Measuring interconnectedness between financial institutions with Bayesian time-varying VARs

Financial Risk & Network Theory

Marco Valerio Geraci [1,2] Jean-Yves Gnabo [2]

[1]ECARES, Université libre de Bruxelles

[2]CeReFiM, University of Namur

Judge Business School, Cambridge
8 September 2015

Interconnectedness

The financial crisis highlighted the importance of **connectedness**

"A bank's **systemic** *impact is likely to be positively related to its* **interconnectedness** *vis-à-vis other financial institutions."*

- Basel Committee on Banking Supervision (2013)

Problem:

- We do not observe true connections
- Plausibly connections are time-varying

Goal:

Develop a framework that accounts for time-varying connections

Previous Studies and Contribution

Prior studies estimate the financial network using **stock return data**

- **Barigozzi and Brownlees (2014); Billio et al. (2012); Diebold and Yılmaz (2014); Hautsch et al. (2014)**
- They estimate the network at a several moments in time

The time-varying network is captured by using **rolling windows**

- Reduces degrees of freedom
- Susceptible to outliers
- Window size ⇒ trade-off bias vs. precision

Our Contribution

Formalize time-dependence of connections by imposing **structure**

- Assumption: Connections evolve smoothly ⇒ realistic
- Does not rely on window size
- Exploits whole length of data ⇒ saving dofs
- Bayesian framework ⇒ offers additional flexibility for large systems

Summary

1. Develop a framework based on **time-varying parameter**
   - Parallels Granger causality methods for estimating networks
   - Estimates the path of the network **ex-post**
2. **Compare** our performance against the rolling window approach
3. Estimate the network of **financial stocks** listed in the S&P 500
   - Covers 1990-2014 at a monthly frequency
4. Show the evolution of **interconnectedness** of key players in the financial sector

Estimating networks by Classical Granger Causaility

We parallel measures of interconnectedness based on **Granger causality testing** **(Billio et al., 2012)**

Let $x_t = [x_{1,t}, \ldots, x_{N,t}]$ be a vector of returns

- Draw a directional edge $i \to j$ if x_i Granger causes x_j

Granger causality can be tested by running

$$x_t = c + \sum_{s=1}^{p} B_s x_{t-s} + u_t,$$

and testing

$$H_0 : B_1^{(j,i)} = B_2^{(j,i)} = \cdots = B_p^{(j,i)} = 0.$$

This is a **conditional** Granger causality test **(Geweke, 1984)**

Methodology

Problem: Granger causality is an insample test, based T observations

If the strength/direction of causality changes in $[0, T]$, the test inference is affected

- Simple solution: rolling windows
  - But this leads to the aforementioned limitations
    - Reduces degrees of freedom
    - Susceptible to outliers
    - Window size $\Rightarrow$ trade-off bias vs. precision

We propose TVP-VAR as in the macro literature **(Primiceri, 2005; Cogley and Sargent, 2005)**

Methodology

$$\textbf{Measurement equation: } x_t = X_t' B_t + u_t \qquad u_t, \sim \mathcal{N}(0, R),$$

where $X_t' = I_N \bigotimes [1,\ x_{t-1}',\ \ldots,\ x_{t-p}']$

$$\textbf{State equation: } B_{t+1} = B_t + v_{t+1} \qquad v_t, \sim \mathcal{N}(0, Q),$$

Test the hypothesis of no link between i and j at t

$$H_{0,t} : \tilde{A} B_t = \mathbf{0}_{p\times 1}.$$

$\tilde{A}$ is the same as for Wald test of Classical **conditional** Granger causality

Simulations

To validate our methodology, we perform **three simulation exercises**

1. **Constant network** with fix edge strength
2. Time-varying network with **Markov switching** link strength
3. Time-varying network with **smoothly varying** link strength

For each experiment, we ran 100 simulations each of which involved $T = 300$ time periods

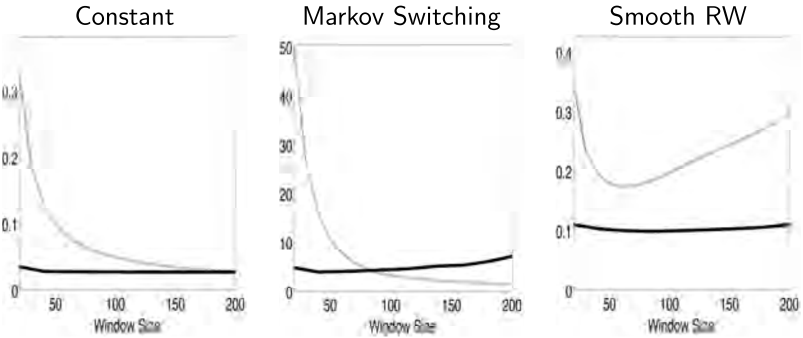
Simulation results show that our framework performs better than the classical rolling windows approach when network is **time-varying**

- In terms of **estimating link strength** *and* **determining link existence**
- For *both* **pairwise** and **conditional** testing

Our framework performs comparatively well when network is constant

Simulations

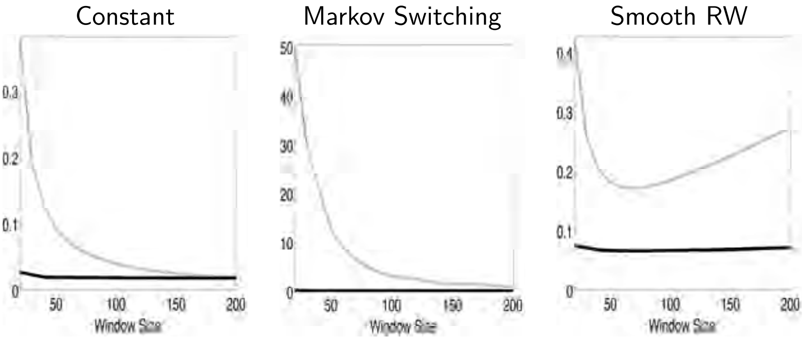
Mean Squared Error - Pairwise testing



Bold solid = TVP; light dashed = rolling windows

Simulations

Mean Squared Error - Conditional testing



Bold solid = TVP; light dashed = rolling windows

Empirical Application

We collected stock prices monthly close of financial institutions

- banks, insurers and real estate companies - SEC codes 6000 to 6799
- components of the S&P 500 between Jan 1990 and Dec 2014
- final sample includes 155 firms

We define monthly stock returns for firm i at month t as

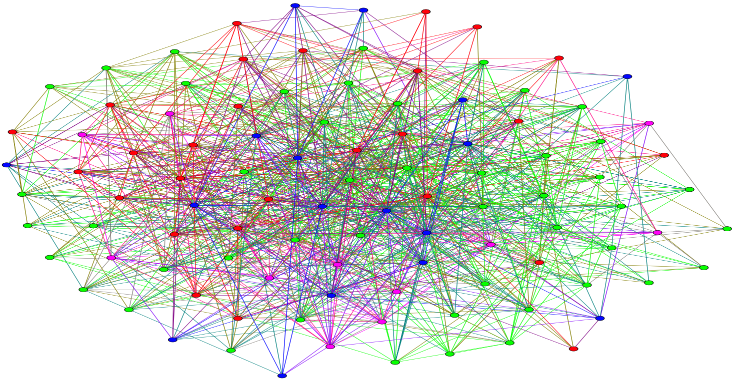
$$r_{i,t} = \log p_{i,t} - \log p_{i,t-1}$$

We estimate the financial network by **pairwise testing** with **TVP-VARs** (recursive Bi-VARs)

- For comparison, we also estimate using classical Granger pairwise testing with **rolling windows of 36 months**

Results: Network Density

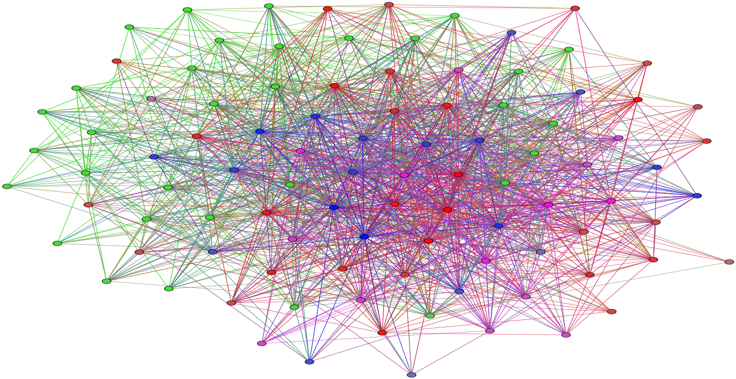
Financial Network estimated by TVP-VARs in **October 2000**



Green = Banks; Magenta = Brokers; Red = Insurers; Blue = Real Estate

Results: Network Density

Financial Network estimated by TVP-VARs in **September 2008**

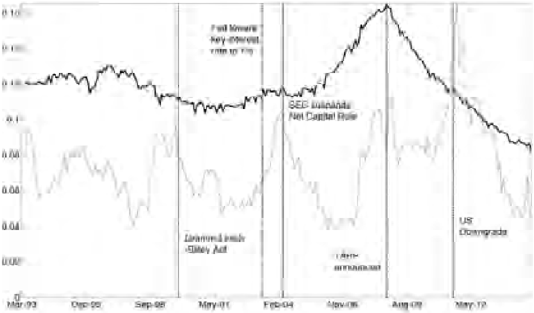


Green = Banks; Magenta = Brokers; Red = Insurers; Blue = Real Estate

Results: Network Density

network density is smoothly varying rather than abrupt changes

$$\text{Density}_t = \frac{1}{n_t(n_t-1)} \sum_{i=1}^{n_t} \sum_{j \neq i} (j \to i)$$



Bold solid = TVP; light dashed = rolling windows

Results: Degree Centrality

- **degree centrality** calculated for the 155 companies

$$\text{In-Degree}_{i,t} = \frac{1}{(n_t - 1)} \sum_{j \neq i} (j \rightarrow i)$$

$$\text{Out-Degree}_{i,t} = \frac{1}{(n_t - 1)} \sum_{j \neq i} (i \rightarrow j)$$

Summarized results with **net out-degree** measure

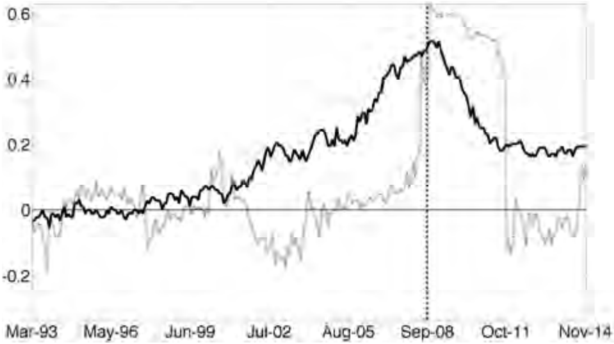
$$\Delta\text{Degree}_{i,t} = \text{Out-Degree}_{i,t} - \text{In-Degree}_{i,t}$$

- Positive net out-degree indicates propagators
- Negative net out-degree indicates absorbers

Results: Degree Centrality

Rolling window approach is **susceptible** to extreme observations

American International Group

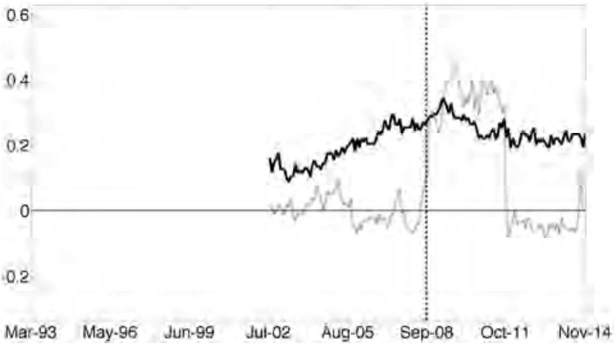


Bold solid = TVP; light dashed = rolling windows

Results: Degree Centrality

Rolling window approach is **susceptible** to extreme observations

Goldman Sachs Group Inc.



Bold solid = TVP; light dashed = rolling windows

Conclusion

Develop a procedure for inferring time-varying connections

- Relies on **Bayesian estimation of time-varying parameter VARs**
- Compared to classical rolling window approach
  - Less susceptible to extreme observations
  - Offers greater **flexibility** than rolling windows
  - Performs well in simulations
- Empirical application reveals limitations of rolling window approach
  - Some **sectors** were acting as propagators prior to crisis
  - At the individual firm level, some **key players** can be identified

References

Matteo Barigozzi and Christian Brownlees. NETS: Network Estimation for Time Series. Working Paper, 2014.

Basel Committee on Banking Supervision. Global systsystemic important banks: updated assessment methodology and the higher loss absorbency requirement. Technical report, Bank for International Settlements, 2013. URL http://www.bis.org/publ/bcbs255.pdf.

Monica Billio, Mila Getmansky, Andrew W. Lo, and Loriana Pelizzon. Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors. *Journal of Financial Economics*, 104:535–559, 2012. doi: 10.1016/j.jfineco.2011.12.010.

John Cochrane and Luigi Zingales. Lehman and the financial crisis. *The Wall Street Journal*, 15 September 2009.

Timothy Cogley and Thomas Sargent. Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US. *Review of Economic Dynamics*, 8(2):262–302, 2005.

Francis X. Diebold and Kamil Yılmaz. On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms. *Journal of Econometrics*, 182(1):119–134, 2014. ISSN 0304-4076.

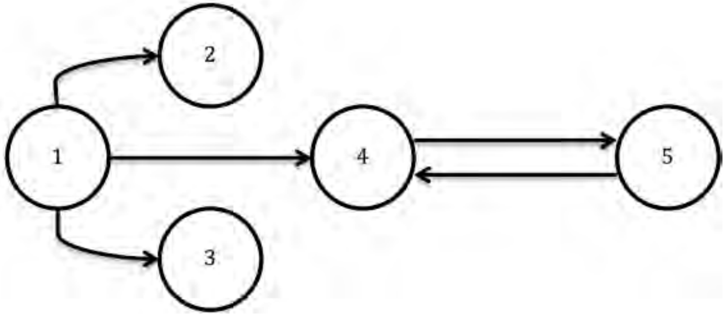
John Geweke. Measurementf Conditional Linear Dependence and Feedback Between Time Series. *Journal of the American Statistical Association*, 79(388):907–915, Dec 1984.

Nikolaus Hautsch, Julia Schaumburg, and Melanie Schienle. Forecasting systemic impact in financial networks. *International Journal of Forecasting*, 30:781–794, 2014.

Giorgio Primiceri. Time Varying Structural Vector Autoregressions and Monetary Policy. *Review of Economic Studies*, 72(3):821–852, 2005.

Anil K. Seth. A matlab toolbox for granger causal connectivity analysis. *Journal of Neuroscience Methods*, 186:262–273, 2010.

Simulations



The Granger Causal Network (Seth, 2010)

Simulations

$$
\begin{aligned}
x_{1,t} &= \alpha_{1,t} + \beta_{1,1,t}x_{1,t-1} + \epsilon_{1,t} \\
x_{2,t} &= \alpha_{2,t} + \beta_{2,1,t}x_{1,t-1} + \beta_{2,2,t}x_{2,t-1} + \epsilon_{2,t} \\
x_{3,t} &= \alpha_{3,t} + \beta_{3,1,t}x_{1,t-1} + \beta_{3,3,t}x_{3,t-1} + \epsilon_{3,t} \\
x_{4,t} &= \alpha_{4,t} + \beta_{4,1,t}x_{1,t-1} + \beta_{4,4,t}x_{1,t-1} + \beta_{4,5,t}x_{5,t-1} + \epsilon_{4,t} \\
x_{5,t} &= \alpha_{5,t} + \beta_{5,4,t}x_{4,t-1} + \beta_{5,5,t}x_{5,t-1} + \epsilon_{5,t}
\end{aligned}
$$

where, $[\epsilon_{1,t} \ldots \epsilon_{5,t}]' = \epsilon_t \sim \mathcal{N}(\mathbf{0}, R)$ and $R = cI_5$ where c was set to 0.01

Experiment 1 - constant linkages

For the first experiment, we fix all regression parameters to constants drawn at the beginning of each simulation.

$$\alpha_{i,t} = a_i \qquad \forall t \in [0, T]$$
$$\beta_{i,j,t} = b_{i,j} \qquad \forall t \in [0, T]$$

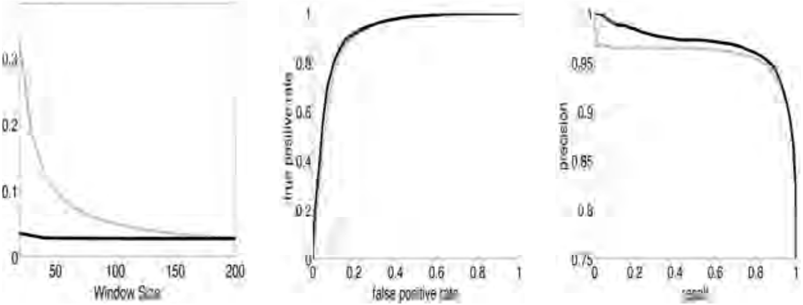
where a_i and $b_{i,j}$ are drawn from a $\mathcal{U}(0,1)$ at the beginning of each simulation
$\forall (i,j) \in \{(2,1),(3,4),(3,5),(4,1),(4,5),(5,4)\} \cup \{i = j \mid i = 1,\dots,5\}$

Go back

Experiment 1 - constant linkages

Pairwise testing

MSE ROC PR Curve

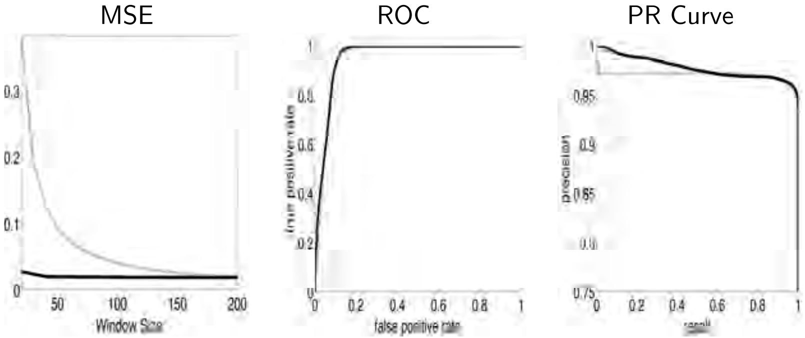


Bold solid = TVP; light dashed = rolling windows

Go back

Experiment 1 - constant linkages

Conditional testing



Bold solid = TVP; light dashed = rolling windows

Go back

Experiment 2 - markov switching linkages

For only the cross terms $i, j \in \{(2,1),(3,4),(3,5),(4,1),(4,5),(5,4)\}$

$$\beta_{i,j,t} = \begin{cases} 0 & s_t^{i,j} = 0 \\ b_{i,j} & s_t^{i,j} = 1 \end{cases}$$

Let $s_t^{i,j}$ follow a first order Markov chain with the follwing transition matrix:

$$\mathbf{P} = \begin{bmatrix} \mathbb{P}(s_t^{i,j} = 0 \mid s_{t-1}^{i,j} = 0) & \mathbb{P}(s_t^{i,j} = 1 \mid s_{t-1}^{i,j} = 0) \\ \mathbb{P}(s_t^{i,j} = 0 \mid s_{t-1}^{i,j} = 1) & \mathbb{P}(s_t^{i,j} = 1 \mid s_{t-1}^{i,j} = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix}$$

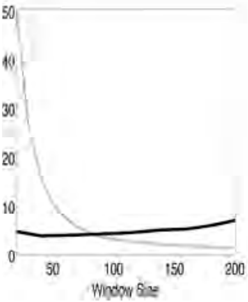
where we set $p_{00} = 0.95$ and $p_{11} = 0.90$

Go back

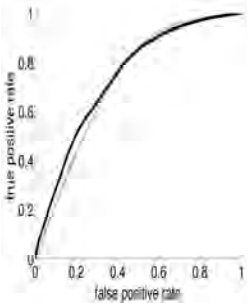
Experiment 2 - markov switching linkages

Pairwise testing

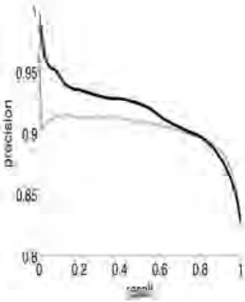
MSE



ROC



PR Curve



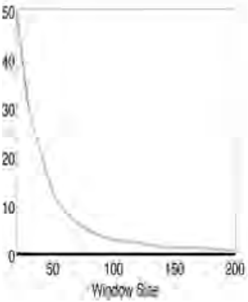
Bold solid = TVP; light dashed = rolling windows

Go back

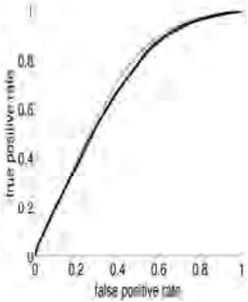
Experiment 2 - markov switching linkages

Conditional testing

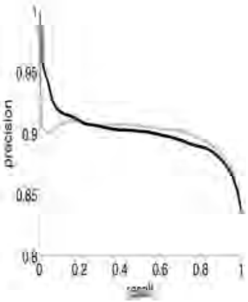
MSE



ROC



PR Curve



Bold solid = TVP; light dashed = rolling windows

Go back

Experiment 3 - random walk law of motion

$$\alpha_{i,t+1} = \alpha_{i,t} + \omega_{i,t} \qquad \omega_{i,t} \sim \mathcal{N}(0, c^2)$$
$$\beta_{i,j,t+1} = \beta_{i,j,t+1} + \zeta_{i,j,t} \qquad \zeta_{i,j,t} \sim \mathcal{N}(0, \tau^2_{i,j})$$

where,

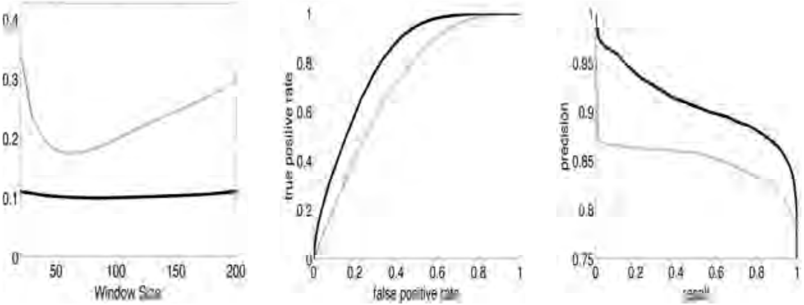
$$\tau^2_{i,j} = \begin{cases} 3 \times c^2 & \text{if } i \neq j \\ 2 \times c^2 & \text{if } i = j \end{cases}$$

Go back

Experiment 3 - random walk law of motion

Pairwise testing

MSE ROC PR Curve



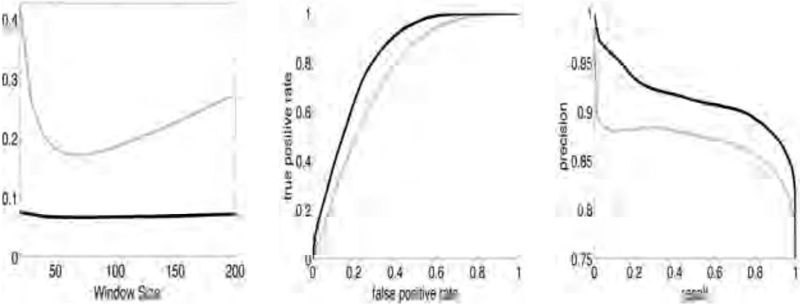
Bold solid = TVP; light dashed = rolling windows

Go back

Experiment 3 - random walk law of motion

Conditional testing

MSE ROC PR Curve



Bold solid = TVP; light dashed = rolling windows

Go back

Results Sectorial Degree

For each sector $m \in \{\text{Banks, Brokers, Insurance, Real Estate}\}$:

$$\text{In-Sector-Degree}_{m,t} = \frac{1}{M_{m,t}(N_t - M_{m,t})} \sum_{s \neq m} \sum_{i \neq j} (i_s \rightarrow j_m),$$

$$\text{Out-Sector-Degree}_{m,t} = \frac{1}{M_{m,t}(N_t - M_{m,t})} \sum_{s \neq m} \sum_{i \neq j} (i_m \rightarrow j_s),$$

number of connections from/to sector m, to/from sectors other than m

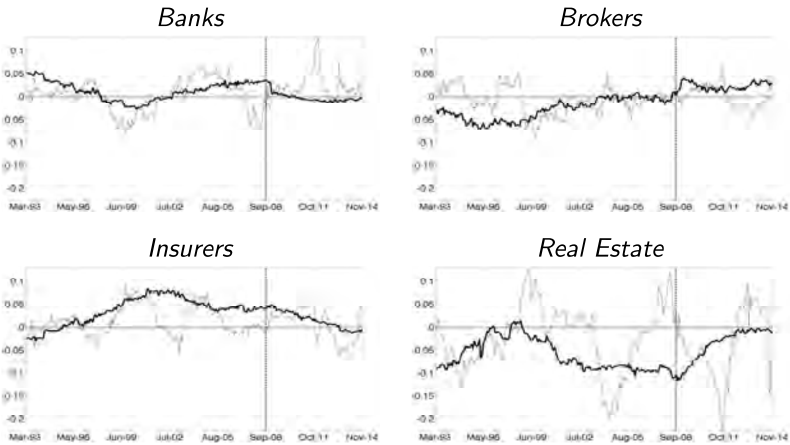
- i_m denotes a node belonging to sector m
- $M_{m,t}$ denotes the number of nodes belonging to sector m in period t

We look at the difference between out- and in-degree

$$\Delta\text{Sector-Degree}_{i,t} = \text{Out-Sector-Degree}_{i,t} - \text{In-Sector-Degree}_{i,t}.$$

Go back

Results Sectorial Degree



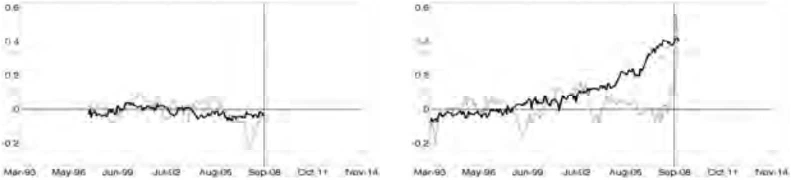
Bold solid = TVP; light dashed = rolling windows

Go back

Results: Degree centrality

Net out-degree:

Lehman Brothers Inc. *Wachovia Corp.*



Bold solid = TVP; light dashed = rolling windows

Lehman did not have high interconnectedness while **Wachovia** was a net propagator of financial spillovers

- Other determinants of systemic risk e.g, size, leverage
- TARP had a more crucial role in the crisis (Cochrane and Zingales, 2009)

Go back