Mapping Change in Complex Financial Markets

Inaugural Conference of the “Journal of Network Theory in Finance” , September 23, 2014. Cambridge Centre for Risk Studies, Cambridge University, UK

The views expressed in this presentation are the authors’ and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System.

Contributing Authors

Martin Rosvall

Department of Physics

Umea University

Carl Bergstrom

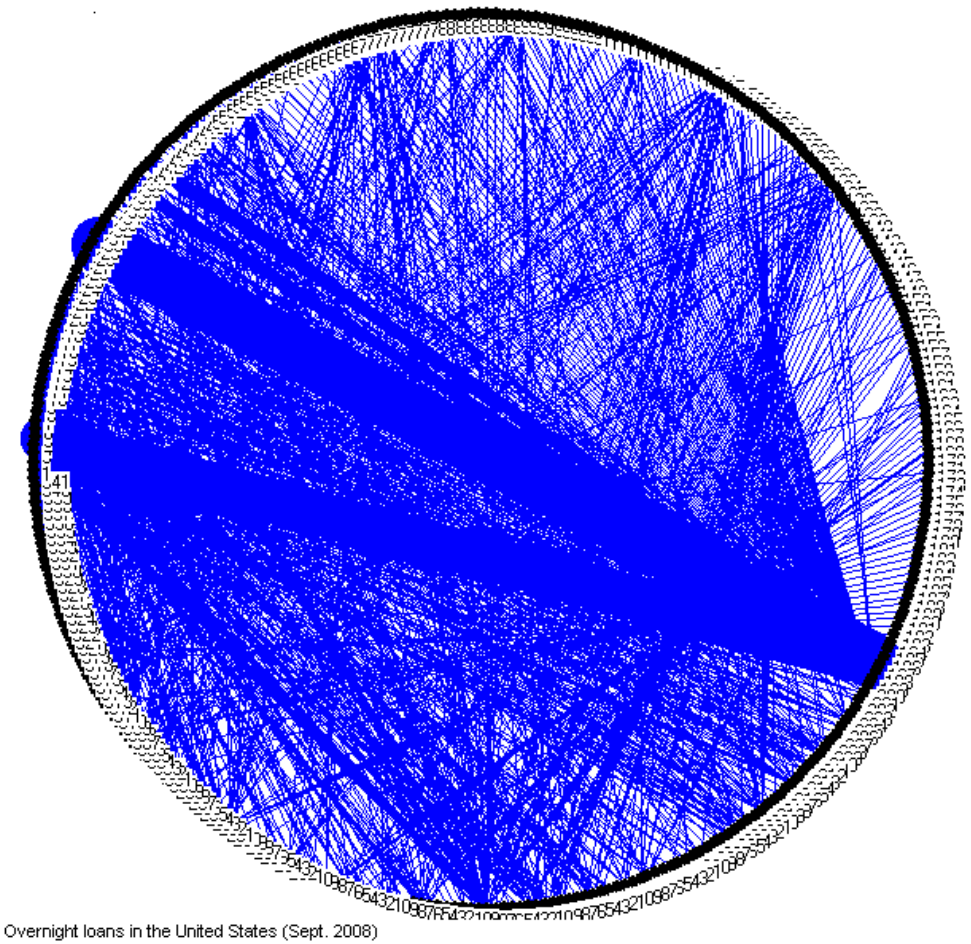
Department of Biology, University of Washington and Santa Fe Instititue

Morten Bech

BIS

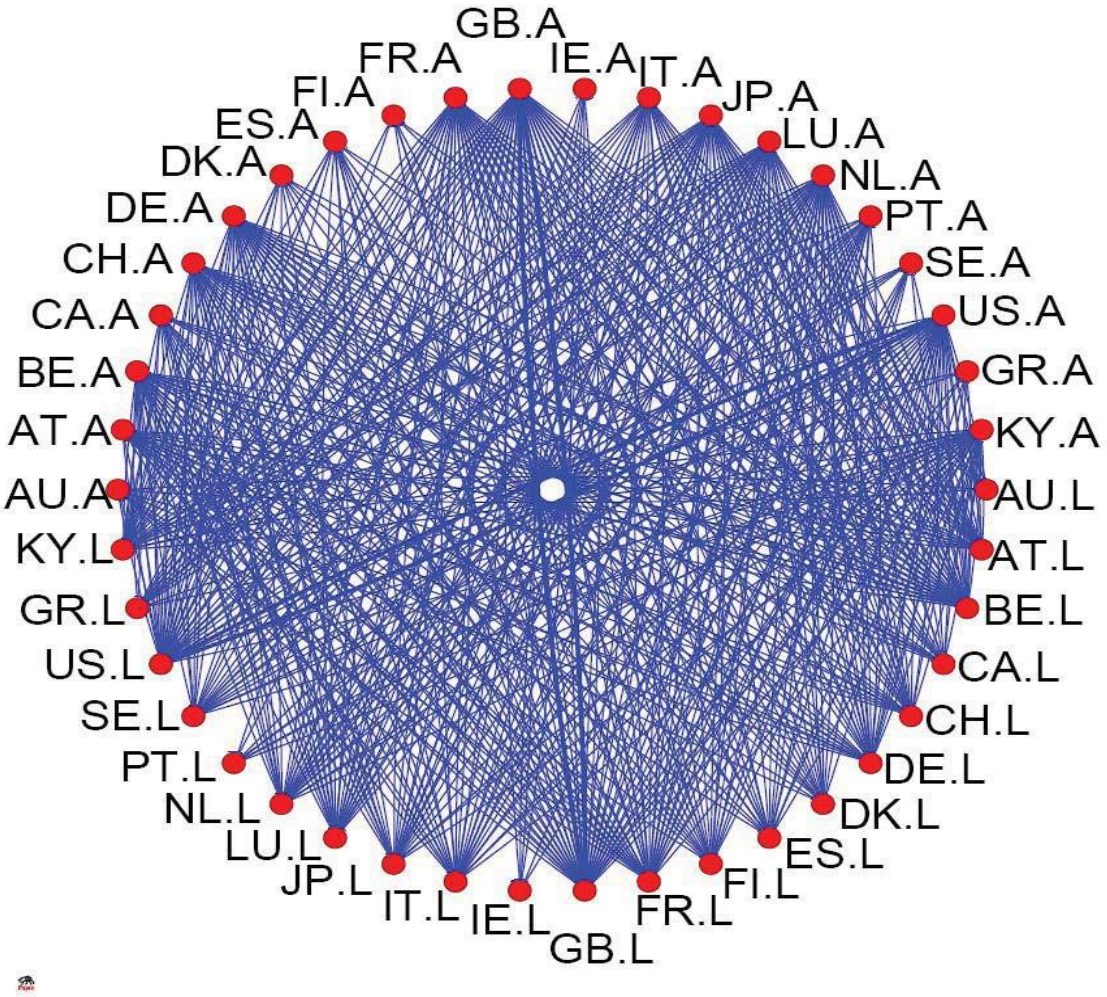
Rod Garratt, FRBNY

U.S. Overnight Money Market



Overnight loans in the United States (Sept. 2008)

Interbank Claims (BIS Locational Data)



We need a way to see the important details of the network structure.


How do we make maps?

good maps simplify


Not helpful!

good maps simplify

and highlight

Regent's Park
Regent's Park Boating Lake
Kings Cross
Grays Inn
City Rd
Ladbroke Grove
Harrow Rd
Carlton Vale
Kilburn Ln
Harvist Rd
Maida Vale
Shirland Rd
Warwick Avenue
St Johns Wood
Maida Hill
Westway
Notting Hill
Marylebone Rd
Park Rd
Baker St
Seymour Pl
West End
Wigmore St
Soho
Coram's Fields
Grays Inn Rd
Rosebery Ave
Goswell Rd
Old St
Bath St
Gower St
Regent St
Strand
Temple
Blackfriars
Bayswater Rd
Hyde Park
The Serpentine
Bayswater
Belgravia
Kensington
Holland Park
Holland Rd
W Cross Rte
Green Park
St James Park
Pall Mall
London
Westminster
Victoria St
Horseferry Rd
Buckingham Palace Gardens
Knightsbridge
Brompton
Victoria
Sloane St
Stamford St
Southwark St
Blackfriars Rd
Borough Rd
Tooley St
Long Ln
Tower Bridge Rd
Newington
Kennington Rd
Elephant And Castle
New Kent Rd
Walworth Rd
Walworth
Lambeth
Millbank
Belgrave Rd
Hammersmith Rd
Earls Ct Rd
Warwick Rd
Old Brompton Rd
Sydney St
King's Rd
Grosvenor Rd
Chelsea Embankment
Edith Grove
Lillie Rd
Dawes Rd
Fulham Palace Rd
Fulham Rd
New King's Rd
West Brompton
Battersea Park
Battersea Park Rd
Parsons Green
South Park
Hurlingham Park
Bishops Park
Wandsworth Bridge Rd
Richmond Rd
Putney
York Rd
Falcon Rd
Latchmere Rd
Queenstown Rd
North St
Wandsworth Rd
Fentiman Rd
S Lambeth Rd
Stockwell Rd
Clapham Rd
Brixton Rd
Kennington Park
Stockwell
Camberwell
Camberwell Rd
Southampton Way
Peckham Rd
Coldharbour Ln
Grove Ln
Burgess Park

good maps simplify

and highlight

relevant structures


Kilburn
Finchley Road & Frognal
Belsize Park
Holloway Road
Brondesbury Park
West Hampstead
Chalk Farm
Caledonian Road
Canonbury
Dalston Kingsland
Camden Road
Kensal Rise
Brondesbury
Finchley Road
Camden Town
Highbury & Islington
Hackney Central
Kensal Green
Swiss Cottage
Caledonian Road & Barnsbury
Kilburn High Road
South Hampstead
St.John's Wood
Mornington Crescent
Queen's Park
Kings Cross St.Pancras
Maida Vale
Great Portland Street
Kilburn Park
Paddington
Edgware Road
Baker Street
Euston
Angel
Warwick Avenue
Royal Oak
Westbourne Park
Old Street
Bethnal Green
Warren Street
Euston Square
Farringdon
Liverpool Street
Paddington
Edgware Road
Marylebone
Regent's Park
Shoreditch
Ladbroke Grove
Russell Square
Barbican
Bayswater
Goodge Street
Chancery Lane
Moorgate
White City
Holland Park
Notting Hill Gate
Lancaster Gate
Bond Street
Oxford Circus
Holborn
Aldgate East
Whitechapel
Shepherd's Bush
Queensway
Marble Arch
Tottenham Court Road
St.Paul's
Bank
Covent Garden
Aldgate
Shadwell
High Street Kensington
Hyde Park Corner
Green Park
Cannon Street
Leicester Square
Piccadilly Circus
Mansion House
Monument
Tower Hill
Tower Gateway
Limehouse
Kensington (Olympia)
Knightsbridge
Charing Cross
Barons Court
Gloucester Road
Sloane Square
St.James's Park
Blackfriars
Wapping
River Thames
Rotherhithe
Temple
London Bridge
West Brompton
Earl's Court
South Kensington
Victoria
Westminster
Embankment
Bermondsey
Canada Water
Fulham Broadway
Waterloo
Surrey Quays
Parsons Green
Putney Bridge
Imperial Wharf
Pimlico
Southwark
Borough
River Thames
Lambeth North
East Putney
New Cross

What is important in our context?

- not trying to get from St John's Wood to King's Cross
- We want to find the structures within the financial network that are important with respect to the flow of funds

Information-theoretic clustering

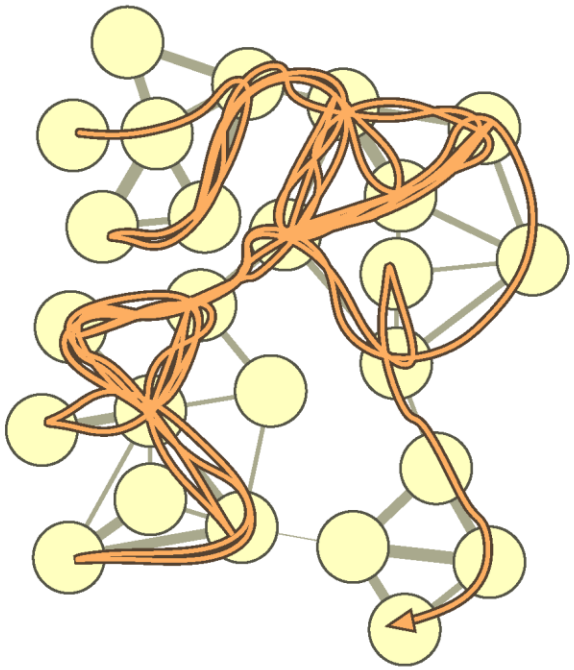
- Consider the overnight money market in the U.S.
- The flow of funds is a large weighted, directed network.
- Individual banks are the nodes.
  - The value of loans from node α to node β determines the weight on the directed link from α to β.
  - Normalized weights become transition probabilities
- In order to understand lending relationships we can use an information-theoretic network clustering technique developed by Rosvall and Bergstrom (2008)

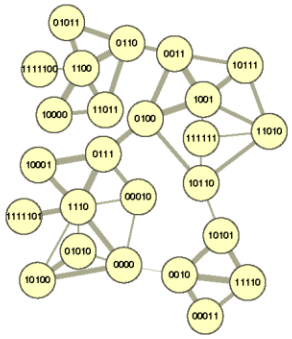
Dual Problem

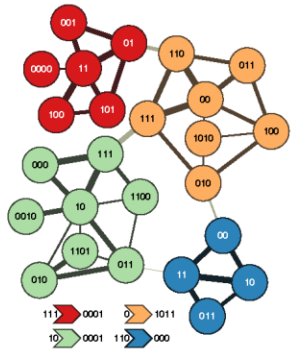
Compressing data ⟷ Finding patterns

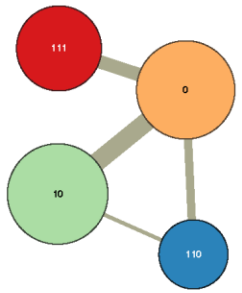
“If we can find a good code for describing flow on a network, we will have solved the dual problem of finding the important structures with respect to that flow.” RB

Maps by compressing flow of information on networks





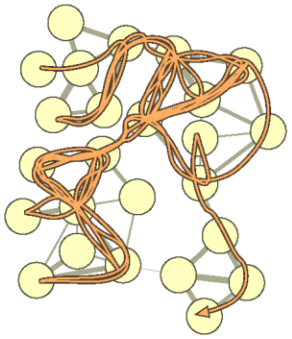


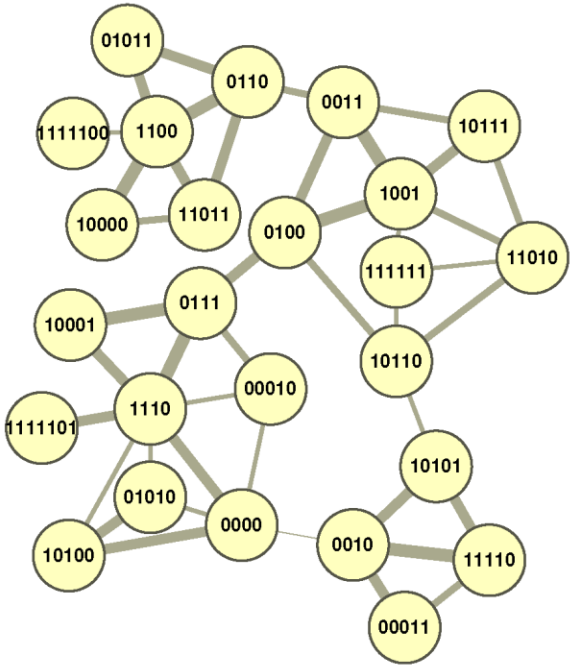


Source: **Maps of random walks on complex networks reveal community structure** by M. Rosvall and C. T. Bergstrom

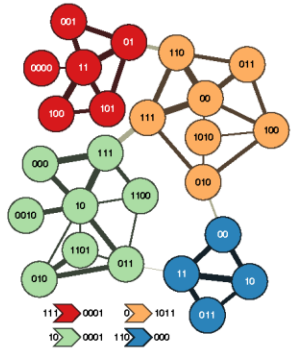
Describing flow is a coding game

— objects deserve unique names

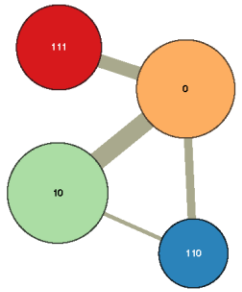




1111100 1100 0110 11011 10000 11011 0110 0011 10111 1001
0011 1001 0100 0111 10001 1110 0111 10001 0111 1110 0000
1110 10001 0111 1110 0111 1110 1111101 1110 0000 10100 0000
1110 10001 0111 0100 10110 11010 10111 1001 0100 1001 10111
1001 0100 1001 0100 0011 0100 0011 0110 11011 0110 0011 0100
1001 10111 0011 0100 0111 10001 1110 10001 0111 0100 10110
111111 10110 10101 11110 00011



111 0000 11 01 101 100 101 01 0001 0 110 011 00 110 00 111
1011 10 111 000 10 111 000 111 10 011 10 000 111 10 111 10
0010 10 011 010 011 10 000 111 0001 0 111 010 100 011 00 111
00 011 00 111 00 111 110 111 110 111 101 01 0001 0 110
111 00 011 110 111 1011 10 111 000 10 000 111 0001 0 111 010
1010 010 1011 110 00 10 011



111 0000 11 01 101 100 101 01 0001 0 110 011 00 110 00 111
1011 10 111 000 10 111 000 111 10 011 10 000 111 10 111 10
0010 10 011 010 011 10 000 111 0001 0 111 010 100 011 00 111
00 011 00 111 00 111 110 111 110 111 101 01 0001 0 110
111 00 011 110 111 1011 10 111 000 10 000 111 0001 0 111 010
1010 010 1011 110 00 10 011

Huffman Code

1. At each step the characters you have seen do not yet correspond to any item, or they correspond to exactly one
2. Encoded message is shortest satisfying 1.

Symbol	Code
a	0
c	10
r	110
t	111

1001110110010111 = cataract

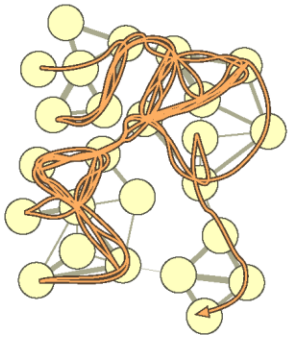
Prefex free code: a receiver can identify each word without requiring a special marker between words.

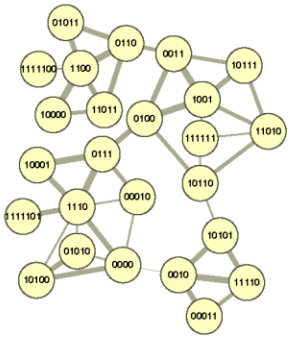
Modular Structure

- RB apply Huffman coding in a “tiered” way, saving code by using two types of code books
  - module codebooks & index codebook
- Can reuse code words in different modules
- Transforms the problem of minimizing the description length of places traced by a path into the problem of how we should best partition the network with respect to this flow
- Trade-off costs and benefits measured in terms of bits

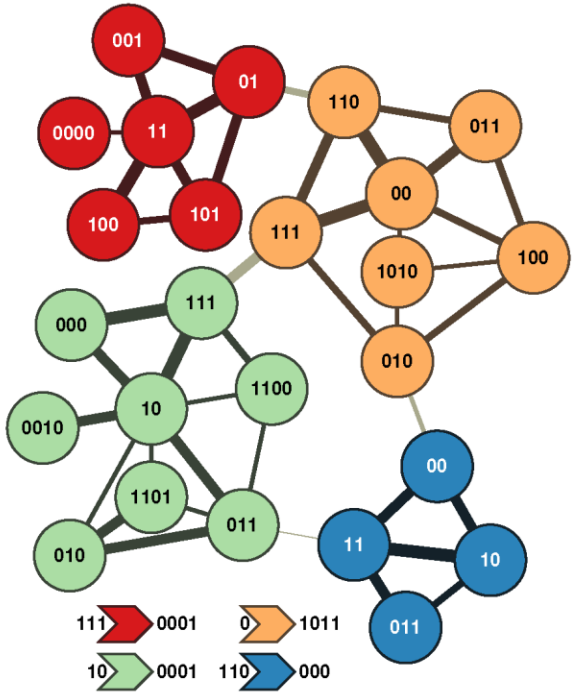
Simplify and highlight

— important objects deserve unique names

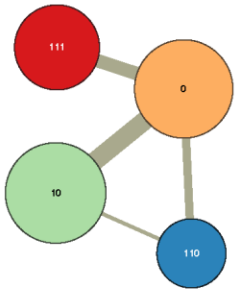




1111100 1100 0110 11011 10000 11011 0110 0011 10111 1001
0011 1001 0100 0111 10001 1110 0111 10001 0111 1110 0000
1110 10001 0111 1110 0111 1110 1111101 1110 0000 10100 0000
1110 10001 0111 0100 10110 11010 10111 1001 0100 1001 10111
1001 0100 1001 0100 0011 0100 0011 0110 11011 0110 0011 0100
1001 10111 0011 0100 0111 10001 1110 10001 0111 0100 10110
111111 10110 10101 11110 00011



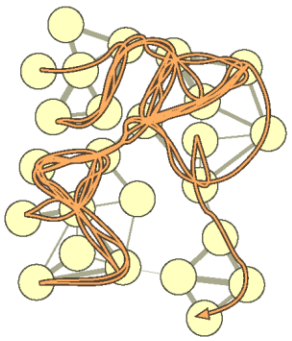
111 0000 11 01 101 100 101 01 0001 0 110 011 00 110 00 111
1011 10 111 000 10 111 000 111 10 011 10 000 111 10 111 10
0010 10 011 010 011 10 000 111 0001 0 111 010 100 011 00 111
00 011 00 111 00 111 110 111 110 1011 111 01 101 01 0001 0 110
111 00 011 110 111 1011 10 111 000 10 000 111 0001 0 111 010
1010 010 1011 110 00 10 011

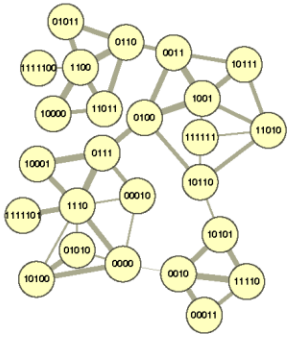


111 0000 11 01 101 100 101 01 0001 0 110 011 00 110 00 111
1011 10 111 000 10 111 000 111 10 011 10 000 111 10 111 10
0010 10 011 010 011 10 000 111 0001 0 111 010 100 011 00 111
00 011 00 111 00 111 110 111 110 1011 111 01 101 01 0001 0 110
111 00 011 110 111 1011 10 111 000 10 000 111 0001 0 111 010
1010 010 1011 110 00 10 011

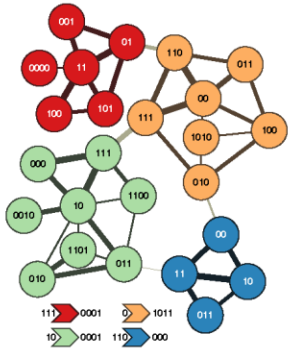
Simplify and highlight

— highlight important objects and filter away details

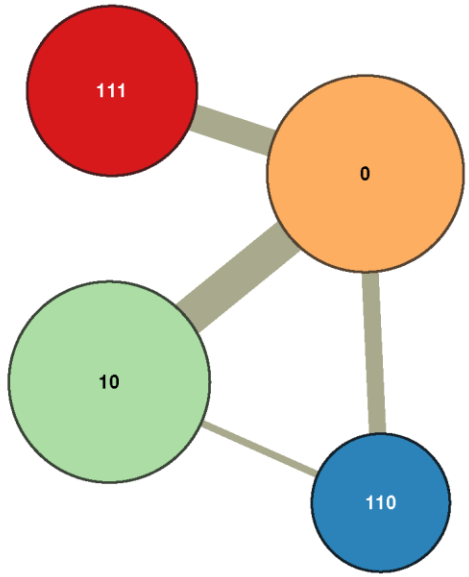




1111100 1100 0110 11011 10000 11011 0110 0011 10111 1001 0011 1001 0100 0111 10001 1110 0111 10001 0111 1100 0000 1110 10001 0111 1110 0111 1110 1111101 11 10 0000 10100 0000 1110 10001 0111 0100 1011 0 1110 10111 1001 0100 1001 10111 1001 0100 1001 0100 0011 0100 0011 0110 11011 0110 0011 0100 1001 10111 0011 0100 0111 10001 1110 10001 0111 0100 10110 111111 10110 10101 11110 00011



111 0000 11 01 101 100 101 01 0001 0 110 011 00 110 00 111 1011 10 111 000 10 111 000 111 10 011 10 000 111 10 111 10 0010 10 011 010 011 10 000 111 0001 0 111 010 100 011 00 111 00 011 00 111 00 111 110 111 110 1011 111 01 101 01 0001 0 110 111 00 011 110 111 1011 10 111 000 10 000 111 0001 0 111 010 1010 010 1011 110 00 10 011



111 0000 11 01 101 100 101 01 0001 0 110 011 00 110 00 111 1011 10 111 000 10 111 000 111 10 011 10 000 111 10 111 10 0010 10 011 010 011 10 000 111 0001 0 111 010 100 011 00 111 00 011 00 111 00 111 110 111 110 1011 111 01 101 01 0001 0 110 111 00 011 110 111 1011 10 111 000 10 000 111 0001 0 111 010 1010 010 1011 110 00 10 011

naming places

Shannon's Source Code Theorem

- We do not need to actually produce code for each partition
- Rather, we can calculate the theoretical limit for all of the different partitions and pick the one that is best (gives shortest description length)
- If you want to describe the states of a random variable X, that occurs with frequency p_i, then the average length of a codeword can be no less than the entropy of X:

$$H(X) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

The map equation

$$L(\mathsf{M}) = q_{\curvearrowright} H(\mathcal{Q}) + \sum_{i=1}^{m} p^{i}_{\circlearrowright} H(\mathcal{P}^{i})$$

The map equation tells us the minimum description length for a particular modular structure

The map equation

frequency of inter-module movements

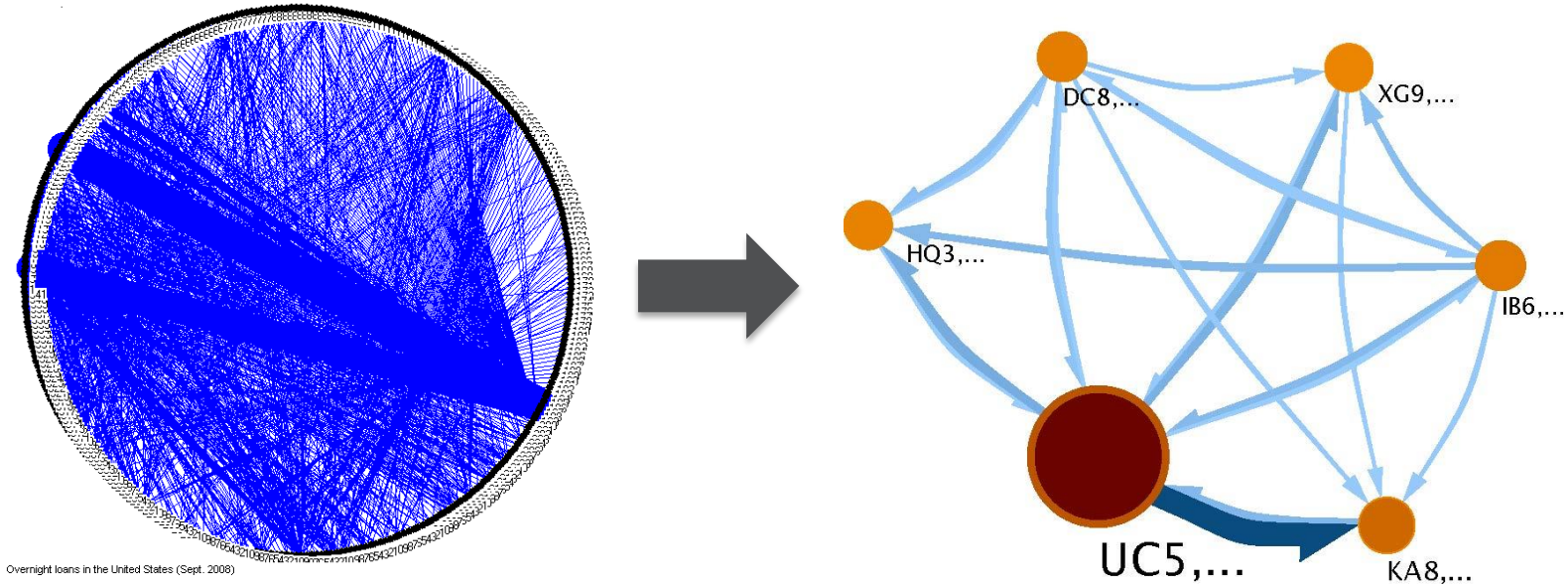
frequency of movements within module *i*

$$L(\mathsf{M}) = q_{\curvearrowright} H(\mathcal{Q}) + \sum_{i=1}^{m} p^i_{\circlearrowright} H(\mathcal{P}^i)$$

code length of module names

code length of node names in module *i*

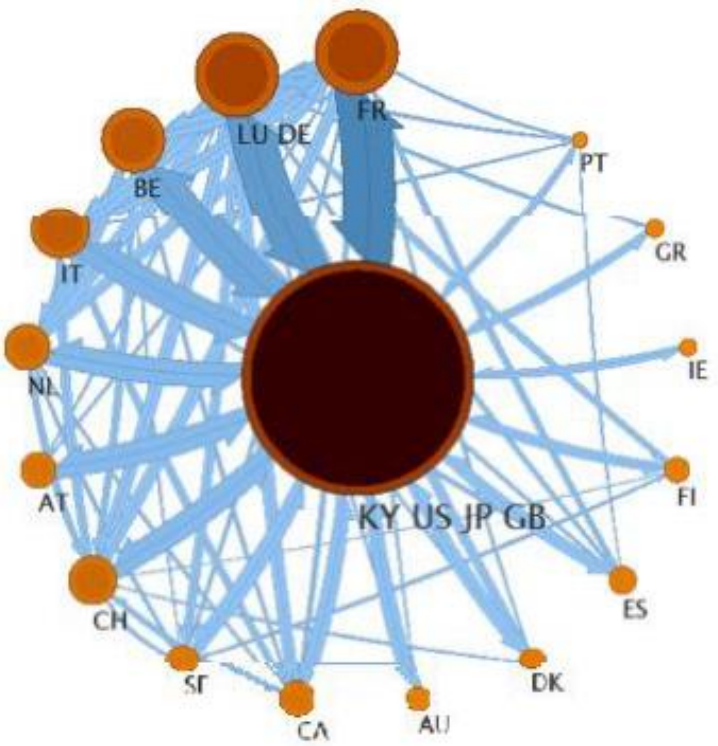
Any numerical search algorithm developed to find a network partition that optimizes an objective function can be modified to minimize the map equation.



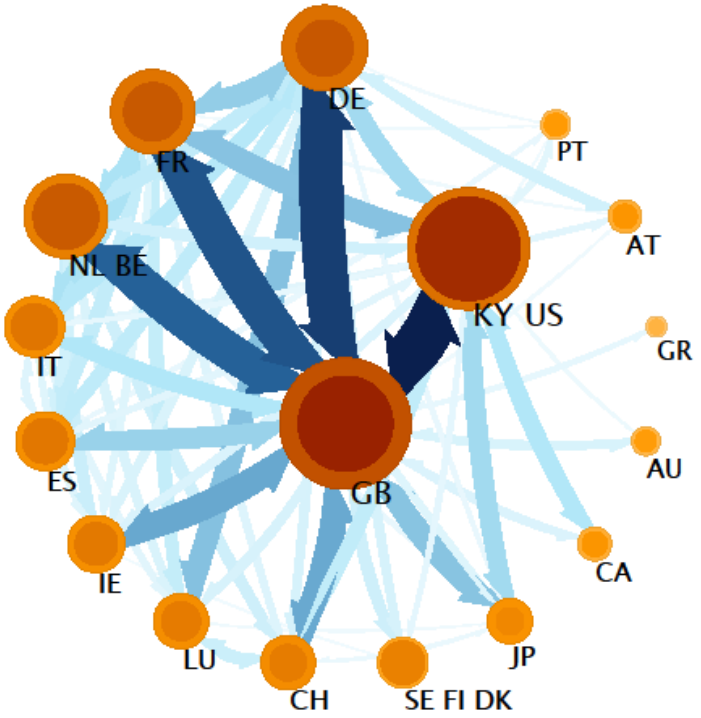
M. Bech, C. Bergstrom, R. Garratt and M. Rosvall, “Mapping Change in the Overnight Money Market,” 2014, mimeo

Interbank Claims

1989 Q3



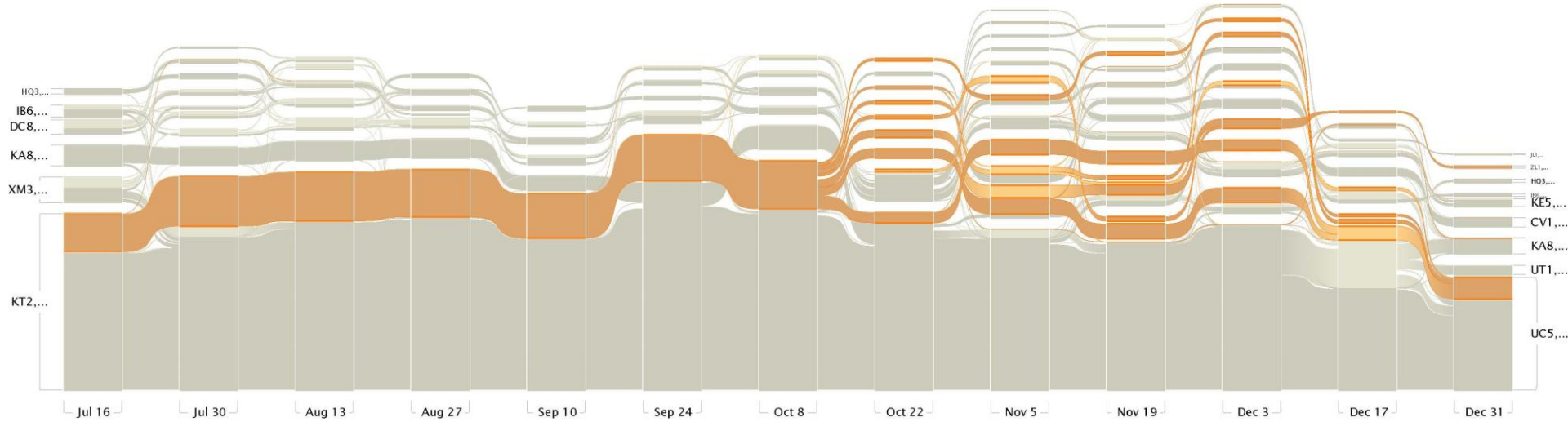
2008 Q3



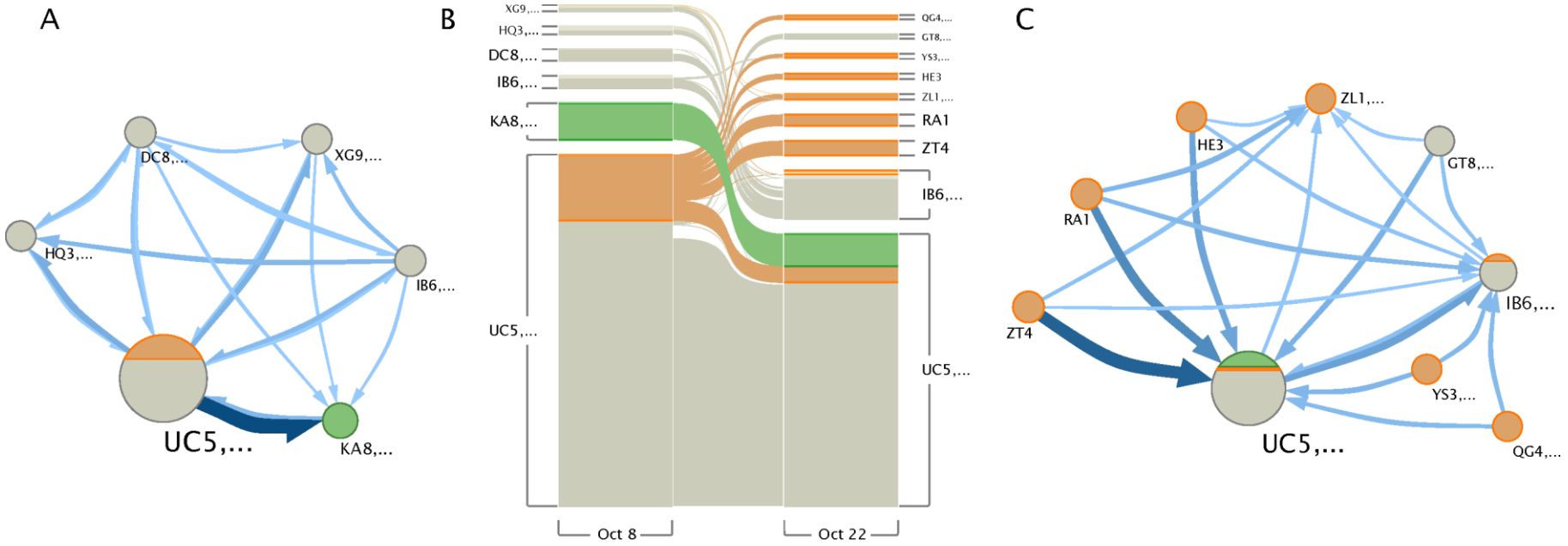
R. Garratt, L. Mahadeva and K. Svirydzenka, “The Contagious Capacity of the International Banking Network: 1985-2009, *JBF, 2014.*

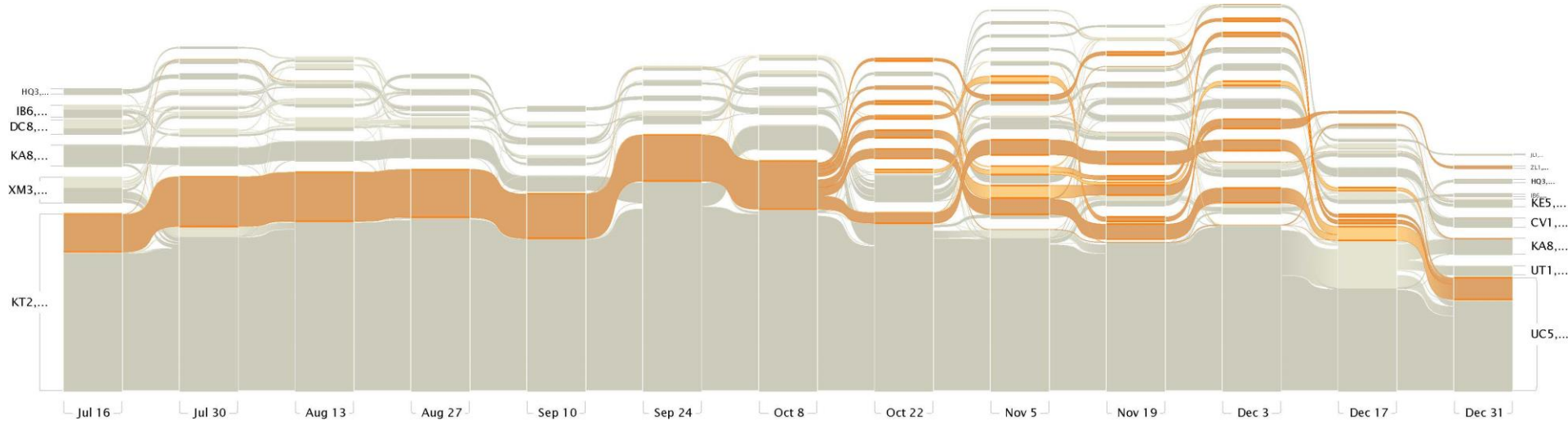
Mapping Change

Alluvial Fan

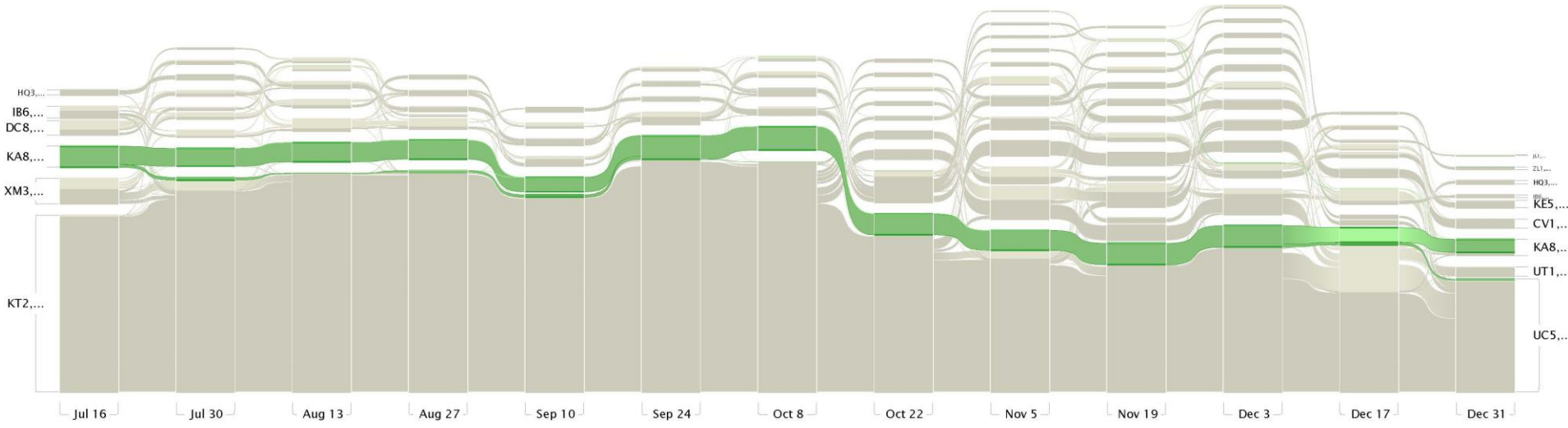


Mapping change of payment flows driven by interbank lending market activity from July 2008 to December 2008.

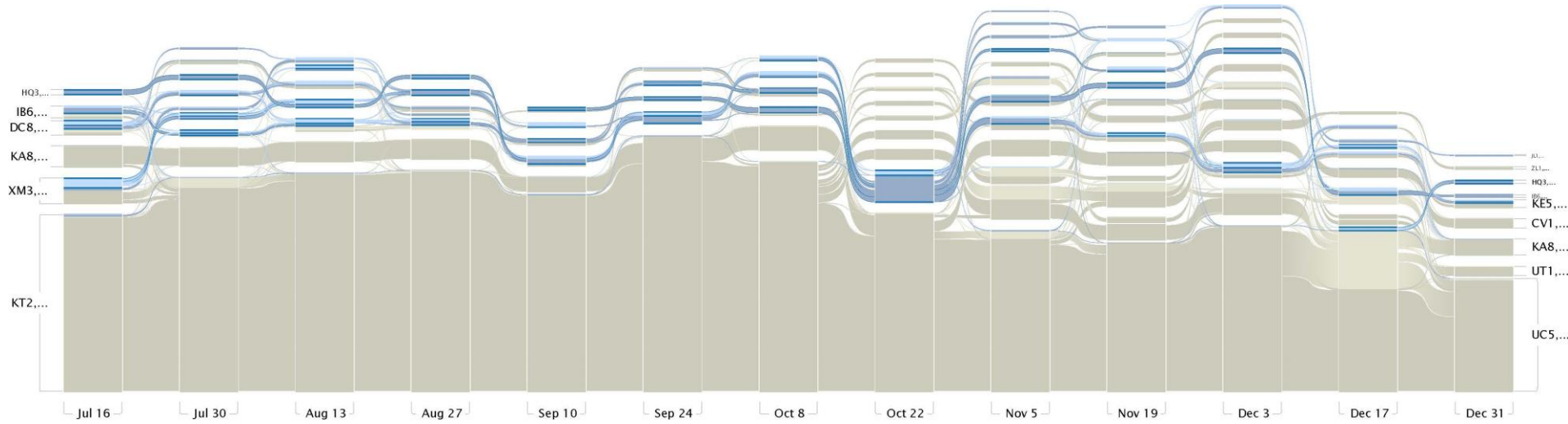




- The orange cluster is dominated by a set of Federal Home Loan Banks and a number of small and medium sized banks.
- "...a combination of financial consolidation, credit losses, and changes to risk management practices has led at least some GSEs to limit their number of counterparties in the money market and to tighten credit lines." (Bech and Klee, *JME*, 2011)



- The fairly stable green cluster which is subsumed into the large cluster after the implementation of interest on reserves, is dominated by a prominent government-sponsored enterprize and one large money center bank.
- The break down of the cluster may reflect the reduction (or even elimination) of the lending relationship to the particular bank by the GSE.

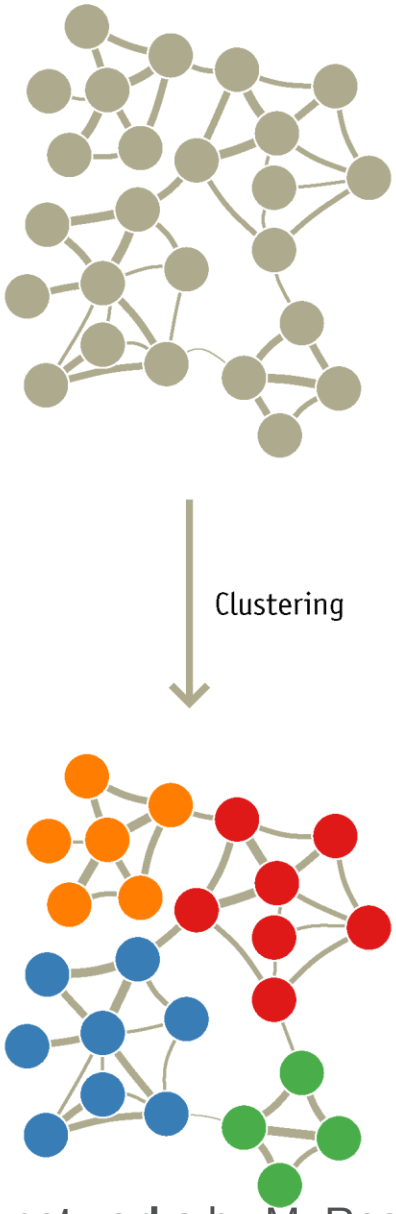


- The blue cluster is comprised of a Federal Home Loan Bank and a number of banks that tend to be located the same geographic region as the Home Loan Bank.
- We speculate that this cluster reflects the fact that the Home Loan bank may have started to intermediate funds between its members by borrowing funds from some and making overnight advances (i.e., collateralized loans) to others during this period.

What are the key differences?

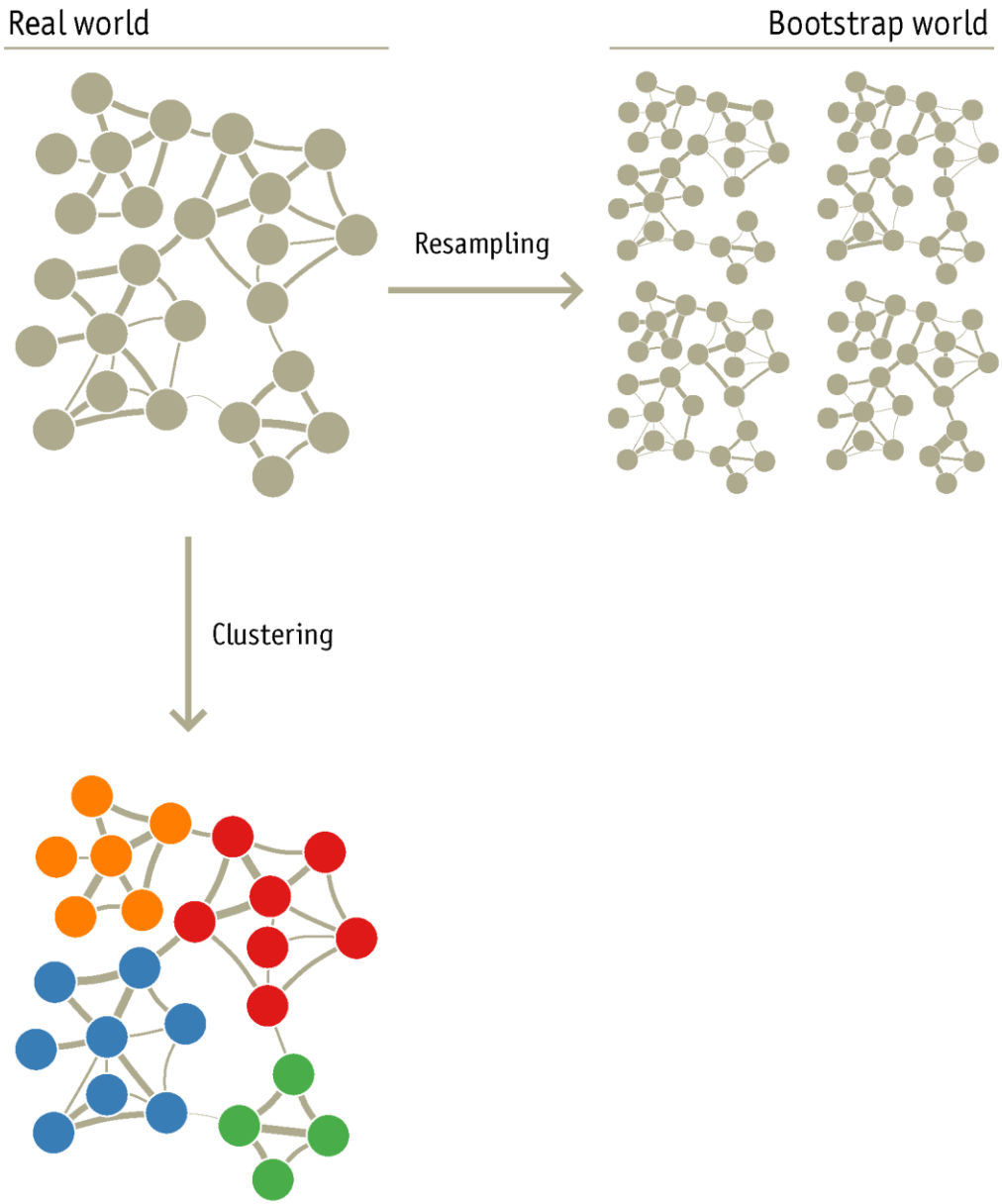
- What is real change and what is mere noise?
- Need to know which structures are statistically significant and which are not.

Significance clustering

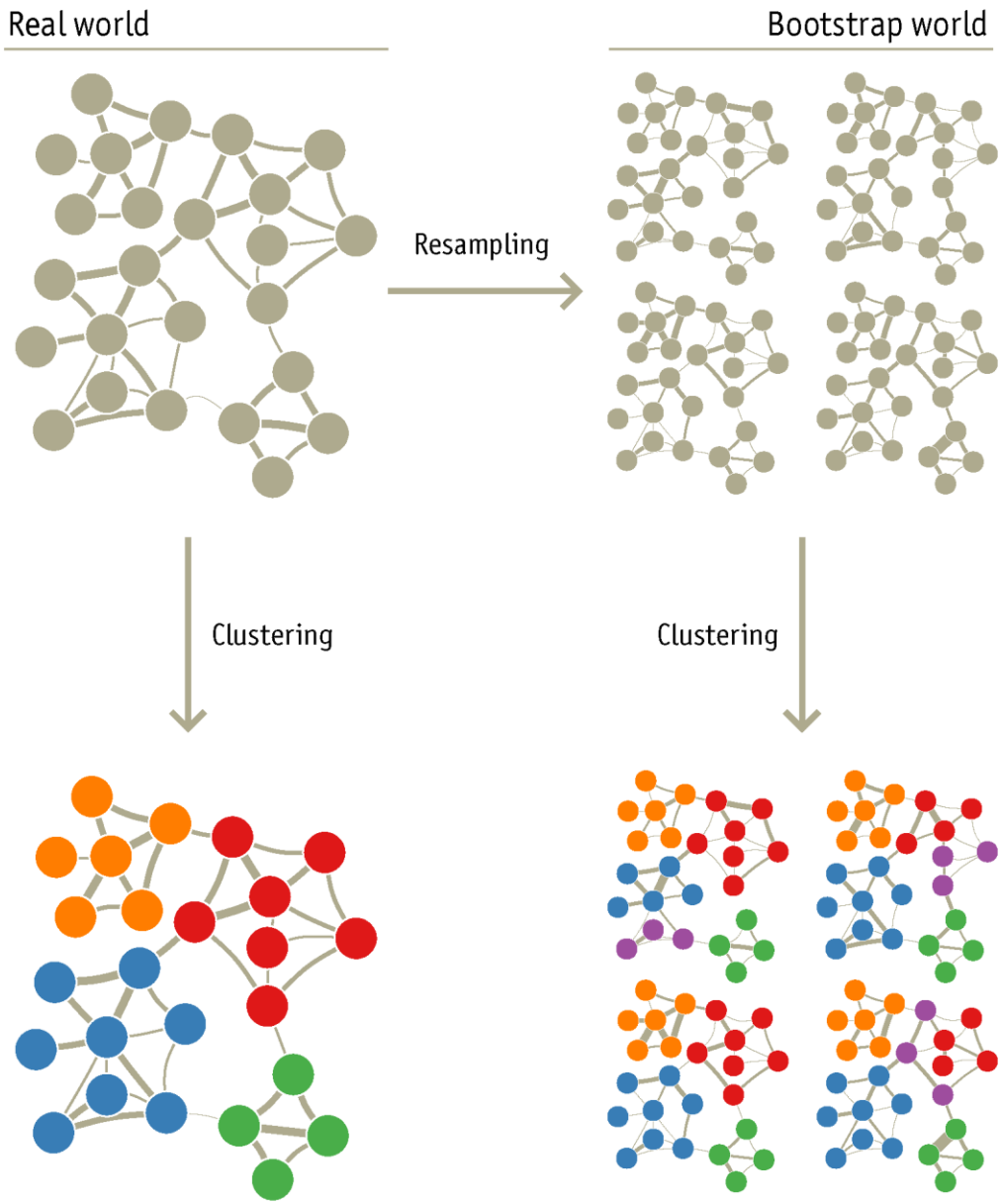


Source: **Mapping change in large networks** by M. Rosvall and C. T. Bergstrom

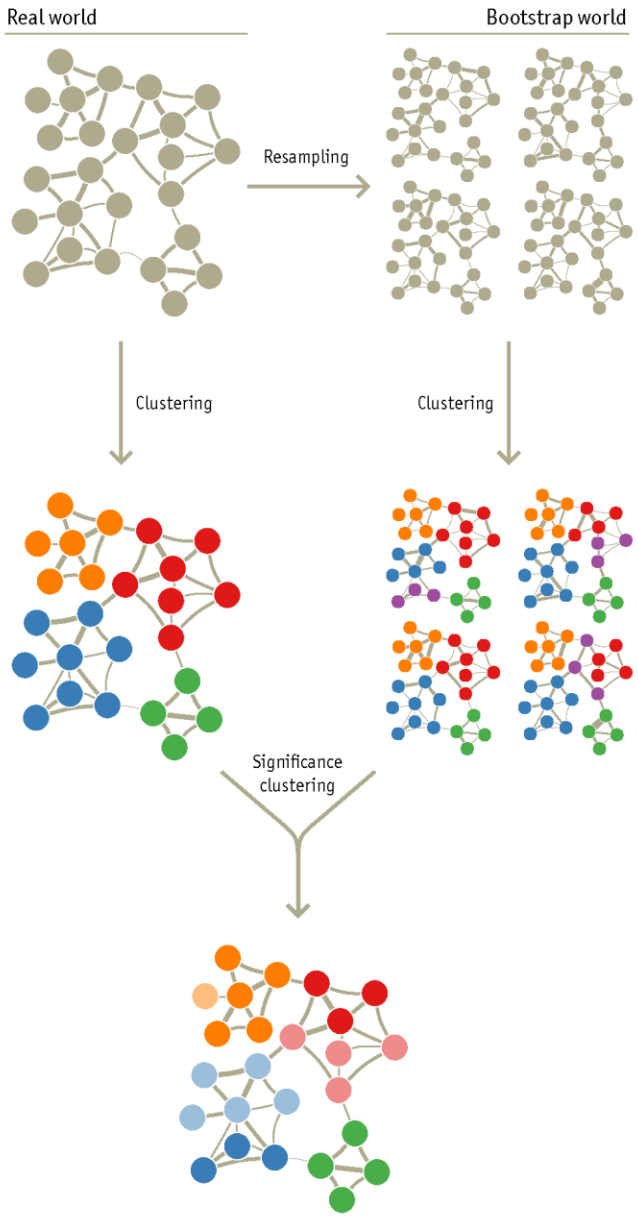
Significance clustering



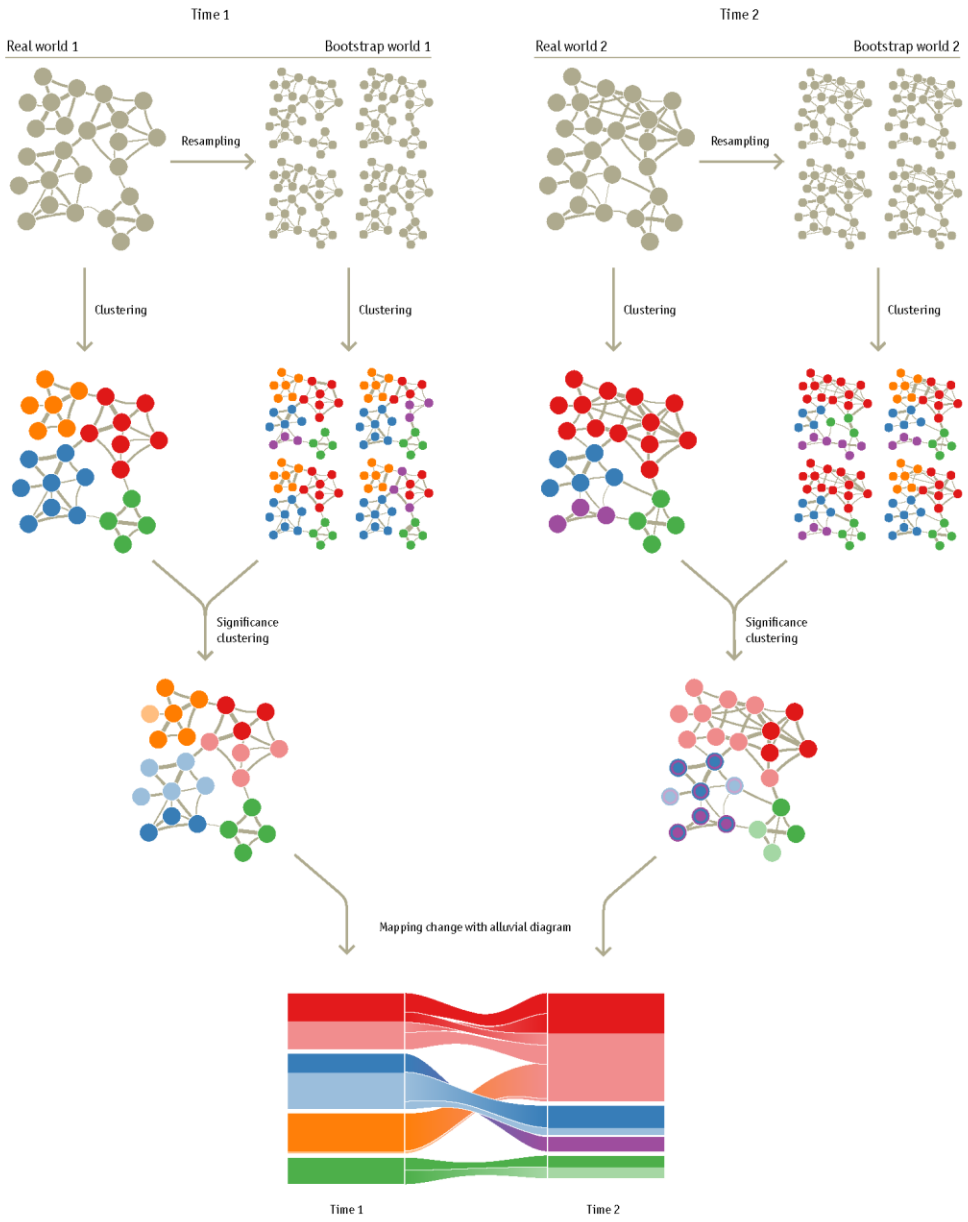
Significance clustering

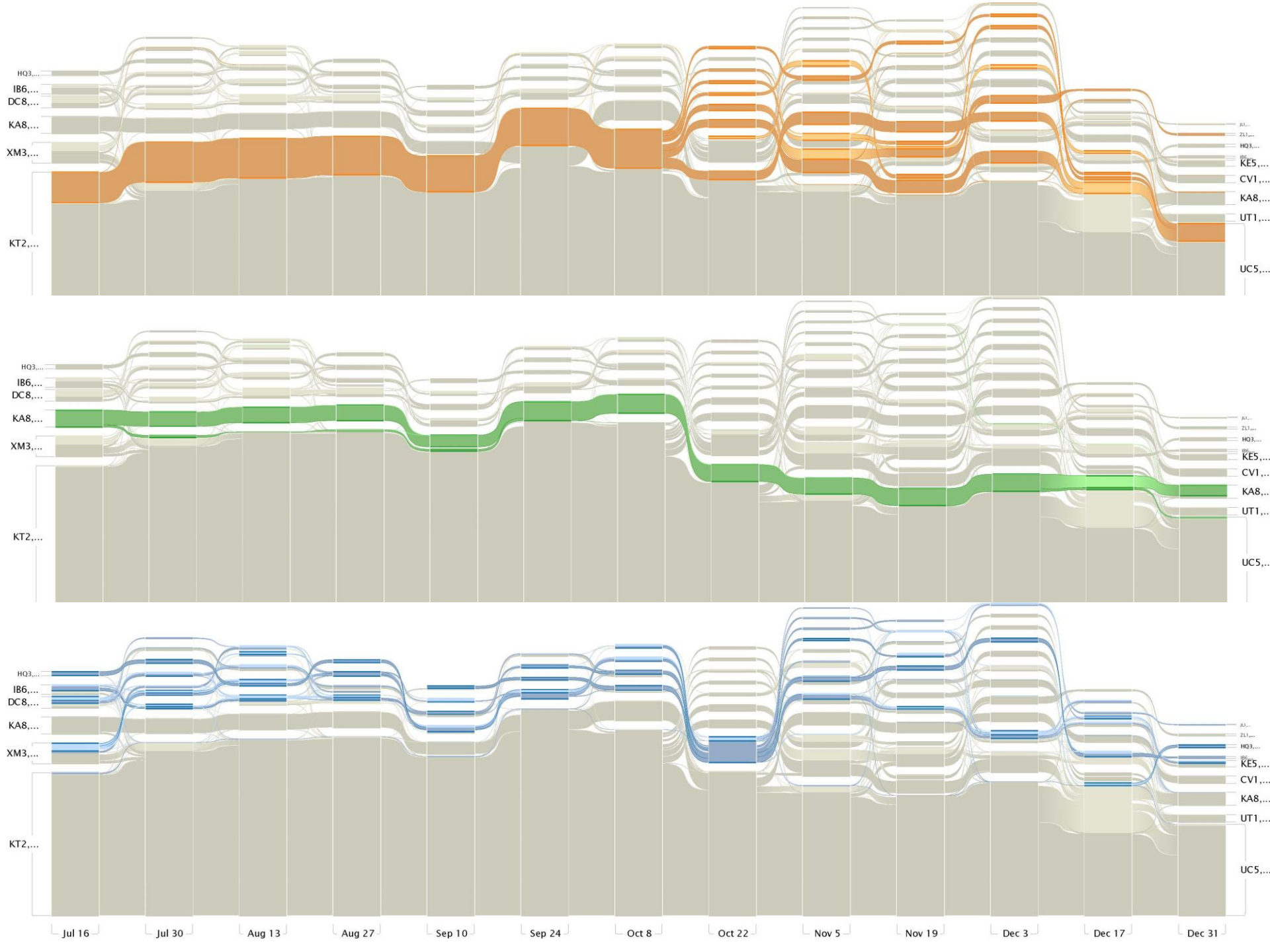


Significance clustering



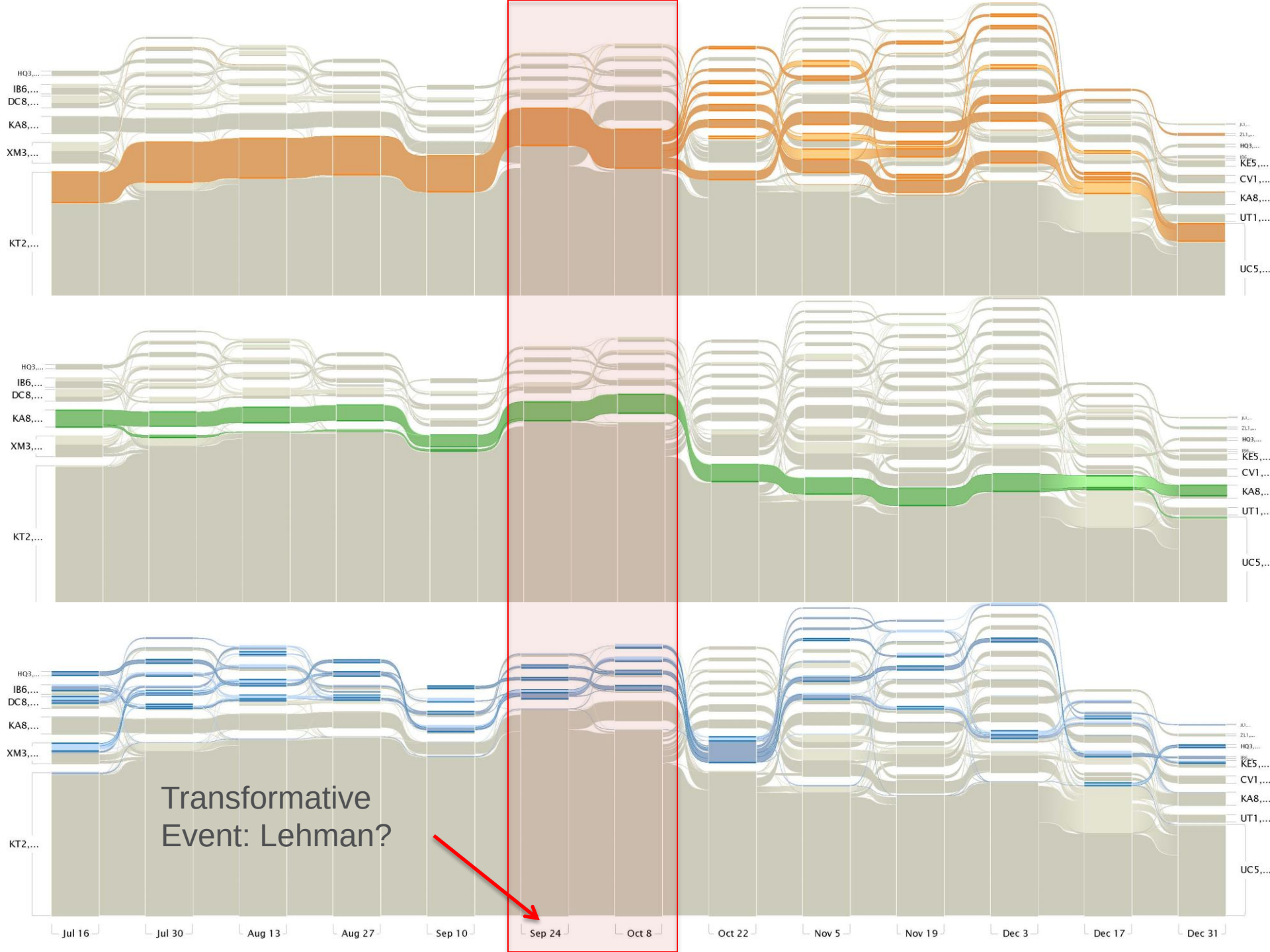
Mapping change

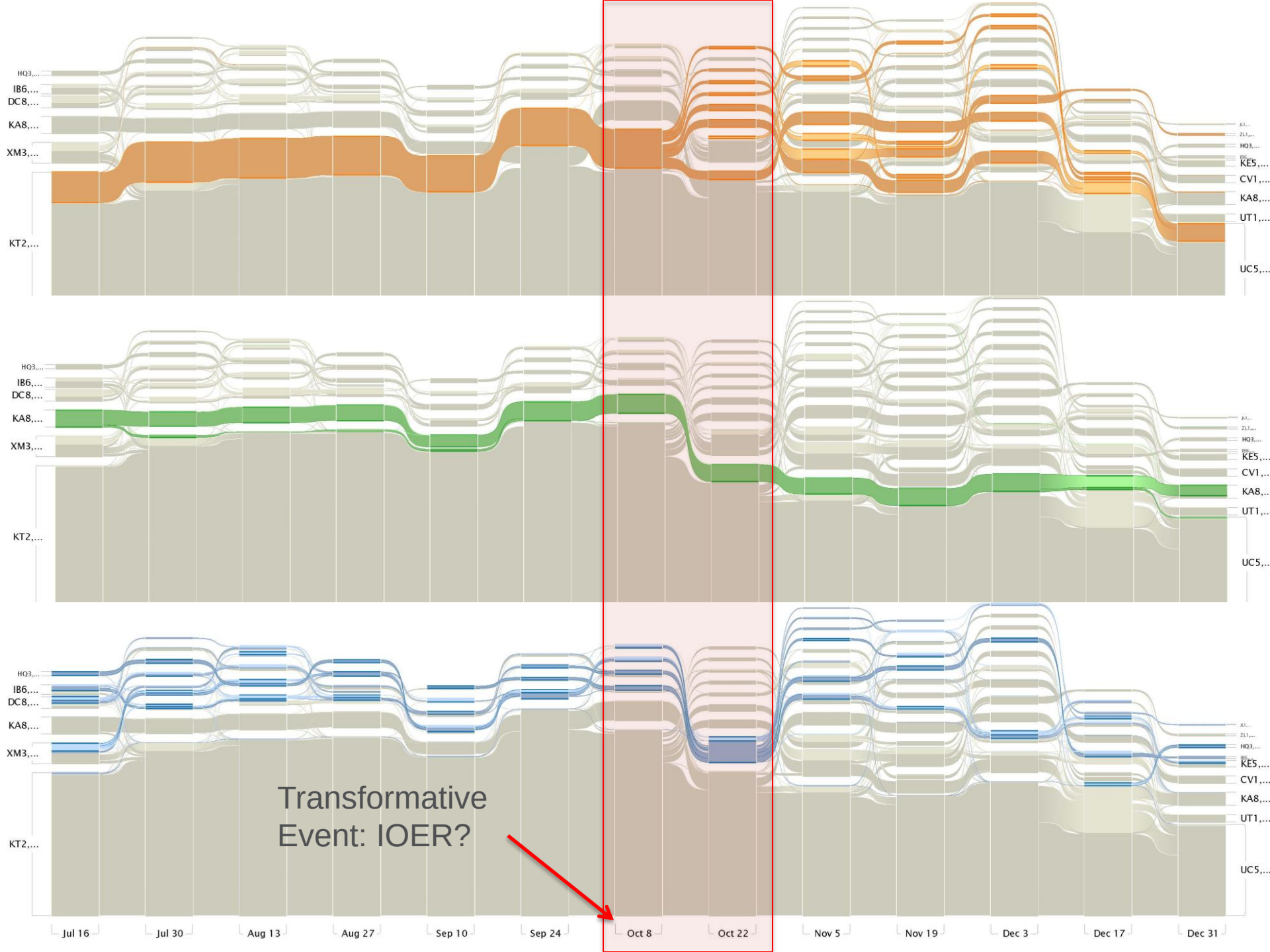



HQ3,...
IB6,...
DC8,...
KA8,...
XM3,...
KT2,...
KE5,...
CV1,...
KA8,...
UT1,...
UC5,...
Jul 16
Jul 30
Aug 13
Aug 27
Sep 10
Sep 24
Oct 8
Oct 22
Nov 5
Nov 19
Dec 3
Dec 17
Dec 31

Analysis of Micro-Scale Rates of Change

- While the alluvial diagrams are very nice for showing the general patterns of how lending takes place in each period, they do not necessarily reveal the onset of change in the system.
- Changes in clustering patterns reflect tipping points at which the cumulative effect of multiple small changes in flows constitute a significant change.


HQ3,...
IB6,...
DC8,...
KA8,...
XM3,...
KT2,...
KE5,...
CV1,...
KA8,...
UT1,...
UC5,...
Transformative
Event: Lehman?
Jul 16
Jul 30
Aug 13
Aug 27
Sep 10
Sep 24
Oct 8
Oct 22
Nov 5
Nov 19
Dec 3
Dec 17
Dec 31


HQ3,...
IB6,...
DC8,...
KA8,...
XM3,...
KT2,...
KE5,...
CV1,...
KA8,...
UT1,...
UC5,...
Transformative
Event: IOER?
Jul 16
Jul 30
Aug 13
Aug 27
Sep 10
Sep 24
Oct 8
Oct 22
Nov 5
Nov 19
Dec 3
Dec 17
Dec 31

Tipping Points

- Suppose that lending configurations a, b, c, d, e, f, g, h all generate a system with module structure of type 1, and configurations i, j, k, l, m, n, o, p all generate a system with module structure of type 2.

T=	config	cluster
0	a	1
1	a	1
2	b	1
3	g	1
4	h	1
5	i	2
6	j	2
7	j	2

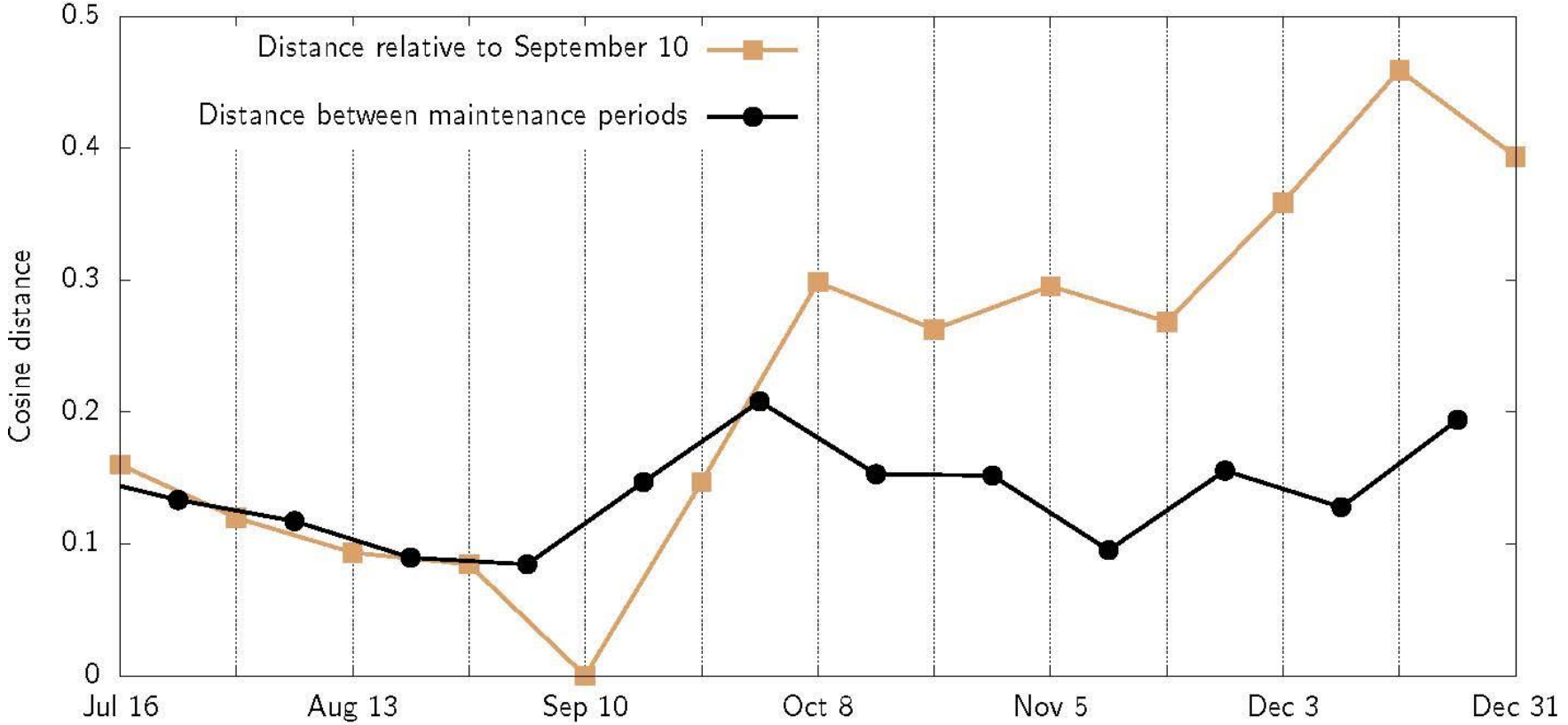
- Most rapid change from period 2 to 3, not 4 to 5.

Micro-Scale Rates of Change

- We would like to be able to differentiate between the two hypotheses that (1) Lehman's failure is associated with a shakeup in lending patterns and (2) paying interest on reserves is associated with the shakeup.
  - An n x n lending matrix is specified by a unique vector of length n^2.
  - each time period corresponds to a vector in n^2 space.
- To measure the amount of change in the system, we look at how much the angle between the vector at each time t changes going to time t+1.

$$\text{similarity} = \cos(\theta) = \frac{A \cdot B}{\|A\|\|B\|} = \frac{\sum_{i=1}^{n} A_i \times B_i}{\sqrt{\sum_{i=1}^{n} (A_i)^2} \times \sqrt{\sum_{i=1}^{n} (B_i)^2}}$$

- This is a standard approach in network theory, known as cosine distance.



- Orange trace compares the distance between each time period and a fixed time period (Sep 10).
- Black trace shows us the velocity of change from one period to the next over time.
- Largest change occurs in the Oct 8 maintenance period, which covers the time period from September 25 to October 8, after collapse of Lehman and before IEOR.

Transformative Event

- While changes in borrowing and lending patterns do not fully reveal themselves in our maps until after the implementation of Interest on Reserves, examination of the micro-scale rates of change strongly suggests that the collapse of Lehman Brothers was the driving force.

Concluding Remarks

- Advanced network techniques can help stakeholders in the financial system to understand its structural features and to analyze the impact of transformative events.
- As illustrated here, the map equation appears to be a very useful tool for understanding funding flows.
  - More appropriate than “competing” techniques that impose community structure or ignore flows
- The lending flows in the interbank lending market changed in a fundamental way between September 10 and October 22, 2008, and the alluvial diagrams reveal this clearly.
- However, there is evidence that the underlying shifts in network flows that led to these structural changes may have been initiated well before the tipping point was reached.

Concluding Remarks

- Considerable caution is required when deriving causal inferences from alluvial diagrams.
  - We advocate the use of two tools for analyzing the structure of financial networks.
  - Each complements the other and neither is sufficient in isolation.
- The map equation reveals community structure and changes can be visualized via alluvial diagrams.
  - However, these maps do not reveal details of micro-scale rates of change that precipitated change.
- The cosine distance analysis is useful in this regard, but itself is not informative: it reveals change, but not the content of that change.

Thank You