An Empirical Analysis of Network Reconstruction Methods using UK CDS Networks

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Outline

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- Reconstruction methods with partial data:
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Motivation

- The assessment of systemic risk in financial systems relies on the full knowledge of networks of bilateral exposures (Gai et al., 2011; Gai and Kapadia, 2010; Amini et al., 2012; Amini et al., 2012; Battiston et al., 2012).
- In practice complete informations on networks of bilateral exposures are rarely available to regulators.
- Most of the time regulators face the problem of reconstructing networks of bilateral exposures from partial data.

Empirical studies of financial networks

- Previous studies: Austrian interbank market (Boss et al., 2004), Italian money market (eMID) (De Masi et al., 2006; Iori et al, 2008), US interbank payment market (Soramaki et al., 2007) and federal funds market (Bech and Atalay, 2010), Brazilian interbank exposures (Cont et al., 2010), UK interbank exposures and interbank funding market (Langfield et al., 2014).
- All these networks present a heterogeneous and sparse structure.
- In particular, the distribution of connections is fat-tailed and scale-invariant—i.e, scale-free networks.

Reconstruction of networks with partial data

- In the literature many methods have been proposed to reconstruct networks of bilateral exposures from partial data.
- The first one is known as Maximum Entropy (Upper and Worms, 2004), and it has been proven that it leads to an underestimation of contagion (Mistrulli, 2007).
- New methods have been proposed to reproduce more realistic networks (Cont and Moussa, 2011; Musmeci et al., 2012; Mastromatteo et al., 2012; Baral, 2013; Halaj and Kok, 2013; Drehmann and Tarashev, 2013; Mastandrea et al., 2014; Anand et al., 2014).
- These methods vary greatly, some of them make use random graph theory whereas others rely on statistical methods. For a comparison of these methods for different types of financial networks see Anand et al., 2015.

Contribution

This work represents the first attempt to tackle the problem of reconstructing networks of bilateral exposures in the OTC derivative markets when the partial data are provided by the new trade reporting rules.

- Trade reporting rules have been introduced by G20 leaders in the aftermath of the crisis to improve the transparency of the OTC market (EMIR and Dodd-Frank act).
- OTC market participants are required to report transaction level data to Trade Repositories (TRs), which are accessible to appropriate regulators.

Overview

In particular, this work concerns the UK CDS market in what follows:

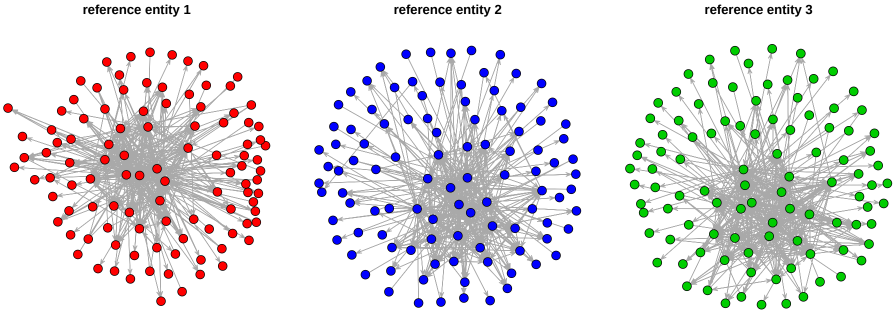
- An empirical study of networks of bilateral exposures of the UK CDS market.
- The application of the Cont and Moussa method for the first time to real data. In order to assess the accuracy of this method, its results are compared with the ones obtained using the Maximum Entropy and the real network itself.
- Propose a modification of the Cont and Moussa method to include new partial data available to regulators due to the introduction of trade reporting rules in OTC markets.

Data set

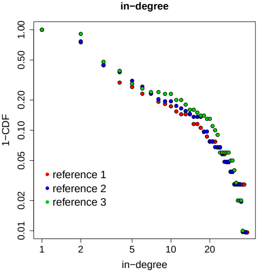
- We use CDS transaction data from 2007-2010 from DTCC's Trade Information Warehouse available to the Bank of England under ODRF and CPSS-IOSCO agreements.
- These transactions include all trades on single corporate name CDS contracts, in this time period, where the underlying assets was a UK supervised firm.
- From this data set it is possible to reconstruct 89 complete networks of gross notional exposures as of the 30th of June 2010, one for each of the available single name entities in the data set.

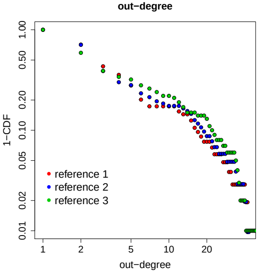
Empirical analysis of UK CDS networks

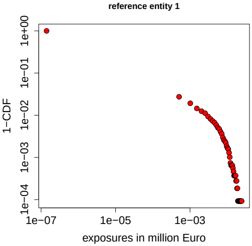
Example: 3 biggest networks in terms of total gross notional and network size

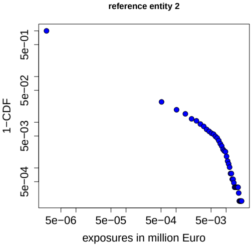


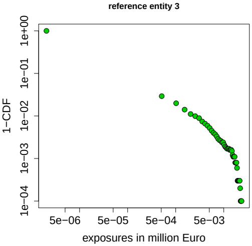
In- and out- degree and exposures











General findings

- The complementary cumulative distribution function $1 - \mathbb{P}(d_{in/out} \leq d)$ of the in- and out- degrees $d_{in/out}$ exhibits a power-law tail suggesting a sparse structure.
- The complementary cumulative distribution function $1 - \mathbb{P}(X \leq x)$ of the exposures X has a Pareto tail, suggesting that exposures are not evenly distributed across counterparties.
- These are the characteristics of weighted directed scale-free networks and confirm the results of previous studies.
- Moreover the highly interconnected component is composed by the dealers and the less interconnected one by their clients.

Network Reconstruction

Network Reconstruction:

reconstruction of the $n \times n$ matrix of bilateral exposures $\boldsymbol{x}$ from the knowledge of the total notional CDS sold and purchased by every institutions.

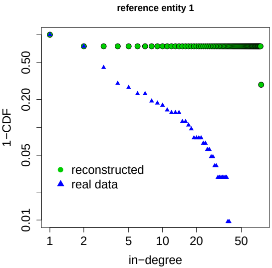
Maximum Entropy:

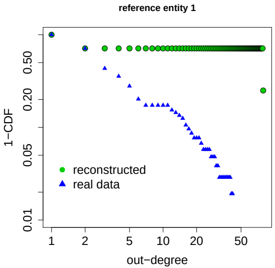
$$\inf_{\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n} \times \boldsymbol{n}}} H(\boldsymbol{x}) = \sum_{ij=1}^{n} x_{ij} \log \frac{x_{ij}}{(x_0)_{ij}}$$

s.t.

$$\sum_{j=1}^{n} x_{ij} = s_i^{out} \quad i = 1, \cdots, n$$

$$\sum_{j=1}^{n} x_{ji} = s_i^{in} \quad i = 1, \cdots, n.$$





Cont and Moussa method

- Generation of an ensemble of M weighted directed scale-free random networks $y^{(1)}, \ldots, y^{(M)}$ using the preferential attachment model of Bollobás et al. (2003) and assigning Pareto distributed weights to the edges.
- Since these networks do not achieve the in- and out-strengths constraints, they are weighted with probabilities $p_1, \ldots, p_M$ in such a way that the constraints posed by the available data are satisfied on the average across these probabilities.
- This method gives a distribution of random networks that satisfies on average the constraints posed by the available data.

The Cont and Moussa method is the following:

$$\inf_{\boldsymbol{p}\in\mathbb{R}^k} H(\boldsymbol{p}) = \sum_{k=1}^{M} p_k \log M p_k$$

s.t.

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = s_i^{out} \quad i = 1, \cdots, n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = s_i^{in} \quad i = 1, \cdots, n.$$

Trade reporting rules in OTC markets

The constraints posed by the new trade reporting rules are:

- knowledge of the total notional CDS sold and purchased by every financial institution (i.e., in- and out- strengths);
- and knowledge of all bilateral exposures of m financial institutions (i.e., knowledge of m rows and columns of the exposure matrix $\boldsymbol{x}$).

$$\overbrace{\left(\sum_{j=1}^{n} x_{1j} \quad \cdots \sum_{j=1}^{n} x_{mj} \quad \cdots \quad \sum_{j=1}^{n} x_{nj}\right)}^{s^{in}}$$

$$\boldsymbol{x} = \begin{pmatrix} x_{11} & \cdots & \cdots & \cdots & \cdots & \cdots & x_{1n} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ x_{m1} & \cdots & \cdots & \cdots & \cdots & \cdots & x_{mn} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \left.\begin{pmatrix} \sum_{j=1}^{n} x_{j1} \\ \vdots \\ \sum_{j=1}^{n} x_{m1} \\ \vdots \\ \sum_{j=1}^{n} x_{jn} \end{pmatrix}\right\} s^{out}$$

Cont and Moussa with TR rules

The Cont and Moussa with TR rules is the following:

$$\inf_{\boldsymbol{p}\in\mathbb{R}^{\boldsymbol{k}}} H(\boldsymbol{p}) = \sum_{k=1}^{M} p_k \log Mp_k$$

s.t.

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ij}^{(k)}\right] = s_i^{out} \quad i = 1, \cdots, n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = \sum_{k=1}^{M} p_k \left[\sum_{j=1}^{n} y_{ji}^{(k)}\right] = s_i^{in} \quad i = 1, \cdots, n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[y_{ij}^{(k)}\right] = \sum_{k=1}^{M} p_k y_{ij}^{(k)} = x_{ij} \quad \text{for } i = 1, ..., m \;\; j = 1, ...n$$

$$\mathbb{E}^{\boldsymbol{p}}\left[y_{ji}^{(k)}\right] = \sum_{k=1}^{M} p_k y_{ji}^{(k)} = x_{ji} \quad \text{for } i = 1, ..., m \;\; j = 1, ...n$$

Existence of a solution

The previous $2n + 2mn$ constraints can be rewritten as

$$\mathbb{E}^{\boldsymbol{p}}\left[C_i\right] = 0 \text{ and } \mathbb{E}^{\boldsymbol{p}}\left[D_i\right] = 0 \text{ for } i = 1, .., n-1$$
$$\mathbb{E}^{\boldsymbol{p}}\left[\rho_j^{(i)}\right] = 0 \text{ and } \mathbb{E}^{\boldsymbol{p}}\left[\gamma_j^{(i)}\right] = 0 \text{ for } i = 1, ..., m \;\; j = 1, ...n$$

where

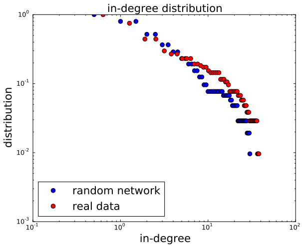
$$C_i(k) := \sum_{j=1}^{n} y_{ij}^{(k)} - s_i^{out} \text{ and } D_i(k) := \sum_{j=1}^{n} y_{ji}^{(k)} - s_i^{in} \text{ for } i = 1, .., n-1$$
$$\rho_j^{(i)}(k) := y_{ij}^{(k)} - \hat{y}_{ij} \text{ and } \gamma_j^{(i)}(k) := y_{ji}^{(k)} - \hat{y}_{ji} \text{ for } i = 1, ..., m \;\; j = 1, ...n.$$

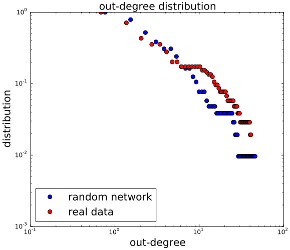
It is possible to prove that

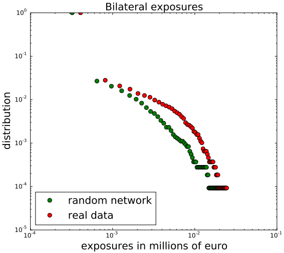
Proposition

If the $2n + 2mn$-dimensional null vector is in the convex envelop $\mathcal{E} = (C_1(k), \ldots, C_n(k), D_1(k), \ldots, D_n(k), \gamma_1^{(1)}(k), \ldots, \gamma_n^{(m)}(k), \rho_1^{(1)}(k), \ldots, \rho_n^{(m)}(k))_{k=1,\ldots,M}$ then the dual problem admits a unique solution, which is also the solution of the primal problem.

Example: weighted directed scale-free random network

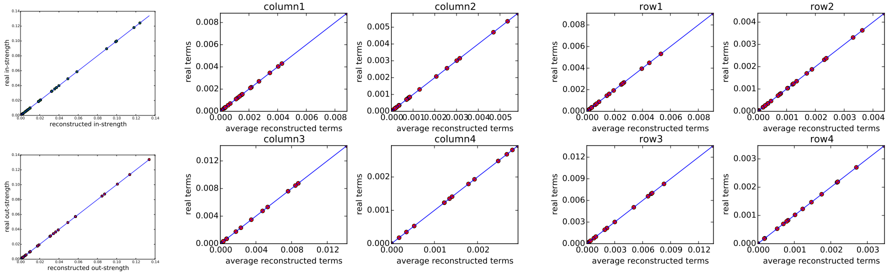






Results

Results when all bilateral exposures of 4 financial institutions are known



Conclusions

- The networks so reconstructed can be used to evaluate any systemic risk indicator ϕ as the average across the network distribution

$$\phi = \mathbb{E}^{\boldsymbol{p}}[\phi] = \sum_{k=1}^{M} p_k \phi(k).$$

- This is the first paper that aims to establish a good method for reconstructing networks of exposures in OTC markets in the presence of trade reporting rules.
- More complexity can be added to explore other cases of incomplete information on bilateral exposure accessible to regulators.
- Policy implications: further work can help to determine the improvements made by trade reporting rules in the ability of regulators to reconstruct networks of bilateral exposures, or to better design data sharing agreements.

Preferential attachment by Bollobás et al. (2003)

It is a model that explains the appearance of directed scale-free graphs.

- A directed scale-free graph is generated by adding at each discrete time step t either a new edge between two existing vertices or a new vertex with a new in- or out-edge.
- This happens through a preferential attachment process: a new institution entering the financial system is more likely to establish financial links with the heavily interconnected institutions.
- It can be proven that in the limit of infinite size the in- and out- degree distributions are power-laws.

Results of power-law fit

Results of the power-law fit for the 10 biggest networks of bilateral exposures in terms of total gross notional.

ref entity	n	α_{in}	x_{min}^{in}	p_{in}	α_{out}	x_{min}^{out}	p_{out}	av degree	α_{exp}	x_{min}^{exp}	p_{exp}
1	104	2.908	13.000	0.224	2.899	13.000	0.505	4.750	2.632	0.0039	0.050
2	103	3.751	16.000	0.296	2.755	13.000	0.520	5.048	2.925	0.0051	0.050
3	100	4.386	19.000	0.137	3.364	18.000	0.092	5.700	1.872	0.0007	0.155
4	83	3.838	18.000	0.246	2.833	13.000	0.017	6.566	2.248	0.0020	0.050
5	46	4.468	16.000	0.181	4.352	15.000	0.190	8.000	2.634	0.0038	0.050
6	79	2.472	9.000	0.094	2.692	9.000	0.290	5.493	2.316	0.0024	0.050
7	59	5.500	18.000	0.315	3.617	13.000	0.294	5.966	2.453	0.0038	0.050
8	73	3.917	15.000	0.008	5.500	21.000	0.788	5.986	2.259	0.0019	0.452
9	83	3.318	13.000	0.059	5.500	23.000	0.841	5.445	2.598	0.0030	0.050
10	57	5.500	16.000	0.801	5.014	16.000	0.779	6.736	2.082	0.0017	0.501