# Inter-bank Network Formation – From Heterogeneity to Systemic Risk

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# Research question(s)

How bankruptcies (failures) spread through a banking system where:

- bankruptcies are endogenous,
- lending decisions (volume, interest) are endogenous,
- trading affects prices,
- banks differ in sizes?

Which factors affect systemic stability the most?

How to efficiently regulate this system?

Does heterogeneity matter?

# Why does it matter?

## Motivation

To delay or ameliorate the next financial crisis we need to examine different approaches to regulate the entire financial system under *dynamically changing* economic conditions. A prerequisite to achieve this objective is a model of a banking system.

Work in progress. All comments welcome!

# Inter-bank (overnight) lending market

*Inter-bank lending is:*

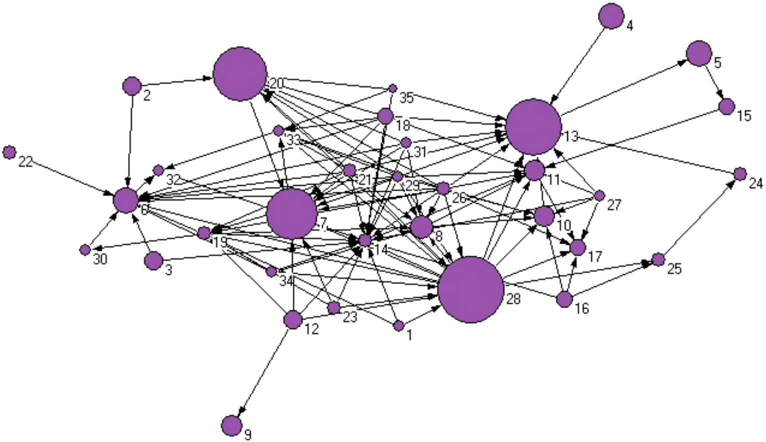
- bilateral, uncollateralized, short-term,
- often represented as a network,
- banks are vertices, loans are edges.

Whether a failure of a single bank causes domino effect does depends on **geometry** of this network.

*Other viable factors:*

- characteristics of borrowers and lenders,
- distress of the system,
- regulations.

# Inter-bank market as a network



# Stylised facts (on lending networks)

*Feature:*

1. scale-free degree distribution
2. network density in certain range
3. disassortative lending
4. persistence
5. small banks are creditors, large banks are debtors
6. large institutions have more links
7. core and periphery

*Source:*

1. (Soramäki, 2007, Physica A)
2. (Becher et al., 2008, BoE)
3. (Cocco, 2009, JFI)
4. (Cocco, 2009, JFI)
5. (Müller, 2006, JFSR)
6. (Müller, 2006, JFSR)
7. (Iori, 2008, JEDC)

# Theoretic contributions

Freixas et al. (2000, JMCB), Allen and Gale (2001, JPE), Babus (2009), Gai and Kapadia (2010, Physica A), Allen et al. (2012, JFE), Zawadowski (2013, RFS), Caballero and Simpsek (2013, JoF), Acemoglu et al. (2015, AER)

*Advantages:*

- exact, rigorous solution
- valid for all admissible parameters

*Typical limitations:*

- fixed: cardinality and market structure, limited risks
- rudimentary assets and liabilities, at most two types of banks
- no dynamics

# Computational contributions

Eisenberg and Noe (2001, MS), Iori et al. (2006, JEBO), Elsinger et al. (2006, MS), Nier et al. (2007, JEDC), Martínez-Jaramillo et al. (2010, JEDC), Gai (2011, JME), Arinaminpathy et al. (2012, BoE), Krause and Giansante (2012, JEBO), Markose et al. (2012, JEBO), Vallascas and Keasey (2012, JIMF), Georg (2013, JBF), Ladley (2013, JEDC), Cohen-Cole et al. (2013)

*(The main) limitation:*

No endogenous network formation – aggregate supply equated to aggregate demand, counterparts matched at random.

*Recent developments:*

Hałaj and Kok (2015), Aldarsolo et al. (2015), Blasques et al. (2015)

# Implications

If inter-bank lending networks are simulated as random:

1) Results conditional on network configurations that may **never** arise in practice
2) Characteristics of the counterparts no longer relevant
3) Aftermath of endogenous bankruptcies distorted
4) No dynamic changes in network geometry
5) No longer a bilateral market
6) Not optimal

*Punchline:* what is required in computational models of banking systems is a protocol for **endogenous** network formation.

# What is done in the presentation

- an endogenous inter-bank network formation protocol
- market structure emerges from optimal interaction of heterogeneous agents
- approximation of a unique network with agreed transaction: prices, volumes and parties involved
- no bank is better of by severing an existing link, no two banks have an incentive to form a link with each other
- contagion: liquidity erosion, fire sales, bankruptcy cascades

# How is the problem solved?

*Run a simulation (experiment):*

1) Initialize population of agents (banks)
2) Equip the agents with assets, liabilities, preferences
3) Derive and implement the rules according to which they borrow from and lend to each other
4) Allow them to interact

If the rules in point 3) are deterministic, exchangeable and the code stops – the problem is solved and has a unique solution.

# Model

*Consumers and regions*

- $T$ periods
- $N$ regions of size $h_k$ with a local bank and $n$ consumers
- who place deposits of $h_k/n$ at $t$ and collect at $t+1+S$, $S \sim \mathrm{P}(\lambda - 1)$

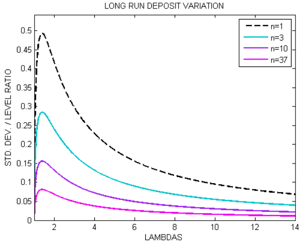
*Banks (all)*

- accept deposits, keep fraction $\rho$ as reserves
- vary in sizes, lending needs, risk perception, risk aversion
- have reservation bid/ask interest rates

*Learning (rolling windows)*

- probability of counterparty default
- realized means and std. deviations of risky asset returns

# Deposit variance process




For large $t+1$ variance of net deposits is (approximately) equal to

$$\mathbb{V}arH_k = \frac{h_k^2}{n}\left(1 - e^{-2(\lambda-1)}\sum_{j=0}^{+\infty}\frac{(\lambda-1)^{2j}}{(j!)^2}\right).$$

# Structure of assets and liabilities

*Assets* = *Liabilities*:

$$a_{k,t} + r_{k,t} + c_{k,t} + l_{k,t} = d_{k,t} + e_{k,t} + b_{k,t}.$$

*Assets:*

1. $a_{k,t}$ – risky asset
2. $r_{k,t}$ – obligatory reserves
3. $c_{k,t}$ – cash
4. $l_{k,t}$ – loans to other banks

*Liabilities:*

1. $d_{k,t}$ – deposits
2. $e_{k,t}$ – equity,
3. $b_{k,t}$ – loans from other banks

*Insolvencies*

- when risk weighted assets fall below 4% of liabilities

# Portfolios

Each bank $k$ has a different portfolio composition:

$$\Delta \ln P_{k,t} = \alpha_0 + \alpha_1 \Delta \ln P_{k,t-1} + Z_{k,t} + \sigma_t Z_{k,t-1}, \quad Z_{k,t} \sim \text{NID}(0,1),$$

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 Z_{k,t-1}^2.$$

*Denote:*

$\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2$ – ARMA(1,1)-GARCH(1,1) parameters
$P_{k,-1}, P_{k,0} := 1$ – boundary conditions
$P_{k,t}$ – price per unit

Not realistic – (large) banks are not price takers.

# Portfolios

Each bank $k$ has a different portfolio composition:

$$\begin{aligned} \Delta \ln P_{k,t} = \alpha_0 + \alpha_1 \Delta \ln P_{k,t-1} + Z_{k,t} + \sigma_t Z_{k,t-1} + I_t, \quad Z_{k,t} \sim \text{NID}(0,1), \\ \sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 Z_{k,t-1}^2, \\ I_{t+1} = \nu (D_t - S_t)/(D_t + S_t). \end{aligned} \tag{1}$$

*Denote:*

$\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2$ – ARMA(1,1)-GARCH(1,1) parameters
$P_{k,-1}, P_{k,0} := 1$ – boundary conditions
$P_{k,t}$ – price per unit

$D_t, S_t$ – aggregate demand/supply of the system
$\nu$ – common price component

# Banks (agents, active)

- have a full information on themselves and their counterparts, but not on DGP
- learn probability of counterparty default and means/std. deviation of risky asset returns from their own past data
- maximize expected utility from a value of a unit portfolio tomorrow, conditional on their own survival
- would like to split their unit investment into risky asset and (seemingly) risk-less interbank loans
- are allowed to pledge loans (volumes, interest)
- may purchase as much risky asset as they want
- to lend/borrow on the market they need a willing counterpart

# Assumptions

*Implementation requirements:*

1) reservation interest rates, deteriorating with trade volume,
2) mapping constraints in volume/interest rate back and forth,
3) formulas for aggregates.

*Network formation protocol:*

1) banks foresee all the steps of the proposed network formation protocol (rationality, consistency)
2) inter-bank lending is concluded at the midpoints of reservation rates (incentives, symmetry)
3) joint beliefs on the probability of counterparts bankruptcy given by $p_{t+1}$ (rare event, empirics)

# Algorithm

1) each bank listed twice, as prospective borrower or lender, both lists sorted in descending/ascending order,
2) agents with best offers active – trade the largest volume such that, in result of current transaction,
   i) best price can not become worse than second best,
   ii) no incentive to switch market sides,
   iii) no lender (borrower) can go from positive to negative surplus (deficit) in a single transaction,
   iv) lending financed with cash or selling risky asset,
   v) supply $=$ demand (also when no other constraints bind)
3) update reservation rates and net cash
4) repeat 1)–3) until no-one wants to trade

# Notation for portfolio problem

*Define:*

$w$ – share of risky asset in a unit portfolio desired by $k$
$\underline{w}$ – share of risky asset at the end of previous period
$\mu_{t+1}, \sigma^2_{t+1}$ – mean and std. dev of returns from risky asset expected by $k$
$\gamma_k$ – CARA parameter
$v, \hat{w}$ – net cash and cash obtained from previous loans to $k$
$\hat{i}$ – aggregate gross interest on previous loans to $k$
$\theta$ – 1 minus loss given default
$p_{t+1}$ – probability that the counterpart default tomorrow
$c$ – trading cost (multiplicative)

*For, respectively, borrower and lender:*

$$\chi_b := c^{-1}\mathbb{1}_{\{w \geq \underline{w}\}}(w) + c\mathbb{1}_{\{w < \underline{w}\}}(w),$$
$$\chi_l := c^{-1}\mathbb{1}_{\{w > \underline{w}\}}(w) + c\mathbb{1}_{\{w \leq \underline{w}\}}(w).$$

# Portfolio problem

*Assumptions*

1) Constant Absolute Risk Aversion (CARA)
2) Gaussian distribution of unconditional returns
3) Multiplicative trading cost ($c$)

It matters how the banks finance their investment.

*Lenders' objective*

Lender $l$ maximizes the unconditional expected utility that he tomorrow derives from his unit portfolio:

$$\mathbb{E}(V_{t+1}(w)).$$

# Portfolio problem

*Tomorrow value of lender's portfolio*

- lender: gets stochastic returns on the risky asset he has
- if he buys a unit of risky asset, obtains (stochastic) returns at the cost of $c^{-1}$ (instantaneous loss)
- if he sells a unit of risky asset, suffers lost opportunity cost and obtains $c$ (instantaneous loss)
- lends the remaining surplus on the interbank market, tomorrow obtains $\theta$ with probability $p$ or $i$ otherwise
- tomorrow repays with interest all the loans, taken before current transaction

$$V_{t+1}(w) = wR_{t+1} + (\theta B_{b,t+1} + i(1 - B_{b,t+1}))\cdot$$
$$\cdot(v + \hat{w} - \chi_I(w - \underline{w})) - \hat{i}\hat{w}.$$

*Borrowers' objective*

Borrower $b$ maximizes the expected utility that he derives from his unit portfolio tomorrow, conditional on his own survival:

$$\mathbb{E}(V_{t+1}(w)|B_{b,t+1} = 0).$$

*Tomorrow value of borrower's portfolio*

- borrower: gets stochastic returns on the risky asset he has
- if he buys a unit of risky asset, obtains (stochastic) returns at the cost of $c^{-1}$ (instantaneous loss)
- if he sells a unit of risky asset, suffers lost opportunity cost and obtains $c$ (instantaneous loss)
- tomorrow repays all the loans with interest

$$V_{t+1}(w) = wR_{t+1} - i(\chi_b(w - \underline{w}) - v - \hat{w}) - \hat{i}\hat{w}.$$

## Proposition (Borrower's behaviour under CARA)

*Assume* $R_{t+1} \sim N(\mu_{t+1}, \sigma^2_{t+1})$, *set* $e_b = \underline{w} + \chi_b^{-1}(v + \hat{w})$.

(i) *Borrowers* **f.o.c.** *is equivalent to*

$$w = \frac{1}{\gamma_b \sigma^2_{t+1}}(\mu_{t+1} - i\chi_b), \quad \text{where } w \neq \underline{w}, \ w \geq e_b.$$

(iii) *Borrower's* **reservation interest rate** $\bar{i}_b$ *is*

$$\bar{i}_b = \chi_b^{-1}\left(\mu_{t+1} - \gamma_b \sigma^2_{t+1}[\underline{w} + \chi_b^{-1}(v + \hat{w})]\right).$$

(iv) *The* **maximum volume of a loan** $\tilde{w}$ *that borrower b would be willing to accept at the interest rate* $\tilde{i}$ *is*

$$\tilde{w} = \gamma_b^{-1}\chi_b^2\sigma^{-2}_{t+1}(\bar{i}_b - \tilde{i}).$$

## Proposition (Lender's behaviour under CARA)

*Assume* $R_{t+1} \sim N(\mu_{t+1}, \sigma^2_{t+1})$, *set* $e_l = \underline{w} + \chi_l^{-1}(v + \hat{w})$.

(i) *Lender* **f.o.c.** *is equivalent to*

$$c_1 + c_2 w - \ln\Big(\frac{c_5}{c_3 + c_4 w} - 1\Big) = 0, \quad \text{where } w \neq \underline{w},\ w \leq e_l.$$

(iii) *Lender's* **reservation interest rate** $\underline{i}_l$ *is*

$$\underline{i}_l = \bar{i}_l \frac{1}{1 - p_{t+1}} + \theta \frac{p_{t+1}}{1 - p_{t+1}}.$$

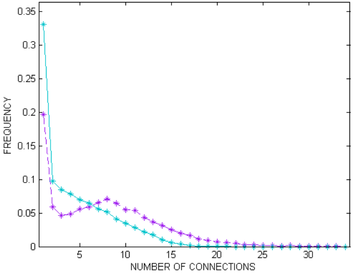
(iv) *The* **maximum volume of a loan** $\underset{\sim}{w}$ *that lender l would be willing to accept at the interest rate* $\underset{\sim}{i}$ *is*

$$\underset{\sim}{w} = \frac{1}{\gamma_l \chi_l^{-2} \sigma^2_{t+1}} [(\bar{i}_l - \underset{\sim}{i})(1 - p_{t+1}) + (\bar{i}_l + \theta) p_{t+1}].$$

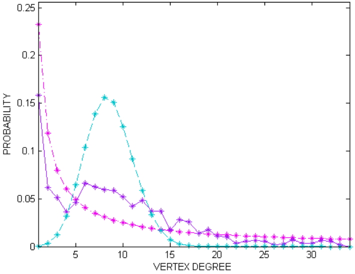
# Calibration

| *Parameters:* | *Represent:* |
| --- | --- |
| 1. $N := 35$ | number of banks |
| 2. $\theta := 0.95$ | 1−loss given default |
| 3. $c := 0.997$ | trading cost (multiplicative) |
| 4. $p := 10^{-4}$ | (prior) probability of default |
| 5. $\rho := 0.10$ | reserve ratio |
| 6. $\zeta := 0.115$ | equity ratio |
| 7. $\lambda := 11.37$ | deposit duration |
| 8. $n := 10$ | no. of customers per region |
| 9. $\gamma_k \sim U(2,3)$ | risk aversion |
| 10. $\alpha_0 := 2{\cdot}10^{-4}, \alpha_1 := -0.05, \alpha_2 := 0.12$ | ARMA(1,1) parameters |
| 11. $\beta_0 := 2 \cdot 10^{-7}, \beta_1 := 0.15, \beta_2 := 0.16$ | GARCH(1,1) parameters |

# Degree distribution

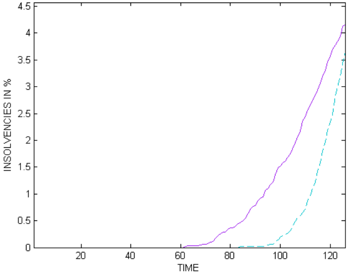



(a) Borrowers (solid cyan) vs. lenders (dashed violet line).

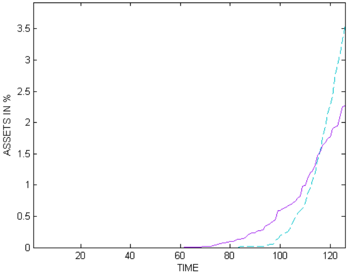



(b) Distributions: simulated and thereti-cal (scale-free, Erdös-Rény).

# Insolvencies under crisis




(c) Number of insolvencies.




(d) Insolvencies in total assets.

Figure: Dashed cyan line – homogeneous bank sizes, solid violet line – heterogeneous sizes.

# Funding liquidity under crises

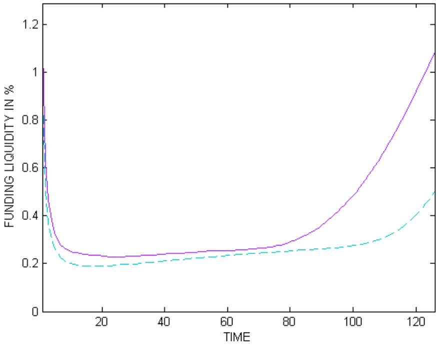



Figure: Dashed cyan line – homogeneous bank sizes, solid violet line – heterogeneous sizes.

# Assortativity and network characteristics

*Assortativity (calibrated assets)*

Typical creditor–debtor pair: risk-averse, small bank who perceives investment risk as high lends to risk-loving large bank who perceives investment risk as low (significant at 0.1% level).

*Network characteristics*

- ✓ (approximately) scale-free degree distribution
- ✓ network density in certain range
- ✓ disassortative lending
- ✓ persistence
- ✓ small banks are creditors, large are debtors
- × core and periphery
- × large institutions have more links

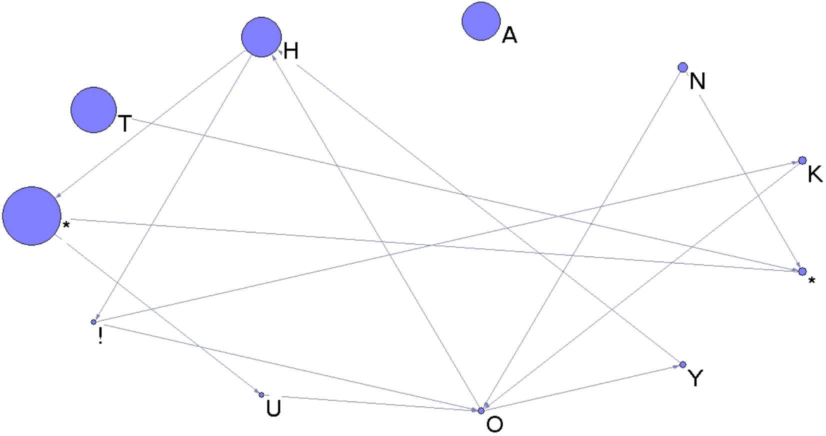
# Limitations and summary

*Limitations*

- $p_{t+1}$ does not depend on counterparty characteristics
- full information, one period – no maturity mismatch
- no analytical tractability

*Summary*

- market structure emerges from interaction of heterogeneous agents, algorithm outputs transactions (volumes, prices, counterparts), outcome (approximately) optimal and stable
- model calibrated to US market, degree distribution in between scale–free and binomial density, instantaneous cascades possible
- three possible channels of contagion, systemic risk may be traced at the transaction and aggregate levels



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