# Liquidity decisions and the timing of payments

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BANCO DE MÉXICO

# Disclaimer

*The views expressed here are those of the author and do not necessarily reflect the position of the Bank of Mexico*

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2. Why? Crucial to understanding risks
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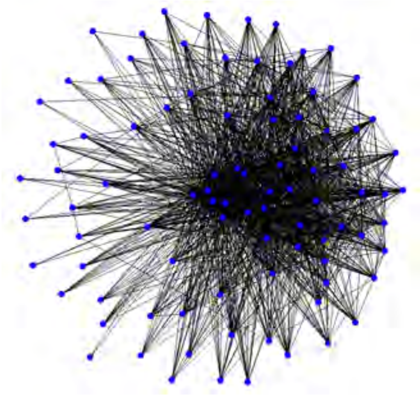
# What?

Figure : SPEI (Mex. RTGS) network



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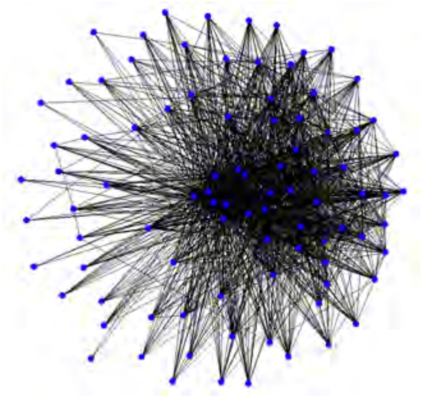

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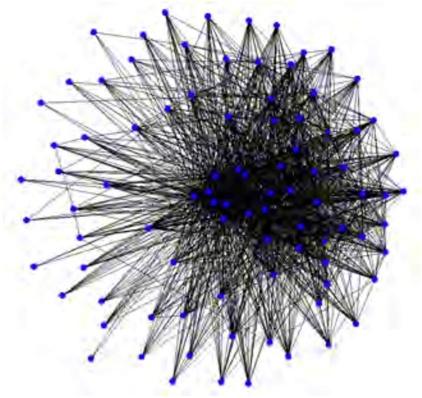

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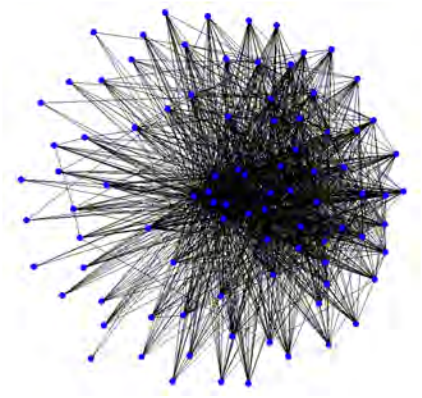

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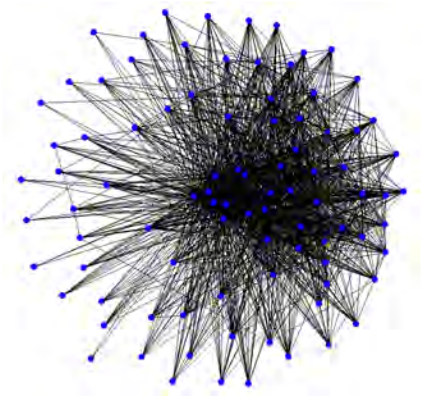

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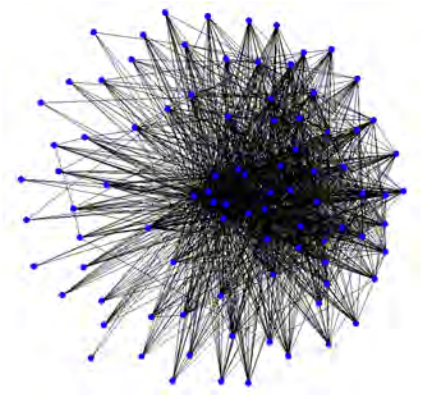

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Output: Simulator of a Real-Time Gross Settlement (RTGS) system

- Timing of payments
- Intraday liquidity demand

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# Why? (1/2)

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Participants' decision-making process: crucial to understanding risks

'Fixed' behavior may not be an adequate assumption:

$$\mathrm{action}_t^i(\mathrm{history}_t) \neq \mathrm{action}_t^i(\mathrm{history}_t{}')$$

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- Deferred Net Settlement (settlement at the end of the day) replaced by Real-Time Gross Settlement
- Provision of intraday liquidity (continuous compensation increases liquidity pressures, private liquidity would be inefficient)

From a central bank's perspective, interesting to measure (and explain) the demand for intraday liquidity and the timing of payments

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Pioneering work: competitive multi-agent model of interbank payment systems (Galbiati & Soramäki, 2011).

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$T$ periods (seconds, minutes, etc.) of a business day

Participant $i$ sends payment $S_t^i$ to other participants on period $t$

Participants face budget constraint. Funding sources:

- Intraday liquidity provided by the central bank $L_t^i$
- Positive account balances they save from previous period $B_t^i$
- Received payments from others on previous period $R_t^i$

$$S_t^i + B_{t+1}^i = R_t^i + B_t^i + L_t^i$$

$$\text{where} \quad B_{t+1}^i \geq 0$$

$$\sum_t L_t \leq \overline{L}^i, \quad L_t^i \geq 0$$

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Reputation costs of period $t$:

$$\text{Reputation}_t^i = \sum_k \kappa O_{t,k}^{ij} \left(1 - x_k^i\right)$$

where $x_k^i = 1$ if payment is settled, $x_k^i = 0$ otherwise, $\kappa$ is the “interest cost” of delaying payments (Galbiati & Soramäki, 2011).

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Settled payments (first-in-first-out algorithm):

$$x_k^i = 1 \iff \sum_{p=1}^{k} O_{t,k}^{ij} \leq B_t^i + L_t^i + R_t^i$$

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Return function for $t \in \{0, \dots, T-1\}$ :

$$\text{Return}_t^i \left( L_t^i, B_t^i, R_t^i, \mathcal{O}_t^i \right) = \sum_k \kappa O_{t,k}^{ij} \left( 1 - x_k^i \right) + qL_t^i + F\mathbb{1}_{\{L_t^i > 0\}}$$

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Terminal return function is the interest payed from missing funds (negative if surplus funds):

$$\text{Terminal}^i\left(\sum_t L_t^i, B_{T+1}^i, R_{T+1}^i\right) = r\left(\sum_t L_t^i - B_{T+1}^i - R_{T+1}^i\right)$$

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## Recursive problem

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Terminal condition:

$$v_T\left(S_T\right) = \beta r \left(L_T^a + \sum_k O_{T,k} - B_T - R_T - \mathrm{E}_T\left(R_{T+1}\right)\right)$$

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Bellman-equation for $t < T$:

$$\begin{gathered} v_t(S_t) = \min_{L_t}\left\{\text{return}\left(S_t, L_t\right) + \beta \mathrm{E}_t\left(S_{t+1}\right)\right\} \\ \text{s.t.} \quad B_{t+1} = B_t + R_t + L_t - \textstyle\sum_k O_{t,k} x_{t,k} \geq 0 \\ L_{t+1}^a = L_t^a + L_t \\ 0 \leq L_t \leq \overline{L} - L_t^a \\ \mathcal{O}_{t+1} = \left(O_{t,k}, \mathcal{O}_{t+1}'\right),\ k : \textstyle\sum_{p=1}^k O_{t,k} > B_t + L_t + R_t \end{gathered}$$

# How?

## Equilibrium and the curse of dimensionality

If we could solve by backwards induction, we would obtain policy functions: $L_t^i\left(S_t^i\right)$, so that $S_0^i \rightarrow L_0^i\left(S_0^i\right) \rightarrow S_1^i \ldots$

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Approximate dynamic programming (ADP) algorithms allow us to perform forward induction (avoid looping through all possible states)

Substitute computational problem for statistical problem: estimate value functions $\overline{v}_{t+1}$

# How?

## Approximate dynamic programming

Estimate value function using basis functions (linear):

$$\mathrm{E}_t\left(\overline{v}_{t+1}\right) = \beta_t + \beta_1 L_t^a + \beta_2 B_t + \beta_3 R_t + \beta_4 \sum_k \left(\mathcal{O}_{t,k}\right)$$

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General idea:

- fix initial parameters in outer loop
- sample different states ($m$) and calculate $m$ parameters in inner loop
- update new parameters using recursive least squares
- use new parameters in outer loop

# How?

## Testing the model

Two tests:

1. Compare the model to observed data. Choose a day at random (January 16th, 2014), and compare two measures (liquidity and timing-skewness)
2. Compare the model to a simulator, under a stress test. Assume that the biggest participant of a given day fails at $t = 0$

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Data:

- 89 participants
- Start at 5:00 am, finish at 5:59:30 pm (1,560 30-second periods)
- Adapt the model to Mexico: two sources of central bank liquidity

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Adapt the model to Mexico: two sources of central bank liquidity

Time-skewness: measures concentration of payments at the end of the day (0: all payments sent during first period, 1: all payments sent in last period): $\sum_t (1 - \text{cum. proportion}) / (T - 1)$

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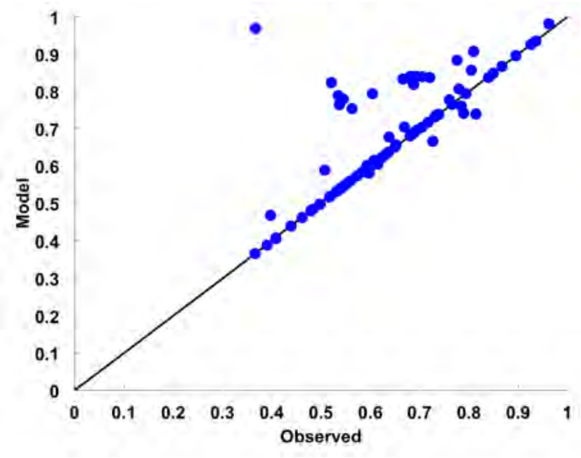
# Lessons? Model vs Reality (1/2)

## The model predicts reasonable time-skewness

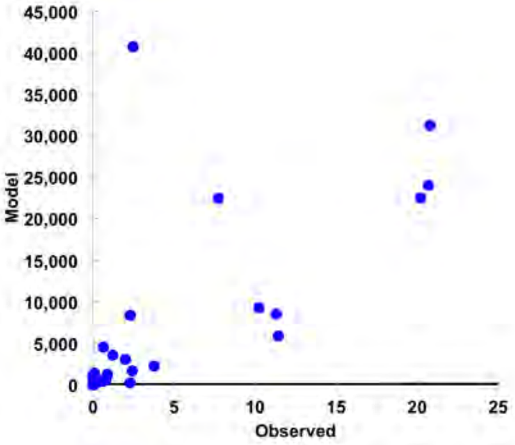
Figure : Time-skewness by participant



# Lessons? Model vs Reality (2/2)

## The model overestimates liquidity (but with some correlation)

Figure : Liquidity demanded by participant



# Lessons? Model vs Simulator (1/2)

The model predicts lower change in skewness

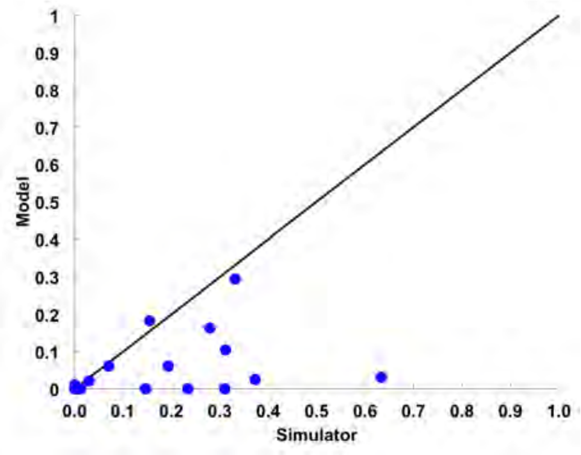
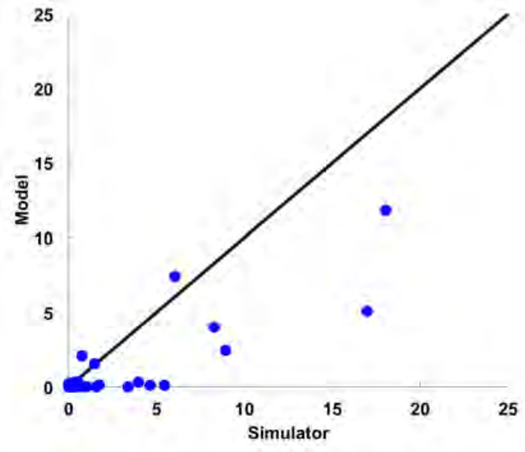



Figure : Skewness(failure)-Skewness(original) by participant

# Lessons? Model vs Simulator (2/2)

The model predicts lower change in liquidity

Figure : Liquidity(failure)-Liquidity(original) by participant



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## Summary

Comparison with reality (typical day):

- the model predicts reasonable time-skewness
- liquidity is overestimated (but with correlation = 0.77)

| | Model | Observed |
|---|---|---|
| Time-skewness | 0.55967 | 0.55787 |
| Liquidity | $1.98 \times 10^{14}$ | $1.23 \times 10^{11}$ |

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Comparison with simulator (failure of biggest participant):

- the model predicts almost no change in time-skewness or liquidity
- simulator: delay in payments and increase in liquidity needs

| | Model | Simulator |
|---|---|---|
| $\Delta$ Time-skewness | 0.00868 | 0.09423 |
| $\Delta$% Liquidity | 0.04% | 163.95% |

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Analyze goodness of fit of the value function estimator

Increase periodicity (from 30-second intervals to 1-second intervals)

Simulate multiple days

# References


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