Measuring interconnectedness between financial institutions with Bayesian time-varying vector autoregressions

Financial Risk & Network Theory, Cambridge

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Motivation

The financial crisis highlighted the importance of **interconnectedness**

"A bank's **systemic** *impact is likely to be positively related to its* **interconnectedness** *vis-à-vis other financial institutions."*

- **Basel Committee on Banking Supervision (2013)**

Knowing how firms are interconnected can help identify potential channels of **contagion**

Problem:

1. We do not observe true connections given by the **network of direct and indirect spillovers** (Adrian and Brunnermeier, 2016)
   - Direct spillovers:
     - Contractual obligations, asset & liability exposures, derivatives
   - Indirect spillovers:
     - Common portfolio holdings, fire sales
2. Connections are **time varying**

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Goal

Develop a framework to estimate interconnectedness that can account for time-varying connections

Previous Studies

Market-based measures of interconnectedness use **stock price data** and measures of **statistical association**

- Contemporaneous dependencies: (e.g. correlation, tail dependence)
  - **Adams et al. (2014); Acharya et al. (2012, 2010); Adrian and Brunnermeier (2016); Brownlees and Engle (2016); Balla et al. (2014); Dungey et al. (2013); Hautsch et al. (2015); Peltonen et al. (2015)**
- Temporal dependencies: (e.g. Granger causality, vector autoregressions)
  - **Barigozzi and Brownlees (2016); Barigozzi and Hallin (2015); Billio et al. (2012); Diebold and Yılmaz (2009, 2014)**

We propose a framework to model both contemporaneous and temporal dependencies

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Previous Studies

Previous studies have used **time-invariant** measures of statistical association to infer interconnectedness

- e.g. **Billio et al. (2012)** use **Granger causality**

Granger causality is an in-sample test, based on T observations

- If the strength/direction of causality changes in $[0, T]$, the test inference is affected

Simple solution: adopt **rolling windows** but this is subject to **limitations**

- Reduces degrees of freedom ⇒ costly in high-dimensional systems
- Susceptible to outliers **(Zivot and Wang, 2006)**
- Window size ⇒ trade-off bias vs. precision **(Clark and McCracken, 2009)**

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Contribution

We propose a market-based framework for measuring interconnectedness

1. The framework accounts for the **time-varying** nature of connections
   - Does not rely on rolling windows
2. The framework models both contemporaneous and temporal dependencies
3. Our TVP-VAR model accounts for the properties of asset returns
   - heteroskedasticity, fat-tails and skewness of asset returns

Main findings

- Assess TVP framework in **simulation exercises** against the classical approach of **Granger causality** testing on **rolling windows** (GC+RW)
  - Our TVP framework performs well vs. GC+RW
    - In terms of the precision in estimating **connection strength**
    - In terms of determing the **presence/absence** of a connection
- Estimate interconnectedness for the US financial system between 1990-2014
  - At the aggregate level: between banks, broker-dealers, insurers, real estate companies
  - At the disaggregated level: between 20 systemically important financial institutions (SIFIs)

Main findings

Estimate interconnectedness for the US financial system between 1990-2014

1. Measures of connectivity and centrality computed using the TVP framework are less volatile than the rolling window approach
   - The rolling window approach is more sensitive to extreme observations
2. Banks were the largest contributors to financial spillovers
   - Whereas real estate companies were the most influenced
3. The time-varying parameter framework produces stable rankings
   - More stable than rankings produced by the rolling window approach
   - More stable than rankings produced by other market-based measures (e.g. Marginal expected shortfall (MES), Beta)
   - More reactive than book-value measures (e.g. Leverage)
4. Key financial institutions were identified
   - American International Group, Goldman Sachs, and Merrill Lynch among largest propagators
   - Bear Stearns among the largest receivers

Empirical methodology

Estimating networks by Classical Granger Causaility

We parallel measures of interconnectedness based on **Granger causality testing** **(Billio et al., 2012)**

Let $R_t = [r_{1,t}, \ldots, r_{N,t}]$ be a vector of returns

- Draw a **directional edge** $(i \rightarrow j)$ if r_i Granger causes r_j

Granger causality can be tested by running the VAR

$$R_t = c + \sum_{s=1}^{p} B_s R_{t-s} + u_t,$$

and testing

$$H_0 : b_1^{(j,i)} = b_2^{(j,i)} = \cdots = b_p^{(j,i)} = 0.$$

This is a **conditional** Granger causality test **(Geweke, 1984)**

Empirical methodology

We adopt the TVP-VAR framework **(Primiceri, 2005)**

$$R_t = c_t + \sum_{s=1}^{p} B_{s,t} R_{t-1} + u_t \equiv X_t' \theta_t + u_t, \quad u_t \sim t_\nu(0, \Xi_t),$$

where $X_t' = I_N \bigotimes [1, \ R_{t-1}', \ldots, \ R_{t-1}']$

$$\theta_{t+1} = \theta_t + v_{t+1}, \qquad v_t \sim \mathcal{N}(0, Q_t),$$

- The off-diagonal elements of Ξ_t capture the time-varying **contemporaneous** dependencies
- The elements of $B_{1,t}, \ldots, B_{p,t}$ capture the time-varying **temporal** dependencies

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$$\theta_{t+1} = \theta_t + v_{t+1}, \qquad v_t \sim \mathcal{N}(0, Q_t),$$

We assume **stochastic volatility** for the diagonal of Ξ_t

- We allow for a **leverage effects** between shocks to **stochastic volatility** and shocks to asset returns u_t
- This allows for skewness in the asset returns

Empirical methodology

Using **Bayes factor**, we evaluate the time-dependent hypothesis of no link between i and j at t

$$H_{0,t} : b_{1,t}^{(j\,i)} = b_{2,t}^{(j\,i)} \cdots = b_{p,t}^{(j\,i)} = 0.$$

We draw a **time-dependent** directional edge $(i \rightarrow_t j)$ if, given the posterior distribution of B_t, there is sufficient evidence against $H_{0,t}$

Empirical analysis

We collected stock prices at monthly close for 155 financial institutions

- banks, insurers, broker/dealers and real estate companies - SEC codes 6000 to 6799
- components of the S&P 500 between Jan 1990 and Dec 2014

We define monthly stock returns for firm i at month t as

$$r_{i\,t} = \log\left(\frac{p_{i\,t} + d_{i\,t}}{p_{i\,t-1}}\right),$$

We estimated the financial network at the **aggregate level** and at the **disaggregated level**

- Aggregate level: four-variable TVP-VAR with sector indices
- Disaggregated level: pairwise bi-variate TVP-VARs between stock returns of 20 SIFIs

Results at the aggregate level: the sectorial network

Network density

Aggregate level: four-variable TVP-VAR(1) with sector indices

- **Network density** is smoothly varying rather than abruptly changing

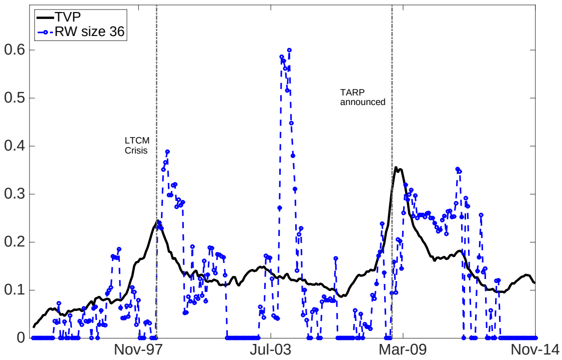
$$\text{Density}_t = \frac{1}{N(N-1)} \sum_{i=1}^{N_t} \sum_{j \neq i} (i \rightarrow_t j) \cdot | \, b_t^{(j\,i)} \, |,$$

with $i, j \in \{\text{Banks, Brokers, Insurers, Real Estate}\}$ and $i \neq j$, where $b_t^{(j\,i)}$ is the cross coefficient connecting i to j, in period t, in the TVP-VAR, and where, in this case, $N = 4$

Results at the aggregate level: the sectorial network

Network density

Sectorial Network Density



Bold solid = TVP; Blue = RW 36M

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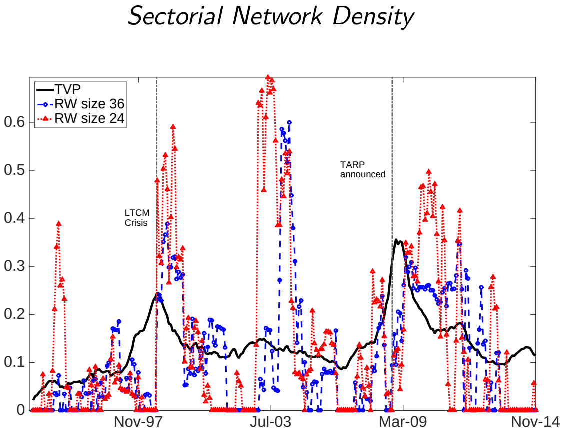

Bold solid = TVP; Blue = RW 36M; Red = RW 24M

Results at the Disaggregated level: the SIFI network

Degree centrality

Disaggregated level: pairwise bi-variate TVP-VARs between 20 SIFIs

- SIFIs selected from FSB and Diebold and Yılmaz (2014)
- We compute in-degree and out-degree measures

$$\text{In-Degree}_{i,t} = \frac{1}{(N_t - 1)} \sum_{j \neq i} (j \to_t i) \cdot \mid b_t^{(i\,j)} \mid,$$

$$\text{Out-Degree}_{i,t} = \frac{1}{(N_t - 1)} \sum_{j \neq i} (i \to_t j) \cdot \mid b_t^{(j\,i)} \mid,$$

where, in this case, $N_t \leq 20$

- We identified key players during the crisis
- We studied interconnectedness based rankings

Results at the Disaggregated level: the SIFI network

Ranking stability

We ranked firms according to their interconnectedness

- $Z_{i,t}^{in}$ is the ranking of institution i at time t in terms of **in-degree**
- $Z_{i,t}^{out}$ is the ranking of institution i at time t in terms of **out-degree**

The ranking can be used for monitoring and policy action

- e.g. the Financial Stability Board (FSB) and the BCBS ranks financial institutions according to their systemic importance
- The ranking is used to determine additional loss absorbency requirements

Results at the Disaggregated level: the SIFI network

Ranking stability

Rankings are unhelpful if they are prone to frequent unmotivated changes

- **Daníelsson et al. (2015)** and **Dungey et al. (2013)**

We computed a measures of **ranking stability**

$$SI_Q^{in} = \sqrt{\sum_{i=1}^{N_t} \frac{(Z_{i,t}^{in} - Z_{i,t-1}^{in})^2}{N_t(T-1)}}, \qquad SI_A^{in} = \sum_{i=1}^{N_t} \frac{\mid Z_{i,t}^{in} - Z_{i,t-1}^{in} \mid}{N_t(T-1)},$$

Results at the Disaggregated level: the SIFI network

Ranking stability

	Stability Indicators			
	quadratic		*absolute*	
	SI_Q^{in}	SI_Q^{out}	SI_A^{in}	SI_A^{out}
Rolling windows	2.4	2.5	1.6	1.7
Time-varying parameter	1.2	1.2	0.8	0.8

Average stability measures 1994-2014

- Rankings based on rolling windows were more unstable

Results at the Disaggregated level: the SIFI network

Ranking stability

Average stability measures across all t

	SI_Q		SI_A	
SRisk	1.3		0.8	
Marginal expected shortfall	3.1		2.3	
Leverage	0.8		0.5	
Market beta	3.1		2.3	
	SI_Q^{in}	SI_Q^{out}	SI_A^{in}	SI_A^{out}
Rolling windows	2.5	2.7	1.7	1.8
Time-varying parameter	1	1.1	0.6	0.7

Average stability measures 2000-2014

- Rankings based on TVP were more stable than MES and Beta (market data)
- Rankings based on TVP were less stable than Lev. (book value data)

Conclusion

Develop a market-based measure of interconnectedness

- Relies on **Bayesian estimation of time-varying parameter VARs**
  - Accounts for time-varying nature of connections
  - Models both temporal and contemporaneous dependencies
  - Accomodates many of the properties of asset returns (heteroskedasticity, skewness, heavy tails)
- Compared to classical rolling window approach
  - Less susceptible to extreme observations
  - Offers greater **flexibility**
  - Performs well in simulations
- Empirical analysis reveals limitations of rolling window approach
  - Rolling window connectivity and centrality measures are susceptible to outliers
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Thank you

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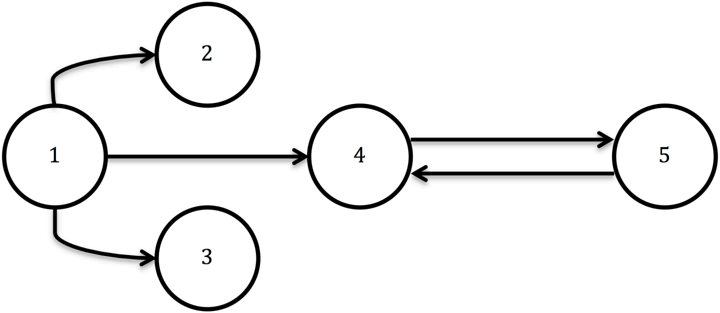
Appendix: Simulations



The Granger Causal Network (Seth, 2010)

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Appendix: Simulations

$$x_{1,t} = \alpha_{1,t} + \beta_{1,1,t} x_{1,t-1} + \epsilon_{1,t}$$
$$x_{2,t} = \alpha_{2,t} + \beta_{2,1,t} x_{1,t-1} + \beta_{2,2,t} x_{2,t-1} + \epsilon_{2,t}$$
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where, $[\epsilon_{1,t} \dots \epsilon_{5,t}]' = \epsilon_t \sim \mathcal{N}(\mathbf{0}, R)$ and $R = cI_5$ where c was set to 0.01

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Appendix: Experiment 1 - constant linkages

For the first experiment, we fix all regression parameters to constants drawn at the beginning of each simulation.

$$\alpha_{i,t} = a_i \qquad \forall t \in [0, T]$$
$$\beta_{i,j,t} = b_{i,j} \qquad \forall t \in [0, T]$$

where a_i and $b_{i,j}$ are drawn from a $\mathcal{U}(0,1)$ at the beginning of each simulation
$\forall (i,j) \in \{(2,1),(3,4),(3,5),(4,1),(4,5),(5,4)\} \cup \{i = j \mid i = 1,\ldots,5\}$

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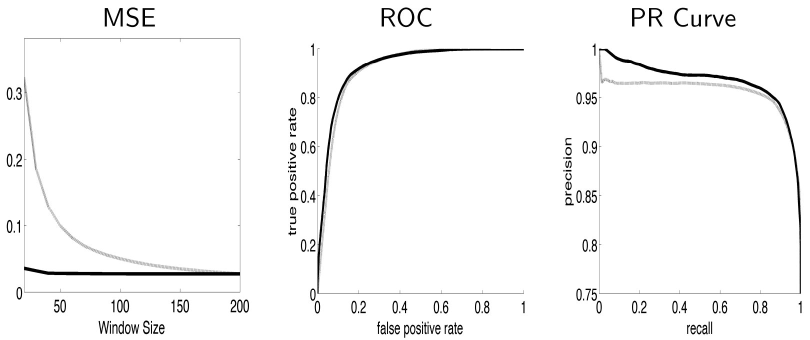
Appendix: Experiment 1 - constant linkages

Pairwise testing



Bold solid = TVP; light dashed = rolling windows

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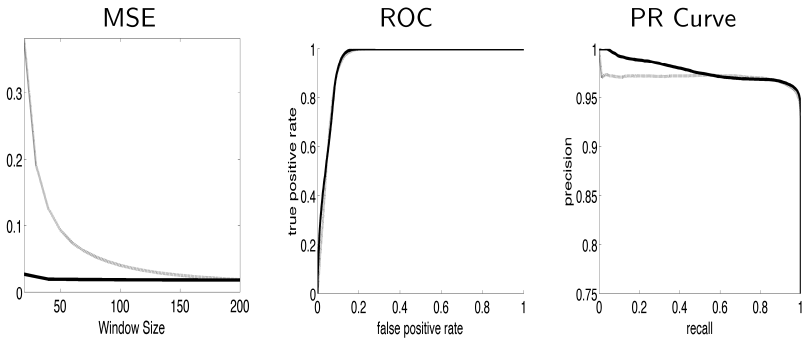
Appendix: Experiment 1 - constant linkages

Conditional testing



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Appendix: Experiment 2 - markov switching linkages

For only the cross terms $i, j \in \{(2,1), (3,4), (3,5), (4,1), (4,5), (5,4)\}$

$$\beta_{i,j,t} = \begin{cases} 0 & s_t^{i,j} = 0 \\ b_{i,j} & s_t^{i,j} = 1 \end{cases}$$

Let $s_t^{i,j}$ follow a first order Markov chain with the follwing transition matrix:

$$\mathbf{P} = \begin{bmatrix} \mathbb{P}(s_t^{i,j} = 0 \mid s_{t-1}^{i,j} = 0) & \mathbb{P}(s_t^{i,j} = 1 \mid s_{t-1}^{i,j} = 0) \\ \mathbb{P}(s_t^{i,j} = 0 \mid s_{t-1}^{i,j} = 1) & \mathbb{P}(s_t^{i,j} = 1 \mid s_{t-1}^{i,j} = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix}$$

where we set $p_{00} = 0.95$ and $p_{11} = 0.90$

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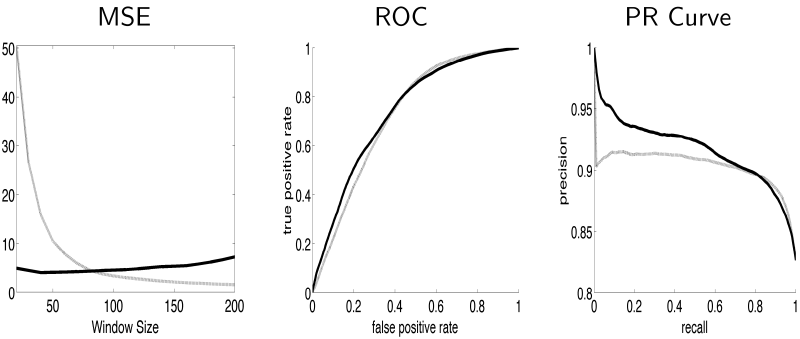
Pairwise testing



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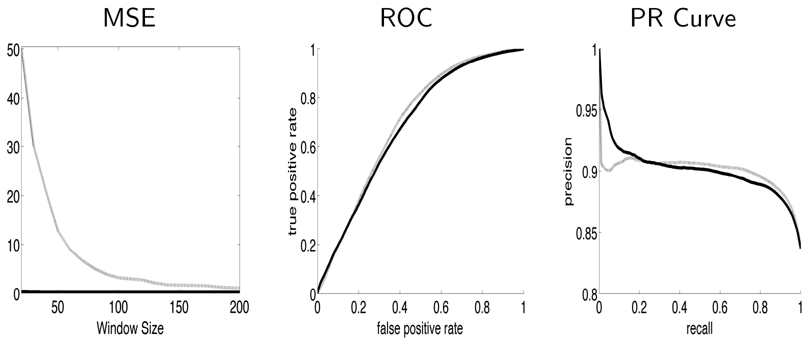

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Appendix: Experiment 3 - random walk law of motion

$$\alpha_{i,t+1} = \alpha_{i,t} + \omega_{i,t} \qquad \omega_{i,t} \sim \mathcal{N}(0, c^2)$$
$$\beta_{i,j,t+1} = \beta_{i,j,t+1} + \zeta_{i,j,t} \qquad \zeta_{i,j,t} \sim \mathcal{N}(0, \tau_{i,j}^2)$$

where,

$$\tau_{i,j}^2 = \begin{cases} 3 \times c^2 & \text{if } i \neq j \\ 2 \times c^2 & \text{if } i = j \end{cases}$$

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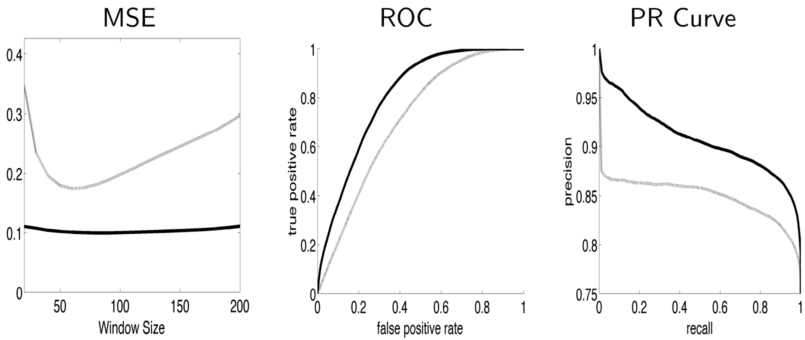
Appendix: Experiment 3 - random walk law of motion

Pairwise testing



Bold solid = TVP; light dashed = rolling windows

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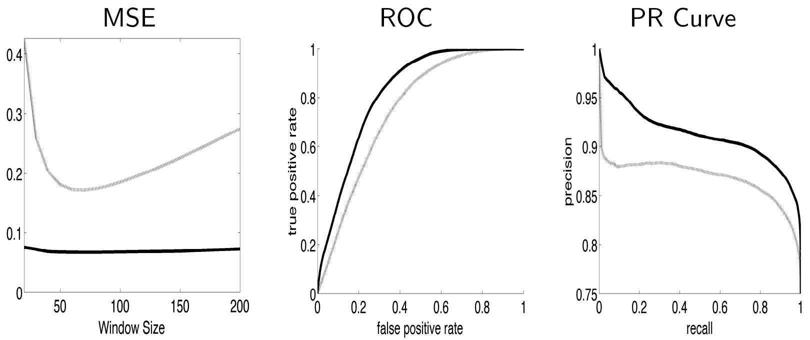
Appendix: Experiment 3 - random walk law of motion

Conditional testing



Bold solid = TVP; light dashed = rolling windows

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Assume the usual **lower triangular factorization** for the variance-covariance matrix,

$$\Xi_t = A_t H_t A_t'$$

and let,

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & \cdots & 0 \\ 0 & h_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_{n,t} \end{bmatrix}, A_t \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \alpha_{2\,1,t} & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \alpha_{n\,1,t} & \cdots & \alpha_{n\,n-1,t} & 1 \end{bmatrix}$$

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Then $h_t = [h_{1,t}, \ldots, h_{n,t}]'$ and $\alpha_t = [\alpha_{1\,2,t}, \ldots, \alpha_{n\,n-1,t}]'$ evolve according to

$$\ln h_t = \ln h_{t-1} + \eta_t$$
$$\alpha_t = \alpha_{t-1} + \tau_t$$

- This allows for stochastic volatility and time-varying contemporaneous dependencies in the shocks to returns

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The error term of the measurement equation is composed of two components,

$$u_t = \Sigma_t \sqrt{\lambda_t} z_t$$

where

- $\nu/\lambda_t \sim \chi^2_\nu$ and
- $z \sim N(0, I_n)$

It follows that,

$$u_t \sim t_\nu(0, \Sigma_t),$$

The errors $[\varepsilon_t, \eta_t, \omega_t, \tau_t]'$ are jointly normal with mean zero and variance-covariance matrix V

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$$V = \begin{bmatrix} I & \Omega & 0 & 0 \\ \Omega & Z_\eta & 0 & 0 \\ 0 & 0 & Z_\omega & 0 \\ 0 & 0 & 0 & S \end{bmatrix}$$

where,

$$\Omega = \begin{bmatrix} \rho_1\sigma_1 & 0 & \cdots & 0 \\ 0 & \rho_2\sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \rho_n\sigma_N \end{bmatrix}, Z_\eta = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_N \end{bmatrix}, \text{and}$$

$$Z_\omega = \begin{bmatrix} \sigma_{\omega,1} & 0 & \cdots & 0 \\ 0 & \sigma_{\omega,2} & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{\omega,N\cdot(1+N)} \end{bmatrix}$$

Ω allows ε_t and η_t, to be contemporaneously correlated row-by-row

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