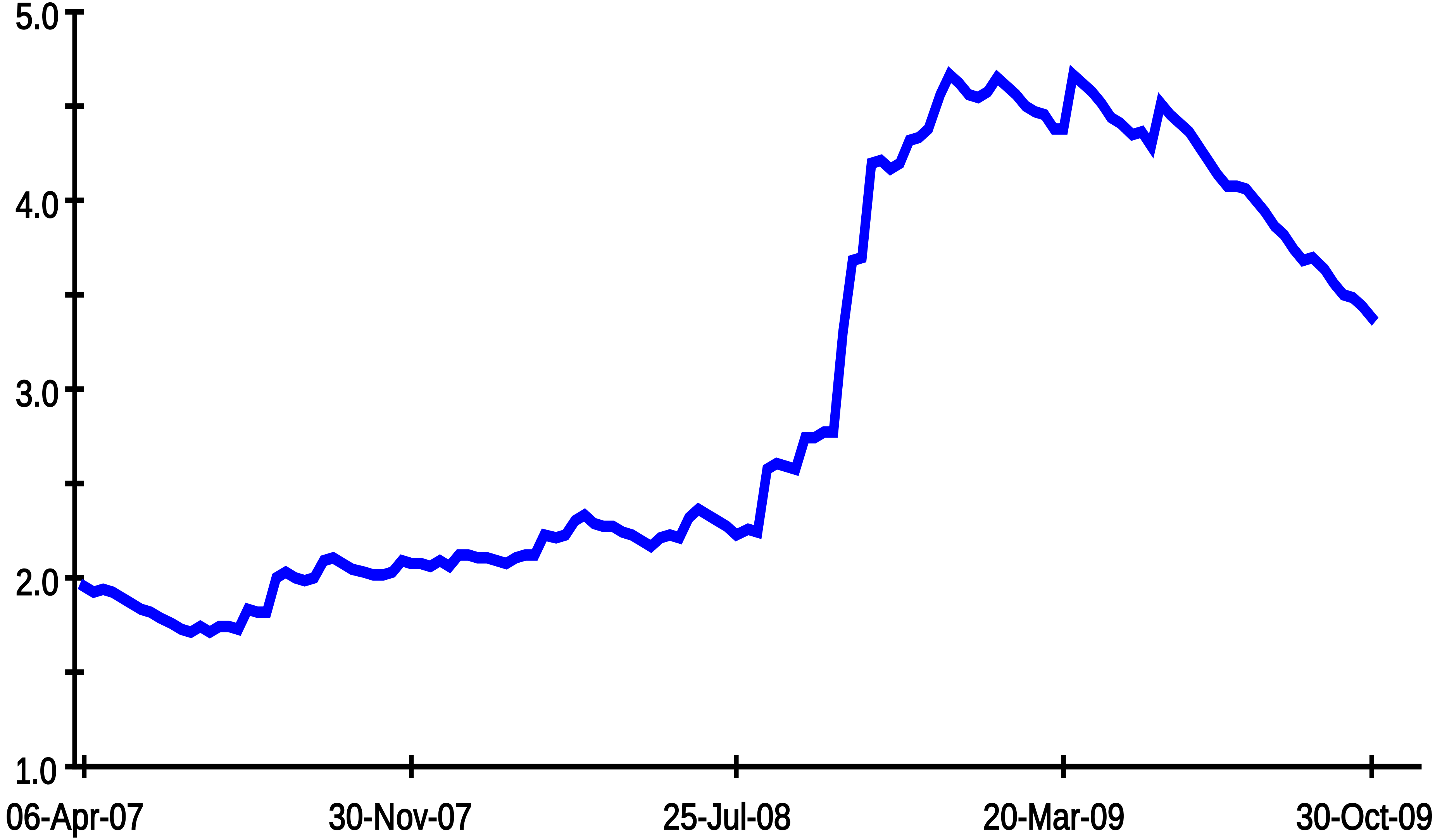
5% Critical value = .11705



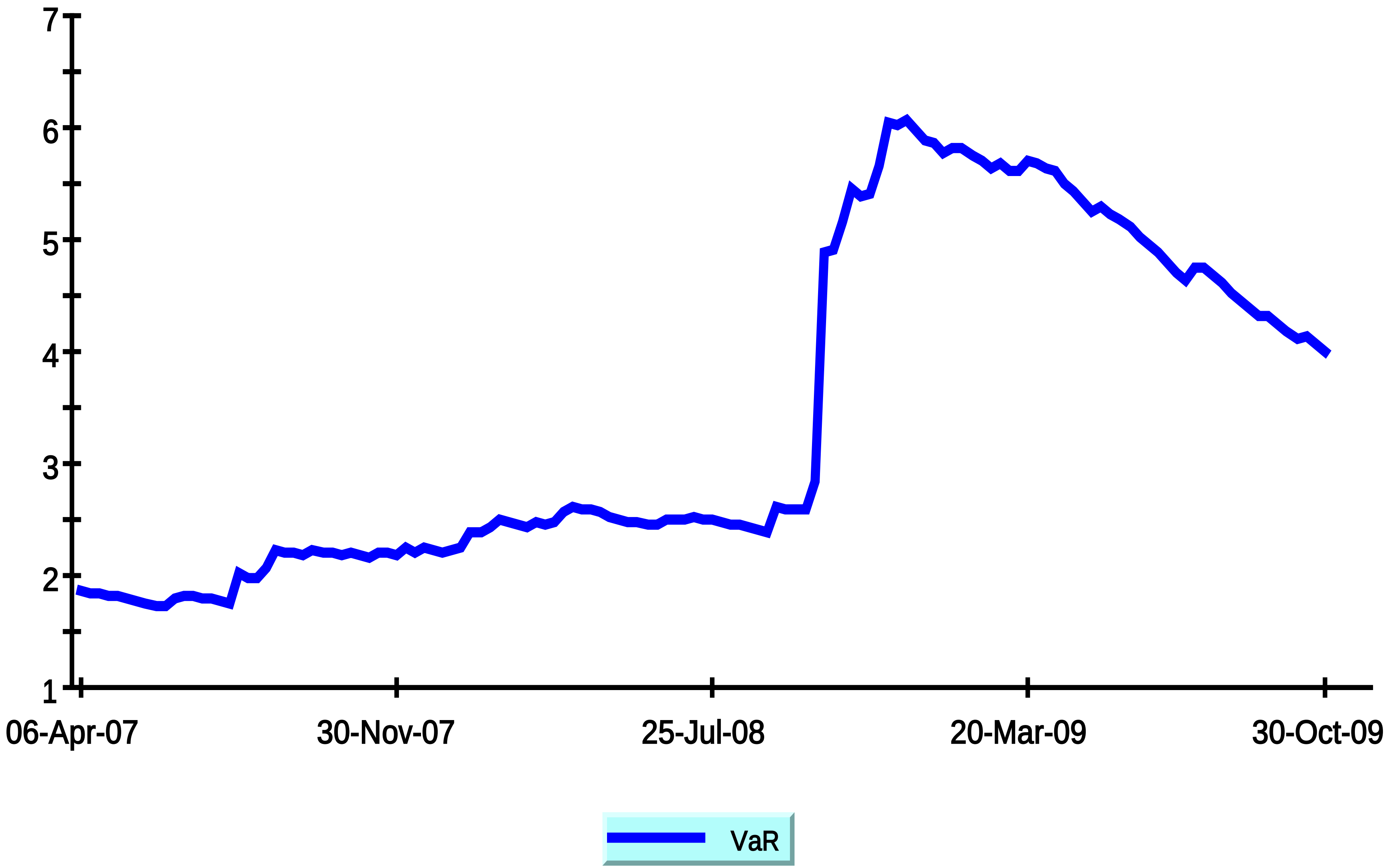

Value at Risk of the Equity Portfolio (Weakly Returns)
16
14
12
10
8
6
4
2
06-Apr-07
30-Nov-07
25-Jul-08
20-Mar-09
30-Oct-09
VaR


Value at Risk for Bond Portfolio (Weakly Return)
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
06-Apr-07
30-Nov-07
25-Jul-08
20-Mar-09
30-Oct-09
VaR

Value at Risk for the FX Portfolio (Weakly Return)



VaR of the Combined Portfolio



Main Features of the Results

- Variations in asset returns have become more volatile - although they have been declining recently
- Asset return correlations have been rising - recent crisis led to further increases.
- The rise in asset return correlations is more reflective of underlying trends - globalization and integration of financial markets.

Main Features of the Results (continued)

- To deal with model uncertainty we advocate the use of 'average' models, and explore their use in optimal portfolio choice.
- Simple decision-based model evaluation tests in terms of VaR performance are proposed
- The test is applicable to individual as well as to average models - the TDCC specification passes the VaR based tests.
- The main problem is how to predict sudden shifts in volatilities and conditional correlations.