Similarity-based & Statistically Validated Networks in Finance

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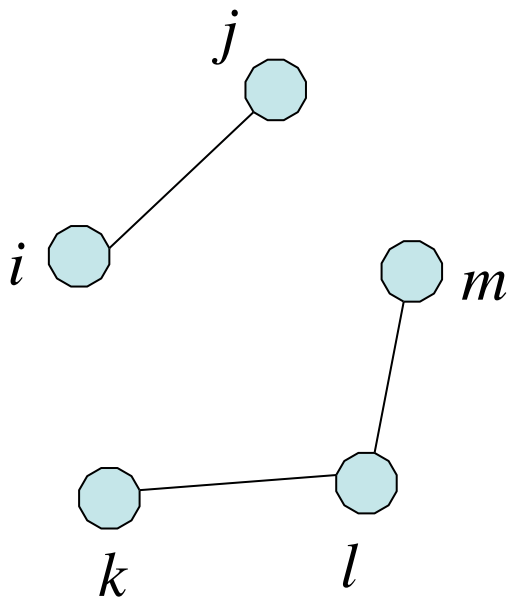
Outline

- I will discuss the concept of similarity based networks and their use in finance;

- I will present the methodology of statistically validated networks by discussing its application to syndicated loans and the interbank market.

Two different approaches in building networks

Event or relation defined networks



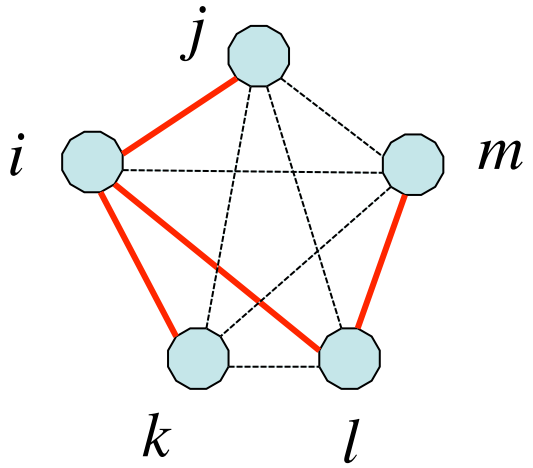
Example:
nodes are banks
links are credit relationships

Similarity-based networks

Example: 1) Consider
Portfolio of bank i
portfolio of bank j
.....
portfolio of bank m

2) Estimate similarity/distance between each pair of banks;

3) Extract a weighted network from a similarity/distance matrix.



The first investigation of a correlation based networks

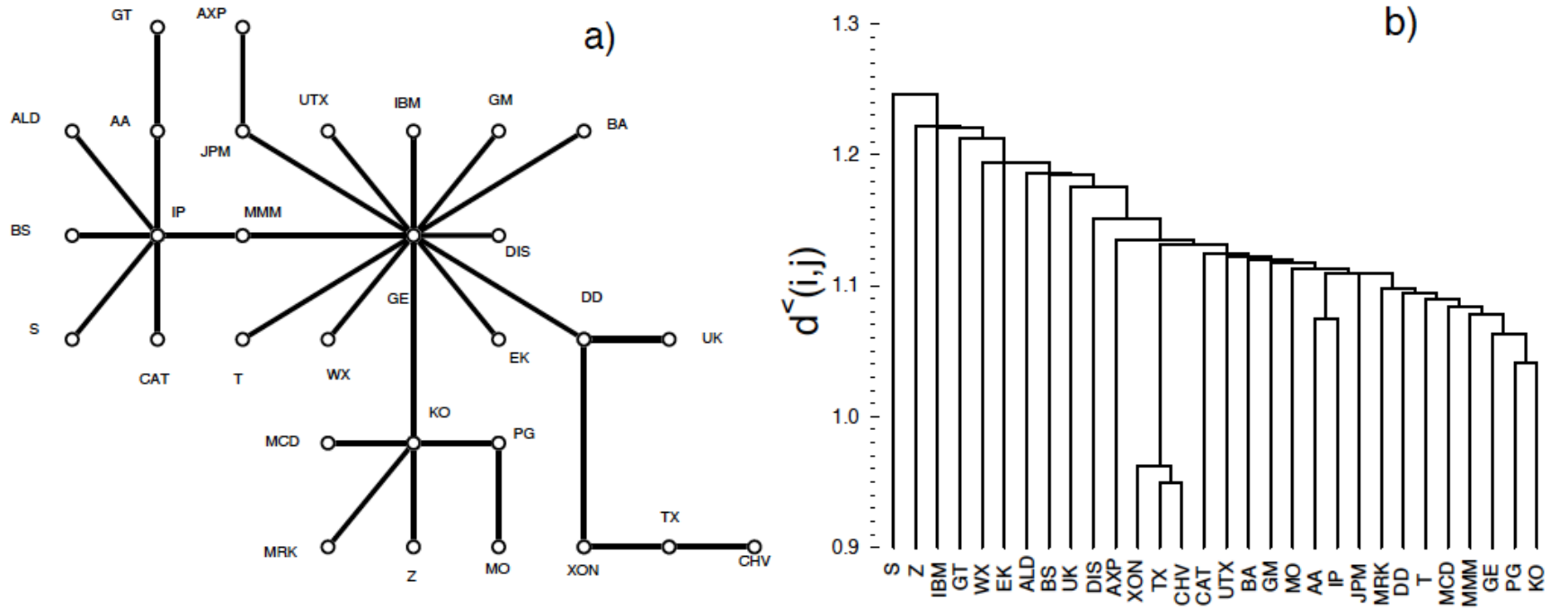


Fig. 1. (a) Minimal spanning tree connecting the 30 stocks used to compute the Dow Jones Industrial Average. The 30 stocks are labeled by their tick symbols. The distance between the stocks is bounded as: CHV-TX $0.90 < d(i,j) \leq 0.95$; XON–TX $0.95 < d(i,j) \leq 1.00$; KO-PG $1.00 < d(i,j) \leq 1.05$; MMM–GE–KO, DD–GE–T, AA–IP and MRK–KO–MCD $1.05 < d(i,j) \leq 1.10$; CAT–IP–MMM, AXP–JPM–GE–GM, BA–GE–UTX, DD–XON and MO–PG $1.10 < d(i,j) \leq 1.15$; DIS–GE–EK, DD–UK, BS–IP–ALD and GE–WX $1.15 < d(i,j) \leq 1.20$; AA–GT, GE–IBM, KO–Z and IP–S $1.20 < d(i,j) \leq 1.25$. (b) Hierarchical tree of the subdominant ultrametric space associated with the minimal spanning tree of a). In the hierarchical tree, several groups of stocks homogeneous with respect to the economic activities of the companies are detected: (i) oil companies (Exxon (XON), Texaco (TX) and Chevron (CHV)); (ii) raw material companies (Alcoa (AA) and International paper (IP)) and (iii) companies working in the sectors of consumer nondurable products (Procter & Gamble (PG)) and food and drinks (Coca Cola (KO)). The ultrametric distance at which individual stocks are branching from the tree is given by the y axis.

R.N. Mantegna, Hierarchical structure in financial markets, Eur. Phys. J. B 11, 193-197 (1999)

Filtering the correlation matrix using single linkage clustering

By starting from a correlation matrix (which is a similarity measure)

	AIG	IBM	BAC	AXP	MER	TXN	SLB	MOT	RD	OXY
AIG	1	0.413	0.518	0.543	0.529	0.341	0.271	0.231	0.412	0.294
IBM		1	0.471	0.537	0.617	0.552	0.298	0.475	0.373	0.270
BAC			1	0.547	0.591	0.400	0.258	0.349	0.370	0.276
AXP				1	0.664	0.422	0.347	0.351	0.414	0.269
MER					1	0.533	0.344	0.462	0.440	0.318
TXN						1	0.305	0.582	0.355	0.245
SLB							1	0.193	0.533	0.592
MOT								1	0.258	0.166
RD									1	0.590
OXY										1

$$d_{ij} = \sqrt{2\left(1-\rho_{ij}\right)}$$

		ρ_{ij}	d_{ij}
AXP	MER	0.664	0.820
IBM	MER	0.617	0.875
SLB	OXY	0.592	0.903
BAC	MER	0.591	0.904
RD	OXY	0.590	0.905
TXN	MOT	0.582	0.914
IBM	TXN	0.552	0.947
AXP	BAC	0.547	0.952
AIG	AXP	0.543	0.956
AXP	IBM	0.537	0.962
SLB	RD	0.533	0.966
MER	TXN	0.533	0.966
AIG	MER	0.529	0.970
AIG	BAC	0.518	0.982
IBM	MOT	0.475	1.025
MOT	MER	0.462	1.037
MER	RD	0.440	1.058
AXP	TXN	0.422	1.075
.....			

The hierarchical tree obtained from single linkage clustering algorithm has information equivalent to a simplified matrix having only n-1 distinct elements. It can be proven that such a matrix is an ultrametric matrix when a distance is defined between each pair of elements.

	AIG	IBM	BAC	AXP	MER	TXN	SLB	MOT	RD	OXY
AIG	1	0.543	0.543	0.543	0.543	0.543	0.440	0.543	0.440	0.440
IBM		1	0.591	0.617	0.617	0.552	0.440	0.552	0.440	0.440
BAC			1	0.591	0.591	0.552	0.440	0.552	0.440	0.440
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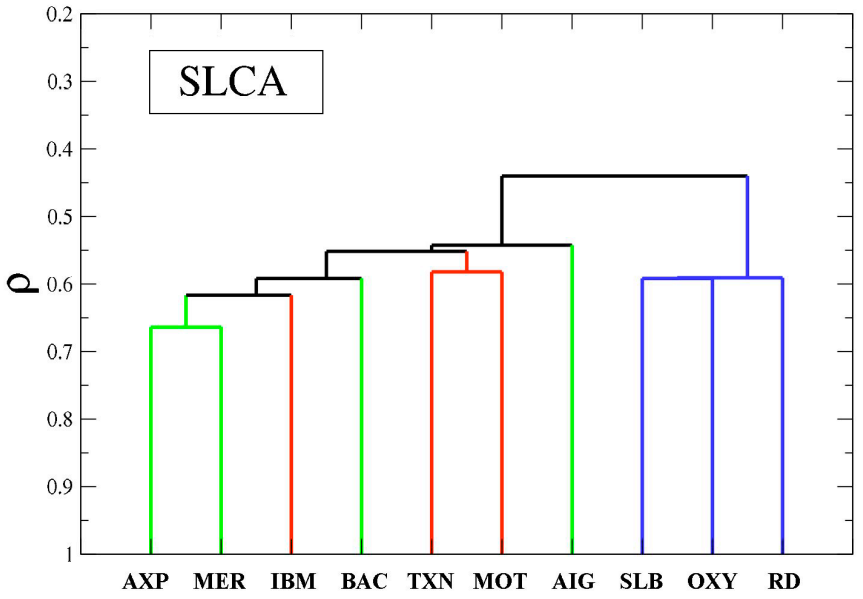
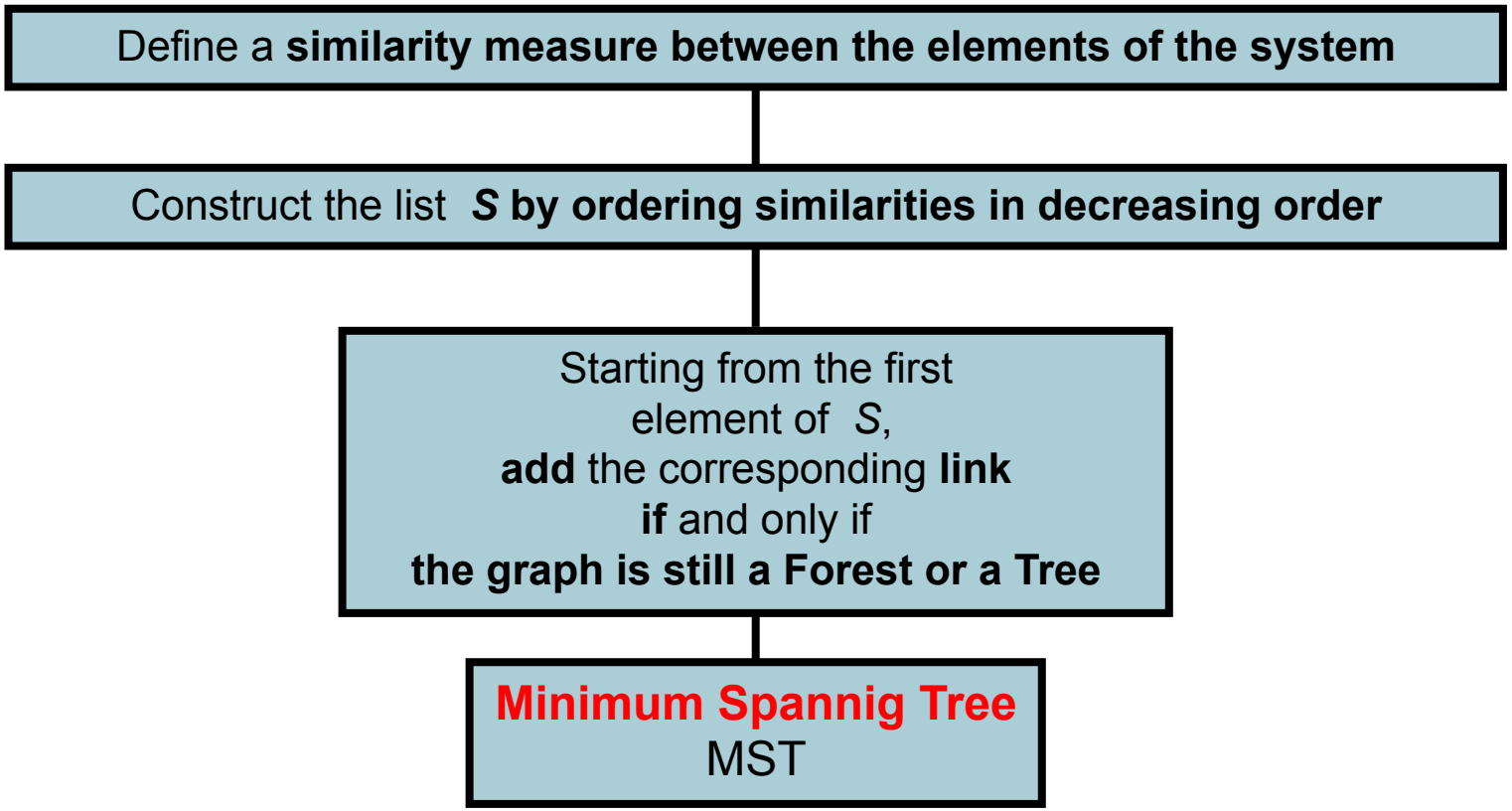
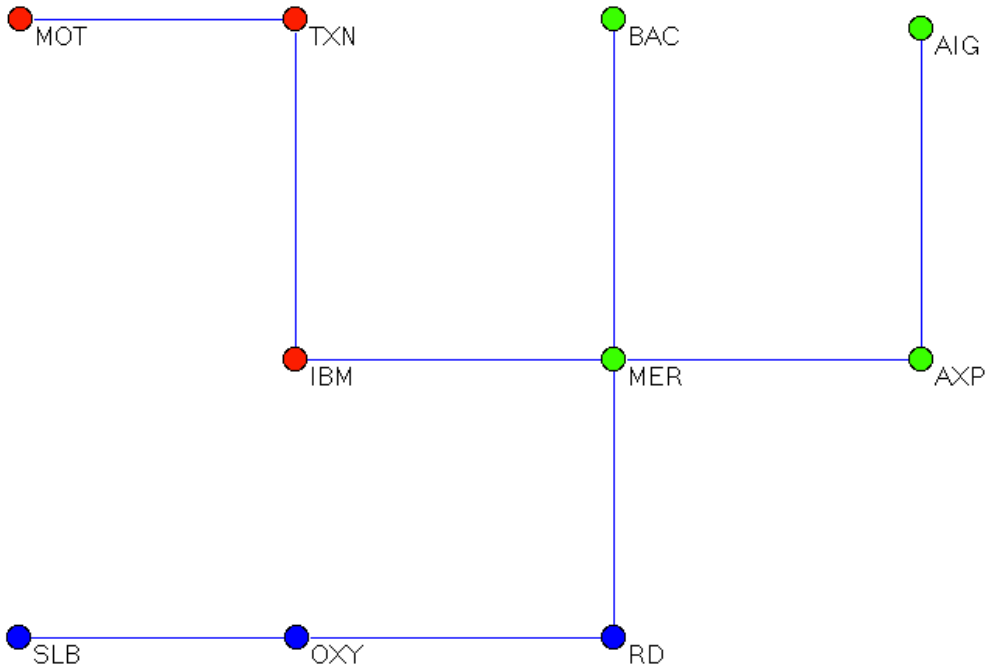


$C^{<}_{SL}$

Kruskal's algorithm of the Minimum Spanning Tree



Correlation based trees and hierarchical trees do NOT carry the same amount of information.



C_{SL}

	AIG	IBM	BAC	AXP	MER	TXN	SLB	MOT	RD	OXY
AIG	1	0.413	0.518	0.543	0.529	0.341	0.271	0.231	0.412	0.294
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$C^{<}_{SL}$

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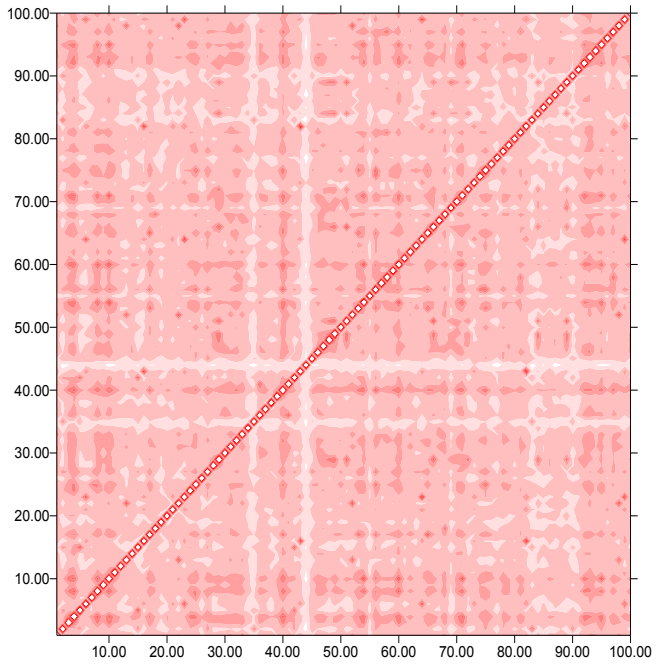
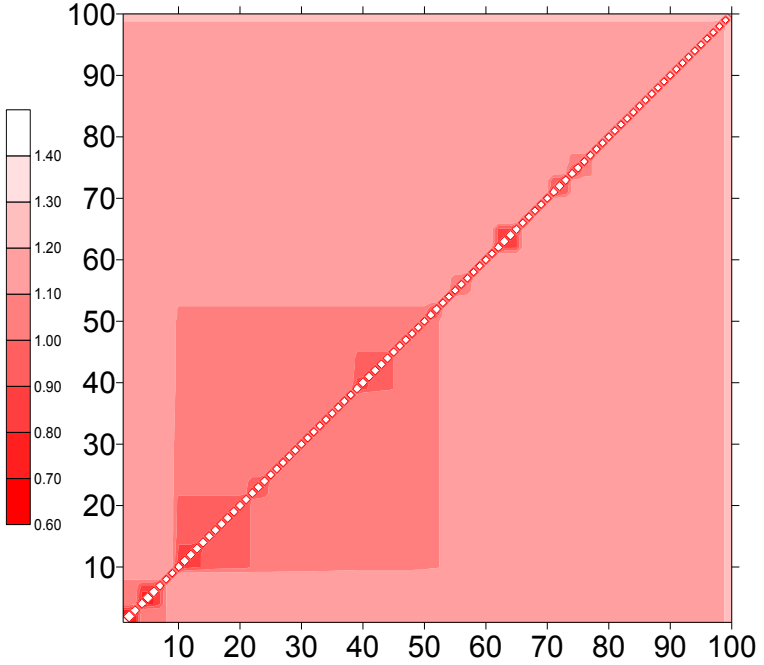
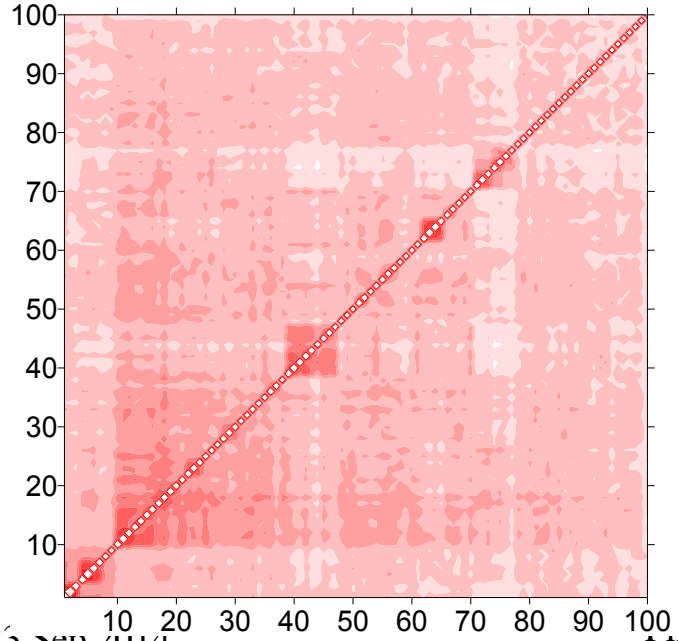
The filtering selects part of the information of a distance matrix

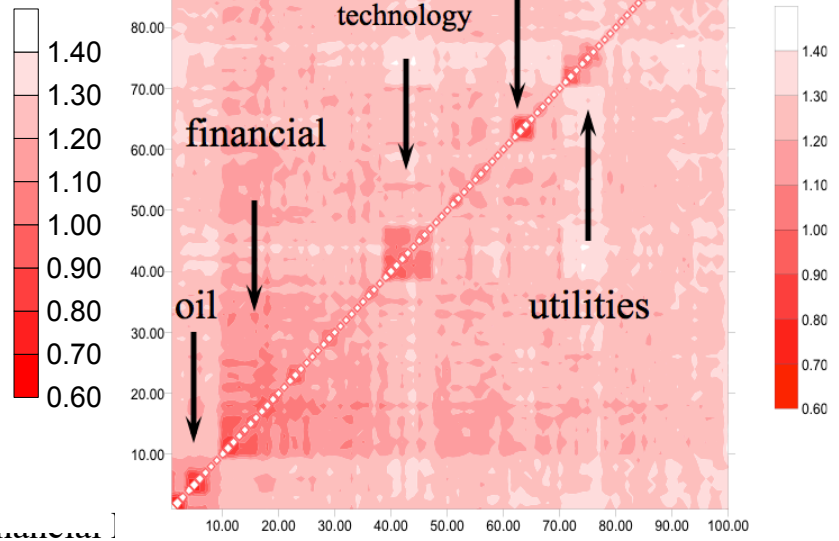




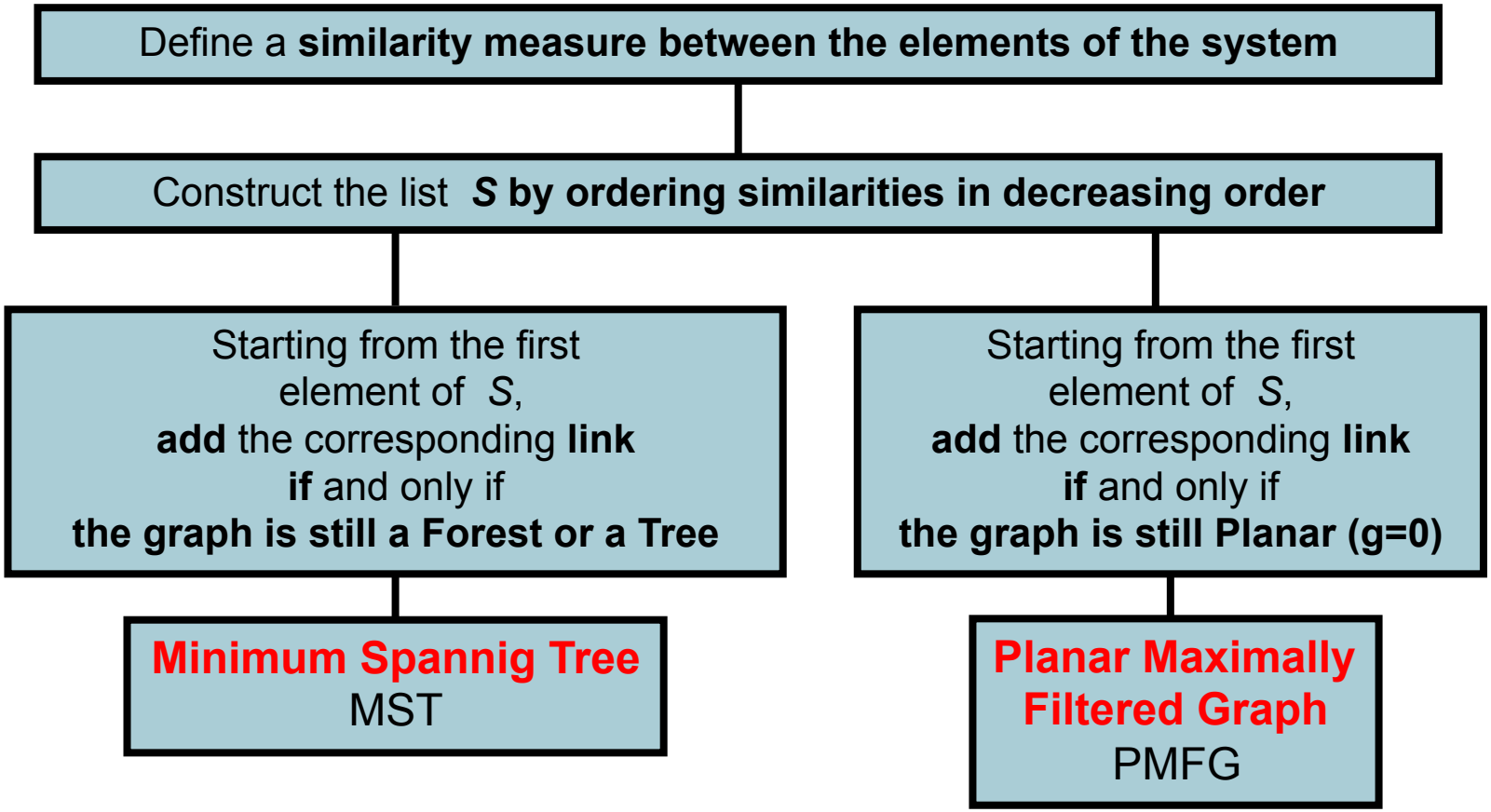
$$d_{ij} = \sqrt{2\left(1-\rho_{ij}\right)}$$

100 highly capitalized stocks of US equity markets 1995-1998 daily data





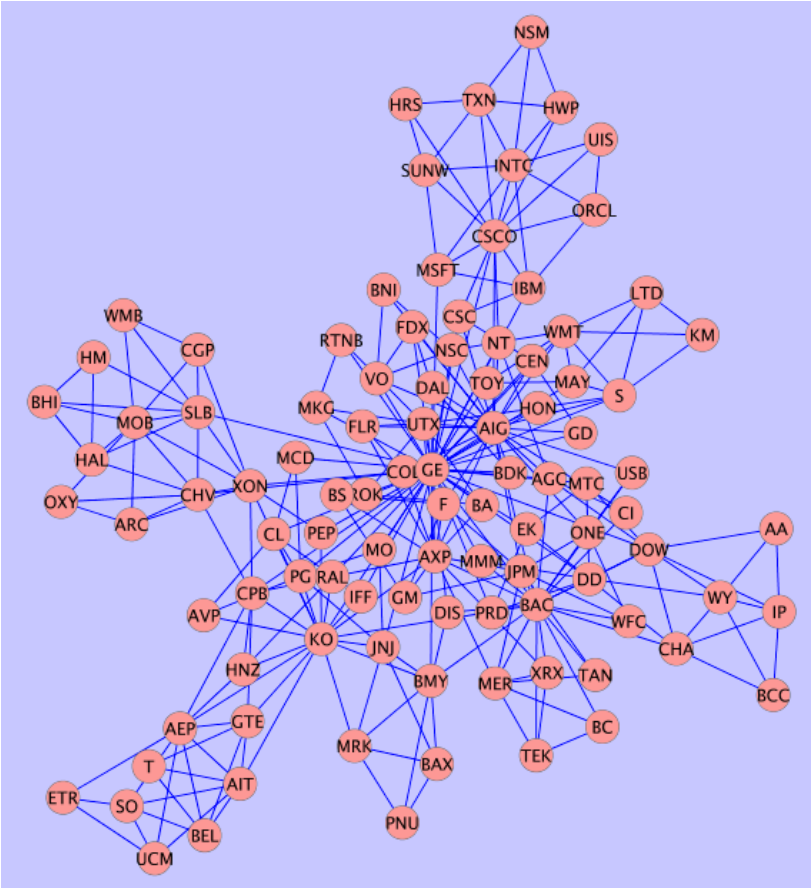
The minimum spanning tree is the most basic and robust way to obtain a similarity based network. There are many generalizations of this basic approach. A prominent one is the Planar Maximally Filtered Graph.

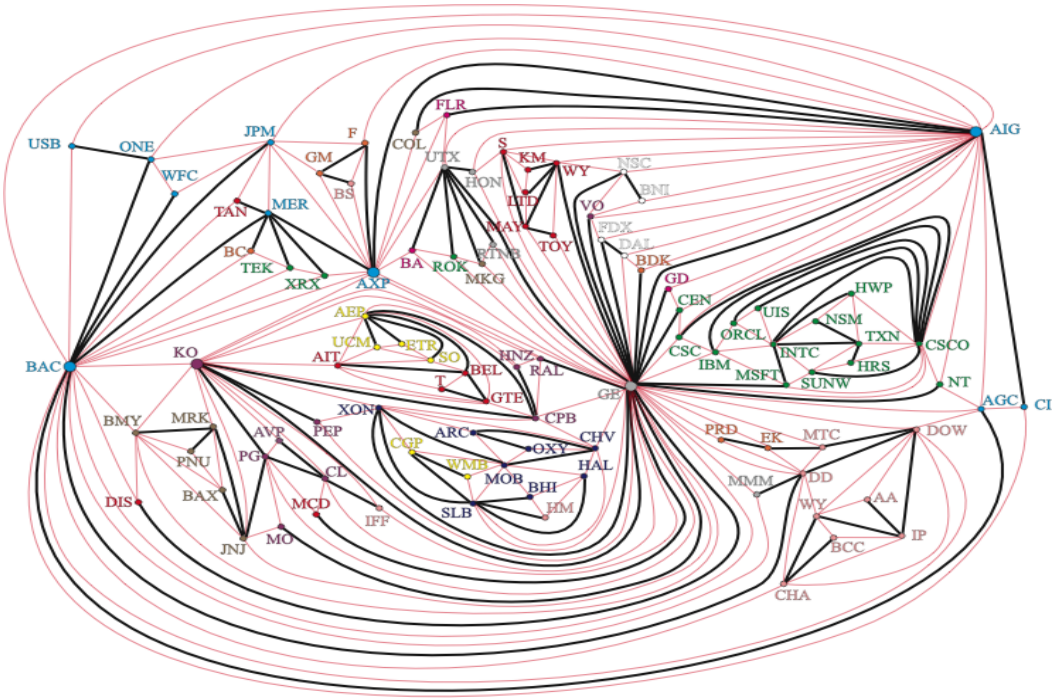


M. Tumminello, T. Di Matteo, T. Aste and R.N.M., PNAS USA 102, 10421 (2005)

Planar graphs

A graph is planar if its edges can be embedded on a surface of genus 0, i.e. a surface like a plane or a sphere, without intersections of the edges.





N=100 (US markets) daily returns
1995-1998 T=1011

Partial correlation network

The partial correlation coefficient

$$\rho(X,Y:Z)$$

between variables X and Y conditioned on the variable Z is the Pearson correlation coefficient between the residuals of X and Y that are uncorrelated with Z

We[¶] investigated the quantity

$$d(X,Y:Z) \equiv \rho(X,Y) - \rho(X,Y:Z)$$

This is an estimation of the correlation influence of Z on the correlation of pair of elements X and Y

It should be noted that *d*(X,Y:Z) assumes non negligible values only when $\rho(X,Y)$ is significantly different from zero.

[¶]Kenett DY, Tumminello M, Madi A, Gur-Gershgoren G, Mantegna RN, et al. (2010) Dominating Clasp of the Financial Sector Revealed by Partial Correlation Analysis of the Stock Market. PLoS ONE 5(12): e15032. doi:10.1371/journal.pone.0015032

The number of d(X,Y:Z) elements is cubic in N.
In fact different elements are N (N-1) (N-2)/2

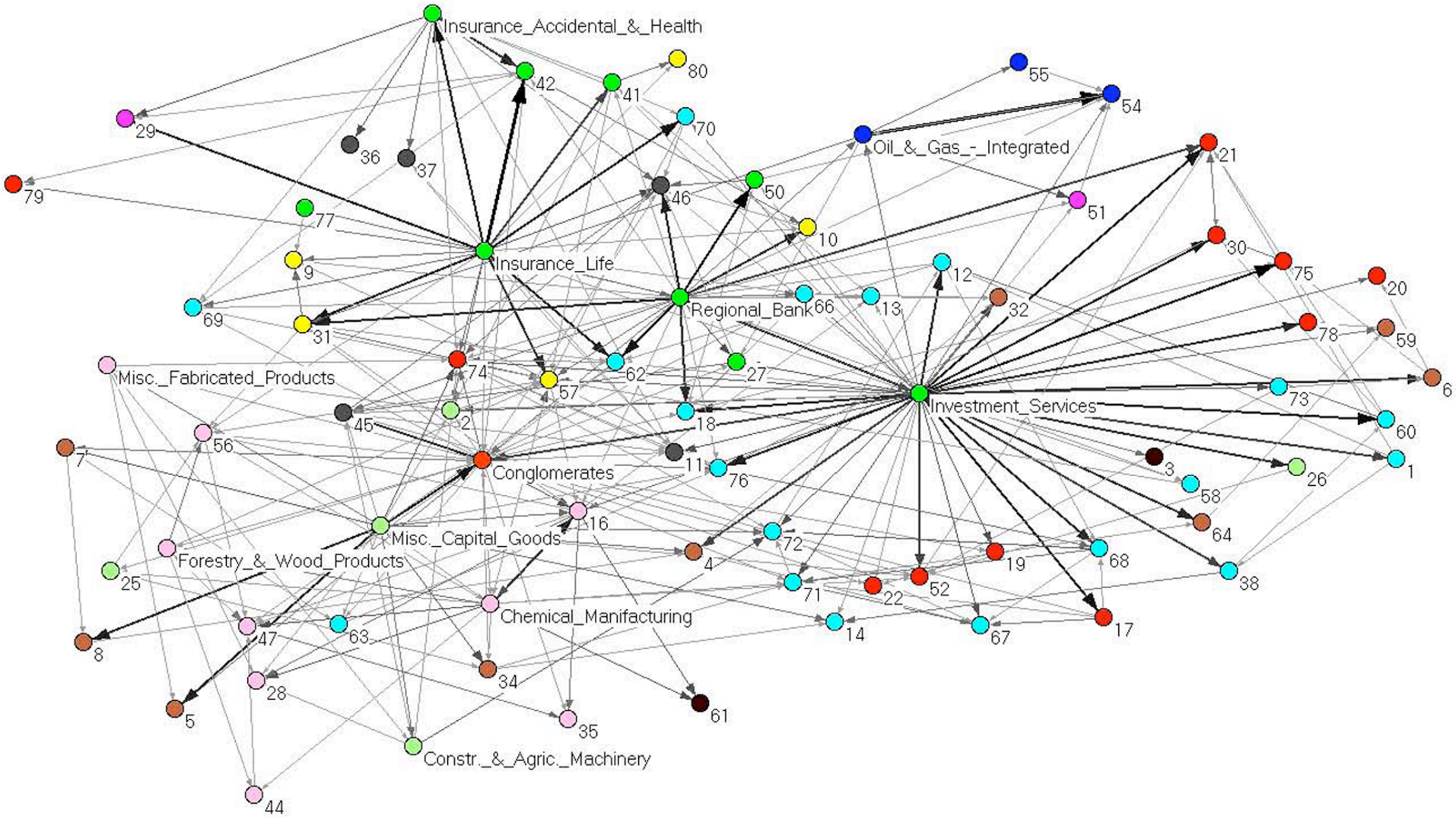
We therefore investigate the overall effect of stock Z on correlation of stock X with all other stocks except Z.

Specifically, we investigate

$$d(X:Z) = \left\langle d\left(X,Y:Z\right)\right\rangle_{Y \neq X,Z}$$

We use this directed similarity measure to obtain a Partial Correlation Planar Graph

The Partial Correlation Planar Graph (economic subsectors)



A statistical assessment of links of similarity based networks can be performed by using bootstrap replicas

Data Set

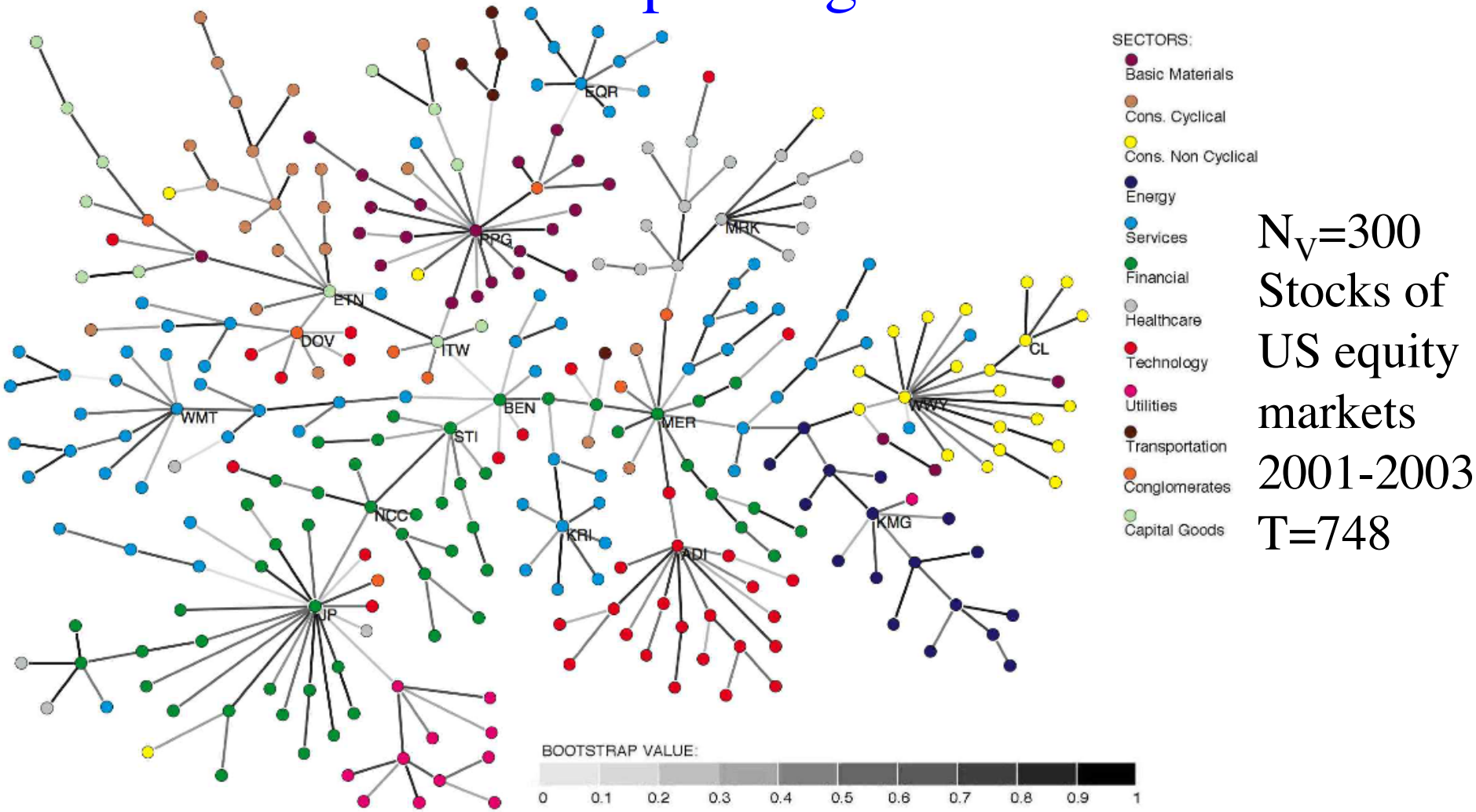
	V_1	V_2	V_3	…	V_n
t_1	0.113	1.123	-0.002	…	0.198
t_2	1.567	0.789	0.842	…	-0.234
t_3	1.065	-1.962	0.567	…	1.785
t_4	1.112	0.998	-0.424	…	2.735
t_5	-0.211	0.312	-0217	…	0.587
…	…	…	…	…	…
T	0.479	-1.828	-2.041	…	-0.193

Pseudo-replicate Data Set

V_1	V_2	V_3	…	V_n
1.567	0.789	0.842	…	-0.234
0.113	1.123	-0.002	…	0.198
1.065	-1.962	0.567	…	1.785
0.113	1.123	-0.002	…	0.198
0.479	-1.828	-2.041	…	-0.193
…	…	…	…	…
0.479	-1.828	-2.041	…	-0.193

M surrogated data matrices are constructed, e.g. M=1000.

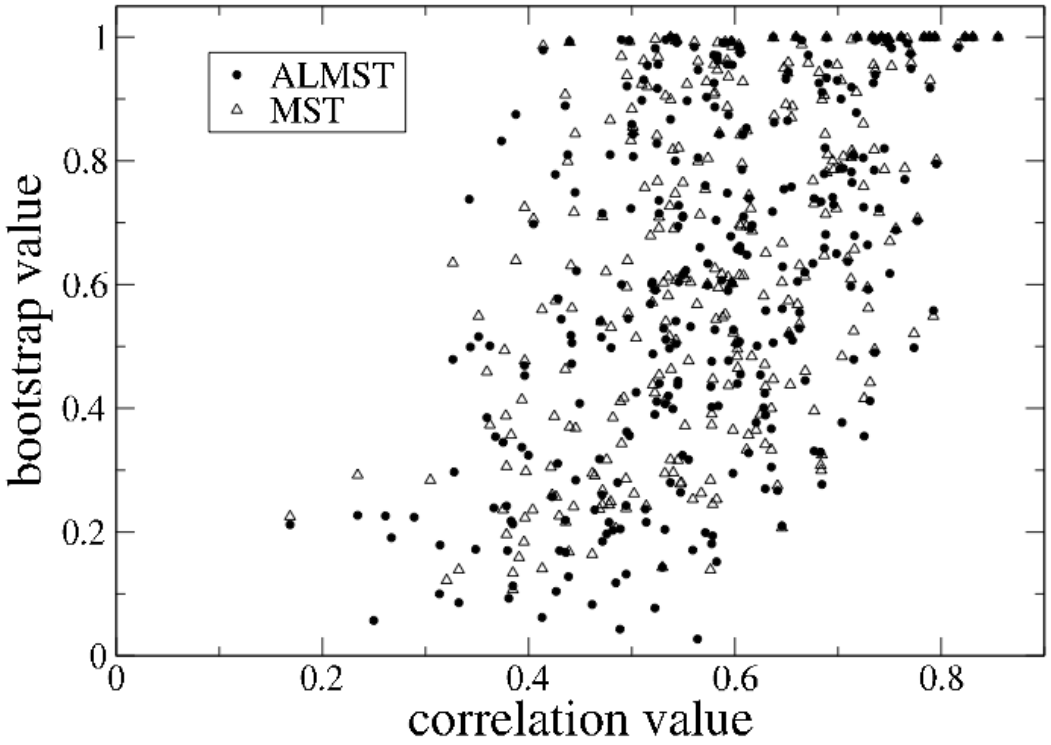
Statistical reliability of the minimum spanning tree



N_V=300 Stocks of US equity markets 2001-2003 T=748

M. Tumminello, C. Coronnello, S. Micciché, F. Lillo and R.N.M., Int. J. Bifurcation Chaos **17,** 2319-2329 (2007).

Bootstrap vs correlation



N_V=300
Stocks of
US equity
markets
2001-2003
T=748

For Gaussian series: $\sigma_\rho = \frac{1-\rho^2}{\sqrt{T-3}}$

Edge filtering is also relevant in networks

Several networks are pretty dense and it is quite difficult to detect their internal structures.

One recent approach[¶] able to detect internal structures of networks is the approach of statistically validated networks.

In statistically validated networks the scientific question is:

Is it possible to detect interaction among nodes of the network that are over- expressed or under-expressed with respect to a null hypothesis taking into account the heterogeneity of the system?

[¶]Tumminello M, Miccichè S, Lillo F, Piilo J, Mantegna RN (2011) Statistically Validated Networks in Bipartite Complex Systems. PLoS ONE 6(3): e17994. doi:10.1371/journal.pone.0017994

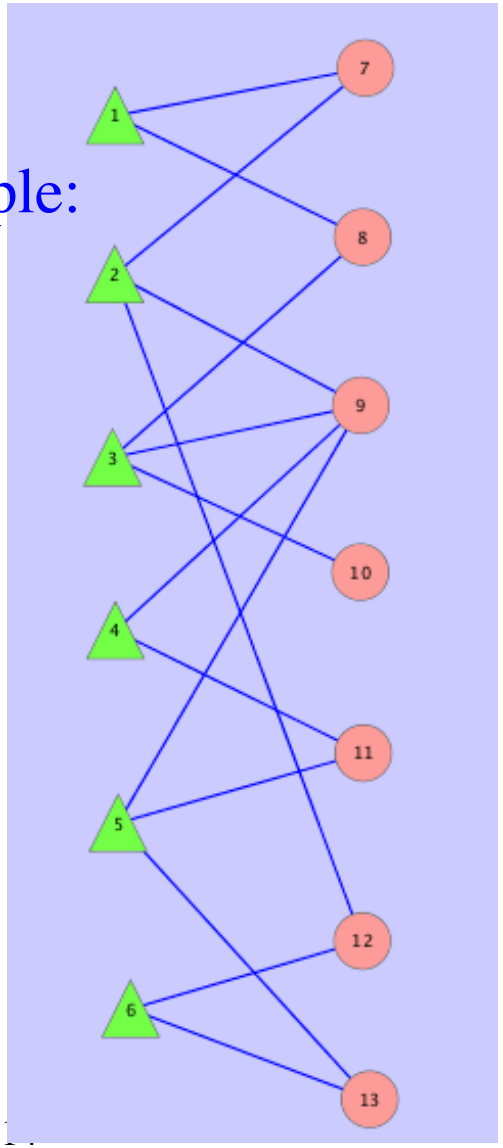
In several cases the problem of statistically validating a link can be mapped into a urn problem

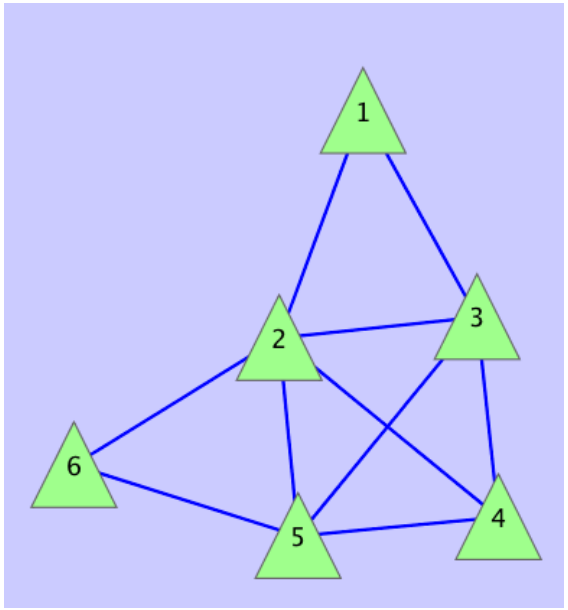
Lending banks

Packages

Projected network of lending banks

Example:



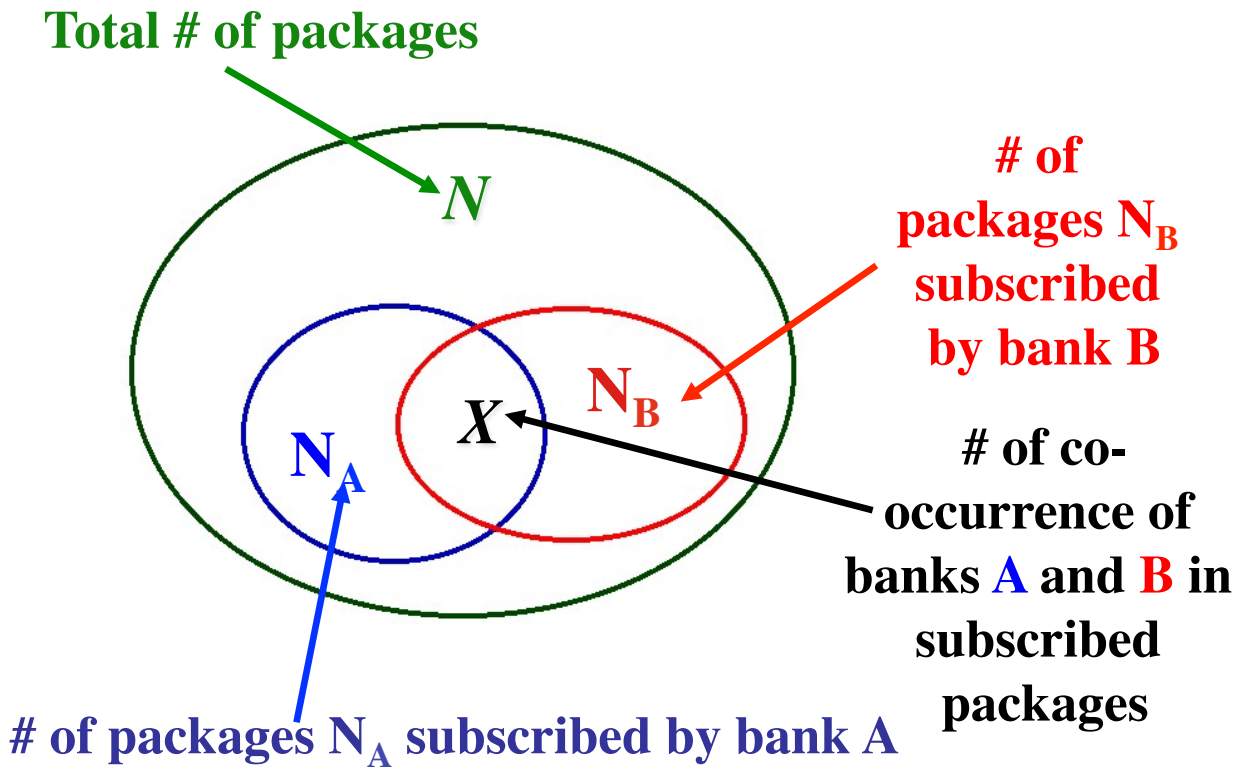


The investigated system concerns syndicated loans.

The database is the DealScan database of Thomson Reuters

A statistical validation of co-occurrence

Suppose there are **N** loan packages in the investigated set. Suppose we are interested to evaluate against a null hypothesis the co-occurrence of lending banks in the same package. Let us call N_A the number of packages that bank **A** has subscribed and N_B the number of packages that bank **B** has subscribed. Let us call X the co-occurrence of the presence of both banks in loan packages.



The question is:
what is the probability of X under the null hypothesis of random matching?

The probability that banks **A** and **B** are both subscribing X packages is given by the hypergeometric distribution

Hypergeometric distribution:

$$P(X \mid N, N_A, N_B) = \frac{\binom{N_A}{X}\binom{N - N_A}{N_B - X}}{\binom{N}{N_B}}$$

Expected number of co-occurrence:

$$\langle X \rangle = \sum x \; P(x \mid N, N_A, N_B)$$

It is therefore possible to associate a p-value to an empirically observed value

p-value associated to a detection of co-occurrence ≥ X:

$$p = 1 - \sum_{i=0}^{X-1} \frac{\binom{N_A}{i}\binom{N - N_A}{N_B - i}}{\binom{N}{N_B}}$$

Corrections for multiple hypotheses testing, and network construction

We can therefore statistically validate a link between two vertices (in the present case two banks) if the associated p-value is below a given threshold showing that the co-occurrence cannot be explained by the heterogeneity of the system taken as a null hypothesis.

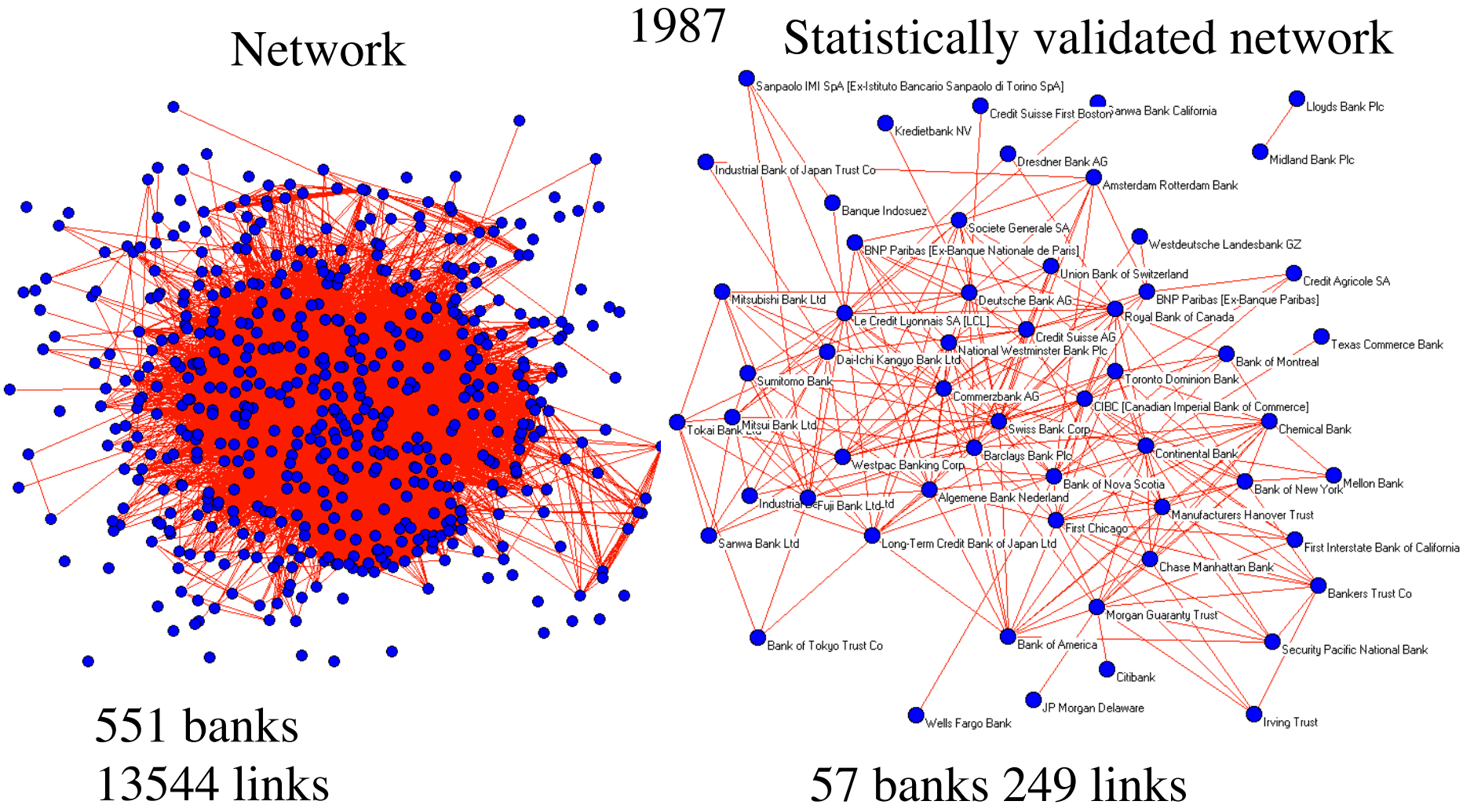
By doing a two tail analysis we can also detect under-occurrence so that detecting the avoidance or minimization of interaction.

To perform the statistical validation of all pairs of vertices a large number of tests need to be performed. One therefore needs a multiple hypothesis test correction.

The most restrictive correction is the Bonferroni correction redefining the statistical threshold as θ=0.01/T where T is the number of tests to be done.

Another type of correction (less restrictive) is the so-called False Discovery Rate correction.

DealScan network of banks performing syndicated loans¶

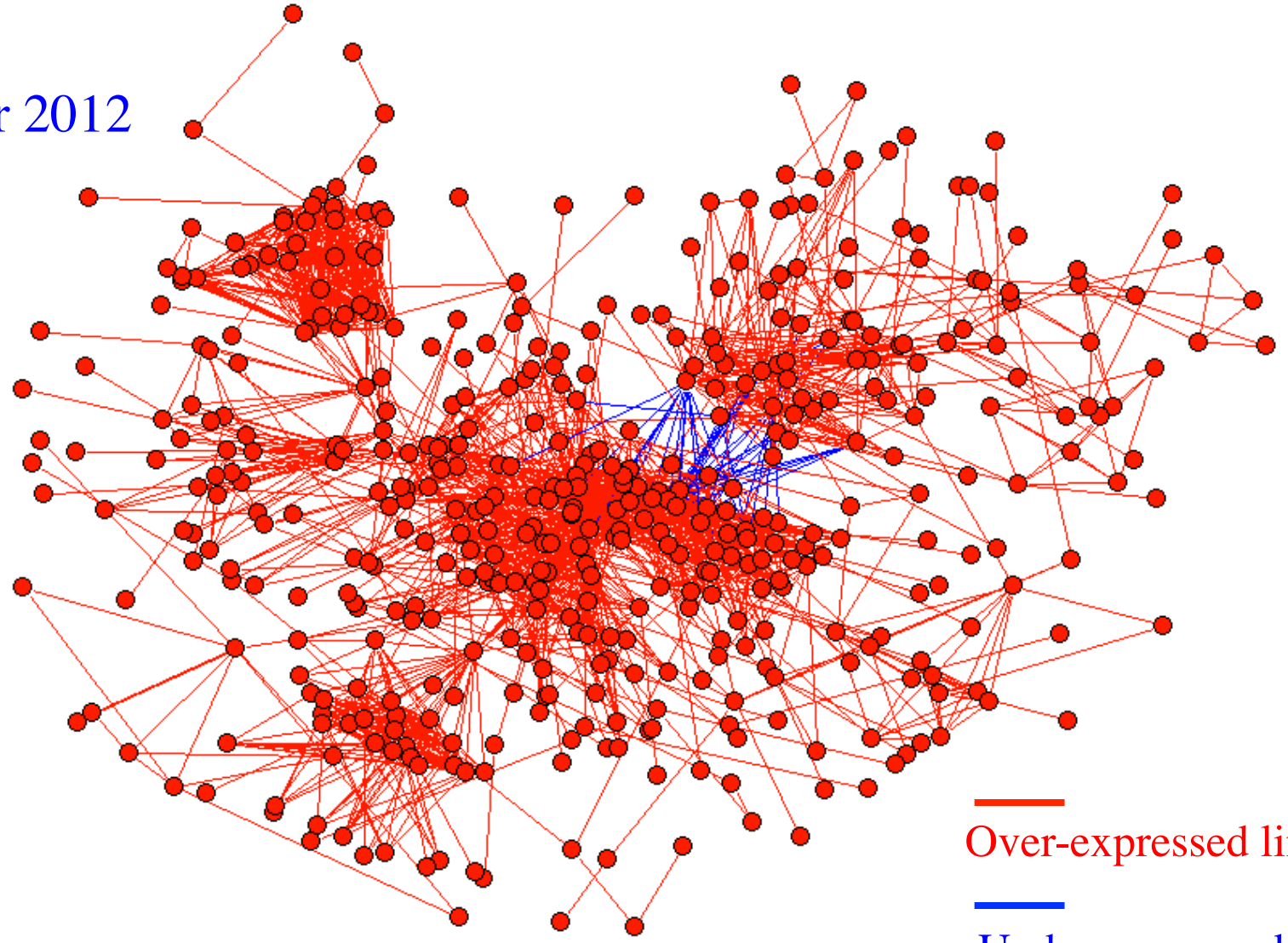


551 banks
13544 links

57 banks 249 links

¶L. Marotta, S. Micciché and R. N. Mantegna, The evolution of the network of banks performing syndicated loans, manuscript in preparation

Statistically validated network of DealScan lending banks

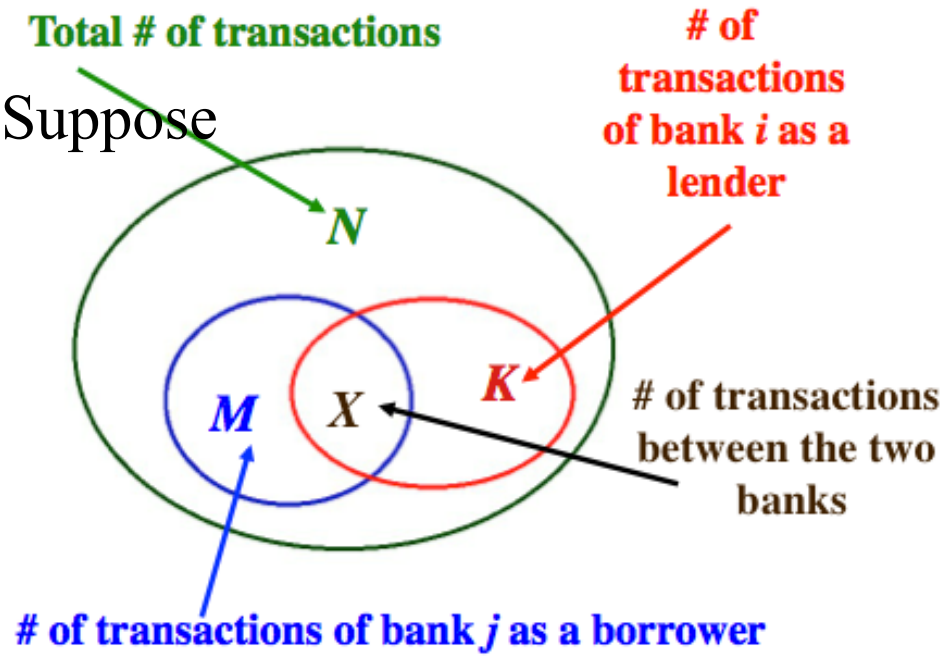


lcc comprising 460 (of 583) banks and 2195 links

The methodology of statistically validated networks is quite flexible and can be easily applied also to directed networks when the underlying network register directional events.

Example: credit relationships in the interbank market.

Suppose there are **N** credit relationships in the investigated set. Suppose We are interested to evaluate the null hypothesis of the co-occurrence of random pairing of lending and borrowing between a pair of banks. Let us call **K** the number of credits relationships of bank ***i*** as a lender and **M** the number of credit relationships of bank ***j*** as a borrower. X is the number of credit relationships with ***i*** lender and ***j*** borrower.



p-value

OVER-expression:

$$p = 1 - \sum_{i=0}^{X-1} \frac{\binom{M}{i}\binom{N-M}{K-i}}{\binom{N}{i}}$$

UNDER-expression:

$$p = \sum_{i=0}^{X} \frac{\binom{M}{i}\binom{N-M}{K-i}}{\binom{N}{i}}$$

By using this approach we[¶] have shown that the e-MID market presents statistically validated links

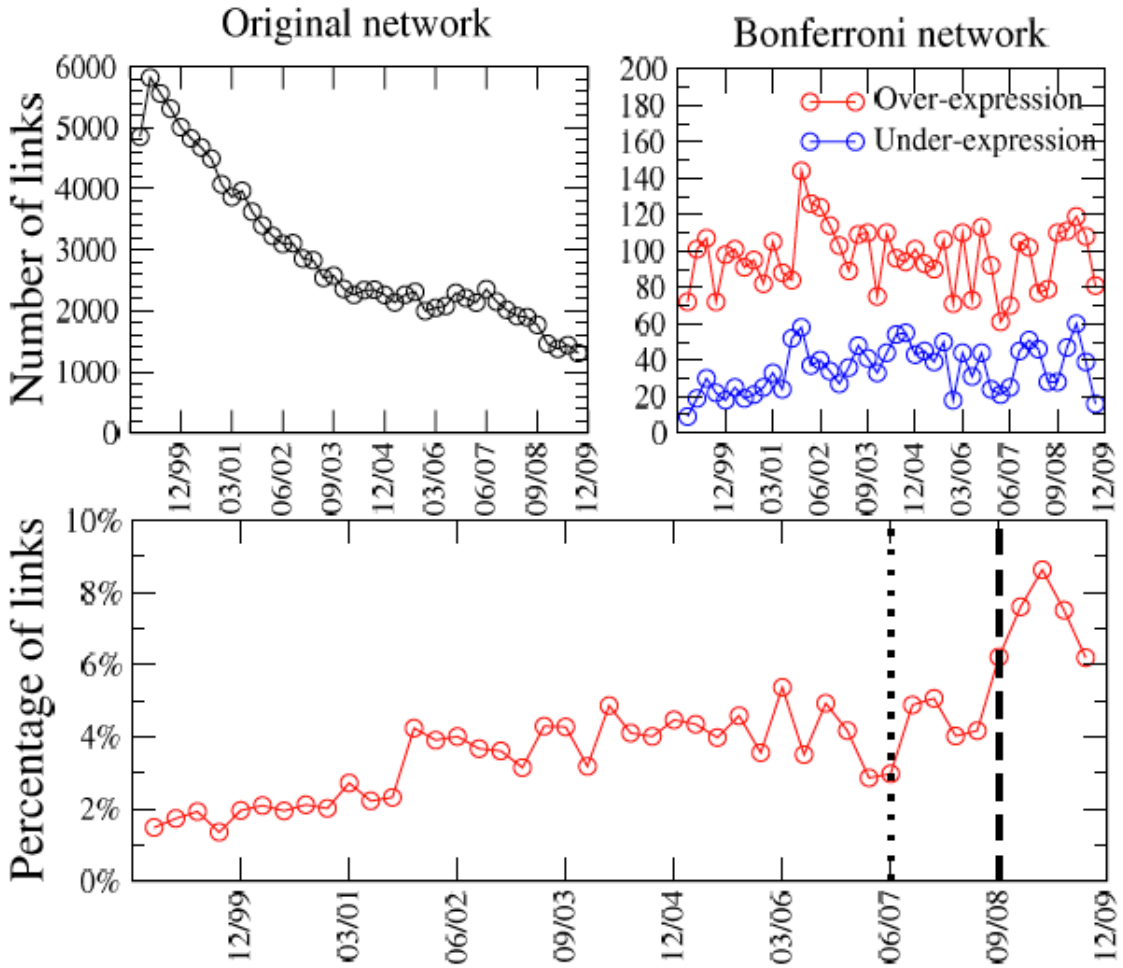


Figure 8. In the top-left panel, we show the number of links observed in the original network. In the top-right panel, the number of over-expressed links (red) and under-expressed links (blue) observed in the Bonferroni network is reported. In the bottom panel, we show the ratio between the number of over-expressed links observed in the Bonferroni and in the original network. The dotted line refers to the August 2007 market freezing, while the dashed line refers to the Lehman's bankruptcy. These data refer to the lender-aggressor dataset. The analysis is performed on the Italian segment of the e-MID market.

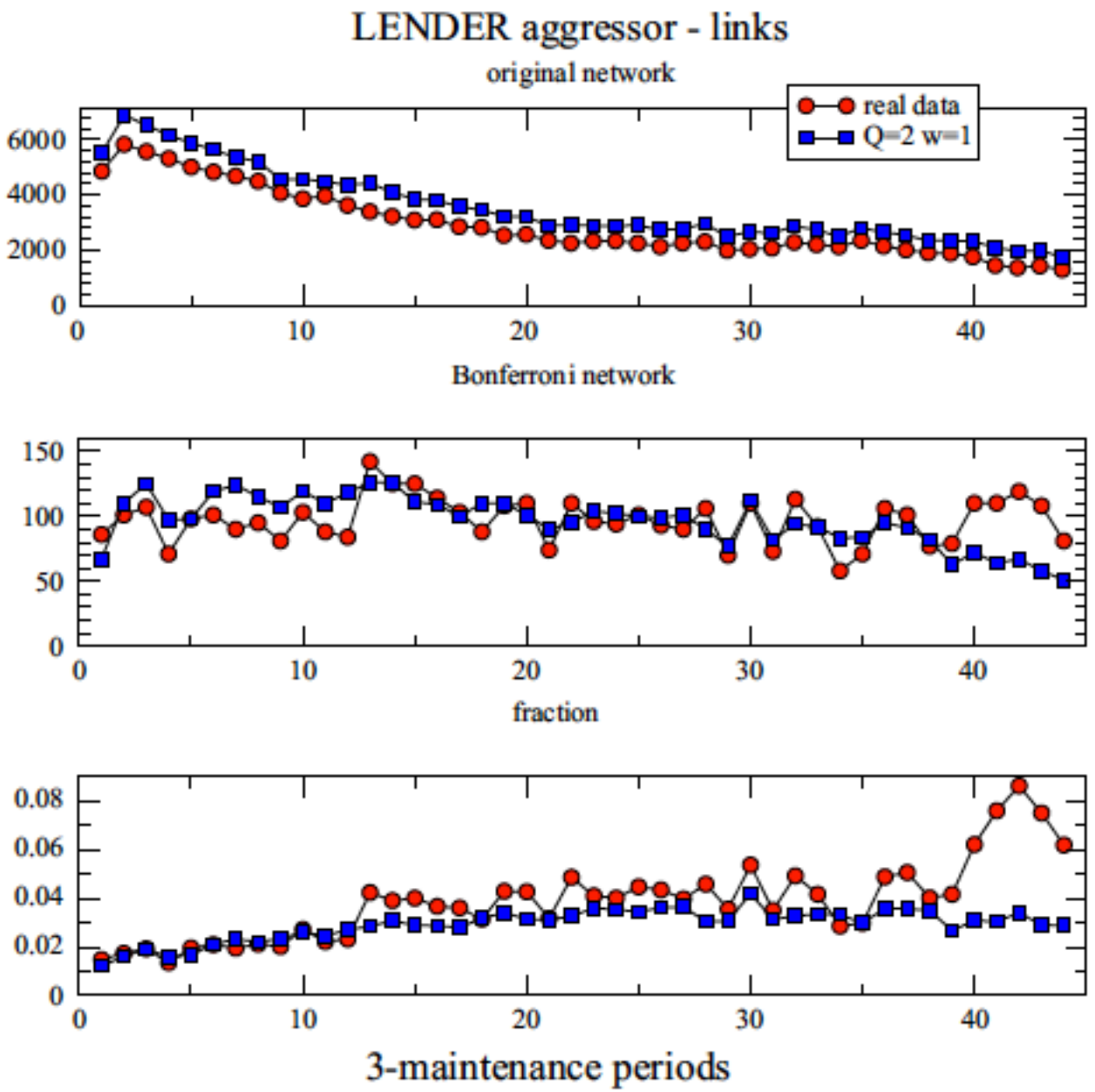
[¶]Hatzopoulos, V. , Iori, G., Mantegna, R. N., Miccichè, S. and Tumminello, M., Quantifying Preferential Trading in the e-MID Interbank Market (October 28, 2013). Available at SSRN: http://ssrn.com/abstract=2343647

A simple model of dynamic network of the interbank market showing statistically validated links[¶].

At each transaction a lender and a borrower that aim to do a transaction are selected. The probability that the selected lender accept the selected borrower is proportional to an attractiveness *w* common to all borrower and to a trust proxy of the specific borrower obtained by considering the number of past credit relationships commonly undertaken. The trust is built within a memory time interval which is a parameter of the model.

[¶]Iori, G., Mantegna, R.N., Marotta, L., Micciché, S., Porter, J. and Tumminello, M. Networked relationships in the e-MID interbank market: A trading model with memory, Journal of Economic Dynamics and Control, (in press) http://dx.doi.org/10.1016/j.jedc.2014.08.016

By calibrating the simulations of this model on the e-MID data we obtains¶



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Conclusions

- Similarity based networks are quite informative in finance;
- Different networks can highlight different information;
- Statistically validated networks are able to detect over-occurrence and under-occurrence of events or relationships and can be useful to highlight the presence of a networked structure of markets.

I acknowledge funding from