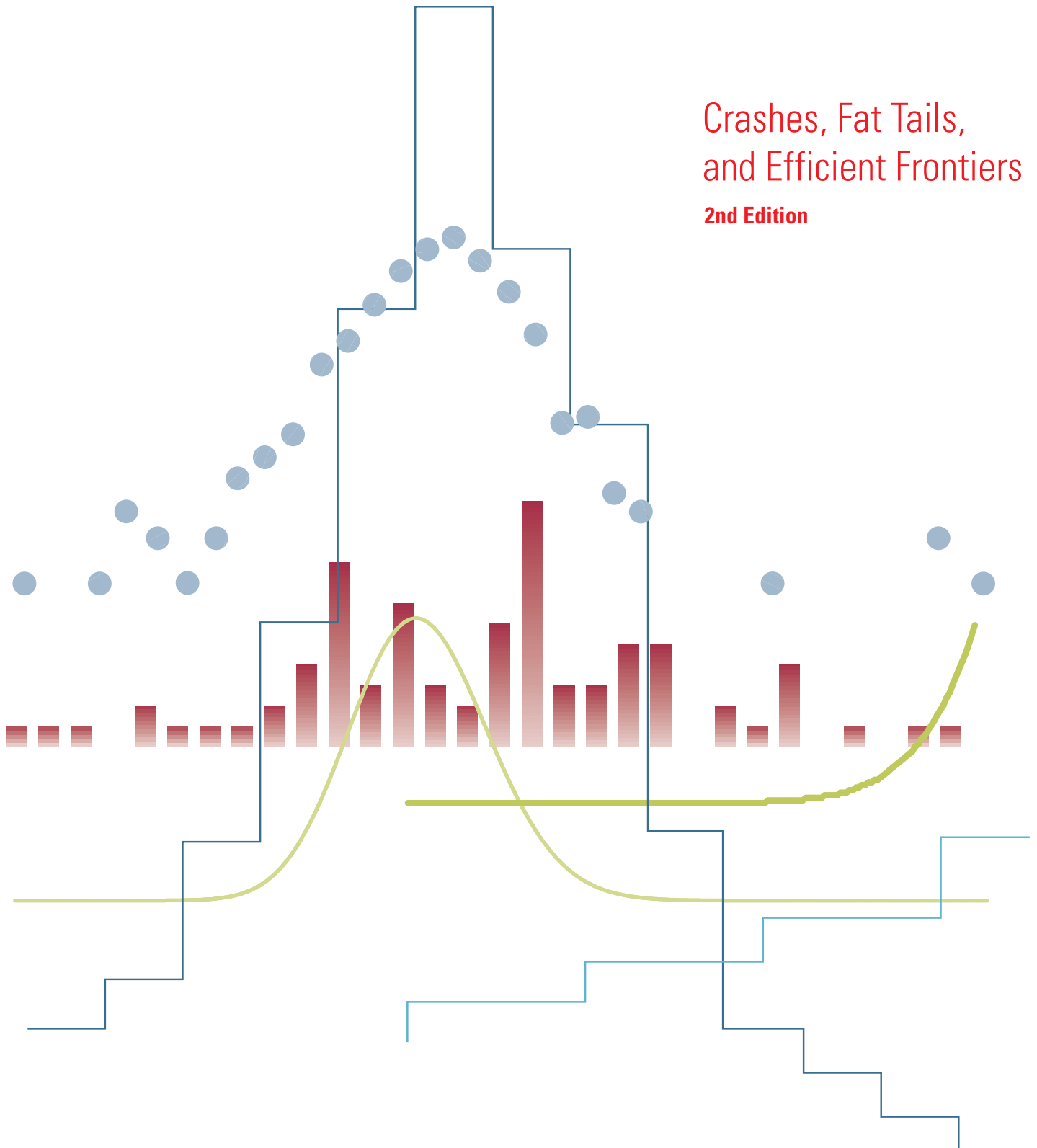


Crashes, Fat Tails, and Efficient Frontiers

2nd Edition



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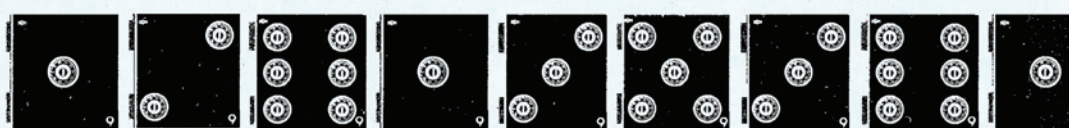
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Déjà Vu All Over Again

By Paul D. Kaplan

When risk models fall short, advisors need to look no further than the historical record to plan for the next 100-year flood.

"We seem to have a once-in-a-lifetime crisis every three or four years."

—Leslie Rahl, founder of Capital Market Risk Advisors¹

The dramatic events on Wall Street and in financial centers around the world that started on "Black Sunday," Sept. 14, have upset many common assumptions about the global financial system. What started as a mortgage crisis spread to nearly every corner of the financial system when Lehman Brothers collapsed, Merrill Lynch sold itself to Bank of America, and AIG became strapped for cash—all in a single weekend. These and the events that followed have shaken investor confidence to the core. As of Dec. 31, the Dow Jones Industrial Average was down 22.4% since Black Sunday. The yield spread on junk bonds over LIBOR reached an unprecedented 16%. The markets for many assets have become illiquid, and credit is dried up for nearly anyone who needs it. The U.S. Federal Reserve, the U.S. Treasury, and their counterparts around the world have taken dramatic steps to restore liquidity to asset markets, stimulate lenders to make loans again, shore up investor

confidence in equity markets, and avoid a deep global recession.

If you need to be reminded how bad things are, listen to our political and fiscal-policy leaders as they describe the crisis with phrases that begin with the ominous words "once in a" As they were pushing their \$700-billion bailout package last fall, members of the Bush administration said that the crisis was a "once-in-a-century event," and this was echoed in November by Henry Paulson, the former secretary of the U.S. Treasury, who said the meltdown was a "once- or twice-in-a-100-year event." Former Federal Reserve chairman Alan Greenspan characterized the crisis as a "once-in-a-century credit tsunami."

There's little doubt that aspects of this crisis are unique and that the economy is facing its hardest challenge since the Great Depression, but are severe economic crises the rare events Paulson, Greenspan, et al., have suggested? A study of capital market history suggests no. To see this, you need to look no further than the Ibbotson Stocks, Bonds, Bills, and Inflation poster from Morningstar hanging on your wall.

Take, for example, the poster's depiction of the compound annual return of the S&P 500 Index, identified on the chart as Large Stocks.^{2,3} The growth of \$1 to \$2,049 over 83 years is impressive (a rate of 9.6% per year), but the record is peppered with several long and severe declines, some in the not-too-distant past.

To illustrate our point, we isolated the S&P 500 line of the poster and added blue areas that show the highest level that the cumulative value of the S&P 500 had achieved as of that date (Exhibit 1). Wherever a blue area is shown, the S&P 500 was amid a decline relative to its most recent peak. The deeper the gap, the more severe the decline; the wider the gap, the longer the time until the S&P 500 returned to its peak. Wherever a blue area is not shown, the S&P 500 was climbing to a new peak.

Not surprisingly, the granddaddy of all market declines started with the Crash of 1929 and did not recover until 1945. The S&P 500 lost more than 83% of its value in about three years and took 12½ years to recover. What may be

¹ As quoted by Christopher Wright, "Tail Tales," *CFA Institute Magazine*, March/April 2007. ² We obtained the historical monthly total returns from Morningstar EnCorr, an institutional asset-allocation software and data package. ³ We use a logarithmic scale for all growth of \$1 charts.

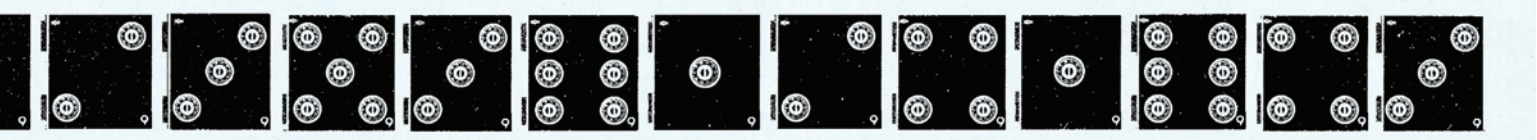
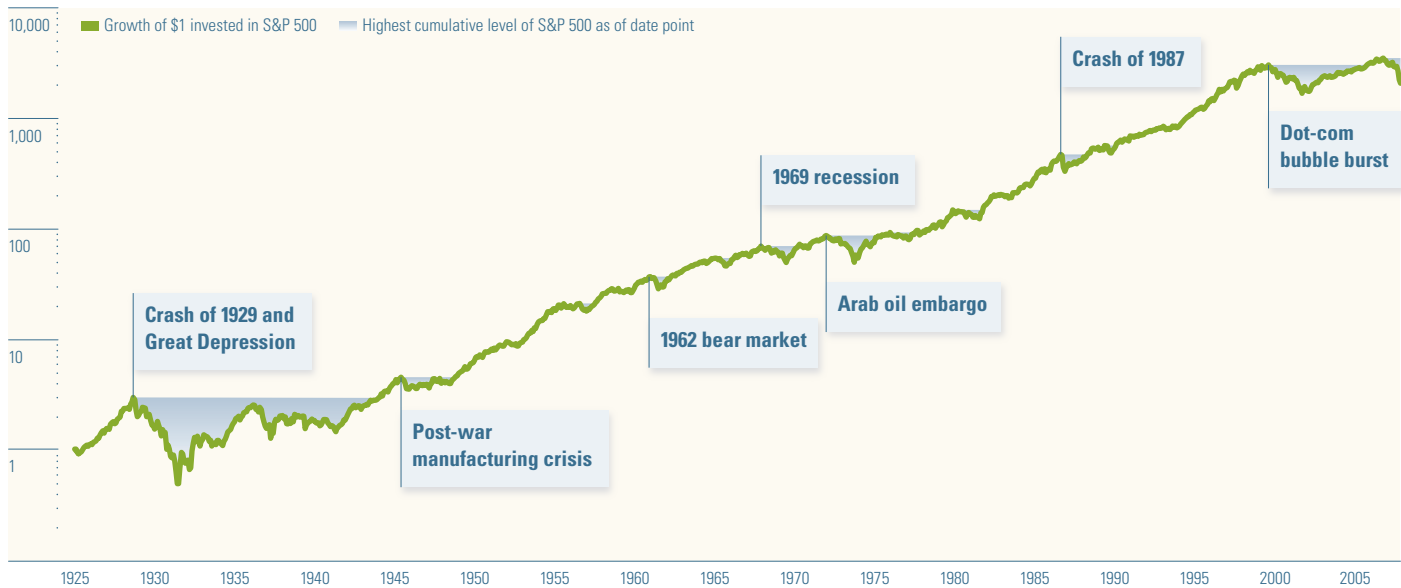


Exhibit 1: Mind the Gaps U.S. large-cap stocks have made impressive gains over the years, but several significant declines have interrupted the S&P 500's trajectory.



Growth of \$1 includes reinvested dividends. Monthly data used to calculate returns.

more sobering, however, is that the second-greatest decline took place within the past decade. With the crash of the Internet bubble in 2000, the S&P 500 lost almost 45% of its value over a two-year period and took four years to return to its peak value.

In all, including the current crisis, the S&P 500 has suffered eight peak-to-trough declines of more than 20% since the mid-1920s. Two of the three greatest declines occurred in the past eight years. To suggest that the current crisis is a once-in-a-century event ignores the record.

Measuring Risk: The Standard Model

With 20% declines occurring, on average, every decade or so, you'd think that the standard risk models that investors use to make their asset-allocation decisions would assign a significant probability that these events will occur. Think again. To see why, we need to look at the history of how these models were formed.

To help make sense of the highly complex capital markets, financial economists in 1960s and 1970s developed a set of mathematical models of the markets that are used to this day

throughout the investment profession. The best known of these models are the capital asset pricing model of expected returns and the Black-Scholes option pricing model. These models' creators have won the Nobel Prize in economics for their path-breaking work. Each of these models starts by making an assumption about the statistical distribution of stock market returns. The CAPM assumes that returns follow a normal, or bell-shaped, distribution. The Black-Scholes model assumes that returns follow a lognormal distribution.⁴

⁴ For returns to follow a lognormal distribution means that logarithm one plus the return in decimal follows a normal distribution.

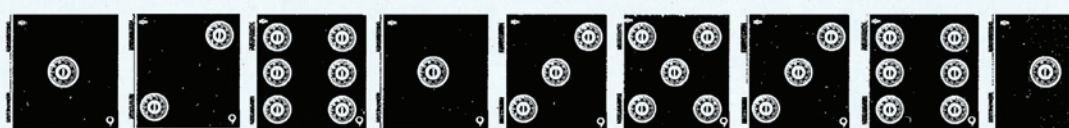
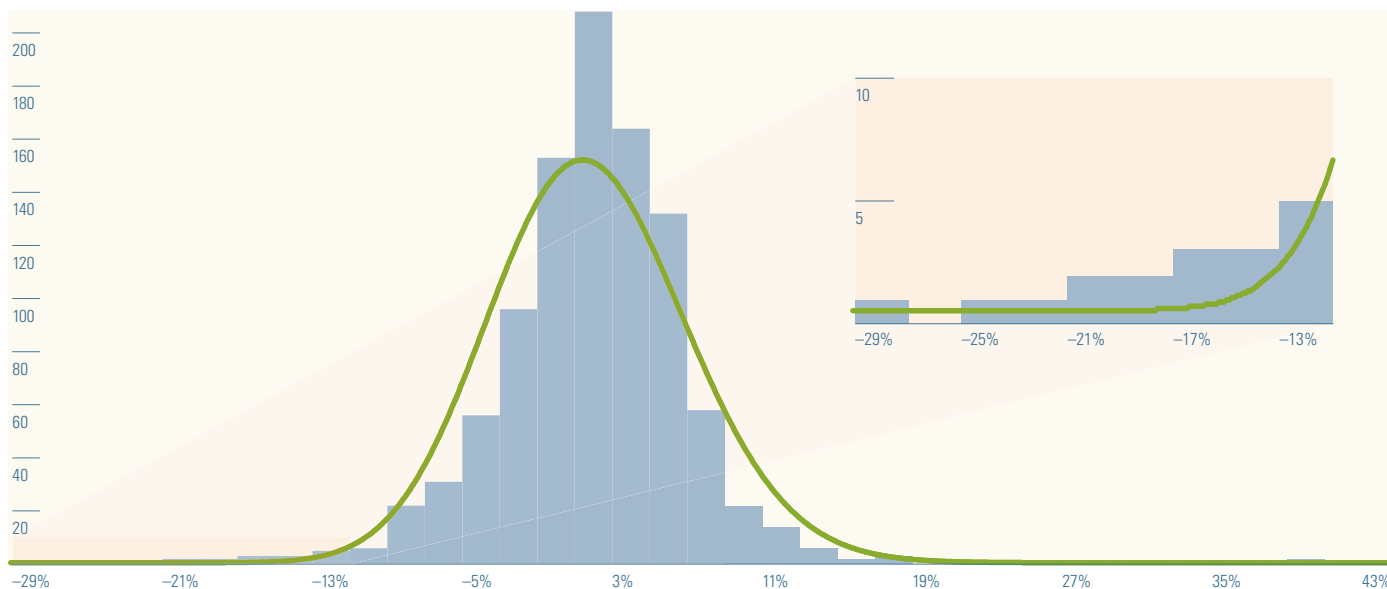


Exhibit 2: Cracks in the Bell Standard risk models assume S&P 500 returns follow a bell-shaped distribution, even though the index has experienced more than 10 declines of at least -13% .



Histogram shows the frequency of monthly returns for the S&P 500 from January 1926 to November 2008.

With these standard models, the primary measure of risk is standard deviation. If returns follow a normal distribution, the chance that a return would be more than three standard deviations below average would be a trivial 0.135%. Since January 1926, we have 996 months of stock market data; 0.135% of 996 is 1.34—that is, there should be only one or two occurrences of such event.

But the record of the stock market tells a different story. The monthly returns of the S&P 500 have been more than three standard deviations below average 10 times since 1926. In other words, the standard models assign meaninglessly small probabilities to extreme events that occur five to 10 times more than the models predict.

We can illustrate the problem further by overlaying a lognormal model of returns on a histogram of monthly total returns on the S&P 500 (Exhibit 2). The model says that declines of more than negative 13% have almost no chance of happening—yet they have occurred at least 10 times since 1926.

An Alternative Approach: Log-Stable Distributions

In the early 1960s, Benoit Mandelbrot, a mathematician teaching economics at the University of Chicago, was advising a doctoral student named Eugene Fama. Mandelbrot had developed a statistical model for percentage changes in the price of cotton that had “fat tails.” That is, the model assigned nontrivial probabilities to large percentage changes. In his doctoral dissertation, Fama applied

Mandelbrot’s model to stock prices and obtained promising results.⁵ Until recently, however, the work of Mandelbrot and Fama had been largely ignored.⁶

In his dissertation, Fama assumed that the logarithm of stock returns followed a fat-tailed distribution called a “stable Paretian distribution,” or stable distribution.⁷ Hence, we refer to the resulting distribution of returns as a “log-stable distribution.”

We can illustrate an example of Fama’s work by using the same S&P 500 histogram in our earlier exhibit but with a log-stable distribution curve overlaying it instead of a lognormal curve.⁸ The log-stable model (Exhibit 3) fits the empirical distribution much closer than the lognormal both at the

⁵ For an account of the work of Mandelbrot and Fama during this period, see Benoit Mandelbrot and Richard L. Hudson, *The (Mis)Behavior of Markets*, New York: Basic Books, 2004. ⁶ The idea of using fat-tailed distributions to model asset returns is starting to gain some traction. FinAnalytica was founded to provide investment analysis and portfolio construction software based on Mandelbrot and Fama’s work. Morningstar added distribution charts and forecasting models based on it to Morningstar EnCorr. ⁷ Strictly speaking, the assumption is that the logarithm of one plus the return in decimal form follows a stable Paretian distribution. ⁸ This chart can be produced in Morningstar EnCorr Analyzer using the log-stable feature.

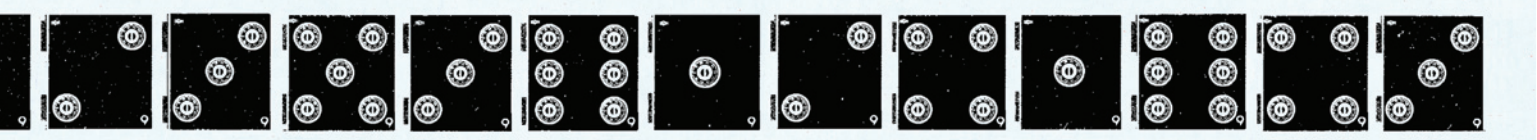
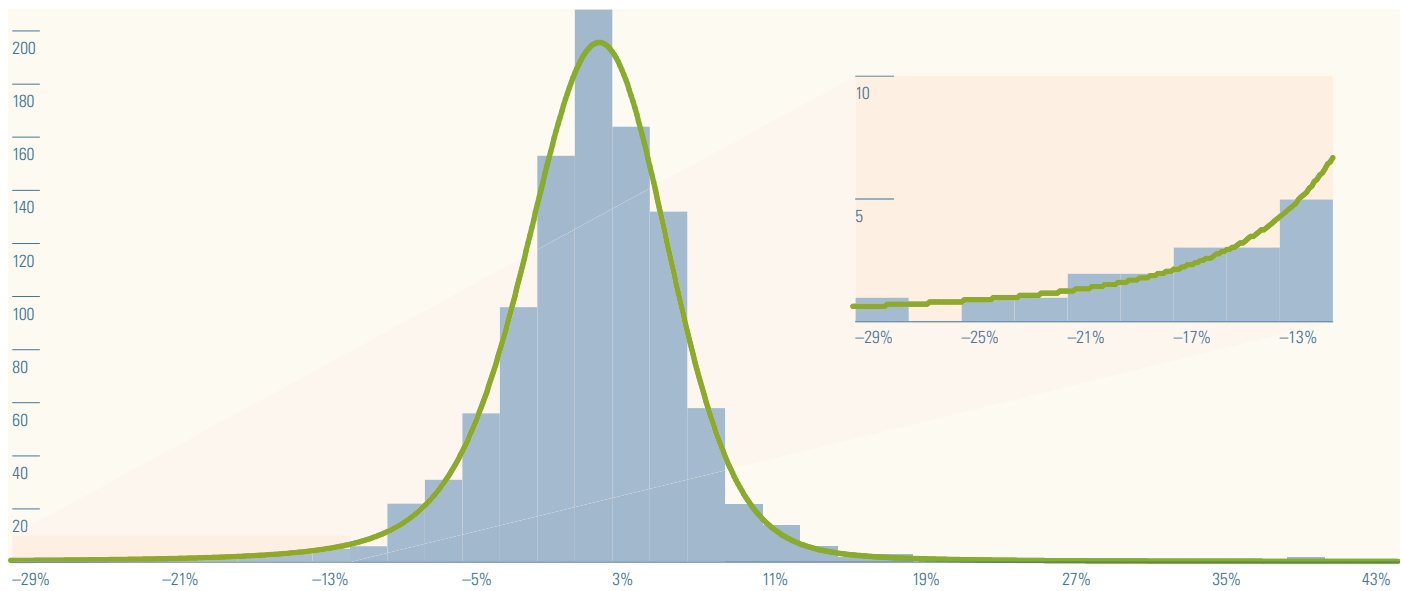


Exhibit 3: It's a Fat-Tailed World, After All A log-stable distribution does a good job of modeling the empirical returns of the S&P 500, especially at the center and the tails.



Histogram shows the frequency of monthly returns for the S&P 500 from January 1926 to November 2008.

center and the tails. In particular, note the close match between the density curve and the histogram between negative 13% and negative 29%.

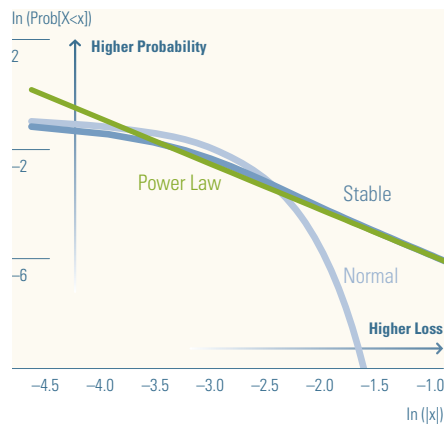
The tails of a stable distribution are so fat that its variance is infinite. In other words, the concepts of standard deviation and variance are not defined for stable distributions. You might find the idea of an infinite variance counterintuitive, because it is possible to calculate a standard deviation for any finite set of data. However, the underlying mathematical distributions that we use to model asset returns assign probabilities over the range from negative infinity to positive infinity.⁹ Some distributions that cover this infinite range assign so little probability out in the tails that variance can be defined. These are “thin-tailed”

distributions, the normal or bell-shaped distribution being the best-known example. Other distributions assign so much probability to the tails that variance is infinite. Such is the case with stable distributions.

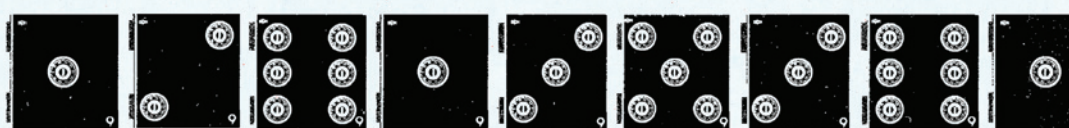
The manner in which a stable distribution assigns probability to its tails is very close to what is known as “power law.” When a distribution of a loss follows a power law, a plot of logarithm of the magnitude of loss (x) versus the logarithm of the probability of the loss turning out to be x or worse is a downward-sloping straight line. Therefore, while the probability of loss decreases with the magnitude of loss, it does so gradually.

In Exhibit 4, we plot the magnitude of loss versus the logarithm of the probability of

Exhibit 4
Power Law Tails: Unlike a normal distribution, a stable distribution approaches the straight line of a power law, indicating that it has “fat tails.”



⁹ That is the probability distribution of one plus the return on an asset return in decimal form. The lowest possible return on an unlevered position in an asset is negative 100%, which is negative 1 in decimal form. Adding one we get 0. The logarithm of 0 is $-\infty$.

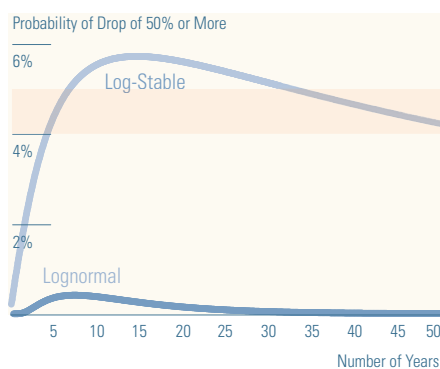


loss for a normal distribution, a stable distribution, and a power law distribution. The line for the normal distribution curves down, indicating that it has thin tails. In contrast, the line for stable distribution approaches the straight line of the power law because it is very similar to a power law for large losses.

These results show that the log-stable distribution does a good job of modeling the empirical returns distribution of the S&P 500. The better fit of the log-stable distribution demonstrates that the S&P 500 has fatter tails than predicted by the lognormal model. It also calls into question commonly used portfolio construction techniques such as the mean-variance optimization, which relies on the assumption of a finite variance.

If the log-stable model does such a better job in describing the distribution of asset returns, why has it not received more acceptance? There are several possible reasons. First, the mathematics is challenging. Second, the variances and all higher moments of stable

Exhibit 5
Role of Time: The log-stable model indicates that there's a 4% to 5% probability that the S&P 500 will lose 50% or more over extended time periods. The lognormal model puts the odds much lower.



Hard Eight

The S&P 500 has suffered eight peak-to-trough declines of more than 20%.

Peak	Trough	Decline %	Recovery
August 1929	June 1932	83.41	January 1945
August 2000	September 2002	44.73	October 2006
December 1972	September 1974	42.64	June 1976
October 2007	November 2008	40.89	To Be Determined
August 1987	November 1987	29.58	May 1989
November 1968	June 1970	29.16	March 1971
December 1961	June 1962	22.28	April 1963
May 1946	November 1946	21.76	October 1949

Table shows the worst cumulative peak-to-trough declines in percentage terms since December 1925. Based on monthly returns.

random variables are infinite. The lack of a finite variance means that most portfolio theories and most portfolio construction techniques are invalid, including those based on alternative risk measures such as “downside risk.” Finally, there is no single obvious way to estimate the parameters of stable distributions as there is with normal distributions.

Risk Measures versus Risk Models

For advisors, the lesson here is not that they should throw away the standard ways of summarizing risk using measures such as standard deviation and downside deviation.¹⁰ Nor should advisors run to embrace Fama’s log-stable models.

Instead, we think advisors should understand the limitations of standard risk measures and have a basic understanding of what Mandelbrot’s and Fama’s work says about describing risk. Rather than solely relying on a few summary statistics to characterize the risks of

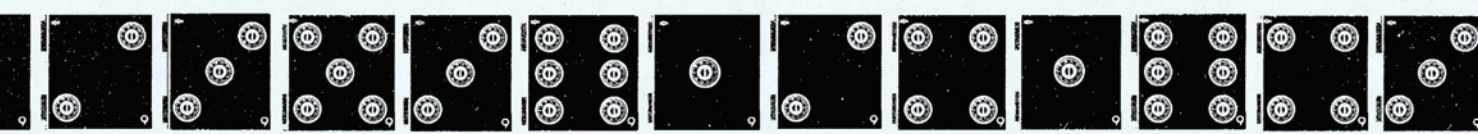
an investment, advisors would benefit by beginning to think about a more complete risk model. A complete risk model allows investors to consider three questions about a potential decline in value simultaneously:

- ▶ How likely might a decline occur?
- ▶ How long might it last?
- ▶ How bad might it get?

It is already common practice in some segments of the financial-services industry to use a risk model to measure “value at risk”—that is, how bad a loss might be over a given length of time and with a given probability.

As you can appreciate through our study of historical stock market declines, time horizon is a key dimension of risk not explicitly addressed by standard risk measures. A complete risk model can be used to explicitly take time horizons into account.

¹⁰ In recognition that return distributions may not be symmetric, measures such as skewness and kurtosis are sometimes presented alongside standard deviation. However, like variance, these measures are not defined for stable Paretian distributions.



For example, in Exhibit 5, we plot the probability of a cumulative loss of 50% or more over various time horizons using the lognormal distribution for the S&P 500 that we show in Exhibit 2 and the log-stable distribution in Exhibit 3. The lognormal model shows that the risk of such a severe decline over an extended period is negligible. The log-stable model, on the other hand, indicates that such a loss over an extended period has a probability of 4% to 5%—numbers significant enough to gain the attention of risk-averse advisors and investors who might want to be prepared for such a scenario.

Conclusion

In every financial crisis, investors relearn the same message—there isn't a magic risk measure or model that can account for or predict every significant drop in the market. Economists and quantitative analysts have made incredible strides over the decades engineering new ways to explain the distribution of returns. These developments provide investors with valuable information to help them decide how to allocate their portfolios for any number of investing scenarios and mitigate risk. But they are not perfect.

As we've shown, the record contains a much bumpier ride than many risk models would suggest. In addition to preparing clients' portfolios for these occasional severe declines and taking other precautions, advisors would do well to keep reminding their clients of the risks they face as investors. Clients should be fully prepared to take on the 100-year floods they will surely face in the future. ■■■

Paul D. Kaplan, Ph.D., CFA, is Morningstar's vice president of quantitative research and a frequent contributor to *Morningstar Advisor*.

We Are Not Alone

The uneven performance of the stock market is hardly unique to the United States. Severe declines—mostly within the past decade—have occurred in developed markets since January 1970. Here are the worst declines for seven countries.

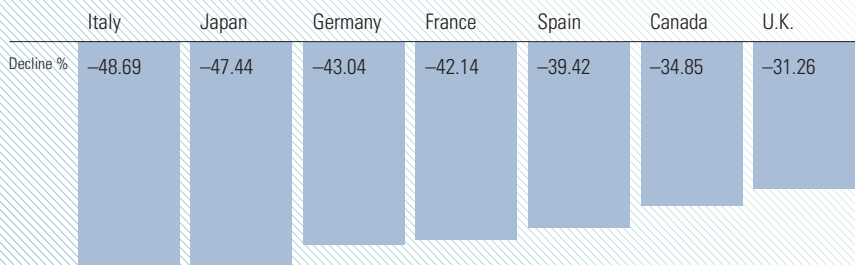
Country	Peak	Trough	Decline %	Recovery
Germany	February 2000	March 2003	67.89	April 2007
Japan	December 1989	April 2003	67.62	To Be Determined
U.K.	August 1972	November 1974	64.73	January 1977
Italy	June 1973	December 1977	59.39	September 1980
Spain	April 1974	November 1979	58.81	March 1984
France	August 2000	March 2003	58.28	March 2007
Canada	August 2000	September 2002	47.11	September 2005

Source: Morgan Stanley Capital International and Morningstar EnCorr. Chart shows monthly return data in local currency for major stock-market index in each country.

The Japanese market has yet to recover from its peak in December 1989.



The markets in four of the seven countries have performed worse since October 2007 than the U.S. market, which has fallen 40%.



Data through December 2008. Based on monthly returns.

Quant Corner: One and a Quarter Centuries of Stock Market Drawdowns

Real stock market returns reveal the true frequency of “once-in-a-century” crashes.



by
Paul D. Kaplan, Ph.D., CFA
Vice President,
Quantitative Research

When former Federal Reserve chairman Alan Greenspan characterized the financial crisis of 2008 as a “once-in-a-century credit tsunami,” I was stunned. Being familiar with long-term data on the U.S. capital markets, I thought a more apropos statement was the one made by Leslie Rahl (founder of Capital Market Risk Advisors) more than year before the crisis when she said, “We seem to have a once-in-a-lifetime crisis every three or four years.”¹ The contrast between Greenspan’s and Rahl’s perspectives inspired me to write an article for *Morningstar Advisor* on the history of market meltdowns, “Déjà Vu All Over Again.”² In that article, I illustrate the frequency and severity of the major drawdown for various countries using time series of stock market total returns. For the U.S., I naturally used the series on the S&P 500 that Morningstar publishes in the Ibbotson® SBBI® Yearbooks and makes

available in its EnCorr® software and data package that starts in 1926. The results clearly demonstrate that Greenspan was in need of a history lesson.

I have recently expanded the analysis into a complete study on global equity market history upon the request of Larry Siegel, director of research at the CFA Institute, as a contribution to his forthcoming book on the global history of market crashes.³ Larry asked me to use monthly *real* total returns⁴ and to go back into history as far as it was possible with reasonably reliable data. The benefit of using real returns is to make meaningful return comparisons, as our study spans such a long period of time. The benefit of going further back in history is, of course, to give us a longer-term and more robust historical perspective on market crashes, in terms of frequency, length, and magnitude.

To complete the study, I needed to find monthly data from before 1925 on both stock returns and inflation, and calculate real returns. Since there was no such return series in existence, I would have to create one out of readable available data.

Professor Robert Shiller of Yale posts a monthly history of U.S. stock market returns and inflation on his Web site that goes back

to 1871. Unfortunately, Professor Shiller’s stock data is based on monthly average prices rather than month-end prices. So I could use his inflation data, but not his stock market data. Separately, Roger Ibbotson and some colleagues created an annual price and total return series for the NYSE that goes back to 1825.⁵ However, annual returns are at too low a frequency to measure the largest drawdowns of the period, such as the large drop in the stock market during the panic of 1907. Fortunately, Larry had a book that contained daily price data on the Dow Jones Averages going back to 1885.⁶ He advised me to estimate the monthly price returns in the broader NYSE price index from the monthly price returns on the Dow Jones Averages and then interpolate the total returns by assuming that the level dividends remained constant during each year.

Following Larry’s advice, and soliciting the help of Morningstar intern Kailin Liu, I produced a time series of real total returns for the U.S. stock markets that runs from 1871 through the present. While for the first 15 years we only have annual returns, we now have more than 123 years of monthly total real returns. This data will appear in future editions of the Ibbotson SBBI Yearbooks, beginning in 2010.

CONTINUED ON NEXT PAGE

Exhibit 1: Real Index and Peak Values of the U.S. Stock Market



Exhibit 2: Largest Declines in U.S. Stock Market History (in real total return terms, from January 1871 to June 2009)

Peak	Trough	Decline %	Recovery	Event(s)
August 1929	May 1932	79.00	November 1936	Crash of 1929, 1st part of Great Depression
August 2000	February 2009	54.00	TBD	Dot-Com Bubble Burst (2000–02), Crash of 2007–09
December 1972	September 1974	51.86	December 1984	Inflationary Bear Market, Vietnam, Watergate
June 1911	December 1920	50.96	December 1924	World War I, Postwar Auto Bubble Burst
February 1937	March 1938	49.93	February 1945	2nd part of Great Depression, World War II
May 1946	February 1948	37.18	October 1950	Postwar Bear Market
November 1968	June 1970	35.46	November 1972	Start of Inflationary Bear Market
January 1906	October 1907	34.22	August 1908	Panic of 1907
April 1899	June 1900	30.41	March 1901	Cornering of Northern Pacific Stock
August 1987	November 1987	30.16	July 1989	Black Monday—Oct. 19, 1987
October 1892	July 1893	27.32	March 1894	Silver Agitation
December 1961	June 1962	22.80	April 1963	Height of the Cold War, Cuban Missile Crisis
November 1886	March 1888	22.04	May 1889	Depression, Railroad Strikes
April 1903	September 1903	21.67	November 1904	Rich Man's Panic
August 1897	March 1898	21.13	August 1898	Outbreak of Boer War
September 1909	July 1910	20.55	February 1911	Enforcement of the Sherman Anti-Trust Act
May 1890	July 1891	20.11	February 1892	Baring Brothers Crisis

Truth in Numbers

The significance of this data is in the lessons that we can learn from it. Over the entire 138½-year period, the Real US Stock Market Index grew from \$1 to \$5,179 in 1869 dollars. This is a compound annual real total of just under 6.4%, almost the same as the post-1925 period. However, as Exhibit 1 shows, it was a very bumpy ride with a number of major drawdowns, some of which can be linked with specific economic and political events.

Exhibit 1 shows the growth of \$1 invested in the U.S. stock market at the end of 1869 through June 2009 in real terms, along with a line that shows the highest level that the index had achieved as of that date. Wherever this line is above the cumulative value line, the index was amid a decline relative to its most recent peak. The bigger the gap, the more severe the decline; the wider the gap, the longer the time until the index returned to its peak. Wherever this line coincides with the index line, the index was climbing to a new peak.

Exhibit 2 lists all of the drawdowns that exceeded 20%. In total, there were 17 such declines, including the present one from which we have yet to recover. Not surprisingly, the granddaddy of all market declines started just before the Crash of 1929 and did not recover until toward the end of 1936. The U.S. stock market lost 79% of its real value in less than three years, and it took more than five years to recover. What may be more sobering, however, is that not only are we currently in the second-greatest decline, but it started nine years ago! The combined effect of the crash of the Internet bubble in 2000 and the financial crisis of 2008 caused the U.S. stock market to lose 54% of its real value from August 2000 to February 2009. Who knows how long it will take to recover from that and when our next crisis will occur?

The history of stock market drawdowns presented here shows that investing in stocks can be very risky business, indeed, and that the current crisis is hardly a “once-in-a-century” event. But to more than just state the obvious, we should use this data to better gauge the potential risks and long-term rewards of investing in risky assets such as stocks. Specifically, we should supplement our traditional measures of risk, such as standard deviation, which relies on a normal distribution, by measures that better capture the fat-tailed nature of the historical returns and drawdowns as presented here. Incorporating fat-tailed distributions into risk measures has become a focus of my research. Stay tuned for more. ■■

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- 1 As quoted by Christopher Wright, “Tail Tales,” *CFA Institute Magazine*, March/April 2007.
- 2 February/March 2009. Available at http://www.nxtbook.com/nxtbooks/morningstar/advisor_20090203/.
- 3 This study will appear in the CFA Institute’s forthcoming book, *Voices of Wisdom: Understanding the Global Financial Crisis*, Laurence B. Siegel, editor.
- 4 That is, returns that include the reinvestment of dividends and are adjusted for inflation.
- 5 Goetzmann, William N., Roger G. Ibbotson, and Liang Peng, “A New Historical Database for the NYSE 1815 to 1925: Performance and Predictability,” *Journal of Financial Markets*, December 2000. The data appear in the *Ibbotson S&P Yearbooks*.
- 6 Pierce, Phyllis, ed. 1982. *The Dow Jones Averages 1885–1980*. Dow Jones Irwin, Homewood, Illinois.

Quant Corner: Stock Market Bubbles and Crashes

A global historical and economic perspective.



by
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Director,
Morningstar Europe

Adapted from "The History and Economics of Stock Market Crashes," by Paul D. Kaplan, Thomas Idzorek, Michele Gambera, Katsunari Yamaguchi, James Xiong, and David M. Blanchett, in *Insights into the Global Financial Crisis*, Laurence B. Siegel, ed., ©2009 Research Foundation of CFA Institute. Portions reproduced and republished with permission from the Research Foundation of CFA Institute. All rights reserved.

According to Shiller (2005), the term "irrational exuberance" is credited to Alan Greenspan, former chairman of the U.S. Federal Reserve Board. In his book, *Irrational Exuberance*, Shiller explains that Greenspan used this term in a 1996 speech:

"But how do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?" [Greenspan] added that,

'We as central bankers need not be concerned if a collapsing financial asset bubble does

not threaten to impair the real economy, its production, jobs, and price stability.'

"Immediately after [Greenspan] said this, the stock market in Tokyo, which was open as he gave this speech, fell sharply, and closed down 3%. Hong Kong fell 3%. Then markets in Frankfurt and London fell 4%. The stock market in the U.S. fell 2% at the open of trade."

Although it is unlikely that Greenspan's simple statement was intended to cause the reaction that it did, the term "irrational exuberance" has now become associated with any period when investors are in a heightened state of speculative fervor. Speculative fervors, or bubbles as they are more popularly known, may be easy to identify with the benefit of hindsight, but they are not nearly as easy to identify when they are occurring. Moreover, they are not by any means new phenomena. Even though the recent market crash beginning in 2007 is likely fresh on the mind of the reader, there have been many others, all around the world, and some far worse.

To place the market meltdown of 2008–2009 in historical perspective, we examine the long-term record of stock market total return indexes¹ for the United States, the United Kingdom, and Japan. We also examine the record of the regional stock market indexes (stated in U.S. dollars) for Asia ex-Japan, Europe, and Latin America from 1988 to the

present and compare them with the indexes for Japan and the United States over that same period to see which of the more recent crashes were regional and which were global in nature. Finally, we look to economic theory to help explain bubbles and crashes and apply these theories to the recent financial crisis. While we don't think bubbles and crashes can be prevented entirely, we believe that necessary steps must be taken to reduce the frequency and magnitude of financial crises.

The U.S. Record

Kaplan (2009) presents the real total return index and the peak values of the U.S. stock market over the period January 1871 through June 2009, a period of just more than 138 years. (See *Morningstar Alternative Investments Observer*, Third Quarter 2009.) Kaplan shows that an investment in a hypothetical index fund of the U.S. stock market held over this period (with all dividends reinvested and no taxes, fees, or other costs) would have grown nearly 5,000-fold in real purchasing power. Nonetheless, a number of significant sharp and/or long declines occurred along the way. The periods where there are gaps between the peak and the index are the times—called "drawdowns"—when the market in question fell below its own immediate past peak and later recovered.

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¹ Total Return Indexes include reinvestment of dividends.

The U.K. Record

The long-term equity returns for the United Kingdom bear a striking resemblance to those of the United States, highlighting how connected the two economies have been. The largest shock to the U.K. stock market over the past 109 years occurred shortly after the collapse of the Bretton Woods system and during the oil crisis that began Oct. 17, 1973, when members of the Organization of Arab Petroleum Exporting Countries, or OAPEC, proclaimed an oil embargo against select industrial governments of the world to pressure Israel during the fourth Arab-Israeli War.² Although the embargo was officially lifted in March 1974, the U.K. stock market did not

regain the peak reached in April 1972 until January 1984, roughly 12 years later.

The 74 percent drawdown in the real total return index of U.K. stocks in the 1970s is much worse than that same market’s decline in the Great Depression, despite the much more severe damage to the real economy in the earlier episode. Thus, markets do not always track real economic events exactly or even somewhat closely, as shown in Exhibit 1.

Japanese Record

The Japanese economy experienced a strong recovery following World War II and had relatively consistent growth through the 1980s,

with the stock market peaking in December 1989. The compound annual real total return of the Tokyo Stock Price Index, or TOPIX, from January 1952 to December 1989 was 13.4 percent.³ The market then declined for much of the subsequent two decades—with stock prices falling 71.9 percent from the 1989 peak, in real terms, by March 2009. Exhibit 2 includes information on the major declines in the Japanese stock market during the past six decades.

It is important to distinguish between market declines caused by business cycles and those caused by sudden unexpected crashes in Japan, as well as in other markets.

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Exhibit 1: U.K. Stock Market History, 1900–2009

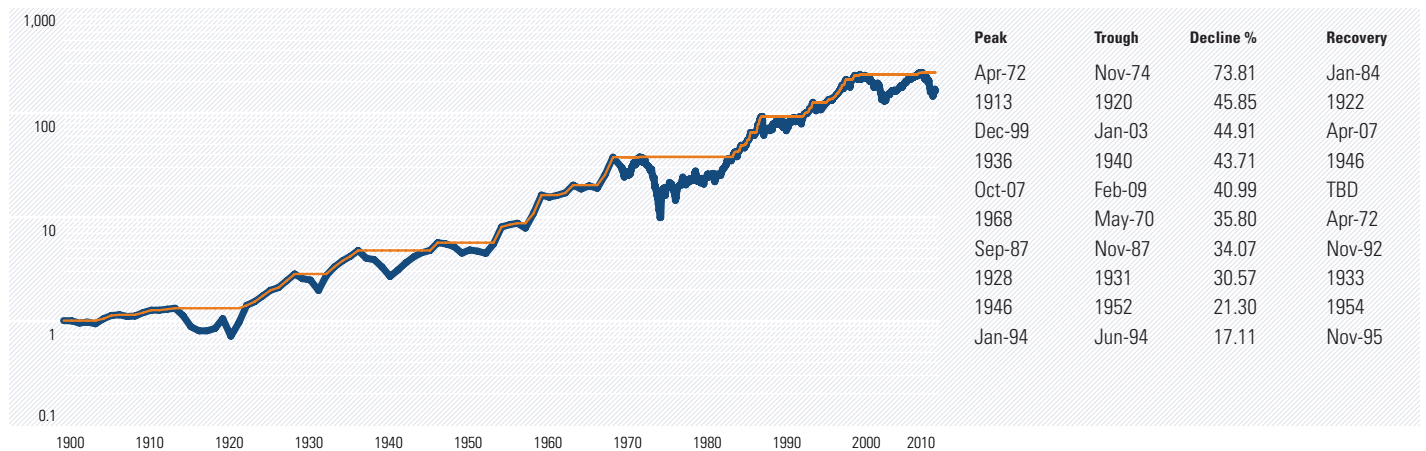
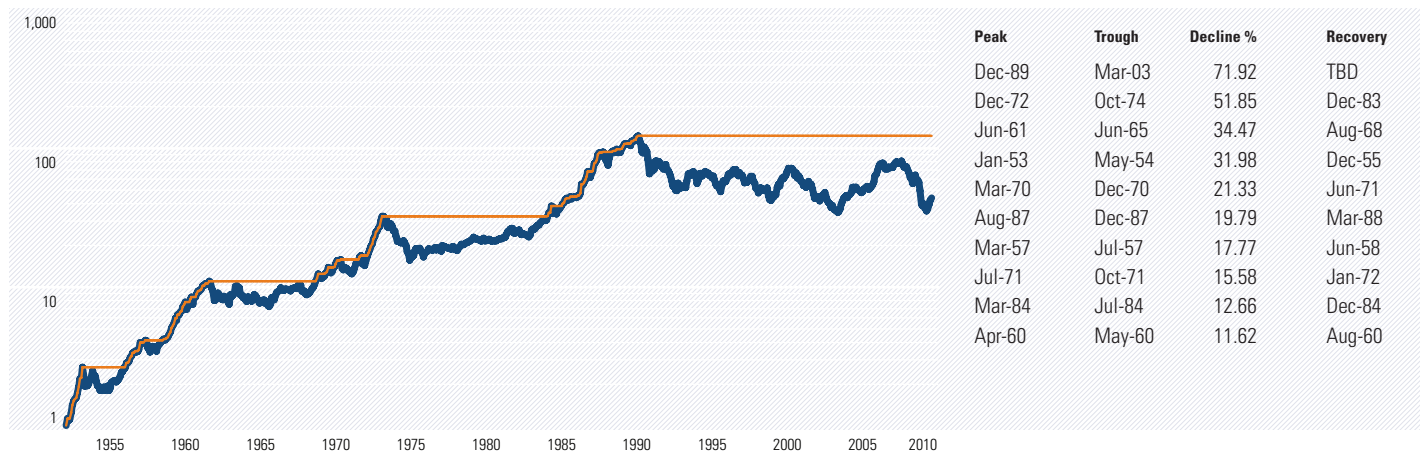


Exhibit 2: Japanese Stock Market History, 1952–2009



² The now better-known OPEC, Organization of Petroleum Exporting Countries, is a separate, overlapping organization.
³ The Tokyo Stock Exchange, TSE, is divided into three markets: the first section, the second section, and Mothers (venture capital market). The first section includes the largest, most successful companies. The TOPIX tracks all domestic companies of the TSE’s first section. See www.tse.or.jp/english/faq/list/general/g_b.html.

For example, the decline that began in December 1972 was triggered by currency instability and rising interest rates following the first oil crisis. The 1961–65 decline was caused, at first, by a tightening of monetary policy and deteriorating corporate earnings, culminating in a financial market crisis that led to a bailout of Yamaichi Securities in 1965. Those are bear markets—continuous declines caused by changes in fundamentals but without a big one-day or several-day “crash.”

Drawdowns During the Long Boom (1982–2007)

Stock markets around the world have experienced a number of large drawdowns over the past 20 years. Most of the period from January 1988 to June 2009 marked a time frame of continued growth for many countries and stock markets, a period often characterized as the “Long Boom.” Drawdowns of more than 50 percent, however, have actually occurred relatively frequently, even during the Long Boom. Generally, they have occurred in emerging-markets nations.

Apart from the crash of 2007–2009, both the Asia ex-Japan and Latin America stock markets have experienced market declines (in some cases experienced as crashes) of more than 50 percent. Exhibit 3 (Page 8) includes information on drawdowns around the world in various markets from January 1988 to June 2009. Unfortunately, we do not have data covering emerging markets in the first years of the Long Boom, 1982–1987.

Why Do Crashes Occur?

Financial crises and bank failures have occurred throughout history. As an example, Calomiris (2008) mentions a bank panic in ancient Rome in A.D. 33. In economies where subsistence farming and barter were widespread, however, banking crises affected only a small part of the population. In today’s world, banks and insurance companies affect a large part of the

economy. As a result, the health of the financial sector is a key factor in the economic cycle.

At the same time, economic theory has devoted increasing attention to the causes of financial crises.

Economic Thought and Financial Crises

Adam Smith stated that the existence of many small banks is a guarantee for the public because, among other things, it limits the systemic effect of the failure of any one bank (Smith 1776, Book II, Chapter II). Apart from Smith’s remarks, bank size was not at the heart of economic theory until recently. The banks at the core of the recent crisis are very large ones. If Smith’s observation is accurate, then something must be wrong with very large banks.

Joseph Schumpeter (1942) brought a new perspective to economic theory related to financial crisis, although his views were not intended as an explanation of one. In his view, technical innovation causes short-term disequilibrium in markets, and that such disequilibrium is a good thing because it fosters product variety and technical efficiency. Moreover, disequilibrium would be limited only to the markets where an innovation has recently occurred.

J.G. Knut Wicksell and Irving Fisher (see, for example, Fisher 1933) introduced a view of disequilibrium that specifically centered on financial markets, particularly the difference between the market interest rate and the equilibrium interest rate. The Walrasian model shows that, in a competitive equilibrium, the interest rate should equal the marginal productivity of capital.⁴ But Wicksell and Fisher pointed to a situation where the market interest rate differs from the equilibrium interest rate. The theory presented by Wicksell and Fisher implies that excessive lending causes financial crises that can stop an entire economy because they cause first a bubble and then a crash in many markets at the same time.

John Maynard Keynes (1936) set forth a theory that markedly differed from those of his predecessors. He argued, loosely speaking, that some special markets are almost never in equilibrium. For example, the labor market is generally in disequilibrium. Financial markets, Keynes quipped, “can stay irrational longer than you can stay solvent.” With this, he meant that financial markets are not perfectly efficient and that government policy, specifically fiscal “stimulus” (deficit spending to accelerate the demand for goods and services), may be a necessary remedy when a serious recession ensues. Not everybody knows that Keynes did not advocate large, persistent government budget deficits; he supported only focused actions against the most serious recessions.

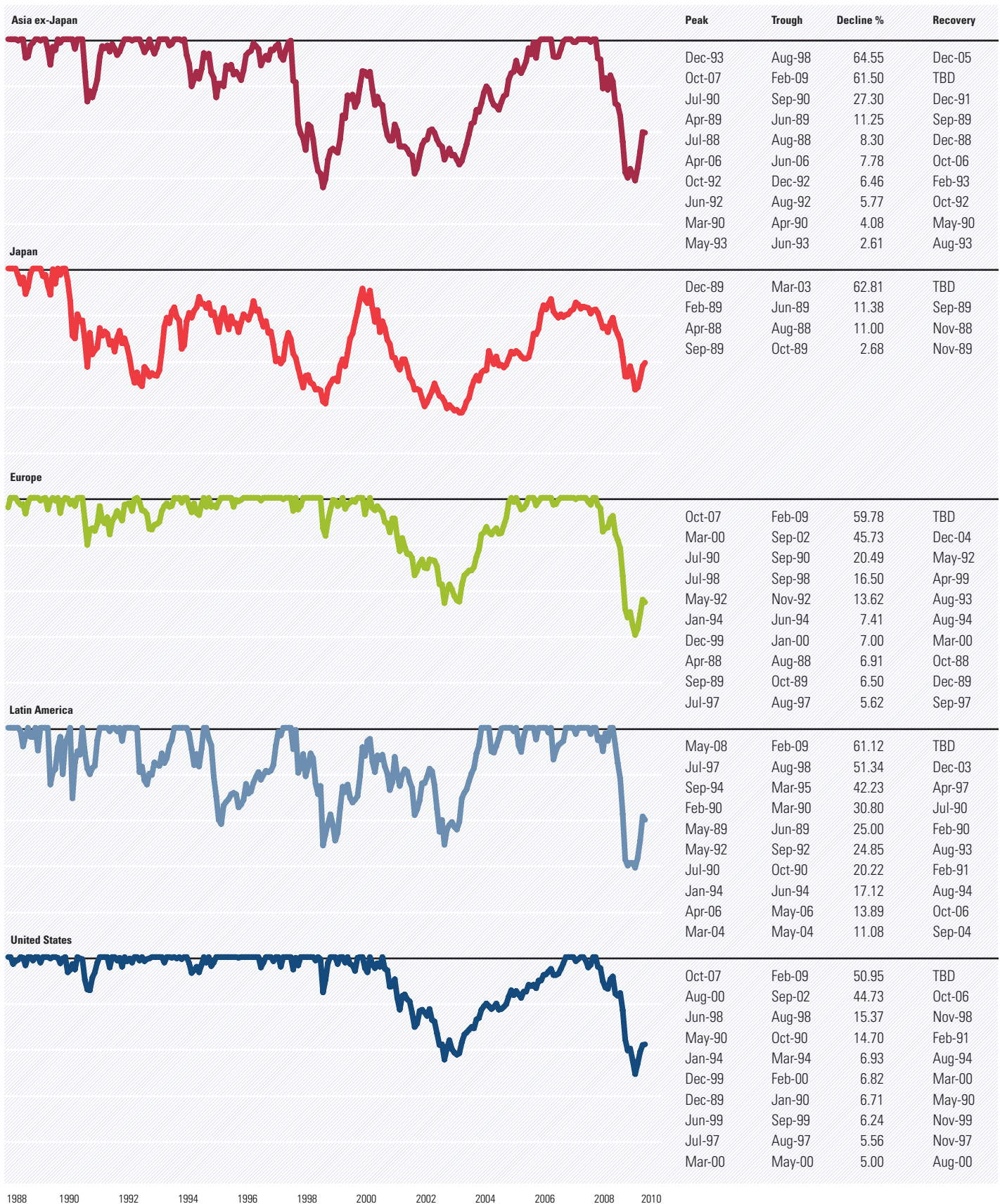
Hyman P. Minsky (1986, 1992) studied why markets are, in Keynes words, irrational, whereas Modern Portfolio Theory relied heavily on market efficiency, which is the exact contrary. Minsky’s insights fit nicely with the findings of behavioral finance. Briefly, Minsky argued that a lack of crises is the cause of future crises; that is, market stability is self-destructing. When market participants have been in a state of calm, they start believing that markets will remain calm for the foreseeable future and, therefore, start underestimating risk. As a result, they behave just like the overoptimistic bankers of Wicksell and Fisher. Minsky suggested some government intervention to prevent this kind of excess.

Finally, Friedrich A. Hayek (1932) believed that government intervention actually triggers a Wicksell-Hayek crash, in which the market interest rate diverges from the natural rate. His view was that when governments and central banks try to expand credit to sustain the economy when a recession is feared, as they typically do, they end up causing a deeper recession. Hayek trusted markets to be efficient enough to take care of themselves; prices and wages would change, and markets would

CONTINUED ON P. 9

⁴ Financial practitioners may be a bit puzzled here because most economic theory relies on just one interest rate, with neither a yield curve (because models often focus on one or two periods) nor a credit spread (because there is no uncertainty). If that is your point of reference, please bear with us because there are useful insights for everyone in the finance viewpoint, which incorporates multiple time horizons and uncertainty.

Exhibit 3: Worst Drawdowns Around the World, January 1988–June 2009 (U.S. Dollars)



go back to equilibrium right away. He thought that workers should accept lower wages when the marginal product of their labor decreased and that governments prevented wage falls for demagogic reasons, which in the end hurt workers.

2007–09 Crash

How do the events of 2007–2009 fit into the aforementioned theories? It is now clear that many financial institutions had taken on too much debt and extended too much credit, thus accumulating an excessive amount of risk. In our opinion, this was a failure in several dimensions:

- ▶ Regulators allowed such accumulation of risk by allowing excessive leverage.
- ▶ Shareholders and boards of directors did not require sound risk management.
- ▶ Market participants underestimated risk.
- ▶ Academics believed too much in market efficiency and were reluctant to admit the possibility of market irrationality, even though some had spent the previous decade analyzing the technology bubble of the 1990s and the subsequent crash.
- ▶ Politicians were all too happy to see the economy grow at an excessive speed because that was good in the short run.
- ▶ Financial company CEOs were also quite happy to see short-term profits swell, hoping that the inevitable crash would occur after they had retired and cashed out of the company.

The events of the residential real estate markets in the United States and in other countries, such as Spain and Iceland, summarize the key points of the crisis. Home prices kept increasing, and people wanted to buy homes, hoping not only to live in them but also to profit from their appreciation in value. Mortgage brokers, whose compensation depended on the number and size of mortgages they originated, gave mortgages to as many people as possible, regardless of whether these people could afford the mortgages. Banks, in a period of low spreads, were looking for fee income and looked for mortgages to be securitized and sold to investors. Investors, frustrated by otherwise low yields, were eager

to purchase higher-yielding mortgage-backed securities, without too much worry about the quality of the securities and, therefore, the sustainability of the yields. Bond-rating agencies, whose income (ironically) comes from bond issuers, made billions of dollars by trusting faulty risk models that gave AAA ratings to questionable mortgage-backed securities. Regulators did not recognize the risk of excessive leverage and allowed banks and other nondepository financial firms—for example, investment banks—to use off-balance-sheet vehicles to hide the risks of securitization from their financial statements.

Therefore, this period saw market inefficiency, inadequate or inconsistent government vigilance, and a Wicksell-Fisher-Hayek-Minsky chain of events leading to excessive lending, a bubble, and a crash (see Cooper 2008). The crash causes a Keynesian aggregate demand drop with ineffective monetary policy because of already low policy interest rates. This is the so-called liquidity trap (see Keynes 1936. For more about the liquidity trap in the current crisis, see Krugman 2008.)

What Have We Learned?

To prevent a repeat of the same type of crisis in the future, we believe that more comprehensive regulation of the financial system is necessary. This does not mean that we advocate red tape, but that supervisors must guarantee transparency and limit leverage. Moreover, this regulation should not only be limited to banks but also apply to insurance companies, investment banks, other nondepository financial institutions, and their holding companies.

When market participants realized that a crash was imminent, they tried to sell all risky assets to take refuge in safe investments, such as short-term government bonds. The leading risk models used by most participants did not consider this possibility. As a result, we believe that risk models must consider scenarios of sudden flight to quality, and financial analysts

should consider this kind of risk when building portfolios and developing their risk models. Moreover, we believe that some aspects of the financial infrastructure, such as the derivatives market, need reform. In particular, a reduction of over-the-counter derivatives transactions would lead to a more transparent and safe financial sector. ■■■

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Déjà vu Around the Word

“We seem to have a once-in-a lifetime crisis every three or four years.”
 --Leslie Rahl, found of Capital Market Risk Advisors¹



by **Paul D. Kaplan, Ph.D., CFA**
 Vice President, Quantitative Research

What started as a mortgage crisis in the United States quickly spread to nearly every corner of the financial system when Lehman Brothers collapsed, Merrill Lynch sold itself to Bank of America, and AIG became strapped for cash—all in a single weekend. These and the events that followed shook investor confidence to the core. Stock markets around the world plummeted as exemplified by the FTSE 100 falling 65% from September to March.

As the markets for many assets became illiquid, and credit dried up for almost everyone who needed it, the Bank of England, the U.S. Federal Reserve, the U.S. Treasury, and their counterparts around the world took dramatic steps to restore liquidity to asset markets, stimulate lenders to make loans again, and shore up investor confidence in equity markets in an attempt to avoid a deep global recession. Political and fiscal policy leaders here in the colonies helped sell their \$700 billion bailout package last fall as an extraordinary remedy for a “once-in-a-century event.” This was echoed in November by Henry Paulson, the former U.S. Secretary of the Treasury, who said the meltdown was a “once- or twice-in-a-100-year event” and former Federal Reserve Chairman Alan Greenspan who characterized the crisis as a “once-in-a-century credit tsunami.”

There's little doubt that aspects of this crisis are unique and that the economy is facing its hardest challenge since the Great Depression, but are severe economic crises the rare events Paulson, Greenspan, et al, have suggested? A study of capital market history around the world suggests no, and perhaps nowhere more clearly than in the United Kingdom. While Americans think of the greatest decline in stock market history as occurring during the 1930s, for British investors, the worst decline was in the 1970s. After taking into account the impact of inflation and even after reinvesting all dividends, the British stock market fell almost 74 percent from April 1972 to November 1972 and took nearly a decade to recover to its previous level.²

Exhibit 1 illustrates the inflation-adjusted growth of £1 invested at the end of 1969 in the MSCI UK Gross Return Index.³ While overall, this investment would have grown to the equivalent of 5.6 times in purchasing power by the end of May 2009, the record is peppered with several long and severe declines. Exhibit 2 lists the worst of these declines.

Exhibit 1: British Record: Disaster, Crisis, Recovers, & Growth

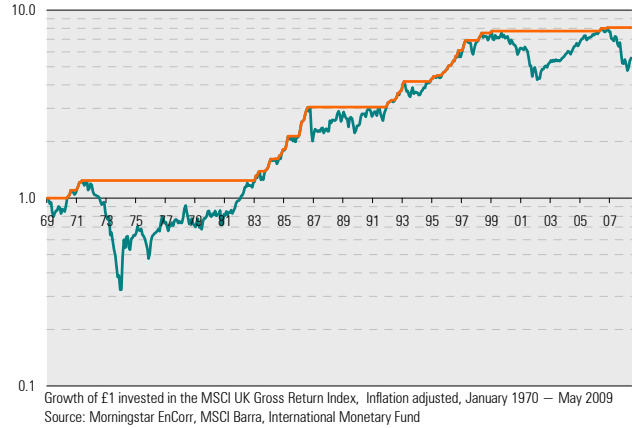


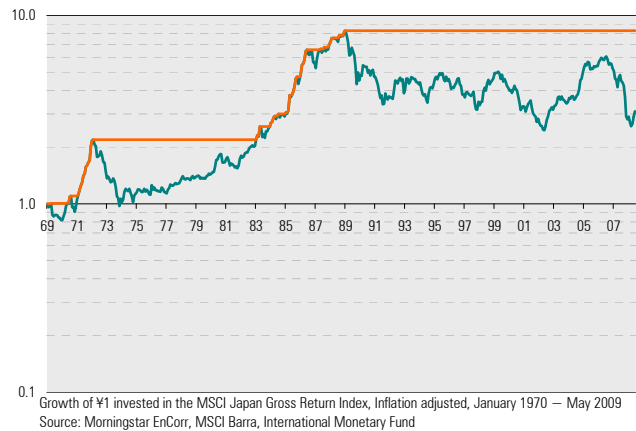
Exhibit 2: Largest Peak-to-Trough Declines for the U.K.

Peak	Trough	Decline	Recovery
April 1972	November 1974	73.81%	January 1984
December 1999	January 2003	44.91%	April 2007
October 2007	February 2009	40.99%	To Be Determined
September 1987	November 1987	34.07%	November 1992
December 1969	May 1970	20.38%	May 1971

Month-end inflation-adjusted results as of May 2009 since 1969
 Source: Morningstar EnCorr, MSCI Barra, International Monetary Fund

Looking at the prosperous island nation at the other side of Eurasia, the story is even more frightening. Exhibit 3 shows that over the same nearly 40-year period, the Japanese stock market is still in its second extended period of decline; and this one began nearly 20 years ago!

Exhibit 3: The Japanese Record: Lightning Can Strike Twice



Furthermore, the capital market histories of the United Kingdom and Japan are not unique. Exhibit 4 depicts the largest inflation-adjusted declines in eight industrialized countries (including the U.K. and Japan) over the past four decades. All of the largest markets suffered a major decline over the period, which clearly illustrates that level of stock risk is high indeed.

Exhibit 4: Largest Peak-to-Trough Declines in Eight Countries Since 1969

Country	Peak	Trough	Decline	Recovery
Spain	April 1973	April 1980	85.36%	December 1996
Italy	January 1970	December 1977	82.58%	March 1986
U.K.	April 1972	November 1974	73.81%	January 1984
Japan	December 1989	April 2003	70.33%	To Be Determined
Germany	February 2000	March 2003	69.44%	To Be Determined
France	August 2000	March 2003	60.52%	To Be Determined
Canada	February 1980	June 1982	51.38%	March 1986
U.S.	December 1999	February 2009	54.84%	To Be Determined

Month-end results as of May 2009 in inflation-adjusted local currency
 Source: Morningstar EnCorr, MSCI Barra, International Monetary Fund

Modeling Risk: The Standard Model

With large prolonged declines occurring with such frequency, you’d think that the standard risk models investors use to make their asset-allocation decisions would assign a significant probability that these events will occur. Think again. To see why, we need to look at how these models were formed.

To help make sense of the highly complex capital markets, financial economists in 1960s and 1970s developed a set of mathematical models of the markets. The best known of these models are the Capital Asset Pricing Model (CAPM) of expected returns and the Black-Scholes Option Pricing Model. Their creators won the Nobel Prize in economics for their ground-breaking work. Each of these models is built on the assumption that the statistical distribution of market returns follows a normal, or bell-shaped, distribution.⁴ And even though the historical data tells a different story, these models are firmly entrenched throughout the investment profession.

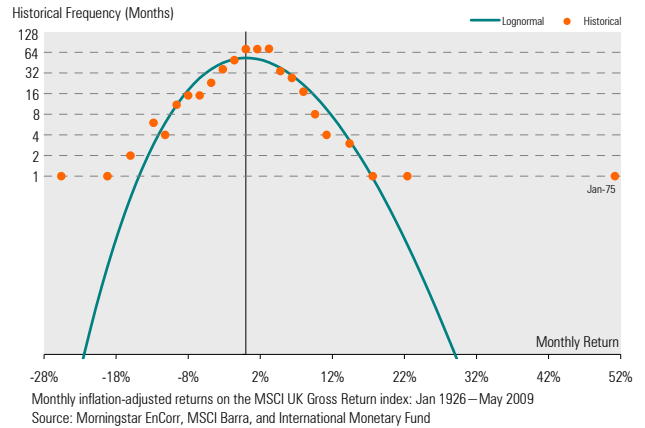
An Alternative Approach: Log-Stable Distributions

Exhibit 5 shows the distribution of monthly real total returns for the UK stock market from January 1970 through May 2009 along with the lognormal distribution curve that best fits the data. (The chart is drawn using a logarithmic scale to emphasis the tails of the distributions.) While in most months, the historical returns closely follow the curve, there are several months that have returns that fall far to the right or left of the lognormal curve. It is these outliers in the tails that constitute both the opportunities and the risks of equity investing. This phenomenon is not unique to the UK market; rather, it is typical of equity markets throughout the world.

In the early 1960s, Benoit Mandelbrot, a mathematician teaching economics at the University of Chicago, was advising a doctoral student named Eugene Fama. Mandelbrot had developed a statistical model for percentage changes in the price of cotton that had “fat tails.” That is, the model assigned nontrivial probabilities to large percentage changes. In his

doctoral dissertation, Fama applied Mandelbrot’s model to stock prices and obtained promising results.⁵ Until recently, however, the work of Mandelbrot and Fama had been largely ignored.⁶

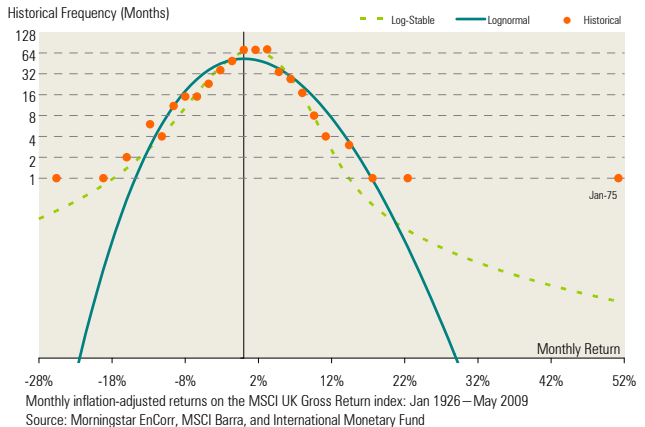
Exhibit 5: Cracks in the Bell Curve – U.K.



In his dissertation, Fama assumed that the logarithm of stock returns followed a fat-tailed distribution called a “stable Paretian distribution,” or stable distribution.⁷ Hence, we refer to the resulting distribution of returns as a “log-stable distribution.”

Exhibit 6 adds the best-fitting log-stable distribution curve to Exhibit 5. While not perfect, the log-stable model fits the historical distribution much closer than the lognormal both at the center and the tails.

Exhibit 6: Modeling Fat Tails – U.K.



Risk Measures

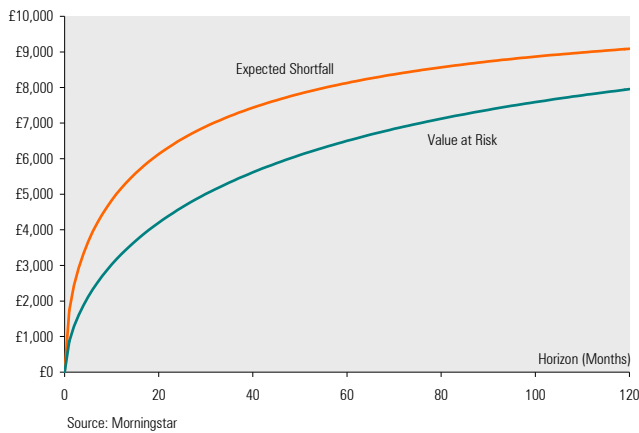
Our analysis of stock market drawdowns and return distributions strongly suggests that summarizing risk with standard deviation omits much of the story. We expect to see modeling tools for advisors come to market in the near future that can account for large, prolonged drawdowns and fat tails.

One modeling approach that is currently used by some institutional money managers and risk analysts is to use fat-tailed models to develop measures of Value at Risk (VaR) and Expected Shortfall.⁸ VaR describes the left tail in terms of how much capital can be lost over a given period of

time. For example, a 5% VaR answers a question of the form: Having invested £10,000 there is a 5% chance of losing X euros in 12 months. What is X? Expected Shortfall is the expected loss of capital should VaR be breached and is therefore always greater than VaR.

As Exhibit 7 shows, VaR and Expected Shortfall depend on the investment horizon. Showing clients charts like this will help better communicate the risks of investing in various asset mixes over various time periods.

Exhibit 7: Value at Risk & Expected Shortfall



Conclusion

In every financial crisis, investors relearn the same message—there isn't a magic risk measure or model that can account for or predict every significant drop in the market. Economists and quantitative analysts have made incredible strides over the decades engineering new ways to explain the distribution of returns. These developments provide investors with valuable information to help them decide how to allocate their portfolios for any number of investing scenarios and mitigate risk. But they are not perfect.

As we've shown, the record contains a much bumpier ride than many risk models would suggest. In addition to preparing clients' portfolios for these occasional severe declines and taking other precautions, advisors would do well to keep reminding their clients of the risks they face as investors. Clients should be fully prepared to take on the 100-year floods they will surely face in the future.

Endnotes

¹ As quoted by Christopher Wright, "Tail Tales," *CFA Institute Magazine*, March/April 2007.

² I obtained the historical monthly total returns and inflation from Morningstar® EnCorr®, an institutional asset-allocation software and data package.

³ We use a logarithmic scale for all growth of \$1 charts.

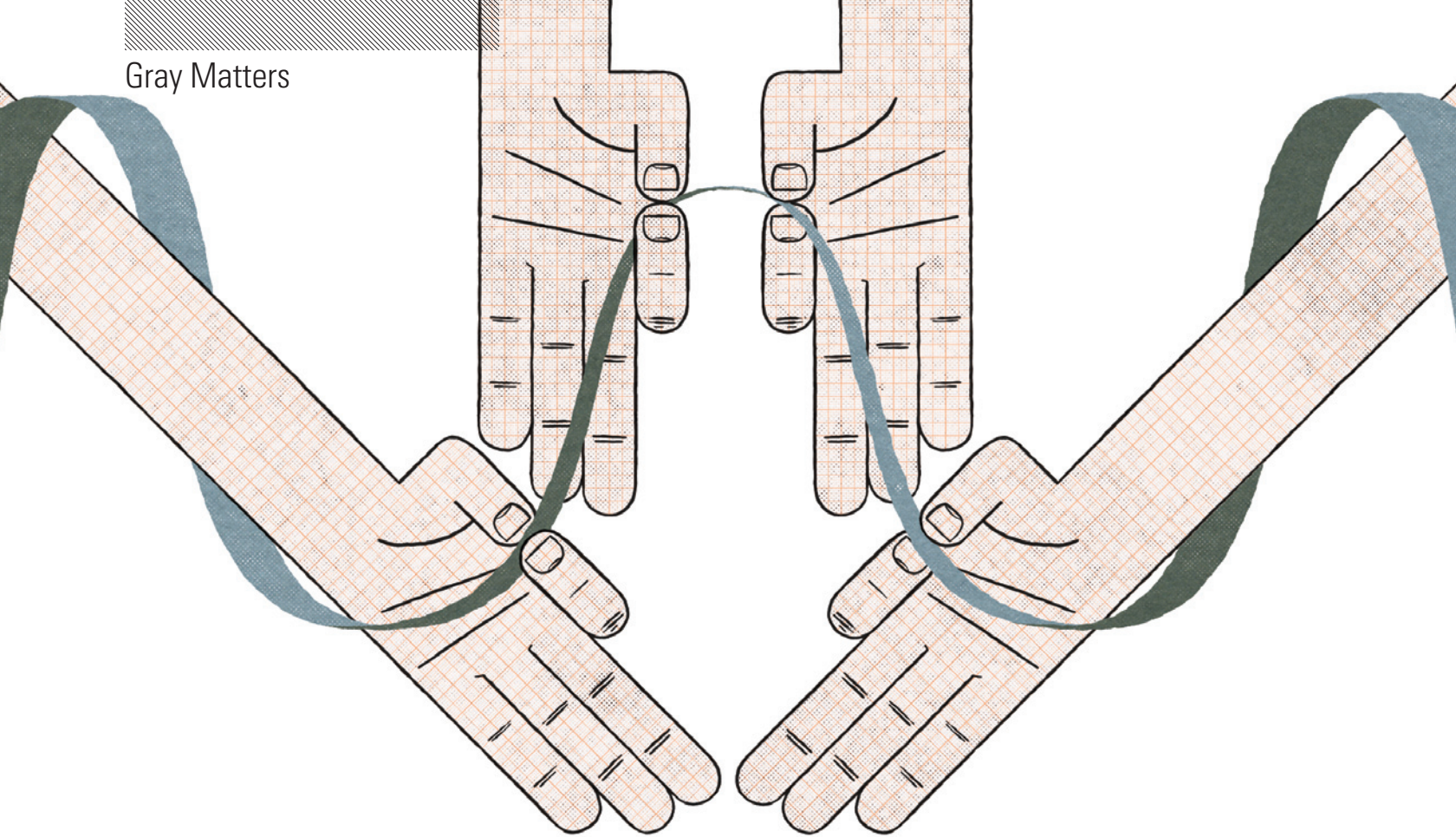
⁴ For returns to follow a lognormal distribution means that logarithm one plus the return in decimal follows a normal distribution.

⁵ For an account of the work of Mandelbrot and Fama during this period, see Benoit Mandelbrot and Richard L. Hudson, *The (Mis)Behavior of Markets*, New York: Basic Books, 2004.

⁶ The idea of using fat-tailed distributions to model asset returns is starting to gain some traction. FinAnalytica was founded to provide investment analysis and portfolio construction software based on Mandelbrot and Fama's work. Morningstar added distribution charts and forecasting models based on it to Morningstar EnCorr.

⁷ Strictly speaking, the assumption is that the logarithm of one plus the return in decimal form follows a stable Paretian distribution.

⁸ Expected Shortfall is also known as Conditional Value at Risk or CVaR.



Nailing Downside Risk

By James X. Xiong

A model called the Truncated Lévy Flight builds on Mandelbrot's work to accurately capture the market's fat tails.

The financial crisis rekindled great interest in "fat-tailed" distributions. (See "Deju Vu All Over Again," by Paul Kaplan, in the February/March 2009 issue.) Investors discovered once again that the odds of experiencing significant losses are much greater than common models of asset returns suggest. Most models assume returns are "normally," or Gaussian, distributed (Bachelier, 1900); when graphed, they look like a bell curve. The ends, or "tails," of the bell curve are thin, meaning that outlier events—the market's extreme gains and losses—should rarely occur.

The historical record presents a different picture: a curve with tails that are fatter than standard models predict. For example, a normal distribution model assumes that an asset return that is three standard deviations below its mean (commonly called a three-sigma event) has only a 0.13% probability of happening, or once every 1,000 return periods. From January 1926 to April 2009, however, the S&P 500 had a monthly mean return of 0.91% and a monthly standard deviation of 5.55%. A negative three-sigma event, therefore, means that the index would suffer a 15.74% monthly

loss. In 83 years, the S&P 500 has suffered 10 monthly returns worse than that amount. The record implies that the probability of a three-sigma event is 1% rather than 0.13%, or eight times greater than an investor would expect from running a normal distribution model. A normal distribution fails to describe the fat tails of possible stock market returns.

Enter Mandelbrot and Fama

That these outlier events occur frequently isn't exactly breaking news. Many academics have created statistical models to account for fat

Three Sigma The S&P 500 has suffered 10 monthly returns worse than three standard deviations below its mean.

Monthly Return S&P 500 (%)	
September 1931	-29.73
March 1938	-24.87
May 1940	-22.89
May 1932	-21.96
October 1987	-21.52
April 1932	-19.97
October 1929	-19.73
February 1933	-17.72
October 2008	-16.79
June 1930	-16.25

Data from January 1926 to April 2009.

tails. Well-known examples are Benoit Mandelbrot's Lévy stable hypothesis (Mandelbrot, 1963), the Student's t-distribution (Blattberg and Gonedes, 1974), and the mixture-of-Gaussian distributions hypothesis (Clark, 1973). Each, however, has its drawbacks.

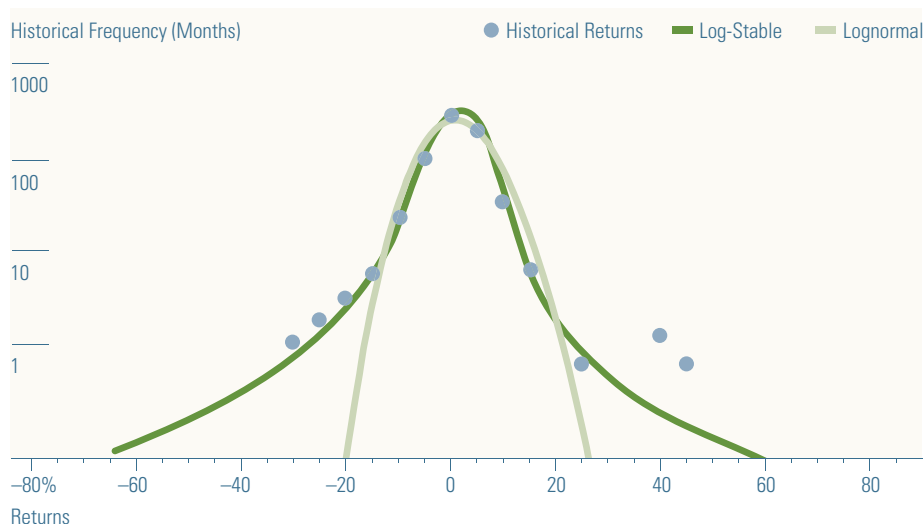
The latter two models possess fat tails and finite variance, but they lack scaling properties. In other words, the models do a good job of capturing the outliers and putting bookends around possible results, but the shapes of their distributions change at different time intervals.

A promising alternative is the Lévy stable distribution model (Lévy, 1925). In 1963, Mandelbrot modeled cotton prices with a Lévy stable process, and his finding was later supported by Eugene Fama in 1965. A Lévy stable distribution model has fat tails and obeys scaling properties, but it has an infinite variance—which greatly complicates things. How can an investor model a portfolio's risk if it has infinite downside or upside returns? In a nutshell, the Lévy model's tails are too fat.

Exhibit 1 displays this problem. In his article,

Exhibit 1 Too Fat The log-stable model does a better job of capturing the S&P 500's extremes than the lognormal model, but its tails are too fat.

S&P 500 Monthly Returns from January 1926 to April 2009



Kaplan uses logarithms to graph stable and normal models over the returns distribution of the S&P 500. He illustrates that a log-stable distribution model fits the tails of the S&P 500 much better than does a lognormal model. In **Exhibit 1**, we change Kaplan's vertical axis to be in log scale with a base of 10. In this scale, it's easy to see the problem with the lognormal distribution. Beyond negative 15% returns (the S&P 500's three-sigma level), the lognormal curve dips toward zero, underneath the S&P 500's historical returns distribution. The log-stable distribution, on the other hand, fits the tail well, but it extends beyond the maximum historical loss of the S&P 500 (negative 30%) with significant probabilities—eventually resulting in an infinite variance and a tail that is too fat to have any practical application for investors.

A Better Model

Do we have a better distribution model? Yes. A simple solution is to truncate the tails of the Lévy stable distribution. The resulting model is what is known as the Truncated Lévy Flight. The TLF distribution has finite variance, fat tails, and scaling properties.

The first graph in **Exhibit 2** compares a log-TLF model with a lognormal model of S&P 500 returns. (See Xiong, 2009, for more details). The log-TLF model provides an excellent fit with S&P 500's returns in all aspects: the center of the curve, its tails, and minimum and maximum monthly returns. In the second graph, we apply the same models to a U.S. long-term government-bond index. Again, the log-TLF does an excellent job in fitting the entire returns distributions of the index.

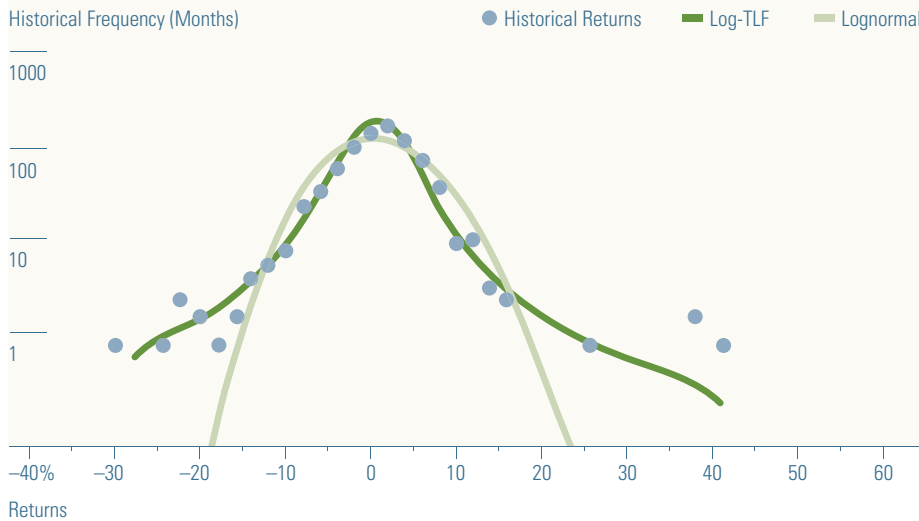
The fact that the log-TLF model does a superior job of portraying the risk of market returns is critical to investors, because many risk estimations rely on the accuracy of the model's tail distributions. Investors who rely on a lognormal model, with its thin tails, will underestimate the market's extreme risks to their own peril. As we will show, using a fatter-tail model has a huge impact on estimates of downside risk and wealth accumulation.

Impact of Fat Tails on Downside Risk

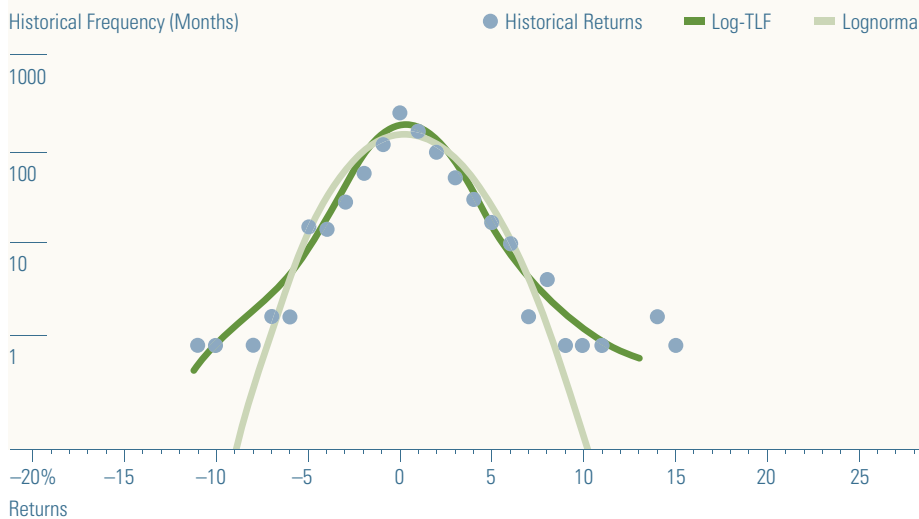
A popular measure of downside risk is called value at risk. Value at risk is the estimate of

Exhibit 2 Log-TLF Versus Lognormal The log-TLF model is an even better fit than the log-stable model for S&P 500 returns. It also does an excellent job of capturing bond returns.

S&P 500 Monthly Returns from January 1926 to April 2009



U.S. Long-Term Government Bond Monthly Returns from January 1926 to April 2009



Capturing the Downside According to CVaR, the log-TLF model captures more of the downside risk of three standard stock/bond portfolios than does the lognormal model.

Portfolios	Mean Return	Std. Dev.	CVaR (Log-TLF)	CVaR (Lognormal)	CVaR Difference (Pct. Pts.)
Conservative	6.8%	10.4%	15.3%	11.8%	3.5
Moderate	8.4%	14.9%	21.7%	17.2%	4.5
Aggressive	10.0%	19.5%	28.7%	23.1%	5.6

the loss on a portfolio that we expect to be exceeded with a given level of probability over a time period. For example, the monthly 5% VaR of the S&P 500 was 7.88% from January 1926 to April 2009. Therefore, the S&P 500 had a 5% probability of losing more than 7.88% in one month.

Conditional value at risk, also called “expected tail risk,” is closely related to VaR, but it takes a more conservative approach because it focuses more on the probability of extreme losses (the left tail of a distribution). CVaR is derived by taking a weighted average between VaR and losses exceeding VaR. By definition, CVaR is always higher than VaR. The monthly 5% CVaR for S&P 500 was 12.29% from January 1926 to April 2009.

Studies (Rockafellar and Uryasev, 2000) have shown that CVaR has more attractive properties than VaR and is a coherent measure of risk. Therefore, we will use CVaR to measure the downside risk of three standard portfolios: conservative (made up of 40% stocks and 60% bonds), moderate (60% stocks/40% bonds), and aggressive (80% stocks/20% bonds).¹ Capital market assumptions are forecast by Ibbotson Associates.

We generated a large sample of 1 million multivariate distributed returns for the asset classes that make up the portfolios. We then calculated the statistics for the three portfolios and compared them using both log-TLF and lognormal distribution models.

We found that the CVaRs for the portfolios under the log-TLF distribution model are 3.5 to 5.6 percentage points higher than the CVaRs under the lognormal distribution model (see table at left). The reason is that the log-TLF model has fatter tails. The differences in CVaRs increase from the conservative to the aggressive portfolios because CVaR not only increases with fatter tails, but it also increases with the portfolio’s volatility.

From a risk-management point of view, these

results are important. The lognormal model can underestimate CVaR by as much as 5.6 percentage points for an aggressive portfolio. That's a huge margin, and it can mislead advisors as they estimate the downside risk of clients' portfolios.

Impact of Fat Tails on Wealth Accumulation

To study the impact of fat tails on the portfolios' wealth accumulation, we next ran two sets of Monte Carlo simulations for each of the three portfolios. The first simulation assumes a lognormal distribution, the second a log-TLF distribution. Each simulation contains 10,000 30-year return scenarios.

The simulated results are similar for the three portfolios, so we only will report the results for the moderate portfolio. Both the log-TLF and lognormal distributions have almost the same wealth at the 50th percentile, but the difference in wealth at the first percentile—the worst-case outcomes—is significant. This makes sense because the log-TLF distribution has a fatter tail and, thus, larger downside risk.

At the first percentile, the moderate portfolio under the log-TLF model can lose 27.5% of its total value in year one. (In other words, the log-TLF model says that in one out of 100 years the moderate portfolio will lose 27.5% of its value.) The lognormal model at the first percentile predicts that the moderate portfolio can lose only 20.1% in one year—a significant difference of 7.4 percentage points. Put slightly differently, the Monte Carlo simulations show that under a log-TLF model it takes the moderate portfolio 40 years to suffer a 20% one-year loss. Under a lognormal model, it takes about 100 years for the moderate portfolio to lose 20% in one year.

To test these results, we observed the returns that the moderate portfolio would have earned since 1926. The portfolio would have lost more than 20% in three calendar years: 1931, 1937, and 2008. Thus, the likelihood of the portfolio losing 20% in one year is about three times in

83 years. The estimate from the log-TLF model (two times in 80 years) is much closer to the historical record than that from the lognormal model (one time in 100 years).

In year six at the first percentile, the moderate portfolio under the log-TLF distribution model can lose as much as 33%; in the lognormal distribution model, the highest loss is 28% in the first six years. These results are particularly important for the wealth accumulation of investors who are six years away from retirement. Such an investor holding a moderate portfolio has a 1% probability of losing one third of his or her total wealth.

With this knowledge, advisors could decide to hedge against this extreme downside risk by using a portfolio insurance product—such as an appropriately priced equity-linked certificate of deposit with a maturity of six years or an insurance product that includes guaranteed minimum withdrawal benefits.

Conclusion

We show that returns models that use a lognormal distribution underestimate the downside risk of a portfolio. Models using a log-TLF distribution are superior, as evidenced by the fact that log-TLF models fit well the entire distribution of historical monthly returns.

These fat tails have further impact on a portfolio's downside risk and wealth accumulation. In general, a diversified portfolio's annualized CVaRs under the log-TLF distribution model are 3.5 to 5.6 percentage points higher than that under the lognormal distribution model. As a result, the lognormal model can mislead advisors and investors when they are considering the risks of their portfolios.

Finally, Monte Carlo simulations using a log-TLF distribution model indicate that investors in a moderate portfolio have a 1% probability that they will lose one third of their portfolio's total value in six years. Therefore, advisors

would be prudent to add a principal hedge against this downside risk for investors nearing their retirement. ■■

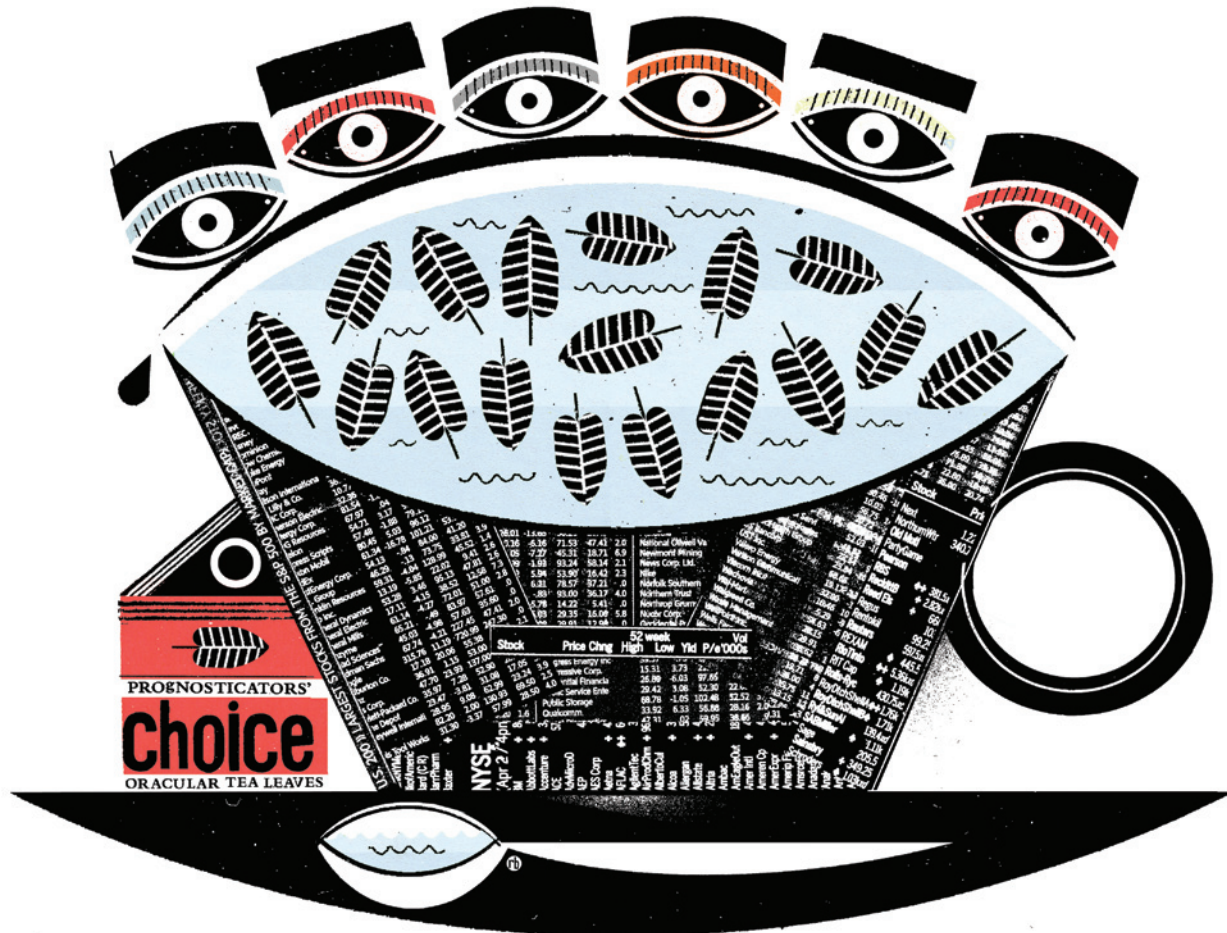
James X. Xiong, Ph.D., CFA, is a senior research consultant at Ibbotson Associates, a Morningstar company. The author thanks Peng Chen, Thomas Idzorek, and Paul Kaplan at Morningstar for their helpful comments.

Footnote

1 Stocks are represented by the S&P 500. Bonds are represented by the BarCap Aggregate Bond, which is backfilled with U.S. intermediate government bonds from 1926 to 1975.

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Getting a Read on Risk

By Paul D. Kaplan

As investors awaken to the risks of investing, our distinguished economics panel debates the value of current risk models.

To investors who lost mightily, the stock market crash of 2008 was a shock to the system. To Drs. Roger Ibbotson, Benoit Mandelbrot, and George Cooper, the decline was just the latest in a long string of the market's fits and starts. And if investors are finally awakening to the risks they incur when they invest in stocks, these three distinguished academics long ago observed that the market is a lot riskier than it may seem.

Ibbotson is the founder of Ibbotson Associates, which is a wholly owned subsidiary of Morningstar, and professor of finance at the Yale School of Management. He is chairman and CIO of Zebra Capital Management, a manager of quantitative equity hedge funds.

Mandelbrot, the inventor of fractal geometry, is Sterling Professor Emeritus of Mathematical Science at Yale and co-author, with R.L.

Hudson, of *The (Mis)behavior of Markets* (Basic Books, 2004).

Cooper, principal of Alignment Investors, is the author of *The Origin of Financial Crises* (Vantage Books, 2008), which *The Economist* calls "a must-read on the origins of the crisis."

From Chicago, we invited them to participate in a conversation, via conference call, about the

It would be very helpful if we recognize that the problems in the 1930s and the problems we're facing today are a result of excessively loose credit policies in the previous decades. That's a missing piece of analysis.

George Cooper

crisis, economy, and the long-term ramifications for investors. On Dec. 17, Ibbotson called from New Haven, Conn., Cooper from London, and Mandelbrot from Boston. The discussion has been edited for clarity and length.

Paul Kaplan: The Fed took a dramatic step yesterday in lowering its funds rate to close to zero. What does that say about the current state of our financial system? How'd we get here?

Roger Ibbotson: Obviously, it's in really bad shape right now. I don't think the Fed funds rate has ever been that low. We are trying to regenerate the economy and save the financial system.

As I look back, it's looking more and more like the 1930s in terms of the financial markets. We haven't seen these large daily price movements in the market since the Great Depression. We had some really bad results in the stock market in the 1970s; we had the crash of 1987; and we were down 45% in 2000-2002.

But why I go back to the 1930s here is that both crises were created by the financial market. Most of the recessions that we have had were not oriented around a breakdown of the financial system. It's only this one and the one in the 1930s that were related to a breakdown in the financial system. In both cases, you had an overleveraged economy with a lack of transparency and a meltdown of various types of financial instruments. In the 1930s, a large number of banks failed and companies were overleveraged. We have that same sort of leverage today, not so much in companies, but both on the household level

and, particularly, in the financial sector. This leverage was packaged and put in complex forms of derivatives, which wasn't always transparent to investors, and sold off around the world. So this crisis is not local to the United States, but it's a global financial crisis in both developed and emerging countries.

What's different this time is that the government is taking action. The government was paralyzed in the beginning of the 1930s, but today, it's acting. Maybe what the government is doing is not coherent or structured enough—there seems to be a lot of one-off actions and some panic—but certainly officials are doing a tremendous amount to try to alleviate this crisis. Part of that was what happened yesterday with the Fed rate.

George Cooper: What the Fed did yesterday is part of the necessary policy response here. They clearly have little choice, other than to use monetary policy and fiscal policy to attempt to prop up the financial markets and the economy more broadly.

What worries me, though, is that we're enacting these very aggressive policy responses without really stepping back and analyzing the problem or the reason that we got into this problem. We should go back a few years to when Ben Bernanke was giving speeches about how he could avoid a deflation problem in America by lowering interest rates and injecting liquidity into the economy; he claimed then that the deflation threat could be offset by stimulating more and more borrowing, which he and Alan Greenspan at the time did by lowering rates to 1% and triggering a boom in the housing market.

What was missed in that analysis was that by generating credit, Bernanke and Greenspan created a temporary boom in the economy. But once the credit needed to be repaid, you created a greater slump in the future. We are now reaping the rewards, if you like, of trying to fix the Nasdaq problem with a housing boom, which has compounded the problem into the current mega-credit cycle.

I think it would be very helpful if we step back and recognize that the problems in the 1930s and the problems we're facing today are a result of excessively loose credit policies in the previous decades. That's a missing piece of analysis. I think we need to fix the problems with the policies being used now, but as we do that, we need to recognize that once the fix is enacted, we need to run monetary policy in a fundamentally different way.

Kaplan: Dr. Mandelbrot, since the early 1960s, you've been building statistical models of asset returns. Your models differ very significantly from the ones that are taught in business schools. You use fat-tailed distributions, long-term memory, and so on. One of your students was Eugene Fama, who wrote his doctoral dissertation based on your research. Today, of course, Fama is very much in the mainstream of financial economics. Please describe your research. Why is it important for financial advisors to be familiar with it?

Benoit Mandelbrot: While working for the IBM Research Center in New York, I became motivated to look very carefully at cotton prices over a fairly ordinary period of five years. I observed that those prices' changes had been always very much dominated by special events

The Participants



Roger Ibbotson

- ▶ Professor of finance at the Yale School of Management and chairman and CIO of Zebra Capital Management, which manages equity long-short hedge funds.
- ▶ Founded Ibbotson Associates, a wholly owned subsidiary of Morningstar.
- ▶ Co-writer of the annual *Stocks, Bonds, Bills and Inflation* (Morningstar, Inc.), the standard reference for information on investment market returns.
- ▶ His many books include *Global Investing: The Professional's Guide to World Capital Markets*, with Gary Brinson (McGraw Hill, 1992).



George Cooper

- ▶ Principal of Alignment Investors, a division of BlueCrest Management, Ltd.
- ▶ Studied at Durham University.
- ▶ Worked as a fund manager at Goldman Sachs and as a strategist for Deutsche Bank and JPMorgan.
- ▶ Author of *The Origin of Financial Crises* (Vintage Books, 2008).



Benoit Mandelbrot

- ▶ Sterling Professor Emeritus of Mathematical Science at Yale.
- ▶ Fellow Emeritus at IBM's Thomas J. Watson Research Center.
- ▶ Inventor of fractal geometry.
- ▶ Received the Wolf Prize in Physics, the Japan Prize in Science and Technology, and awards from the U.S. National Academy of Sciences, the IEEE, and numerous universities.
- ▶ His many books include *The (Mis)behavior of Markets* (Basic Books, 2004).

that provoke sharp, even overwhelming, discontinuities. Then, I found that the same is true for the prices of wheat, stocks, and a multitude of other price series. The standard theory of price variation assumed continuity, but the data were very discontinuous. I became hooked on this problem and have worked on it ever since.

By training, I am a mathematician, but a very peculiar one, for an easily identifiable reason. During World War II, I studied by myself, up in the mountains and not in proper school. Therefore, I read many things that nobody else read, and I didn't learn many things other people learned, consciously or not. So I decided to look more and more carefully at price changes and see whether the fact that anything close to the simplest random walk fails to catch the variability of the process was something particular to the data I dealt with, or more widespread.

Kaplan: What you're saying, Dr. Mandelbrot, is that when you began to look at financial data, you observed that contrary to the standard models, which say that returns follow a bell-curve distribution and move in a continuous fashion, the data were dominated by lurches and discontinuities. Today, of course, there's a lot of talk about "black swans," and you've coined the term "gray swans" to indicate events that differ significantly from the norm and should be planned for.

Mandelbrot: That's correct. When asked to comment about this, I always say that I've been studying gray swans, just because the problem is not just with one specific extreme event. You may say there are swans of every level of blackness, from almost white to completely black, and "completely black" has no limit.

Kaplan: Dr. Mandelbrot's thinking is very different from what is taught in business

schools. Nobel Prizes have been awarded for mean-variance analysis, the capital asset pricing model, the Black-Scholes model of options pricing—all of which are based upon this notion that prices move in a continuous fashion. Should Dr. Mandelbrot's work be taught in business school?

Ibbotson: It's fine to teach it in business school, but let me say that I don't think you have to throw out all the standard deviation work because there are jumps and discontinuities in return series. I don't think there's any doubt that we have jumps and discontinuities and special events.

If you think of the implied volatility in the Black-Scholes model, that implied volatility takes on widely different numbers at different times. It's not a constant, and in fact, where we typically have standard deviations of, say, 20% implied in the stock market, or even 15%

I don't know when it's going to start to straighten out, but ultimately, in the long run, stocks are a good investment.

Roger Ibbotson

in more recent years, it's reached as high as 80% in this crisis. So I think you can resurrect this standard deviation framework, the mean-variance framework, but you have to recognize that the variance itself is stochastic; it's changing.

Mandelbrot: I am very pleased to see that, after many years of denial, discontinuities are now allowed into mainstream economics.

In a way, what you describe is an unmanageable way of interpreting my latest model. There, the observed function is an ordinary Brownian motion [the standard bell-curve model], but time itself is suitably compressed or decompressed; sometimes it runs lightning fast and you get a discontinuity, and sometimes it runs very slowly. But, of course, my representation doesn't help unless the process ruling the intrinsic time—hence the discontinuities—is represented mathematically in manageable and realistic form. This is what I achieved with the concept of multifractal.

The job is not by any means finished, but I did show how a small number of assumptions and intrinsic parameters can represent—and, hopefully, in due time, master—a great deal of complexity. This is a clear advance.

Ibbotson: Benoit Mandelbrot, you deserve a lot of credit for all your work on this field, but I'm not ready to throw out all these other models, because I think they still have a lot of use. For example, in the options framework, if you're valuing something over a relatively short time period, allowing for a very different standard deviation can often roughly correspond to what would be a good valuation of these options.

Mandelbrot: In that case, if you say that the probability of whatever, 2% or 1%, that everything is going to blow up, you can't always do it. Every curve, if you say that you don't have to follow it with all the zigzags, can be represented by a much smoother curve. Local averaging is a very common procedure. I have nothing against it.

Cooper: Could I step in? I think we're on a very interesting topic here. I became very interested in Professor Mandelbrot's work when I was trying to make sense of how the financial markets were behaving and reading a lot of work by an economist called Hyman Minsky. I became fascinated when I saw that Minsky was suggesting a model of the financial markets and, particularly, the credit markets, that behaved at times in a manner exhibiting self-reinforcing phenomena, meaning that there was a dependence in behavior in the way that Professor Mandelbrot was talking about earlier.

These self-reinforcing phenomena could produce sudden jumps with very non-normal distributions. It struck me that there was quite a close parallel between what Professor Mandelbrot had discovered in the data and what Minsky was proposing for his financial instability hypothesis. It seemed to me that fusing the two together would lead to quite a substantial improvement in the way we look at things.

With respect to Dr. Ibbotson on the idea that we can model the market with conventional Brownian motion and conventional Gaussian distributions, yes, you could do that in a piecemeal manner. But in practice, what we have seen is that those sort of models that give

us a relatively benign view of how markets might behave have in large part led us into this financial crisis; those models suggest very much lower levels of real risk in the system relative to what can be delivered.

If these events don't teach us to revisit the statistics that we're using for financial markets, then really we are not adhering to the scientific principle of allowing the data to force the theories to be corrected when they're proven to be wrong.

Mandelbrot: Yes, thank you. I second your opinion very strongly, and I very much regret that I didn't know about Minsky until very recently. In the past few years, friends have been pointing out his work and I hope to read his books soon to get a feeling of his thinking. I understand, however, that it is largely qualitative.

Kaplan: Dr. Cooper, please explain Minsky's theories.

Cooper: The essence of Minsky's theories are really very simple. He claims—and I think the evidence supports him very strongly—that there are self-reinforcing processes operating within our economy, largely because of the way our economy is financed through debt. Those self-reinforcing processes mean that a credit expansion, when it starts, can act through what is known as positive feedback, which means that an effect intends to self-reinforce itself. If you can imagine as asset prices start inflating, you're able to borrow more money against those higher asset prices and you're able to then use that money to buy more assets, which creates higher asset prices again.

This asset inflation and credit creation can spiral on the upside and create, for example, a housing boom, as we've just seen, or a Nasdaq boom, as we saw in the previous decade. But equally, when they go in the opposite direction, they can spiral in a negative manner and create asset price deflation with credit destruction, as we're witnessing now.

The essence of Minsky's theory—and I would say Minsky's theory is really just an extension of Keynes' theories—is that the financial economy is fundamentally unstable. This is directly opposite to what I would describe as mainstream economic thought, which is that our economic system is fundamentally self-stabilizing.

The reason that I find this fascinating, and what is a lot of the topic of my recent book, is that if we examine what the central banks are doing—which is trying to manipulate and control the economy under conventional economic theory—those actions should not be necessary if the economy is self-stabilizing.

We have a quite fascinating confusion at the moment in that we have a theory that says the economy is self-stabilizing and we don't need central banks, but then we have these central banks attempting to stabilize it. Unfortunately, because they are operating to this efficient market theory, the central banks are getting that stabilization process wrong because they're working to the wrong paradigm.

Kaplan: Dr. Ibbotson, is the economy fundamentally unstable or does it self-stabilize? It is curious that economists of every stripe right now are calling for aggressive government action regardless of what theory they seem to normally subscribe to.

Ibbotson: The economy has lots of self-stabilizing features, and it has other features that are destabilizing. Most of the time the economy is stabilizing, but certainly, I won't argue that the situation is stable now;

instead, we have discontinuities here of an extreme sort.

But there are also behavioral aspects of this. I think the risks are definitely much higher than you might think of just looking at standard deviation, not only from the mathematical aspects of other measures of risk, but also from the way people react when they have the bad result. People often have the bad result at the same time they are losing their human capital income. They're losing all of their wealth at the same time, so they tend to be much more risk-averse than standard economics would show them to be. There is a lot of risk, and there's more risk than we think. I agree with both Benoit and George on these points.

Kaplan: If you were to receive a phone call from President Obama asking for your advice, what should policy be going forward, both in terms of fiscal policy and monetary policy?

Ibbotson: This process is so complicated that I don't have a ready solution as to how he should organize all these things. There's not much monetary policy left to be played here because they've already cut the rates to near zero. There's a lot of fiscal policy in Obama's plan, but I worry about the government being involved too much in the private sector. I think the likelihood of the government being able to straighten out this situation completely is not high; there will be a vast amount of waste in how they spend that money. Putting money into failing companies may be temporarily stabilizing, but it creates long-run problems.

Cooper: Well, like Roger Ibbotson, I don't believe that there is a quick, painless fix available. I think there are different routes that can be taken, but none of them are going to be pain-free. If I were advising President Obama, I would suggest that he acknowledge that we've had an excess accumulation of debt and that we now have little choice but to alleviate the burden of that debt through controlled monetization. That is, to inflate away the debt.

As we recognize that, however, we must recognize that this also represents a failure of previous monetary policy, and that once we have monetized the debt away, we will have to enact a radically different approach to monetary policy—one that pays close attention to credit cycles and not just to managing consumer price inflation.


Kaplan: Our readers are getting a lot of questions from their clients about what they should do. What kinds of things should advisors be discussing with their clients?

Ibbotson: I would be saying that when markets pull out of calamities, they often have their highest returns. We had the highest return ever in 1933 in the midst of a severe depression. You get the extreme pullout when things start to get a bit better. The markets in general move ahead of what's actually happening in the economy. The risk premium on stocks has gone way up because of the fact that investors now recognize that there is much more risk in the market than they had recognized. Stocks may not be done dropping, especially in light of what's happened to the financial system, and I don't know when it's going to start to straighten out, but ultimately, in the long run, stocks are a good investment.

Cooper: What I would say is that if we look back through history, yes, we had a crisis in the 1930s, especially in America, and look what happened to America afterwards. America was the most powerful, strongest growing economy for many, many decades.

I think if we step back from the financial side of things, and we focus on what really generates wealth for people in the long run, which are technological improvements, we'll realize that we're still living through a fantastic environment. Emerging markets are opening up to inward investment and adopting free market principles, allowing investment and allowing human potential to be used more efficiently.

I think there's very good reason to believe that,

 In the 1960s, I found that the bell-curve models concerned only a part of nature. [They] failed to apply to the real world of finance. 

Benoit Mandelbrot

for example, China has entered its own industrial revolution. I think over the coming decades, there's very good reason to believe that the growth of the emerging markets will be a genuine powerhouse to improve the living standards of everybody on the planet. But there's no doubt about it; we're facing a very tough few years in the near term.

Kaplan: Dr. Mandelbrot, in your book, *The (Mis)behavior of Markets*, you point out that the truly risky nature of stock market investing, which is not really adequately captured by the standard models, could provide an explanation for the so-called "equity risk premium puzzle." As Dr. Ibbotson has documented, stocks over the past century have garnered enormous returns compared with fixed income. Yet, it's a puzzle because we can't square the theory with the data using models based on standard deviation.

You suggest, however, that investors, without the mathematical training that you have, do have some notion that stock markets are risky; they are aware that these crises occur and that the market moves erratically. Therefore, if you're going to be a long-term stock investor, you deserve to get a high equity risk premium. So the problem is with our models.

Mandelbrot: Indeed, the problem resides in the models. They began more than 100 years ago in the works of a man named Louis Bachelier. Little is known about him, but in 1900, he earned a Ph.D. in mathematics with a dissertation that put forward a theory of speculation. Unfortunately, his model for price variation was already very elaborate and I am sure far too mathematical for his time, so it fell into a black hole.

Soon afterwards, however, the same process was reinvented in physics by Norbert Wiener, and a huge theory developed on this basis. In a certain sense, it came to be viewed as the most basic and manageable model of variability that one can have. It was taught everywhere, and for reasons that are too complicated to explain, it became known as the Brownian motion.

Now, I have the greatest admiration for Bachelier and Wiener. But the only data Bachelier mentions concerned a very peculiar and highly controlled market. He had limited experience in running some very small investments. He was so isolated that no one knew him well.

In the 1960s, I found that the bell-curve models concerned only a part of nature. In particular, the standard Brownian models failed to apply to the real world of finance. Therefore, very thorough rethinking was necessary. I wrote a great deal on this topic, but, clearly, I did not speak loudly or convincingly enough.

Kaplan: But financial advisors need some way of explaining to the ordinary investor what are the risks of different kinds of investments. Is there a way to explain the risks and rewards of the market to an investor who has no mathematical training, so that the next time a crash happens, it won't be such a surprise?

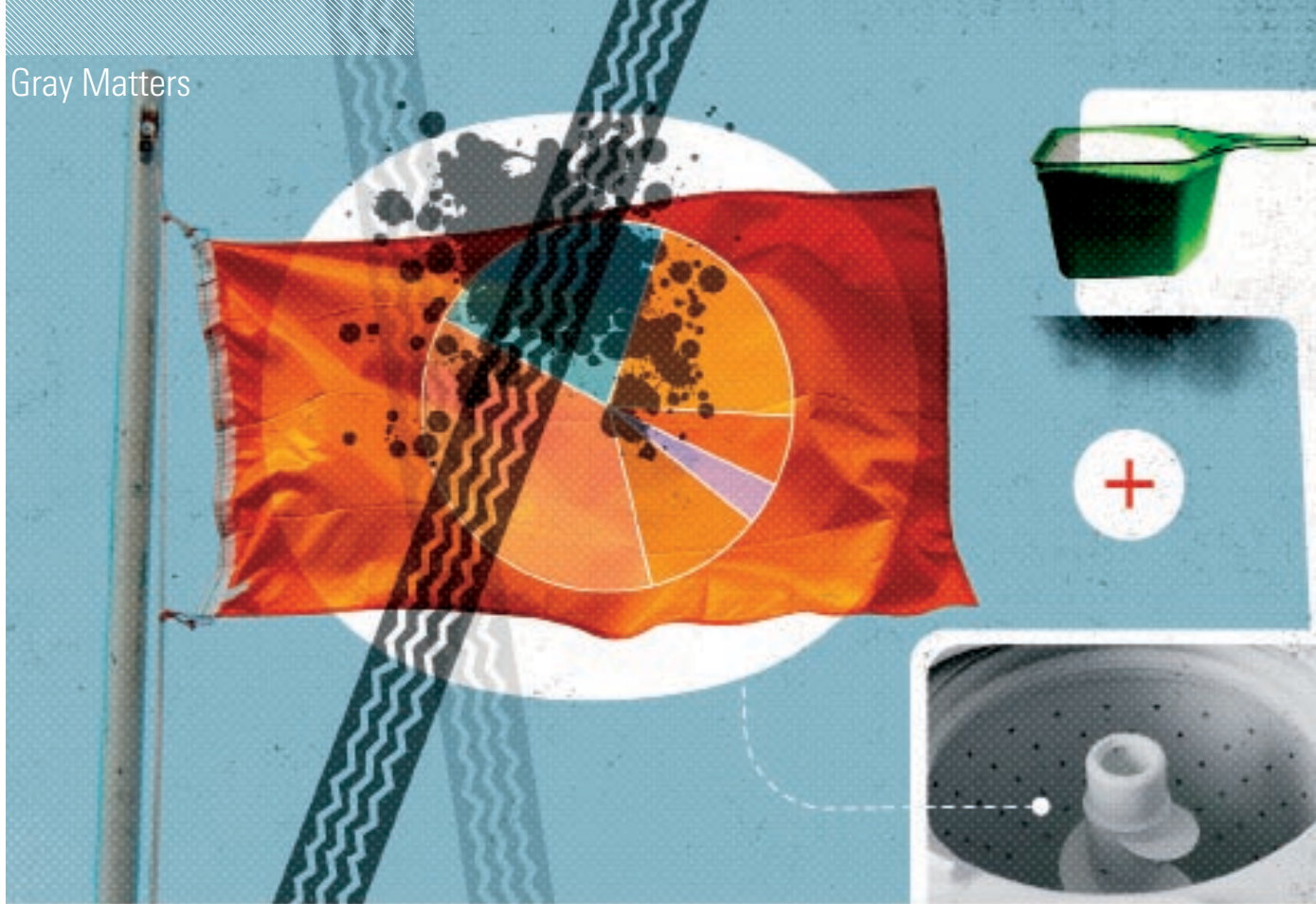
Ibbotson: I think the simple message is that there's much more risk than there appears to be and that the standard deviation doesn't capture all the risk. Whether it's creating more-sophisticated statistical measures or whether it's just using behavioral economics and seeing the way people behave in

crisis situations versus how they react on a questionnaire—all these sorts of things suggest that there's much more risk and much more risk aversion in markets than is revealed in the ordinary way we look at economics.

To me, there's never been a risk premium puzzle, because I've always thought that the risks are much higher and that there should be a payoff for this kind of risk. We'll see that people will be much more averse to risk going forward and much more averse to the stock market going forward.

If there is a positive equity risk premium, and I certainly think that's the case, this crisis will make it more obvious. The crisis itself is creating a big negative return, but going forward in the long run, I'm quite confident that stocks will outperform bonds. ■■■

Paul D. Kaplan, Ph.D., CFA, is Morningstar's vice president of quantitative research and a frequent contributor to *Morningstar Advisor*.



MPT Put Through the Wringer

By Paul D. Kaplan

After another market crash, advisors question whether Modern Portfolio Theory is the best way to tackle asset allocation. We asked two experts to debate its merits.

On the heels of the financial crisis and market crash, advisors are asking whether Modern Portfolio Theory is a valid approach to asset allocation and portfolio management. To shed light on this issue, I asked Steven Fox, director of capital markets research at Russell Investments, and Michael Falk, vice president and chief investment officer of ProManage LLC and an adjunct professor at DePaul University in its Certified Financial Planner program, to debate this question on May 28 at the 2009 Morningstar Investment Conference. Here is an edited transcript of our discussion.

Paul Kaplan: Steven, the current market environment reminds us again that capital markets can be very risky. Less than a decade ago, we had the tech bubble burst. We had the crash of October 1987, we had the bear market of the 1970s, and we had the great crash of 1929. Nearly every time there's been a crash, years pass before the market reaches its previous peak. Yet, the way that Modern Portfolio Theory models risk implies that these sorts of events never happen. In light of capital market history, why should investors use Modern Portfolio Theory?

Steven Fox: This question reminds me of the debates we've had in the past decade or so. We had the death-of-beta debate, which was probably premature, and the debate about the equity risk premium, which we could argue still exists. The common characteristic of all these debates is that a market event caused stress on investors. We're all looking around for explanations of something that in advance of the event seemed impossible.

Today, we're calling into question one of the most powerful, intuitive, and accessible



Steven Fox, Paul Kaplan, and Michael Falk

Because it is intuitive and elegant in its simplicity, MPT has attracted a great number of followers. The greater the following became, the fewer questioners debated its merits. The dogma ate their homework.

Michael Falk

tools that advisors have for financial planning, which is Modern Portfolio Theory. Embedded in MPT is a very concise way to measure the trade-off of risk and return and the trade-off of commonality measured as correlation among assets. Those trade-offs, and the results of the theory, tell us some very powerful things about how we should put portfolios together, such as: 1) how much you hold of an asset is inversely related to your perspective on risk; 2) similar assets should have similar status within a portfolio; and 3) diversification mitigates risk.

What you call into question here, Paul, is more the relevance of capital markets' history for making our planning decisions. If we look at the historical record, the average annual return of U.S. equities since the 1920s through 2007 is something like 12% and the standard deviation about 20%. Investors have a one in four chance in any one year of earning a negative return. They have a one in six chance of seeing a return that is less than one standard deviation, or minus 8%. Conditional upon both of those events occurring, we

end up with a fairly serious expectation for a negative outcome. If we're in a world where we're seeing less than zero for an equity return, the average outcome is about minus 12%, based on these numbers. If we're down below one standard deviation, we should expect something substantially less than that, around minus 18%. So there's room in the capital markets' record, even if you distill it down to summary statistics, for these once-in-a-lifetime events to occur.

Kaplan: Michael, isn't diversification good advice? Shouldn't investors hold less of an asset the more risky that they think it is?

Michael Falk: MPT has two parts, the way I define it. One is diversification. The other is the math for how you decide how you allocate the assets/asset classes you have selected to use in your diversified portfolio.

Diversification, absolutely, is a good idea. The way I talk about it with students is that when you are properly diversified you will at all times have a dog in your portfolio. Think about

that for a second. You want to ensure that at no given time will your entire portfolio bark at you. That is the objective of diversification.



Now, the math part. Because it is intuitive and elegant in its simplicity, MPT has attracted a great number of followers. The greater the following became, the fewer questioners debated its merits. The dogma ate their homework. The assumptions baked into MPT are perceived as certainties: They're mathematical, and we trust numbers. Maybe we shouldn't. Because the issue is simple: The math doesn't work. The markets are not normally distributed. Therefore, standard deviation cannot function. We need accurate ex-ante predictions for the inputs for returns, standard deviations, co-variances, and the future is unknown. It assumes that all investors at all times are risk-averse.

How do you argue with Dr. Harry Markowitz, a Nobel Laureate? You use another Nobel Laureate—2002, Daniel Kahneman, economics. Kahneman says that investors are not risk-averse; they're loss-averse. They have an S-shaped utility curve. They are risk-averse with gains, and risk-seeking with losses.

Why are we having these 100-year events every few years? Has anybody thought that maybe the definition is just wrong? The market is not normally distributed. If the market is not normally distributed, standard deviation doesn't hold as a definitional statistic. Garbage in, garbage out.

Kaplan: Steven, you spend a good deal of your time at Russell coming up with assumptions. Garbage in, garbage out?

Fox: I certainly hope not! I hope that my assumptions are an informed view of what could possibly happen. As soon as you put a portfolio together, you're taking a viewpoint about what's going to happen in the future. We can't disengage our portfolio decisions from that. MPT gives you a structured set of rules by which to make that decision,

 The minute you say that return to asset class X is going to be 10%, as a forecaster, I know that's going to be wrong. The beauty of MPT is that I have some idea about how much uncertainty there is around the expectation of that forecast. 

Steven Fox

given a certain set of assumptions. Nobody knows what's going to happen in the future. But I think we can make some fairly educated guesses and form some expectations based on good information. They may not always come true, but in the absence of anything else, I think we have to do that.

Kaplan: Michael, with the models that Steven works with, asset-allocation weights explicitly come out of assumptions about expected returns, standard deviations, and correlations. You have a very different point of view on how to come up with asset-allocation weights.

Falk: I've learned that the only thing forecasts do for you is that they make you wrong. So why not just avoid forecasting? Let's start with diversification and the building blocks of a portfolio. We could all probably agree that there are six to 12 basic asset classes. Which ones you choose is the investment diversification selection. What's the allocation or weighting of those selections? One choice would be to market-weight them, and another is one-over-N. People may know this as the "naïve diversification theory." Equal-weight asset classes? You're probably thinking, how did this guy get on this panel?

In 1988, in an interview for *Money* magazine, Jason Zweig asked Dr. Markowitz how he invested his retirement dollars. His answer was, "I have half of my money in stocks, and I've got half of my money in bonds." Sounds like one-over-N to me. Zweig then asked Dr. Markowitz how he came to that allocation. Was it through a quadratic calculation? He said, "Jason, I probably should have done some form of calculation to decide my weights. But the

reality is, I do not know which one is going to perform better in the future, and I do not want to regret making the wrong choice."

The man whose theory so many are following was a one-over-N man. That's how I got on this panel!

Kaplan: So Steven, what is it that you're doing at Russell again?

Fox: I'd argue that Michael is also doing forecasts. There is a view embedded in one-over-N that in the MPT space would come out as: Every asset class has the same marginal contribution to total portfolio risk. So you'd allocate equally across all those assets.

But I also have to acknowledge that forecasts can be wildly wrong. The minute you say that return to asset class X is going to be 10%, as a forecaster, I know that's going to be wrong. The beauty of MPT is that I have some idea about how much uncertainty there is around the expectation of that forecast. That's where the concept of risk gets introduced in MPT. It's standard-deviation proxies, both investor preference for risk and my uncertainty surrounding what I think the future is going to hold. You're really a forecaster, Michael. Just admit it.

Falk: As long as you remember that the standard deviation—which is a measure of uncertainty, not a measure of risk—only fits in if it's a normal distribution. If it's not a normal distribution, standard deviation has no merit in terms of the calculus.

Fox: Suppose for a moment that, in fact, the world is not normal. It has slightly fatter tails than standard deviation would allow you to believe. Question: How much weight do you want to put to that part of the outcome space in your portfolio decision, if instead of a one over six chance of there being a minus 8% return, it's one in four?

The alternative to using MPT is not well specified. So you have to be very careful with assuming that, well, the model offers a horrible approximation. But it may be that it's good enough, in the sense that you don't want to strongly overweight low probability events. You might want to take a different approach to mitigating that type of risk.

Falk: Which really goes to the other side of the one-over-N, which is it doesn't have to be the complete portfolio. It can be within a core/explore framework.

The core, 60% to 80% of the portfolio, is the one-over-N, or market. These are liquid, typical asset classes. Let's start with the macros: domestic equities, foreign equities, U.S. Treasuries, U.S. corporate bonds, foreign bonds, commodities, and cash. If you want to throw real estate in there, feel free. But if you're going to use active management instead of passive in the portfolio, just know your active managers are buying REITs at times, hopefully the right times. On the equity side, you can get more granular. U.S. equity can be large, mid, small. On the foreign side, you could do large and "smid"—because we don't have a well-defined mid and small—and if you want to, throw in emerging markets. But if you're using active managers, again, know they may use emerging in their portfolios. What we're really talking about is six to 12 asset classes for core. The reason why I go a little more granular myself is because I do still hold regression to the mean near and dear, and I want a little bit more to rebalance with.

The 20% to 40% explore is what I refer to as the TAIL: Tactical. Active risk tilting. Insurance.

Leverage. This is where you bring in your active strategies to express a view. Tactical could be a hedge fund play or a go-anywhere manager. The insurance is Vineer Bhansali's tail-risk hedging strategies at PIMCO, or it could be an advance life-delayed annuity contract for a retiree. My point is that if you want to express views, it's OK. We're human. Our clients, ourselves, the PMs we give money to, all have biases built into their decision process. But we should try to sequester the lion's share of our portfolios away from these biases. Don't let your views overarch the core.

Kaplan: Moshe Milevsky of York University in Toronto has a book called *Are You a Stock or a Bond?* What Milevsky is saying is that whether your human capital—the present discounted value of all of your future income—is more correlated with stocks or with bonds depends on the nature of sources of income. A tenured professor is a “bond” because like a bond her future income is at a known fixed level. In contrast, a stock trader is a “stock” because the value of his career is highly correlated with the stock market. Is MPT a general-enough framework for us to deal with that adequately?

Fox: To the extent that you can define something in terms of measurable return and risk, it can fit into the MPT framework. As soon as you introduce multiperiod structures and cash-flow requirements, you're stressing its limits. But in general, to get a broad, adequate picture of what a portfolio should look like, sure there's room.

Kaplan: What about for a one-over-N'er?

Falk: Human capital is critical to the equation. When you expand the asset picture from paper assets, stocks and bonds, and real assets, you think of the human capital, Social Security, and pension assets. What we find is that people have a much bigger portfolio than they think they do, and it is heavily weighted towards fixed income.

Kaplan: Is that a problem for MPT?

Falk: All depends on how you create the inputs and the value of those securities. It's one thing to model Social Security, which is fairly consistent. If we're considering an individual a stock or a bond, what happens when the person loses his or her job or changes career paths or starts a company? You'll have to make changes in how you calculate that.

Kaplan: Nevertheless, when those changes occur, would you change the asset mix or not?

Falk: Typically, I don't change the asset mix. I use it as a tool in the rebalancing argument, because the reality is, we're not selling anywhere nearly as much bond to buy stock as clients think we are, because their bond asset is significantly larger than they think it is.

Fox: Did I hear you right? Did you say that you don't change a portfolio allocation when a client's circumstances change?

Falk: One-over-N is the core. Why would you reweight a purposely equally weighted allocation? The explorer portion of the portfolio is where you will reshape a portfolio based upon clients' changes. The challenge here, as we think of classic risk tolerance, is how we then produce allocations. Risk tolerance is two things: risk preference and risk capacity. Risk preference may be consistent, other than the fact that investors are loss averse. Risk capacity is what we're talking about when clients' goals change. Do we change the portfolios? Do we change the definition of the goal? How much should they be saving? This gets into managing expectations.

Fox: Because the university professor who goes from MIT to Wall Street thinking he can be a trader suddenly moves from being a bond to a stock. I think that should fundamentally shift your perspective of what the portfolio allocation should be. That's an easy translation to make, in terms of the structured decision you'd get from MPT.

Falk: ... if you think he's going to stay on Wall

Street. Because again, we're talking about something that we think we can count on. It's a challenge. The human capital asset can be very big, and the accuracy of forecasts about its value is critical due to the inordinate impact they can have on the allocation. The allocation framework is where you get to a lot of the art of this business.

Kaplan: When we do asset allocation, we're always looking for what else can we put in a portfolio that's not going to go down at the same time that stocks and bonds go down. We've just gone through an event where pretty much most of the asset classes went down. Is there really such a thing as a non-correlated asset class?

Fox: An uncorrelated asset is a myth. It doesn't exist. An example I use is oil and large-cap equity. It goes through fits and starts where correlation goes between plus 0.5 and minus one. It depends on which window you're looking at, as to how big that number is.

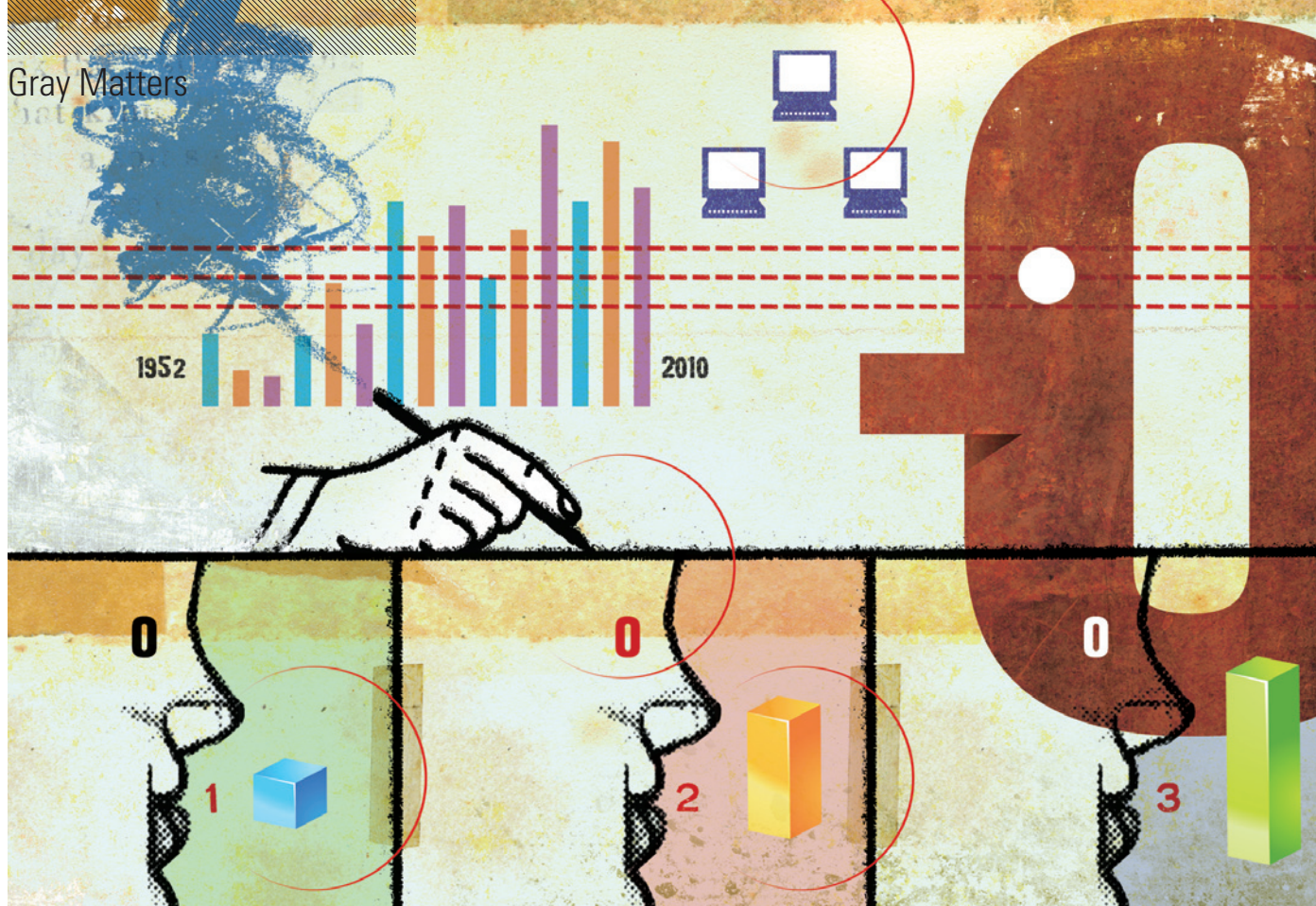
Falk: Not only doesn't it exist, but if it's ever actually found, we would ruin it. Think about this for just a second—the pursuit of alpha, assuming that actually does exist, versus beta. We've moved into esoteric asset classes. We've moved into potential illiquid asset classes that the early adopters could extract additional returns, alpha. Once those asset classes have reached a level of popularity and have become more liquid through other investment vehicles—ETFs, ETNs, etc.—the pursuit becomes beta. It is no longer alpha.

Fox: Your ability to gain returns, disproportionately to the risk inherent in it, goes away.

Falk: Yes.

Kaplan: On that point of agreement, let's conclude the session.

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Markowitz 2.0

By Paul D. Kaplan and Sam Savage

How Markowitz's portfolio-construction tool can be enhanced for the 21st century.

When the Wright Brothers pioneered powered flight in 1903, their genius lay in conquering the three axes of control: pitch, yaw, and roll. Over the years, technologies advanced, planes crashed, and aviation evolved to compensate. By 1952, the Wrights' original airplane was barely recognizable in a world of jets and supersonic aircraft, which nonetheless were still governed by the same three principles of control.

In 1952, another pioneer, Harry Markowitz, invented portfolio optimization. His genius was

also based on three principles: risk, reward, and the correlation of assets in a portfolio. Over the years, technologies advanced and markets crashed, but portfolio-optimization models did not evolve to compensate. This is surprising. Markowitz himself was a pioneer of technological advancement in the field of computational computer science. Furthermore, he did not stand idly by in the area of portfolio modeling; he continued to improve his models and to influence the models of others. Few of these improvements, however, were broadly picked up in practice.

Going Supersonic

Because Markowitz's first effort was so simple and powerful, it attracted a great number of followers. The greater the following became, the fewer questioners debated its merits. Markowitz's original work is synonymous with Modern Portfolio Theory; it has been taught in business schools for generations and, not surprisingly, is still widely used today.

Then came the crash of 2008, and people are starting to ask questions. The confluence of the recent economic trauma and the technological

advances of the past few decades make today the perfect time to describe the supersonic models that can be built around Markowitz's fundamental principles of risk, reward, and correlation. We assert that Markowitz's original work remains the perfect framework for applying the latest in economic thought and technology. We dub our updated model Markowitz 2.0.

The Flaw of Averages

The 1952 mean-variance model of Markowitz was the first systematic attempt to cure what Savage (2009) calls the "flaw of averages." In general, the flaw of averages is a set of systematic errors that occurs when people use single numbers (usually averages) to describe uncertain future quantities. For example, if you plan to rob a bank of \$10 million and have one chance in 100 of getting away with it, your average take is \$100,000. If you described your activity beforehand as "making \$100,000," you would be correct, on average. But this is a terrible characterization of a bank heist. Yet, as Savage writes, this very mistake is made all the time in business practice. It helps explain why everything is behind schedule, beyond budget, and below projections, and it was an accessory to the economic catastrophe that culminated in 2008.

Markowitz's mean-variance model attempted to fix the flaw of averages by distinguishing between different investments with the same average (expected) return, but with different risks, measured as variance or its square root, standard deviation. It was a breakthrough that ultimately garnered a Nobel Prize for its inventor. The use of standard deviation and covariance, however, introduces a higher-order version of the flaw of averages, in that these concepts are themselves versions of averages.

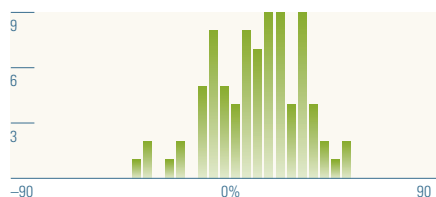
Adding Afterburners

By taking advantage of the very latest in economic thought and computer technology, we can, in effect, add afterburners (more

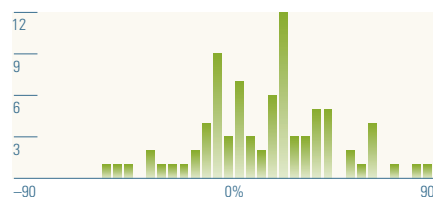
Exhibit 1 Skewed Returns

The return distributions of different asset classes don't always follow a symmetrical bell-shaped curve.

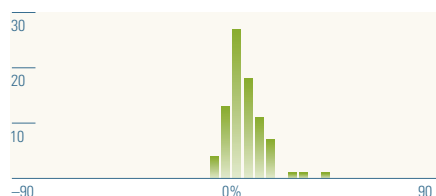
Large Company Stocks



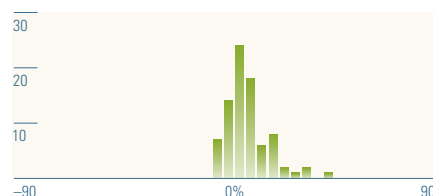
Small Company Stocks*



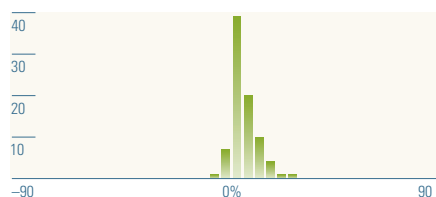
Long-Term Corporate Bonds



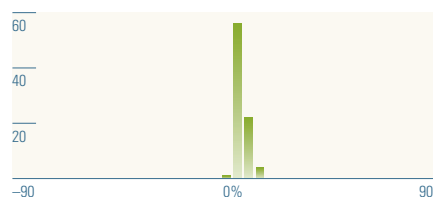
Long-Term Government Bonds



Intermediate-Term Government Bonds



Treasury Bills



Annual total returns 1926–2008. *The 1933 small-company stocks total return was 142.9%

thrust) to the original framework of the Markowitz portfolio-optimization model. The result is a dramatically more powerful model that is more aligned with 21st-century investor concerns, markets, and financial instruments (such as options).

Traditional portfolio optimization, commonly referred to as mean-variance optimization, or MVO, suffers from several limitations that can easily be addressed with today's technology. Our discussion here will focus on five practical enhancements:

First, we use a scenario-based approach to allow for "fat-tailed" distributions. Fat-tailed return distributions are not possible within the context of traditional mean-variance optimization, where return distributions are assumed to be adequately described by mean and variance.

Second, we replace the single-period expected return with the long-term forward-looking geometric mean; this takes into account accumulation of wealth.

Third, we substitute conditional value at risk, which only looks at tail risk, for standard deviation, which looks at average variation.

Fourth, the Markowitz model used a covariance matrix to model the distribution of returns on asset classes; we replace this with a scenario-based model that can be generated with Monte Carlo simulation and can incorporate any number of distributions.

Finally, we exploit new statistical technologies pioneered by Savage in the field of probability management. Savage invented the Distribution String, or DIST, which encapsulates thousands of trials as a single data element or cell. It eliminates the main disadvantage of the scenario-based approach—the need to store and process large amounts of data.

The Scenario Approach

One of the limitations of the traditional mean-variance optimization framework is that it assumes that the distribution of returns for the assets in the optimization can be described simply by mean and variance alone. The most common depiction of this assumption is to draw the distribution of each asset class as a symmetrical bell-shaped curve. As illustrated in [Exhibit 1](#), however, the return distributions of different asset classes don't always follow a symmetrical bell-shaped curve. Some assets have distributions that are skewed to the left or right, while others have distributions that are skinnier or fatter in the tails than others.

Over the years, various alternatives have been put forth to replace mean-variance optimization with an optimization framework that takes into account the non-normal features of return distributions. Some researchers have proposed using distribution curves that exhibit skewness and kurtosis (that is, ones that have fat tails), while others have proposed using large numbers of scenarios based on historical data or Monte Carlo simulation.

The scenario-based approach has two main advantages over a distribution-curve approach. One, it is highly flexible. Nonlinear instruments such as options, for example, can be modeled in a straightforward manner. Second, it is mathematically manageable. For example, portfolio returns are simply weighted averages of asset-class returns within the scenarios. In this way, the distribution of a portfolio can be derived from the distributions of the asset classes without working complicated equations that might lack analytical solutions; only straightforward portfolio arithmetic is needed.

In standard scenario analysis, there is no precise graphical representation of return distributions. Histograms serve as approximations, such as those shown in [Exhibit 1](#). We augment the scenario approach by employing a smoothing technique so that smooth curves represent return distributions. [Exhibit 2](#) shows the distribution curve of annual returns for large-company stocks under our approach. Comparing [Exhibit 2](#) with the large-company-stock histogram in [Exhibit 1](#), we can see that the smooth distribution curve retains the properties of the historical distribution while showing the distribution in a more aesthetically pleasing and precise form. Furthermore, our model makes it possible to bring all of the power of continuous mathematics (previously enjoyed only by models based on continuous distributions) to the scenario approach.

In [Exhibit 2](#), the green line is what we get when we use mean-variance analysis and assume that returns follow a lognormal distribution. The blue line is what we get when we use our smoothed scenario-based approach. The area under the blue solid line to the left of the red vertical segment shows that the 5th-percentile return under our model is negative 25.8%, meaning there is 5% probability of a return of less than negative 25.8%. Under the lognormal model, however, the probability of the return being less than negative 25.8% is only 1.6%.

This illustrates how a mean-variance model can woefully underestimate the probability of tail events.

As Kaplan et al. (2009) discuss, tail events have occurred often throughout the history of capital markets all over the world. Therefore, it is important for asset-allocation models to assign nontrivial probabilities to them.

Reward Over the Long Term

The second enhancement we make to MVO is to use geometric mean. In MVO, reward is measured by expected return, which is a forecast of arithmetic mean. Over long periods of time, however, investors are not concerned with simple averages of return; rather, they are concerned with the accumulation of wealth.

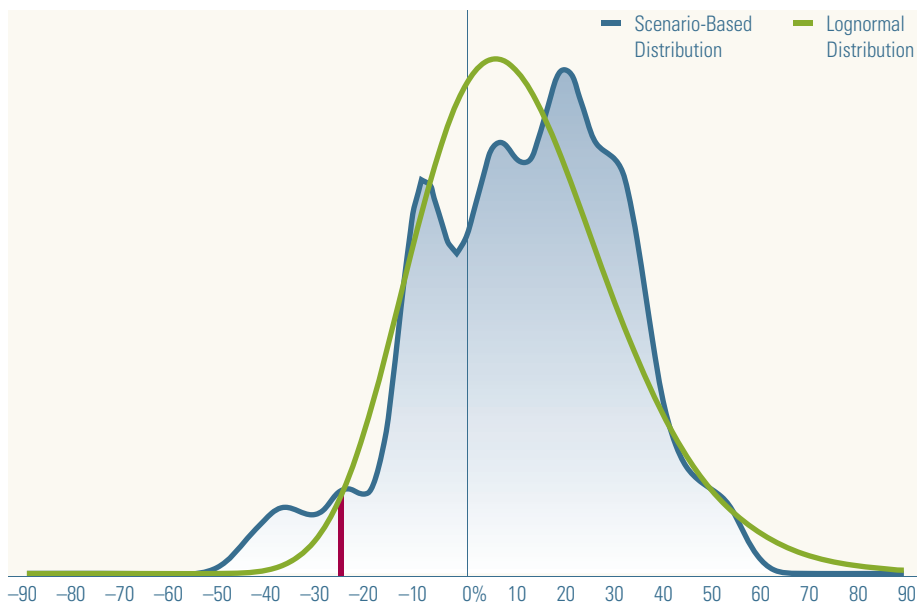
We use forecast long-term geometric mean as the measure of reward, because investors who plan on repeatedly reinvesting in the same strategy over an indefinite period would seek the highest rate of growth for the portfolios as measured by geometric mean.

Downside of Standard Deviation

Our third enhancement deals with risk. Much has been written about how investors are not concerned merely with the degree of dispersion of returns (as measured by standard deviation), but with how much wealth they could lose. Many "downside" risk measures have been proposed to replace standard deviation as the measure of risk in strategic asset allocation. While any one of these could be used, our preference is to use conditional value at risk (CVaR).

CVaR is related to value at risk (VaR). VaR describes the left tail in terms of how much capital can be lost over a given period of time. For example, a 5% VaR answers a question of the form: Having invested \$10,000, there is a 5% chance of losing \$X or more in 12 months. (The "or more" implications of VaR are sometimes overlooked by investors, with serious consequences.) Applying this idea to returns, the 5% VaR is the negative of

Exhibit 2 Smooth Approach As the area to the left of the red vertical line shows, the smooth scenario-based approach does a better job of estimating the probability of tail events than the mean-variance model.



Annual returns of large-company stocks 1926–2008.

the 5th percentile of the return distribution. For example, as we mentioned, the 5th percentile of the distribution shown in Exhibit 2 is negative 25.8%, so its 5% VaR is 25.8%. This means there is a 5% chance of losing \$2,850 or more on a \$10,000 investment. CVaR, however, accounts for possible losses beyond VaR; it is the expected or average loss of capital should VaR be breached. Therefore, CVaR is always greater than VaR. The 5% CVaR for the distribution shown in Exhibit 2 is 35.8%, or \$3,580, on a \$10,000 investment.

Scenarios Versus Correlation

Next, we model the distribution of returns differently. In mean-variance analysis, the covariation of the returns of each pair of asset classes is represented by a single number, the correlation coefficient. This is mathematically equivalent to assuming that a simple linear regression model is an adequate description of how the returns on the two asset classes

are related. In fact, the R-squared statistic of a simple linear regression model for two series of returns is equal to the square of the correlation coefficient.

For many pairs of asset classes, however, a linear model misses the most important features of the relationship. For example, during normal times, non-U.S. equities are considered to be good diversifiers for U.S. equity investors. But during global crises, all major equity markets move down together. Furthermore, suppose that the returns on two asset-class indexes were highly correlated, but instead of including direct exposures to both in the model, one was replaced with an option on itself. Rather than having a linear relationship, we now have a nonlinear relationship that cannot be captured by a correlation coefficient.

Fortunately, these sorts of nonlinear relationships between returns on different investments

can be handled in a scenario-based model. For example, in scenarios that represent normal times, returns on different equity markets could be modeled as moving somewhat apart from each other while scenarios that represent global crises could model the markets as moving downward together.

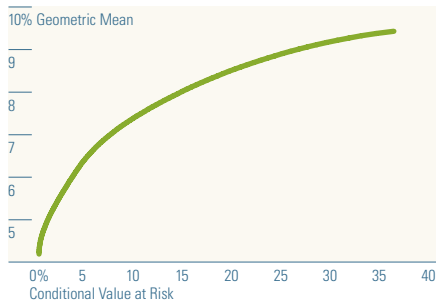
Ultrasonic Statistical Technology

Finally, we make use of new technology. Because it could take thousands of scenarios to adequately model return distributions, a disadvantage of the scenario-based approach has been that it requires large amounts of data to be stored and processed. Even with the advances in computer hardware, the conventional approach of representing scenarios with large tables of explicit numbers remained problematic. That is, until recently.

The phenomenal speed of computers has given rise to the field of probability management, an extension of data management to probability distributions rather than numbers. The key component of probability management is DIST—Savage’s Distribution String, which can encapsulate thousands of trials as a single data element. The use of Distribution Strings greatly saves on storage and speeds processing time—a Monte Carlo simulation consisting of thousands of trials can be performed on a personal computer in an instant. While not all asset-management organizations are prepared to create the Distribution Strings needed to drive the geometric-mean-CVaR optimization, some outside vendors, such as Morningstar’s Ibbotson Associates, can fulfill this role.

Another facet of probability management is interactive simulation technology, which can run thousands of scenarios through a model before the sound of your finger leaving the Enter key reaches your ear. These supersonic models allow much deeper intuition into the sensitivities of portfolios and encourage users to interactively explore different portfolios, distributional assumptions, and potential “black swans.”

Exhibit 3 Markowitz 2.0: Using new statistical technology and a scenario approach, a new efficient frontier is formed.



A working sample of such an interactive model is available for download from ProbabilityManagement.org.

The New Efficient Frontier

Putting it all together, we form an efficient frontier of forecast geometric mean and

CVaR as shown in [Exhibit 3](#), incorporating our scenario approach to covariance and new statistical technology. We believe that this efficient frontier is more relevant to investors than the traditional expected return versus standard deviation frontier of MVO because it shows the trade-off between reward and risk that is meaningful to investors; namely, long-term potential growth versus short-term potential loss. ■■

Paul D. Kaplan, Ph.D, CFA, is director of quantitative research with Morningstar Europe. Sam Savage, Ph.D, a consulting professor at Stanford University, is author of *The Flaw of Averages* (John Wiley & Sons, 2009).

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