DebtRank: a microscopic foundation to shock propagation

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Complex systems approach to financial contagion

- Financial markets as interacting systems
- Systemic risk as an emergent phenomenon
- Interactions between banks can be modeled through networks

how are exogenous shocks amplified by endogenous dynamics?

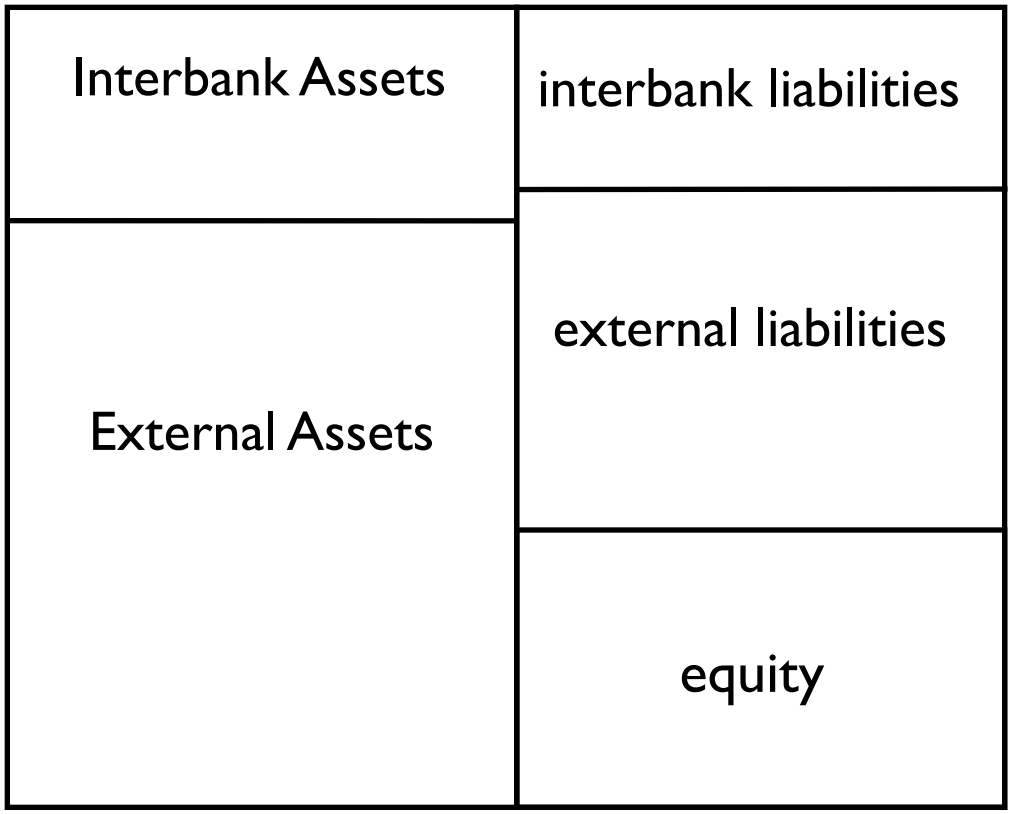
Statistical mechanics*: "The wide perspectives opening up if we think of applying this science to the statistics of living beings, human society, sociology and so on, instead of only to mechanical bodies, can here only be hinted at in a few words"*

(L. Boltzmann 1904)

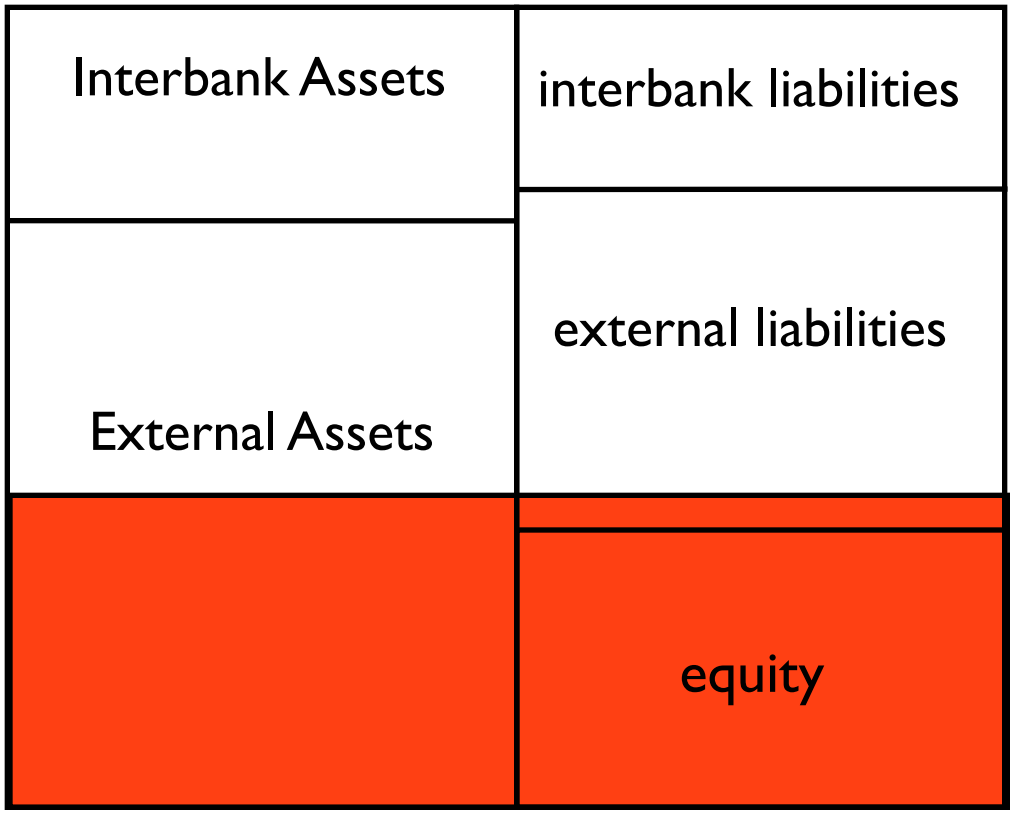
Financial contagion due to counterparty default risk

- Network of interconnected balance sheets
- Links represent interbank loans
- When a bank defaults its creditors suffer losses
- If these are big they default as well, and so on

Stylized balance sheet

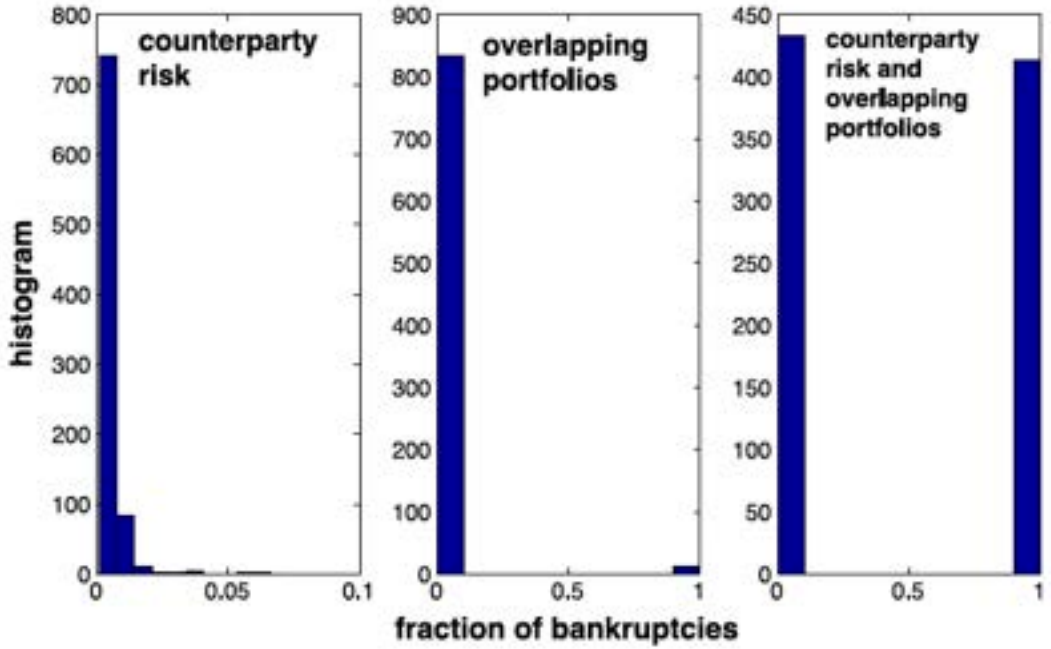


Threshold dynamics



Financial contagion due to counterparty default risk

Caccioli, Farmer, Foti and Rockmore (JEDC 2015)

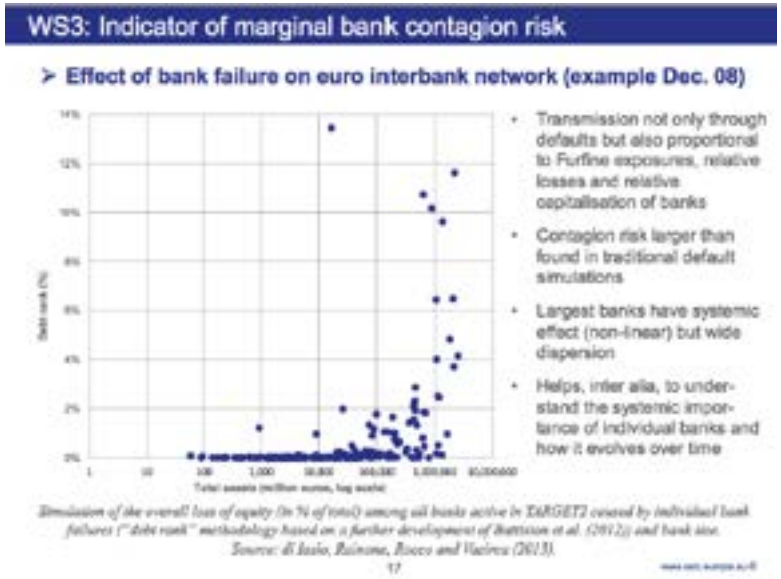


no large cascades unless other contagion channels are considered

DebtRank

Battiston et al. Scientific Reports (2012)

- Iterative algorithm inspired by google page-rank
- Stress propagates even in absence of defaults
- Tool to see the build-up of systemic risk
- Ranking of banks in terms of their systemic importance



DebtRank

level of distress

impact of bank j on bank i

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_j W_{ij} h_j(t-1)\sigma_j(t)\right\}$$

only active banks propagate shocks

- Each bank can propagate shocks only once (the first time they are hit)
- This may lead to an underestimation of systemic risk

we propose a generalized dynamic that

1) accounts for further rounds of shock propagation

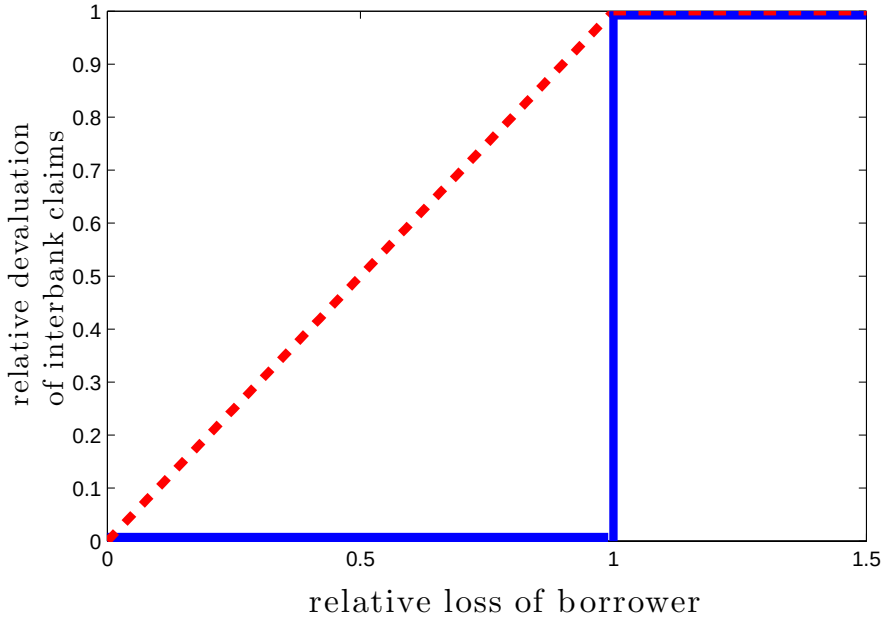
2) allows some analytic characterization

Generalized debtRank

- Relative equity loss $$h_i(t) = \frac{E_i(0) - E_i(t)}{E_i(0)}$$

- Linear propagation of shocks (strong assumption!)

$$\frac{E_j(t) - E_j(t-1)}{E_j(t-1)} \rightarrow \frac{A_{ij}(t+1) - A_{ij}(t)}{A_{ij}(t)}$$



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- Matrix of interbank leverage $$\Lambda_{ij} = \frac{A_{ij}}{E_i}$$

- Iteration map

$$h_i(t+1) = \min\left\{1, h_i(t) + \sum_j \Lambda_{ij}\left(h_j(t) - h_j(t-1)\right)\right\}$$

Stability properties of the generalized debtRank

- The stability of the dynamics depends on the largest eigenvalue of the matrix of interbank leverage (see also Markose et al. JEBO 2012)
- If the eigenvalue is larger than one shocks are amplified by the dynamics
- If the eigenvalue is smaller than one shocks are progressively damped

An application to EU banks

- 183 European banks publicly traded between 2008 and 2014
- Information on total interbank assets, liabilities, and tier 1 capital
- Network reconstruction using a fitness model coupled to a RAS algorithm

Fitness model

- Share of interbank assets hold by i

$$a_i = \frac{\text{interbank assets of bank i}}{\text{total interbank assets}}$$

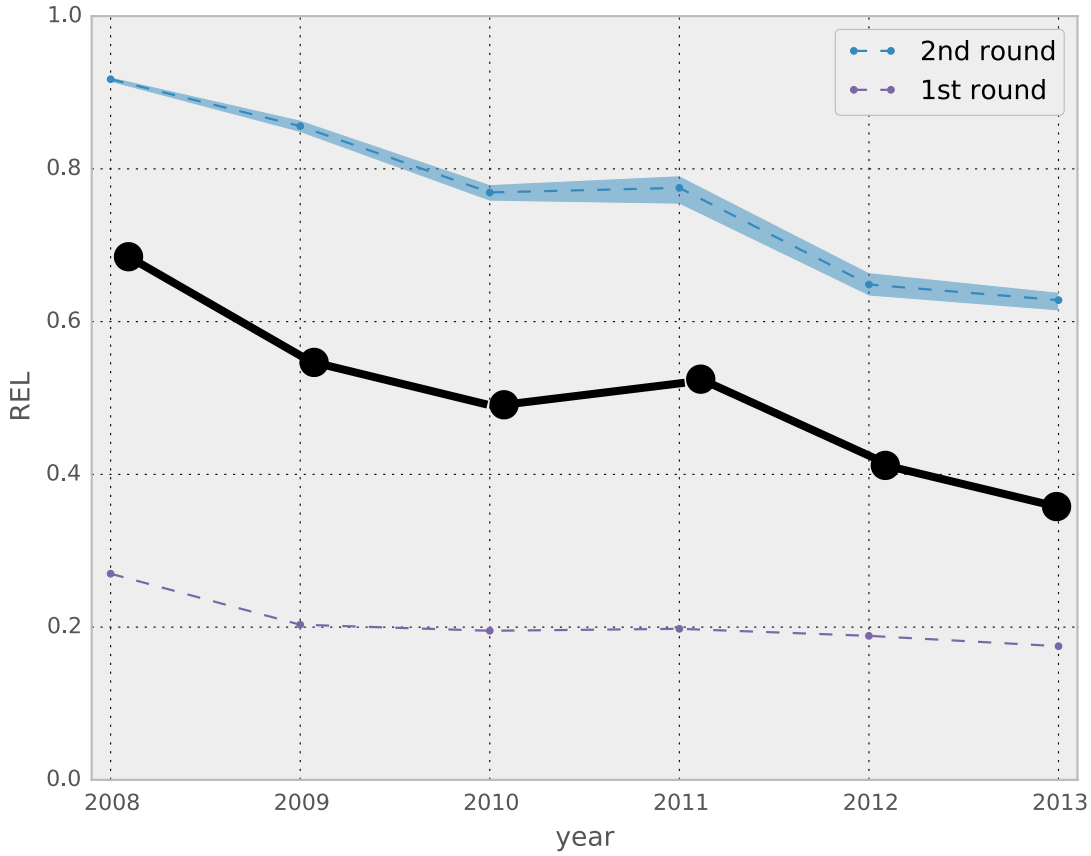
- Share of interbank liabilities hold by i

$$\ell_i = \frac{\text{interbank liabilities of bank i}}{\text{total interbank liabilities}}$$

- Probability that bank i lends to bank j

$$p_{i \to j} = \frac{z a_i \ell_j}{1 + z a_i \ell_j}$$

Stress test exercise



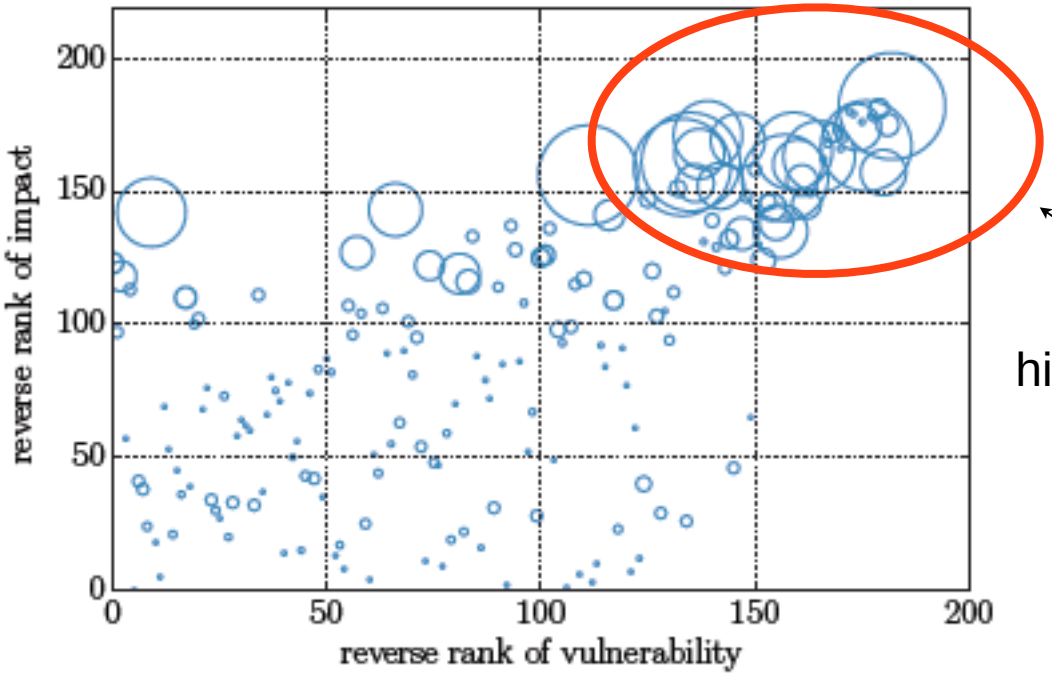
generalized debtRank

original debtRank

exogenous shock

Secondary rounds of shock propagation significantly contribute to systemic risk

Ranking banks: Impact and vulnerability



highest impact and vulnerability

- The most impactful banks are also the most vulnerable
- Among these there are also small banks

Conclusion

- Generalization of debtRank to account for further rounds of shock transmission
- The stability of this contagion dynamics is determine by interbank leverages

- Secondary waves of contagion can induce significant losses
- The most impactful banks are also the most vulnerable

Stability properties of the generalized debtRank

- Before the first default the dynamics is

$$\Delta\vec{h}(t) = \mathbf{\Lambda}\Delta\vec{h}(t-1)$$

- Fixed point: $\Delta\vec{h} = 0$

- Shocks will be amplified if $\lambda_{max} > 1$

- If $\lambda_{max} < 1$

$$\vec{h}(\infty) = (1 - \mathbf{\Lambda})^{-1}\vec{h}(0)$$

final level of distress

initial exogenous shock