BANK FOR INTERNATIONAL SETTLEMENTS

Filling in the Blanks: Interbank Linkages and Systemic Risk

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Cambridge – September 2014

Motivation

- Interbank contagion is central, but bilateral linkages often unknown
- Standard: estimate counterparty exposures by **maximum entropy**
- Yet spreading exposures as evenly as possible can be misleading:
  - Conceals "true" structure of linkages in network analysis
  - Diversification assumption causes bias in systemic stress tests
- This short paper proposes opposite benchmark: **minimum density**
- Produces a highly concentrated sparse network that
  - Retains some of the original network structure, and
  - Provides useful robustness bounds for systemic stress tests

Actual Data

True Network

	A	B	C	D	E	F	G	A_i
A	0	3	1	0	0	1	2	**7**
B	2	0	2	0	0	0	1	**5**
C	1						0	**3**
D	1						0	**1**
E	0	0	2	0	0	0	1	**3**
F	0	0	0	0	0	0	0	**0**
G	0	1	0	0	0	0	0	**1**
L_i	**4**	**5**	**5**	**0**	**0**	**2**	**4**	**20**

Density 33%

Observable Interbank Market

	A	B	C	D	E	F	G	A_i
A								**7**
B								**5**
C				?				**3**
D								**1**
E								**3**
F								**0**
G								**1**
L_i	**4**	**5**	**5**	**0**	**0**	**2**	**4**	**20**

Estimated Networks

Maximum Entropy Solution

	A	B	C	D	E	F	G	A_i
	0	2.53	2.18	0	0	0.74	1.55	**7**
	1.72	0	1.6	0	0	0.54	1.14	**5**
	0.98	1				1	0.65	**3**
	0.25	(				8	0.17	**1**
	0.75	0.81	0.7	0	0	0.24	0.5	**3**
	0	0	0	0	0	0	0	**0**
	0.3	0.32	0.28	0	0	0.09	0	**1**
	4	**5**	**5**	**0**	**0**	**2**	**4**	**20**

Density 62%

Minimum Density Solution

	A	B	C	D	E	F	G	A_i
	0	3	0	0	0	0	4	**7**
	3	0	2	0	0	0	0	**5**
	0						0	**3**
	0						0	**1**
	0	0	3	0	0	0	0	**3**
	0	0	0	0	0	0	0	**0**
	1	0	0	0	0	0	0	**1**
	4	**5**	**5**	**0**	**0**	**2**	**4**	**20**

Density 21%

Roadmap

- Part I: The minimum density approach (MD)
- Part II: Network features of estimated benchmarks (ME, MD) versus the "true" interbank network
- Part III: Performance in a systemic stress test

Part I: Maximum Entropy vs Minimum Density

- Notation:
  - Interbank network: $\mathbb{X} \in [0, \infty)^{N \times N}$
  - Bilateral exposures: $X_{ij} \geq 0$ $(X_{ii} = 0)$
  - Interbank assets: $A_i = \sum_j X_{ij}$
  - Interbank liabilities $L_i = \sum_j X_{ji}$ = "marginals"

- Suppose we *only know* the marginals A_i and L_i for each of the *N* banks
  → estimate max entropy $\mathbb{E}$ and min density solution $\mathbb{Z}$ on marginals

- **Entropy:** find matrix $\mathbb{E}$ that satisfies the marginals, given "prior" $Q_{ij} = A_i L_j$:

$$\min_{\mathbb{E}} \sum_{i,j} E_{ij} \log \frac{E_{ij}}{Q_{ij}} \text{ s.t. } \sum_j E_{ij} = A_i \text{ and } \sum_j E_{ji} = L_i$$

The Maximum Entropy solution $\mathbb{E}$

- Advantages:
  - Implementable using a standard iterative algorithm (RAS)
  - Yields a unique solution for $\mathbb{E}$
- Disadvantages:
  - Is optimal only if *nothing else* is known about a network
  - Completeness contradicts facts of real interbank networks:
    - Sparsity (Bech Atalay 2010), Tiering (Craig von Peter 2014), relationships (Cocco et al 2009), with disassortative features.
  - When diversification reduces contagion, entropy under-estimates systemic risk (Mistrulli 2011, Markose 2012)
  - → Case for an alternative benchmark

The Minimum Density method

- **Premise**: Network linkages are costly and based on relationships

- **Efficiency:** Minimally connected network s.t. satisfying marginals:

$$\min_{\mathbb{Z}} \quad \sum_{i=1}^{N} \sum_{j \neq i}^{N} c \times \mathbf{1}_{[Z_{ij}>0]} \quad \text{s.t.}$$

$$\sum_{j=1}^{N} Z_{ij} = A_i \quad \forall i = 1,2,\dots,N$$

$$\sum_{j=1}^{N} Z_{ij} = L_i \quad \forall i = 1,2,\dots,N$$

$$Z_{ij} \geq 0 \quad \forall i,j$$

- Analogous to transport network design problems: NP-hard (O'Kelly 2012)

Approach guided by two main ideas

- Robust choice under uncertainty → multinomial logit function

$$\max_{p\in\Delta} \left[\mathbf{v}'\mathbf{p} - \delta v(\mathbf{p},\mathbf{q})\right] \quad \Longrightarrow \quad p_i^* = \frac{q_i \, \mathrm{e}^{v_i/\delta}}{\sum_{j\in\mu} q_i \, \mathrm{e}^{v_j/\delta}}$$

- Prior ~ economic incentives: focus on disassortative relationships matching large surpluses with small deficits and v.v.

$$Q_{ij} \propto \max\left\{\frac{AD_i}{LD_j}, \frac{LD_j}{AD_i}\right\} \quad \forall i,j \in \mu.$$

- i → j if big lender to small borrower, or small lender to big borrower
- Algorithm identifies probable links to load to the maximum extent.

The Minimum Density Algorithm

Complexity rises exponentially (2^N) even before allocating value → algorithm

1) Compute current deficits $AD_i = \left(\sum_j Z_{ij} - A_i\right)$, $LD_i = \left(\sum_j Z_{ji} - L_i\right)$
2) Select link (i, j) according to probability $Q_{ij} \propto \max\left\{\frac{AD_i}{LD_j}, \frac{LD_j}{AD_i}\right\} \quad \forall i, j$
3) Load exposure $Z_{ij} = \lambda \times \min\{AD_i, LD_j\}$ with $\lambda = 1$, or less*
   If $V\left(\mathbb{Z}' = \mathbb{Z} + \mathbb{Z}_{ij}\right) \geq V(\mathbb{Z})$, then accept link Z_{ij}
4) Update set of priors Q_{ij} as in steps 1-2)
5) Iterate until 100% volume is allocated $\sum\sum Z_{ij} = \sum\sum X_{ij} = \sum A_i$
   - Interbank assets matched: $\sum_j Z_{ij} = A_i \quad \forall i$
   - Interbank liabilities matched: $\sum_j Z_{ji} = L_i \quad \forall i$

* We can generate "low density" solutions using $\lambda < 1$.

Part II: Comparing benchmarks with the original network

- The "true" interbank network $\mathbb{X}$ constructed from Bundesbank data
  - "*Gross- und Millionenkreditstatistik*" between 2000+ banks
  - All large (≥€ 1.5m) or concentrated (>10% K) exposures
  - Consolidated at the bank holding company level ("*Konzern*") and excluding cross-border linkages

- Basic features: large market ($N = 1779$), considerable volume (>$1 trillion), sparse (density=0.59%) core-periphery structure

- Maximum Entropy (ME) blurs network structure (density 93%)

- Minimum Density (MD) solution is "too" efficient (density 0.1%)

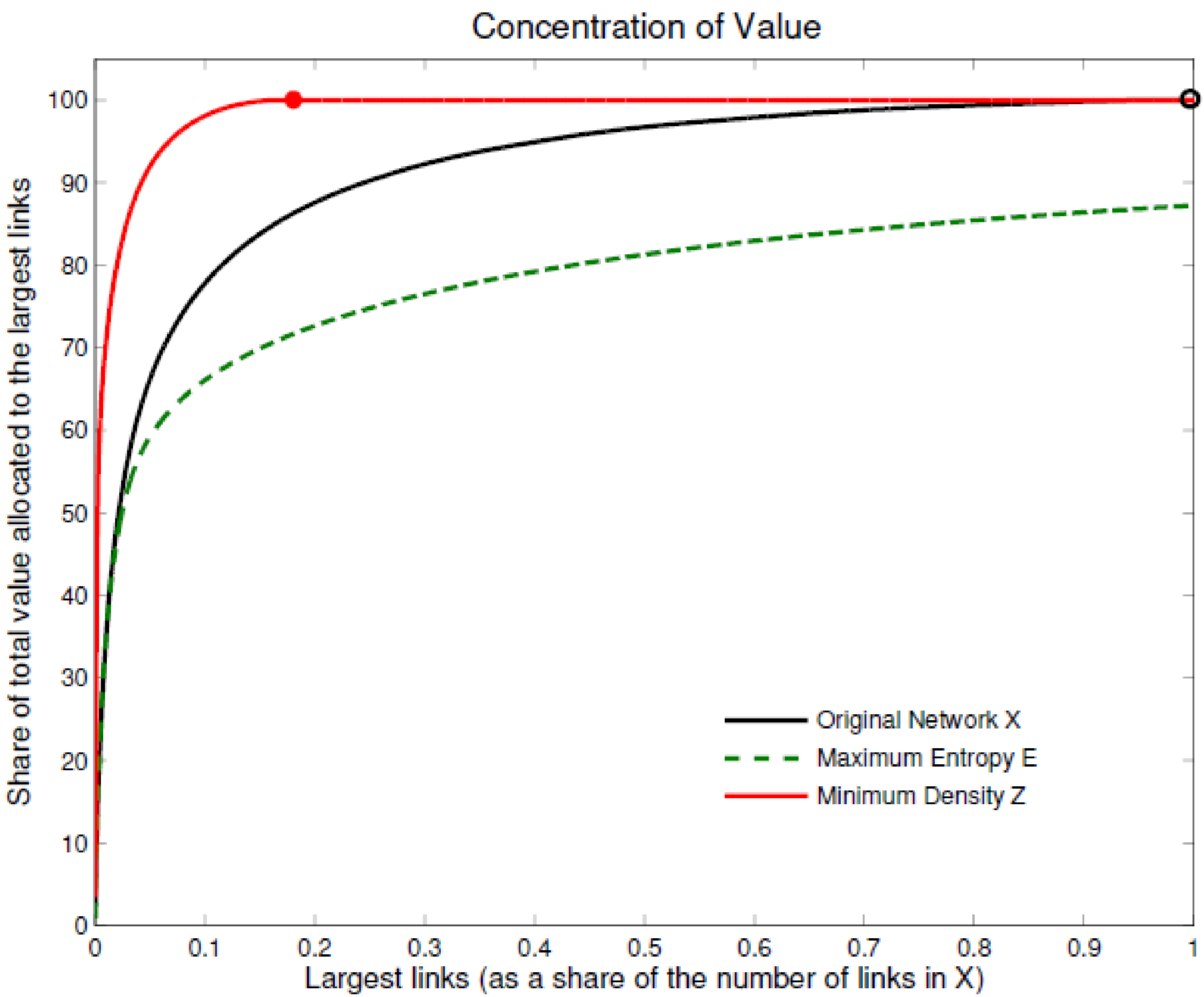


Figure 2: The figure shows the concentration of value on the largest links for the different networks. The x-axis ranks bilateral linkages (in descending order of size) and expresses the first n links as a share of the total number of links in the original network $\mathbb{X}$ (18624). The y-axis shows the cumulative share of value allocated to the largest n links, relative to total interbank volume. The dots indicate at which point 100% of volume has been reached. For $\mathbb{X}$ this is at unity, for $\mathbb{Z}$ this occurs at 0.185, whereas $\mathbb{E}$ needs 158 times the number of links in $\mathbb{X}$ before reaching 100% of interbank volume.

MD preserves *some* structural features – ME fails to do so

Network Characteristic	$\mathbb{E}$ Max Entropy	$\mathbb{X}$ True Network	$\mathbb{Z}$ Min Density	$\mathbb{Y}$ Low Density
Density, in %	92.8	0.59	0.11	0.61
Degree (average)	1649	10.5	1.94	10.9
Degree (median)	1710	6	1	4
Assortativity	-0.03	-0.53	-0.40	-0.32
Dependence when borrowing, %	12.2	84.7	97.3	93.4
Dependence when lending, %	7.2	45.1	97.4	87.2
Clustering local average, %	99.8	33.4	0.03	7.62
Core size, % banks	92.6	2.5	1.1	2.1
Error score, % links	14.6	9.2	41.2	35.7

Table 1: Comparing basic network features of benchmark estimates with those of the original German interbank network.

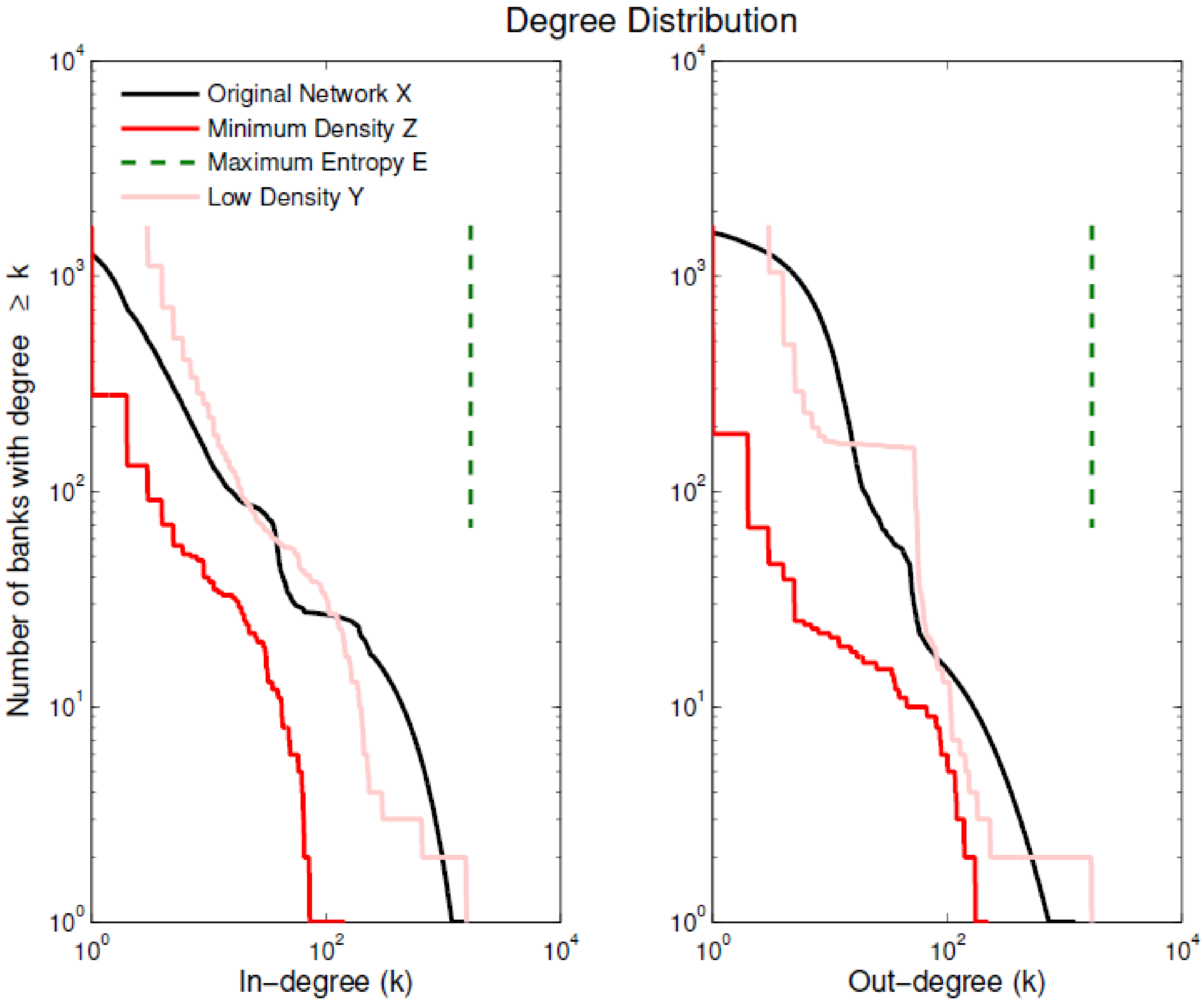


Figure 3: The figure displays the degree distribution in its cumulative form, showing the number of banks with degree greater than the number shown on the x-axis, on a double log scale. A straight line would indicate a Pareto cumulative indicative of a power law distribution. The degree distribution of the original network $\mathbb{X}$ has been smoothed to preserve the confidentiality of individual bank data and shows averages at the end-points instead.

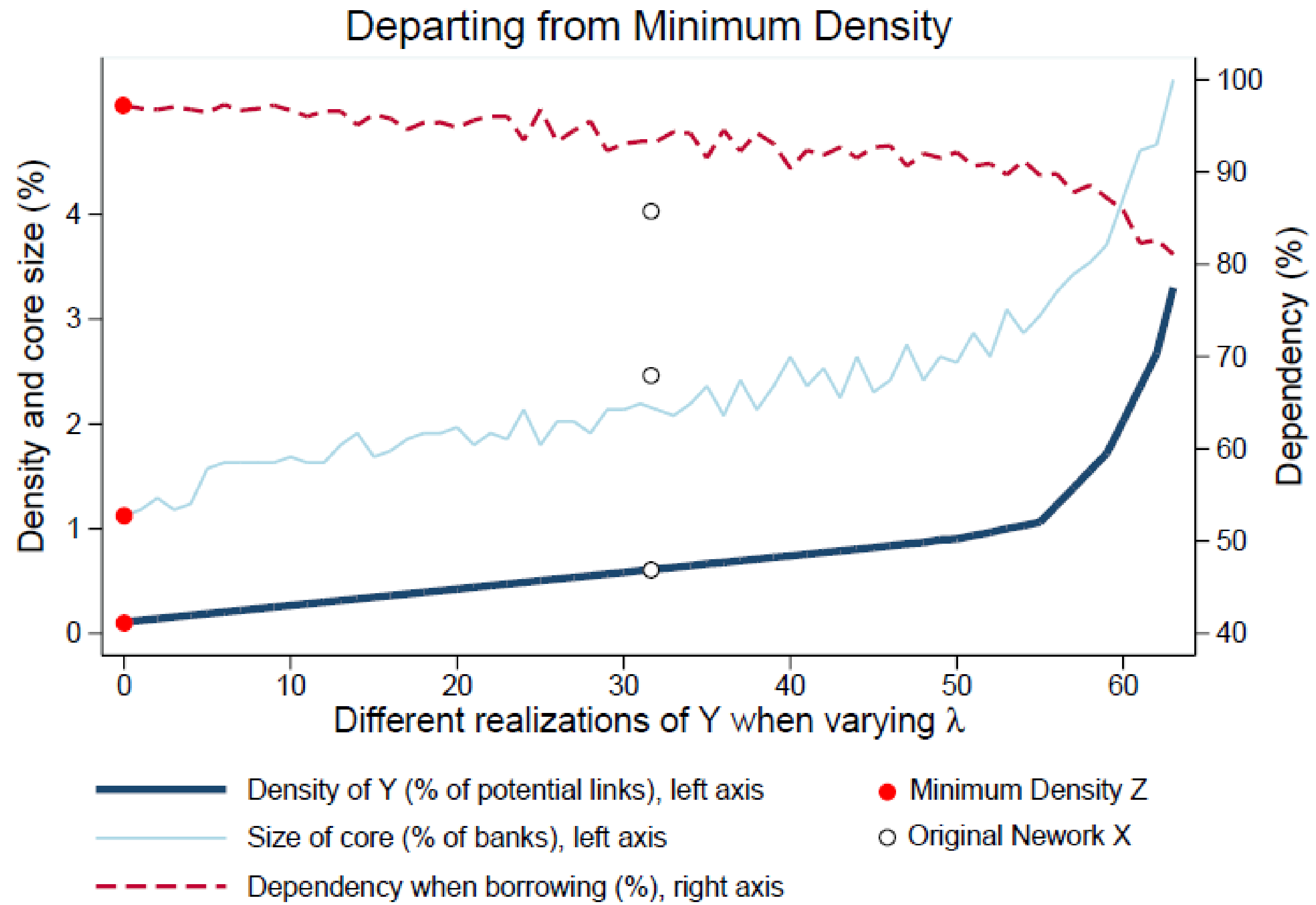


Figure 4: This figure shows three network features for 65 different low density solutions $\mathbb{Y}$. The implementation here sets $\lambda = 0.5$ for the first k links being filled by the algorithm and $\lambda = 1$ thereafter, with k raised from 0 to 100,000 in 65 (unequally spaced) steps. The first realization (at $k = 0$) is the minimum density network $\mathbb{Z}$ with the network features shown as red dots (as in Table 1). The black circles indicate the values for the original network $\mathbb{X}$, plotted at the point where a comparable low density network $\mathbb{Y}$ reaches a density similar to $\mathbb{X}$ (at k=16,000).

Part III: Performance in systemic stress tests

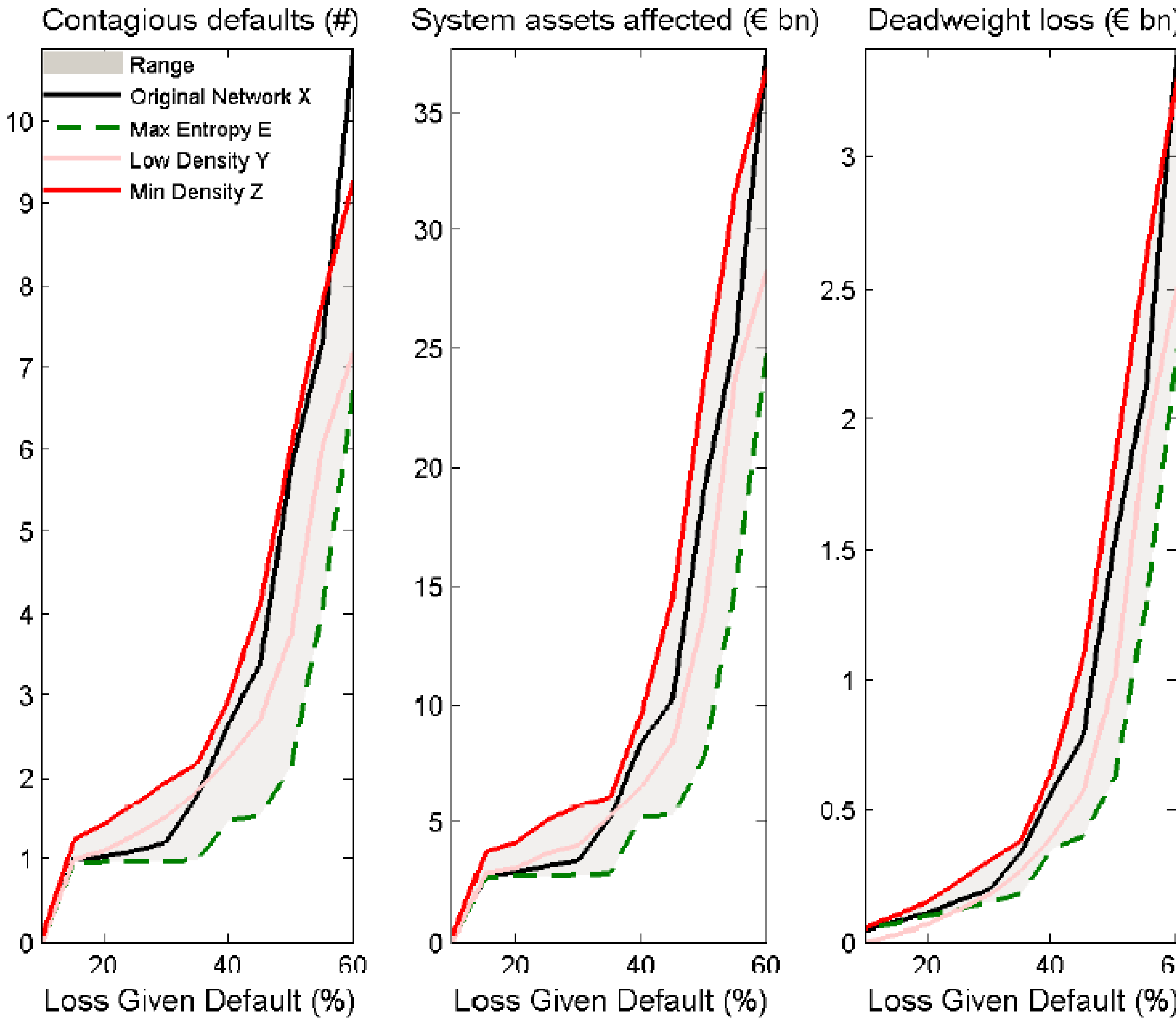
- Run stress tests to compare ME, MD with “true” network in practice

- Standard simulation methodology – ingredients:
  - Trigger: single bank failure (+ a capital shock)
  - Mechanism: (1) sequential default algorithm, and
  - (2) Eisenberg-Noe clearing vector (endogenous LGD + cost β)

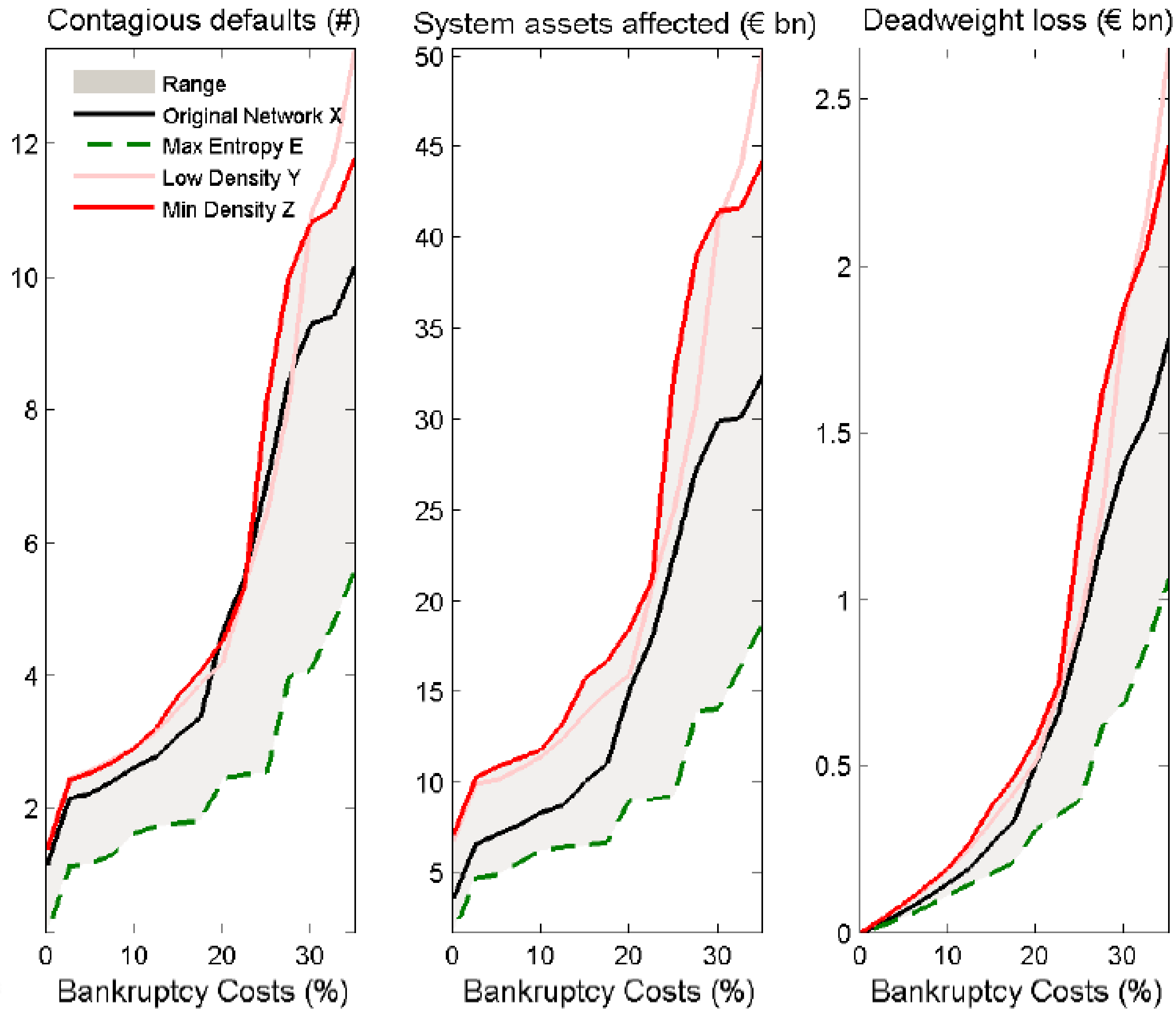
- Let each of 1779 banks fail 1x1 and measure contagion (if any):
  - # banks in default as a consequence of contagion (excluding i)
  - System assets affected, and deadweight loss (€ bn)
  - Report average over i's, and repeat for higher LGD or costs β.

- Evaluate how close contagion in $\mathbb{E}$, $\mathbb{Z}$ is to “true contagion” in $\mathbb{X}$.

Test 1: Sequential default algorithm



Test 2: Clearing vector methodology (Eisenberg-Noe)



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Interpretation

- Results are similar across contagion methodologies…
- … but differ across interbank networks $\mathbb{E}$, $\mathbb{Z}$, $\mathbb{X}$ (the inputs)

- Max entropy $\mathbb{E}$ underestimates systemic stress substantially:
  - Diversified exposures, smaller losses can be absorbed
- Min density $\mathbb{Z}$ overstates contagion for most of parameter space
  - + Concentrated exposures, failure may kill the counterparty
  - − Sparsity: fewer conduits for the propagation of losses
  - Former dominates due to negative assortativity in $\mathbb{Z}$ and $\mathbb{X}$

- In line with earlier findings on bias (Mistrulli 2011)…
- …but no tipping point in sight (Nier et al 2007, Gai et al 2011).

Conclusion

- The paper had a simple goal: to provide a meaningful alternative to the maximum entropy benchmark for estimating counterparty exposures

- Min density solution uses information theory and economic rationale

- The solution retains more structural features of the original network

- In stress testing applications:
  - ME understates contagion, whereas MD generally overstates it
  - Using ME & MD jointly delivers a useful confidence interval, and
  - MD also allows for many interior outcomes (low density solutions).

The broad interval shows: pattern of linkages matters for systemic risk!

Thanks for your attention.