Characterization of Lower-bound and Upper-bound of Required Settlement Fund under Real-Time Gross Settlement

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Interbank Settlement Systems

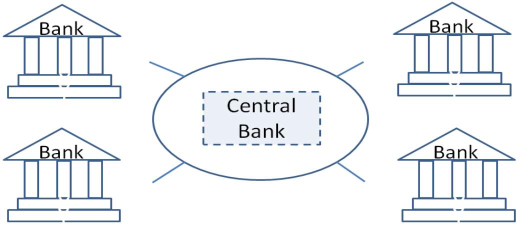
Interbank Settlement System: TARGET2, CHAPS, BOJ-Net, Fedwire

"real-time gross settlement" (RTGS) system

- small risk, large requirement of settlement fund

↔ "designated-time net settlement"(DTNS) system

- large risk, small requirement of settlement fund



Research Question

- How much settlement fund is to be required under RTGS?
  - in the face of complex network of obligations

    (about **500** participants, **50, 000** daily transactions in the BOJ-NET)

which and how network structure matters?

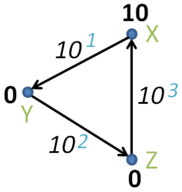
- theoretical approach
  formalize as network problem (, omitting incentive)
  characterize the problem with network factors

What this paper does

- settlement in interbank settlement systems
  - banks settle obligations on behalf of non-financial customers
    - given obligations in the morning, all settled within a day
    - zero balance in the morning
    - intraday lending by central bank
    - borrowed amount returned at the end of the day

- required amount of settlement fund under RTGS
    - minimum required amount for each realized order of settlement

      ← no redundant lending, no reserve

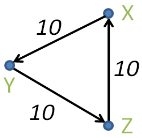


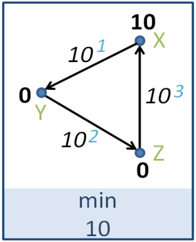
required amount: 10 + 0 + 0 = 10

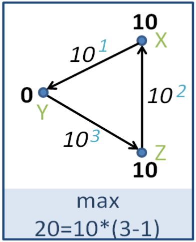
What this paper does

- Formalize a pair of minimization/maximization problem
  - for given "distribution of obligations" with settlement unit constraint,
  - evaluate lower-bound and upper-bound of required settlement fund
  - in relation to physically available order of settlement
- Characterize the problems in terms of network factors
  - Propose original concepts on twisted nature of payment network : arrow-twisted, vertex-twisted

example





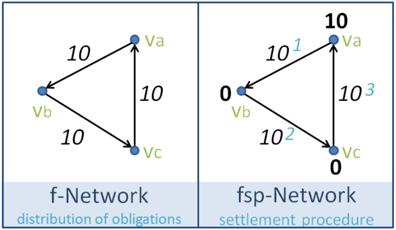


Framework

- base elements $\boldsymbol{V, A}$
  - $< \boldsymbol{V, A} >$: directed graph (vertices, arrows)
- additional elements $\boldsymbol{f, s, p}$
  - $\boldsymbol{f : A \to R_+}$ (flow) : amount of obligation
  - $\boldsymbol{s : A \to \{1, 2, .., |A|\}}$ (sequence) : order of settlement
  - $\boldsymbol{p : V \to R_{0+}}$ (potential) : allocation of settlement fund
- Networks
  - f-Network : $< \boldsymbol{V, A, f} >$: **distribution of obligations**
  - fsp-Network : $< \boldsymbol{V, A, f, s, p} >$: **settlement procedure** (for $< \boldsymbol{V, A, f} >$)

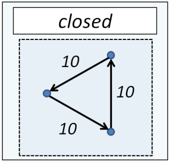
Example

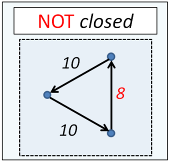
$V=\{v_a, v_b, v_c\}$
$A=\{(v_a, v_b),(v_b, v_c),(v_c, v_a)\}$
$f(a)=10, \ \forall a \in A$
$s((v_a, v_b))=1, s((v_b, v_c))=2, s((v_c, v_a))=3$
$p(v_a)=10, p(v_b)=p(v_c)=0$



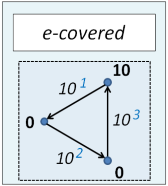
Basic Properties for Networks

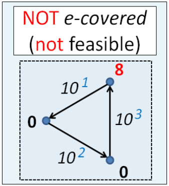
- closed $< V, A, f >$: balanced distribution of obligations
  - $\forall v \in V$, sum of payments indicated by f are balanced

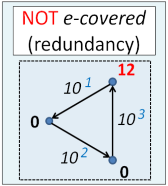




- e-("exact-")covered $< V, A, f, s, p >$: proper settlement procedure
  - (feasible) For payments of $< V, A, f >$, all the payments are executable at any point under the order indicated by s, and also
  - (no redundancy) there exists no other feasible $< V, A, f, s, p' >$ such that $p(v)' \leq p(v)$ for every $v \in V$, and $\exists v \in V, p'(v) < p(v)$







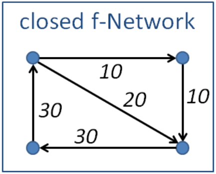
Formalization of our problem

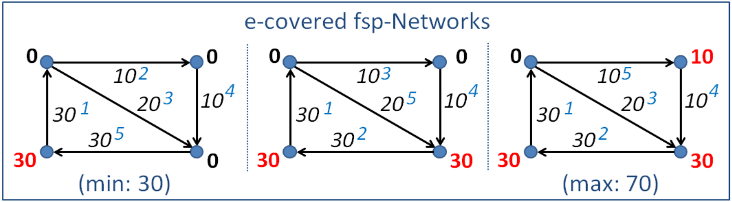
Given a closed f-Network (balanced distribution of obligations) $N^f = < V, A, f >$,

min $_{s,p} \sum_{v \in V} p(v)$

max $_{s,p} \sum_{v \in V} p(v)$

s.t., $< V, A, f, s, p >$ is e-covered (proper settlement procedure)





Approach: Closed Cycle Decomposition

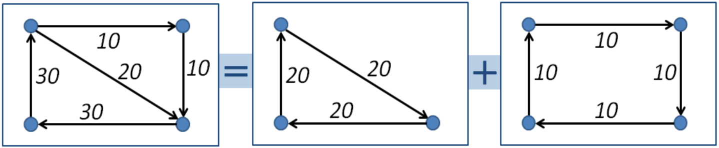
Theorem (Closed Cycle Decomposition (fulkerson, 1962))

For any closed f-Network $\mathbf{N^f} = < \mathbf{V}, \mathbf{A}, \mathbf{f} >$, *there always exists a closed cycle decomposition.*

$\mathbf{N^f} = \sum_{c \in C} < \mathbf{V^c}, \mathbf{c}, \mathbf{f^c} >$

$= \sum$*(1 cycle closed f-Network)*

- Example for closed cycle decomposition



Problem rewritten with Closed Cycle Decomposition

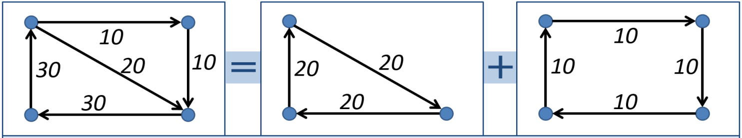
Given a closed f-Network N^f, with some closed cycle decomposition $N^f = \sum_{c \in C} < V^c, c, f^c > = \sum_{c \in C'} < V^c, c, f^c >$,

$$x^{min}(N^f) = \sum_{c \in C} f^c + R^{min}(N^f, C, \{f^c\}_{c \in C})$$

$$x^{max}(N^f) = \sum_{c \in C'} f^c * (|c| - 1) + R^{max}(N^f, C', \{f^c\}_{c \in C'})$$

(**cycle oriented amount**) + (**residual**)

Example for $R^{min}(N^f, C, \{f^c\}_{c \in C}) = R^{max}(N^f, C', \{f^c\}_{c \in C'}) = 0$

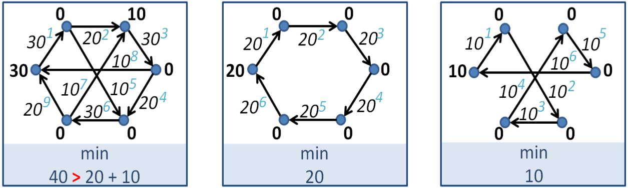


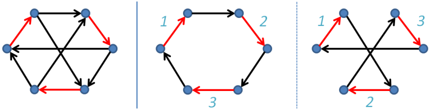
$x^{min}(N^f)$	=	20	+	10
$x^{max}(N^f)$	=	20 *(3-1)	+	10*(4-1)

Theorem (arrow-twisted)

Given a closed f-Network $\boldsymbol{N^f}$,
there exist *arrow-twisted* *cycles* $\Leftrightarrow \exists(C,\{f^c\}_{c\in C}),\ \boldsymbol{R^{min}(N^f, C, \{f^c\}_{c\in C}) > 0}$,

Example for $\boldsymbol{R^{min}(N^f, C, \{f^c\}_{c\in C}) > 0}$





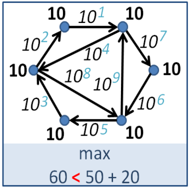
Economic interpretation: synchronization inconsistency regarding each payment

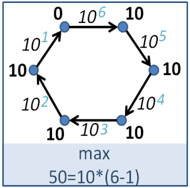
Major Network Factors : vertex-twisted

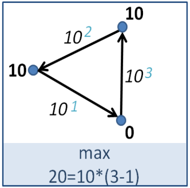
Theorem (vertex-twisted)

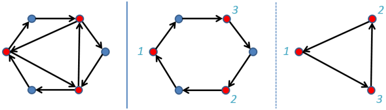
Given a closed f-Network $\boldsymbol{N^f}$,
there exist vertex-twisted cycles $\Leftrightarrow \exists(C, \{f^c\}_{c \in C}),\ \boldsymbol{R^{max}}(\boldsymbol{N^f}, C, \{f^c\}_{c \in C}) < 0$

Example for $\boldsymbol{R^{max}}(\boldsymbol{N^f}, C, \{f^c\}_{c \in C}) > 0$









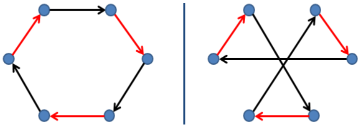
Economic interpretation: synchronization inconsistency regarding each subject

arrow-twisted and vertex-twisted

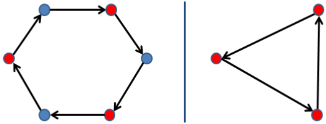
Theorem (arrow-twisted and vertex-twisted)

*Given a set of cycles **C**,*

$\boldsymbol{C}$ is arrow-twisted $\Rightarrow$ $\boldsymbol{C}$ is vertex-twisted



arrow-twisted, so vertex-twisted



vertex-twisted, but not arrow-twisted

Economic interpretation: scenario dependent feature of *arrow-twisted*

Summary

- Introduced a mathematical framework for "RTGS"
  - order of settlement
    - timing of payment in addition to balance-sheet linkage
- Provided **a general analysis** on which and how network structure matters for required amount of settlement fund
  - "arrow-twisted"
    - synchronization inconsistency regarding each payment
  - "vertex-twisted"
    - synchronization inconsistency regarding each subject
  - relation between "arrow-twisted" and "vertex-twisted"
    - scenario dependent feature