

Financial Networks as Probabilistic Graphical Models (PGM)

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What PGMs are

- ▶ A set of **random variables** can be given a graphical representation which encodes the conditional independencies between them in a visually appealing form
- ▶ The name of this representation is Probabilistic Graphical Models (PGM)
- ▶ A graphical representation consists of
 - ▶ Nodes – the random variables
 - ▶ Edges – the probabilistic interactions between them

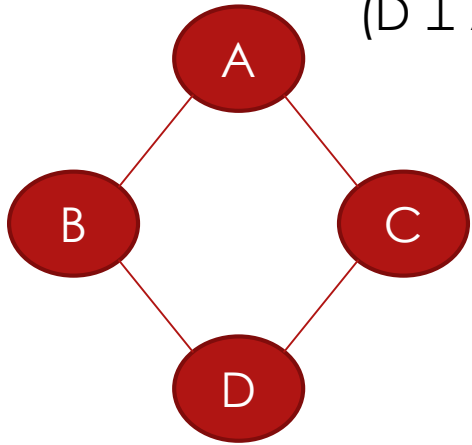
A short taxonomy

- ▶ There are many types of Probabilistic Graphical Models
- ▶ Some of them are suitable for studying networks of firms such as:
 - ▶ Bayesian Nets (BN)
 - ▶ Markov Random Fields (MRF)
 - ▶ Chain Graphs (CG)
 - ▶ Directed Cyclic Graphs (DCG)

Probabilistic Graphical Models

Markov Random Field (MRF)

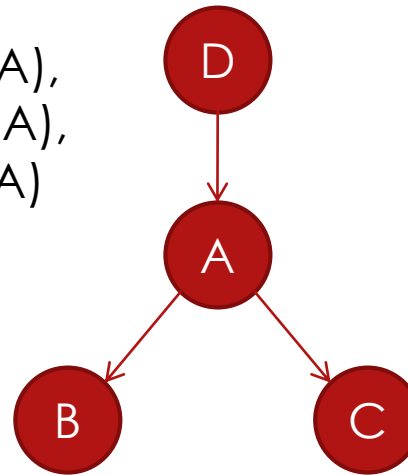
$$(B \perp C \mid A, D)^1$$
$$(D \perp A \mid B, C)$$



1. The symbol \perp denotes independence relationship. The symbol $|$ denotes "given". We more compactly want to say **B** is independent of **C** given **A** and **D**.

Bayesian Net (BN)

$$(B \perp C \mid A),$$
$$(C \perp D \mid A),$$
$$(B \perp D \mid A)$$

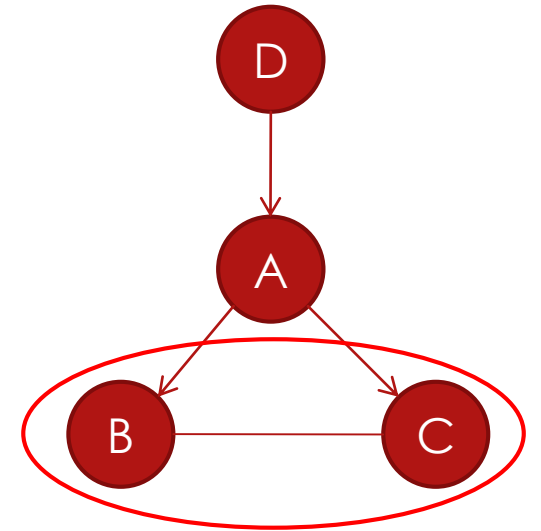


Chain Component

The set of variables which remain connected by undirected edges after removing the directed edges

NB Also **D** and **A** are chain components formed each by 1 node

Chain Graph (CG)



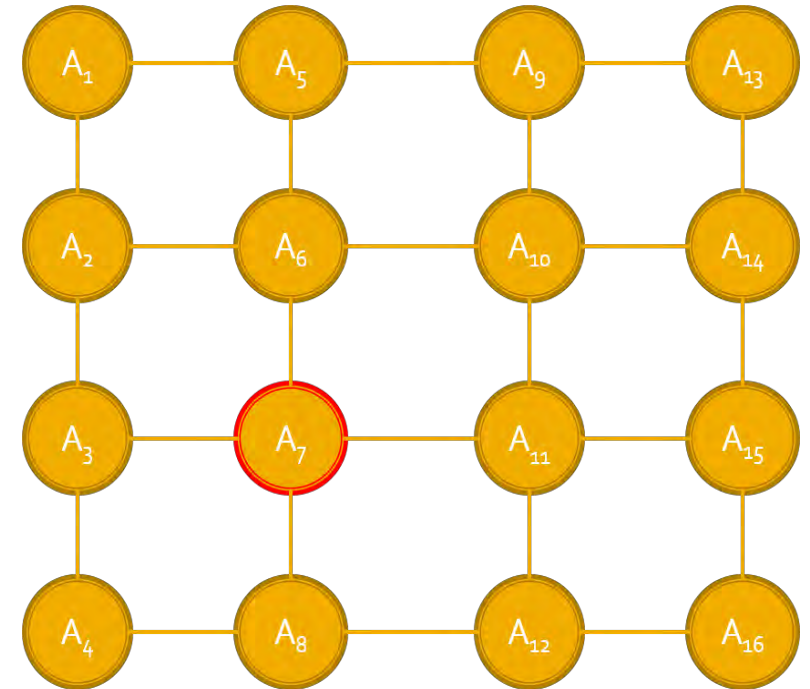
$$(B, C \perp D \mid A)$$

PGM and Financial Networks

- ▶ The nodes in a PGM can represent random variables characterising a set of financial firms and the edges how these variables influence each other
- ▶ Typical examples of random variables are:
 - ▶ Probabilities of default
 - ▶ Asset returns
 - ▶ Equity returns
 - ▶ Etc

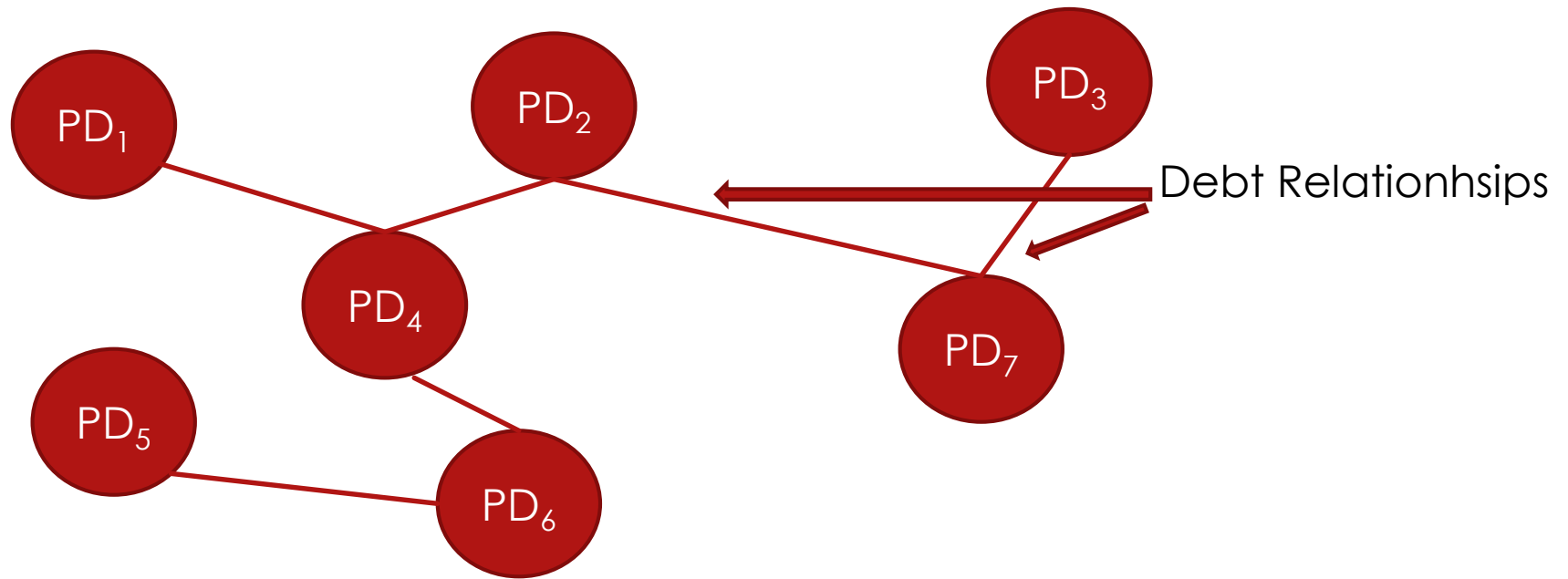
MRF and Default Configurations

- ▶ A network of firms resembles (with an oversimplification) a grid of atoms whose debt interactions are, on the face of it, similar to the those of the Ising model usually modelled through a MRF
- ▶ In such a description, firms themselves (and the interactions between them) can be seen as atoms, where the default of a set of debtors to firm i can “flip” i into default
- ▶ MRFs provide a natural representation of a network of debt relations, as they are endowed with desirable screening properties. In fact, if two firms are not indebted with each other, we have no reason to believe that they should exert any direct influence on each other's probability of default



PGM and Networks of Defaults

Markov Random Field (MRF) representation of a network of probabilities of default



Populating the MRF

- ▶ When specifying a MRF we must define the affinity between the two variables and we shall do this through a potential $\phi(A, B)$
- ▶ A potential is a positive real-valued function over a discrete domain Ω
 $\phi(\Omega): \Omega \rightarrow \mathbb{R}^+$
- ▶ For the boolean MRF below we have to define four values for the potential $\phi(A, B)$, $\phi(\neg A, B)$, $\phi(A, \neg B)$ and $\phi(\neg A, \neg B)$



Example

\bar{A}	\bar{B}	50
\bar{A}	B	10
A	\bar{B}	20
A	B	1

Normalizing

- ▶ The joint probability is given by:

$$P(A, B) = \frac{1}{Z} \varphi(A, B)$$

- ▶ With the normalization constant:

$$Z = \sum_{A, B} \varphi(A, B)$$

Example: Joint Probability Table

\tilde{A}	\tilde{B}	61.7%
\tilde{A}	B	12.3%
A	\tilde{B}	24.7%
A	B	1.2%

The Joint Probability Table

- ▶ When extending to more complex networks what we need to provide is only the potential encoding the interaction of a random variable with its neighbors
- ▶ From local assignments of potentials we build a global characterization of the network given by the joint probability table

Example: JPT for 3 nodes

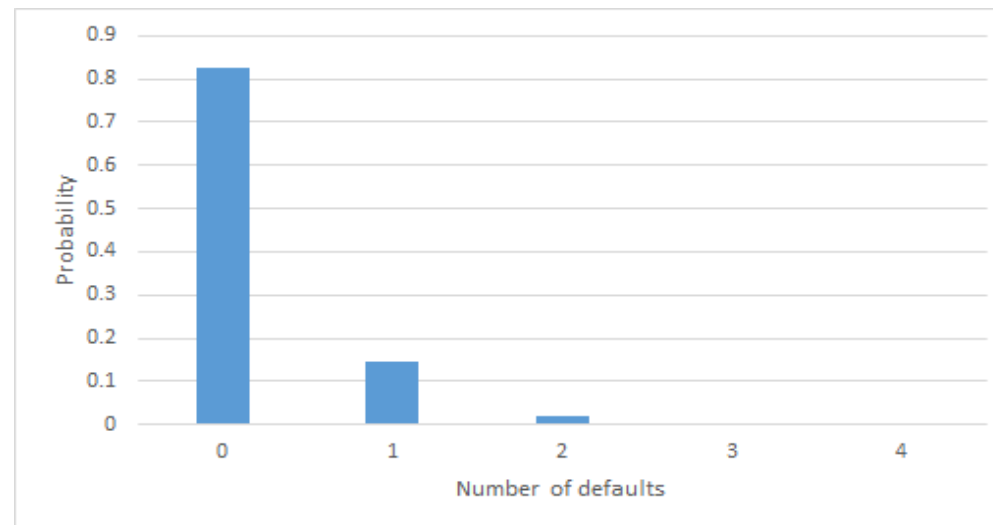
V_1	V_2	V_3	Joint Probability
0	0	0	p_0
1	0	0	p_1
0	1	0	p_2
0	0	1	p_3
1	1	0	p_4
1	0	1	p_5
0	1	1	p_6
1	1	1	p_7

Calibration

- ▶ Such network can be calibrated by knowing 2 sets of quantities
 - ▶ For each X_i the marginal probability $P(X_i)$
 - ▶ For each pair $\{i, j\}$ the correlation $\rho(X_i, X_j)$
- ▶ Or:
 - ▶ For each X_i the marginal probability $P(X_i)$
 - ▶ For each pair $\{i, j\}$ the joint probability $P(X_i, X_j)$
- ▶ Or:
 - ▶ For each X_i the marginal probability $P(X_i)$
 - ▶ For each pair $\{i, j\}$ the conditional probability $P(X_i|X_j)$ (or $P(X_j|X_i)$)

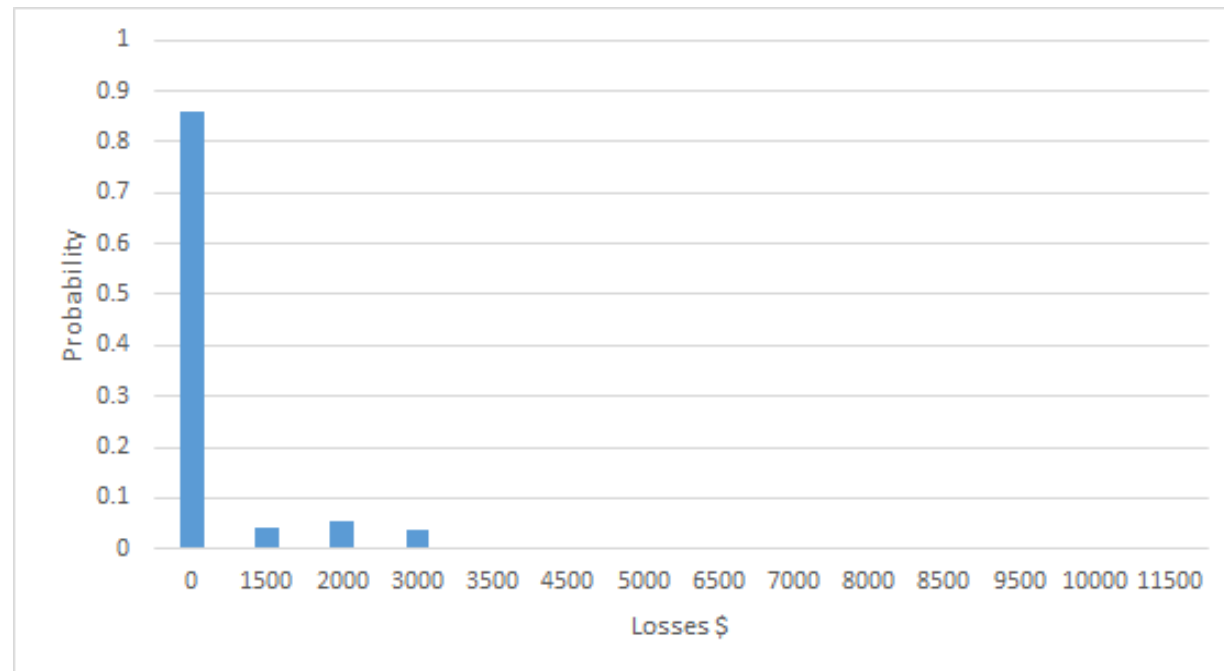
Distribution of Defaults

- ▶ Once the network is calibrated then a network default distribution can be calculated



Distribution of Losses

- ▶ ...and a distribution of losses in the system



Distribution of Losses

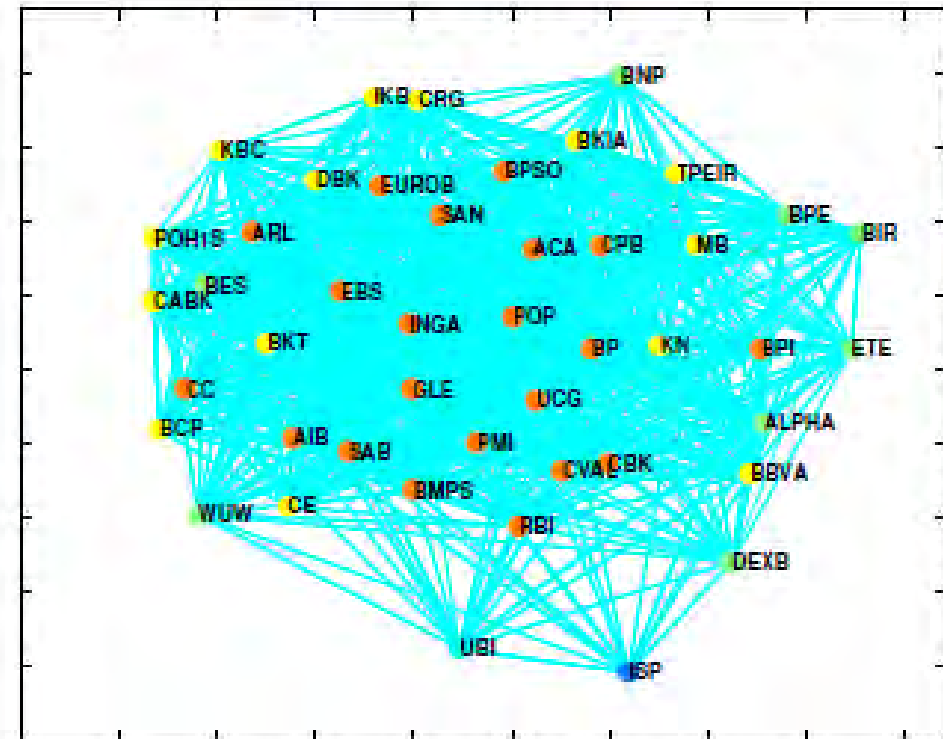
- ▶ Systemic risk indicators can be computed from the distribution of losses e.g.:
 - ▶ VaR
 - ▶ Conditional VaR
- ▶ The contribution of a single firm to the the distribution of losses can be measured as:
 - ▶ Component VaR ($VaR = \sum_{i=1}^N VaR_i$)
 - ▶ Component Conditional VaR ($CVaR = \sum_{i=1}^N CVaR_i$)

Other Networks

- ▶ The previous setup assumes that the structure of default relationships between firms is known
- ▶ Sometimes obtaining such information may be very difficult even for central banks
- ▶ Other types of network relationships can be still studied e.g. equity returns

Training a Continuous MRF

- ▶ If a variable in a network represents the equity return of a firm one can train a MRF on a dataset containing the equity returns of a set of firms¹
- ▶ The learning algorithm will automatically find the conditional independencies and detect the significant edges
- ▶ One can then transform the equity returns in network default probabilities by introducing a default threshold and discretising

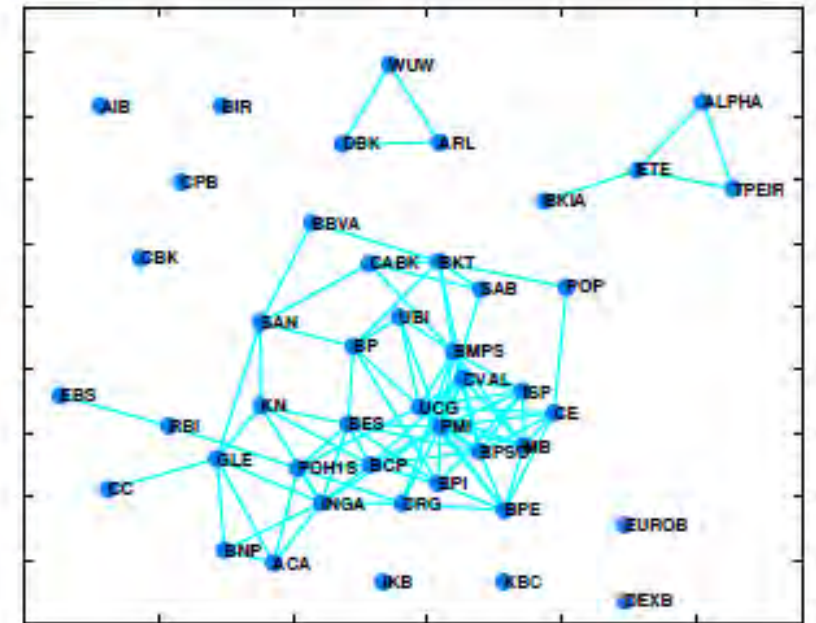


(a) Non-Hierarchical Network (Total Links : 711)

¹ An example from Ahelegbey and Giudici (2014)

Reducing the network

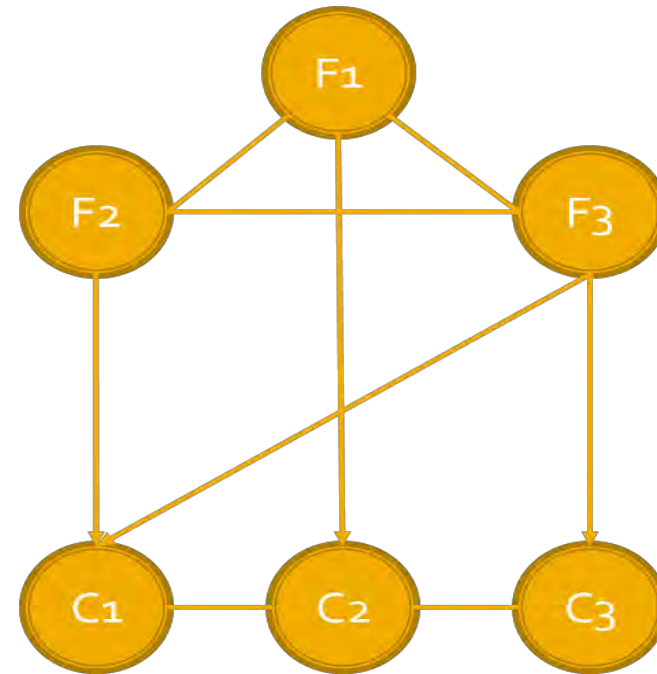
- ▶ One can condition the equity returns on a global market risk factor (e.g. the S&P 500 or a liquidity index) and train a PGM on the returns residuals which are non-explained by the risk factors
- ▶ The number of links is greatly reduced but not eliminated



(b) Hierarchical Network (Total Links : 95)

A chain graph approach

- ▶ The example of the two previous slides are essentially a Chain Graph PGM



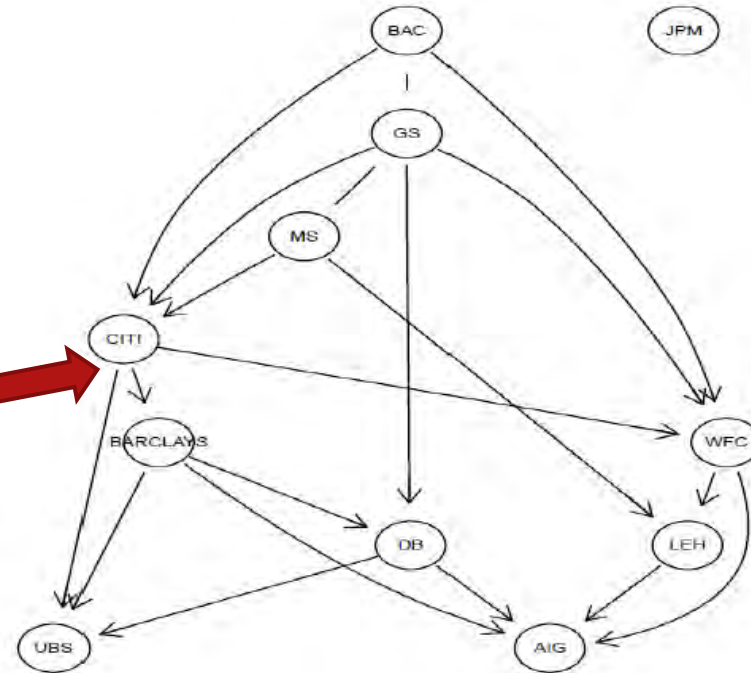
A Chain Graph of 3 factors $F=\{F_1,F_2,F_3\}$ and 3 firms $C=\{C_1,C_2,C_3\}$

A Bayesian Net approach

- ▶ As an alternative one can choose to train a Bayesian Net on the firms' residuals as done in Kitwiwattanachai (2014) on CDS data of large banks

$$\Delta \log S_{i,t} = \alpha_i + \beta_m R_{m,t} + \beta_v \Delta VIX_t + \varepsilon_{i,t}$$

where Δ denotes weekly changes of the CDS $S_{i,t}$ of institution i , $R_{m,t}$ is the S&P500 weekly returns at time t , and VIX is the CBOE implied volatility index

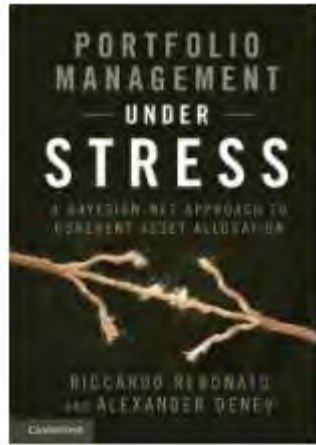


Source: C. Kitwiwattanachai (2014)

Conclusions

- ▶ PGMs are a good framework to modelling financial networks as they can take into account the inherent stochasticity of complex systems of interacting entities
- ▶ PGMs can express conditional independencies as the ones observed in real network of interacting entities
- ▶ PGMs can be trained automatically on data to unveil the underlying structure of the network at hand

Books



Portfolio Management under Stress
A Bayesian Net Approach to Coherent Asset Allocation
Riccardo Rebonato, Alexander Denev

Probabilistic Graphical Models in Finance
Alexander Denev

