Incentivizing Resilience in Financial Networks

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(joint with Stefan Thurner)

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Introduction

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  $\Rightarrow$ Failure to manage systemic risk (SR) can be extremely costly for society (e.g. financial crisis of 2007-2008)

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- This happens in financial (i.e. interbank) systems:
  ⇒ Failure to manage systemic risk (SR) can be extremely costly for society (e.g. financial crisis of 2007-2008)

- Regulations proposed fail to address the fact that SR is a network property (BASEL III. e.g. Tobin taxes, capital requirements)

Insolvency Cascades in Networks

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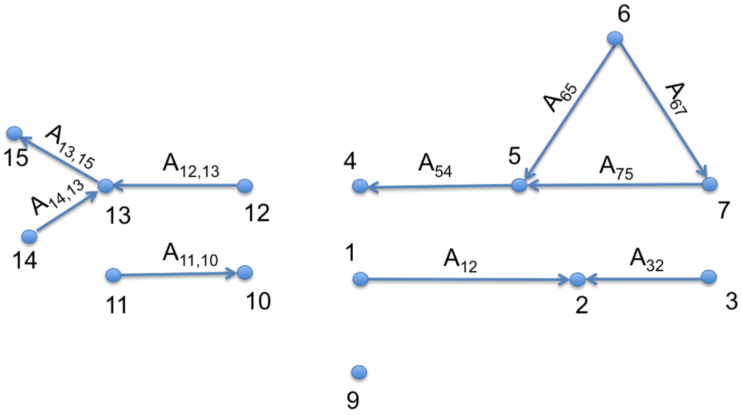

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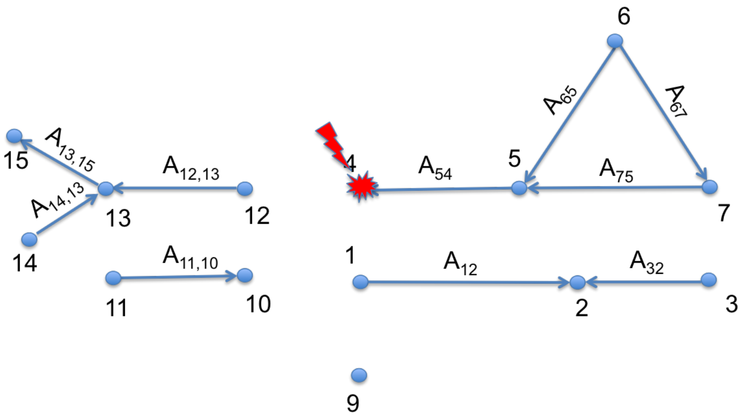

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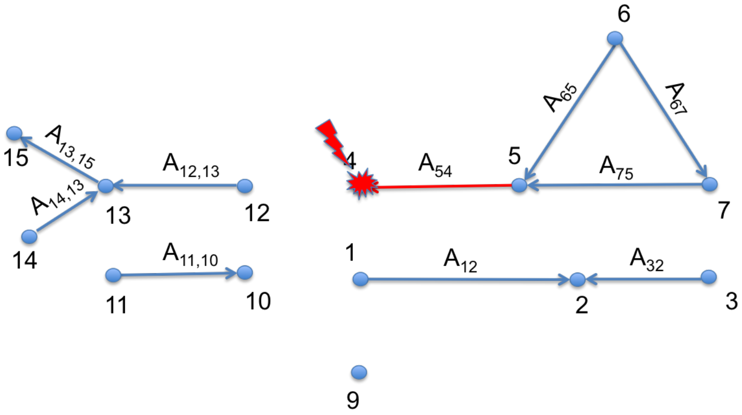

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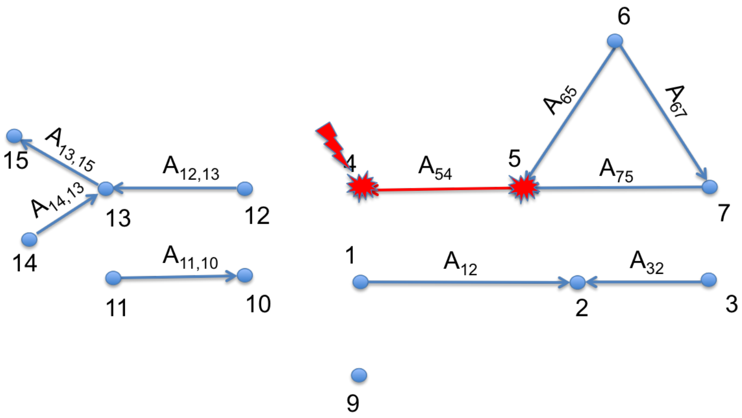

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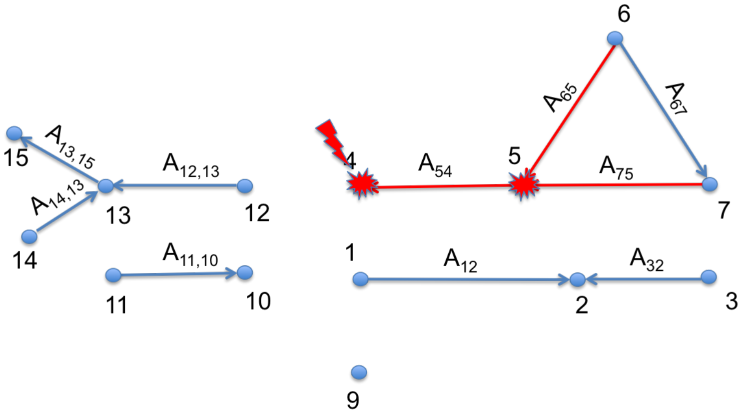

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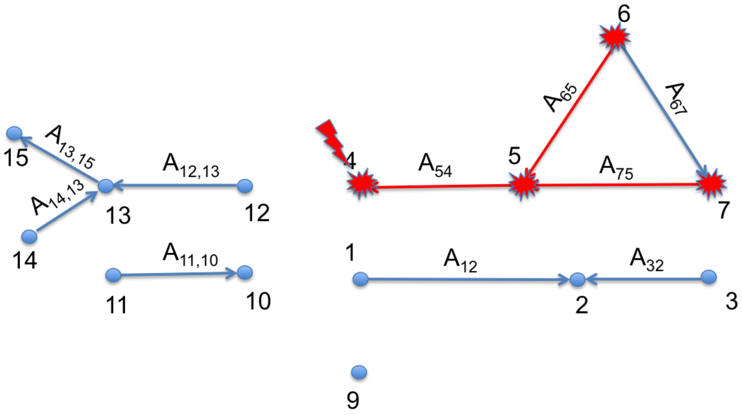

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Quantifying Systemic Risk

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- The systemic impact of bank i at time t:

$$SI^i(\bar{A}_t, \vec{E}_t) = \sum_{j \neq i} \mathbb{1}_{\{j \ fails \ | \ i \ fails\}} E_t^j.$$

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- An algorithm can compute $\mathbb{1}_{\{j \ fails \ | \ i \ fails\}}$

  $\rightarrow$ à la DebtRank (Battiston et al. (2012), Thurner and Poledna (2013))

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- Expected Systemic Loss:

$$ESL(\bar{A}_t, \vec{E}_t) = \sum_{j=1}^{n} \mathbb{P}\{j\ defaults\} \cdot SI^j(\bar{A}_t, \vec{E}_t)$$

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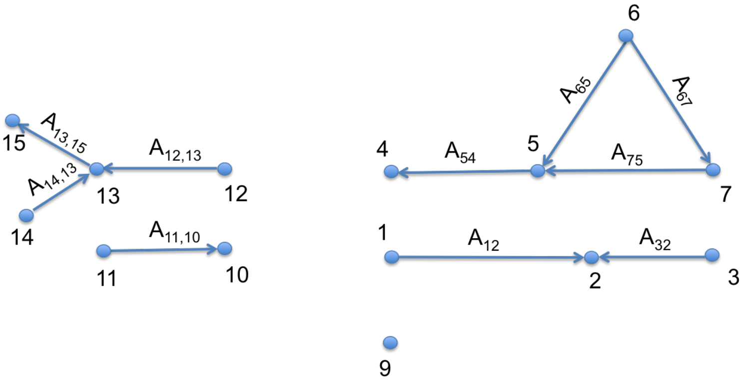
- Different topologies have different effects on size of insolvency cascades (e.g. Boss et al. (2004), Gai & Kapadia (2010), Amini et al. (2013), Poledna et al. (2015))
- Some work focuses on minimizing $ESL(\bar{A}_t, \vec{E}_t)$, given a certain topology (e.g. by injecting capital in a certain set of banks)
- Less work focuses on controlling the network topology (e.g. Poledna & Thurner (2016))

Controlling Formation of a Financial Network

- **Question**: How can we control formation of a financial network?

  Assume network is formed dynamically by interbank loans



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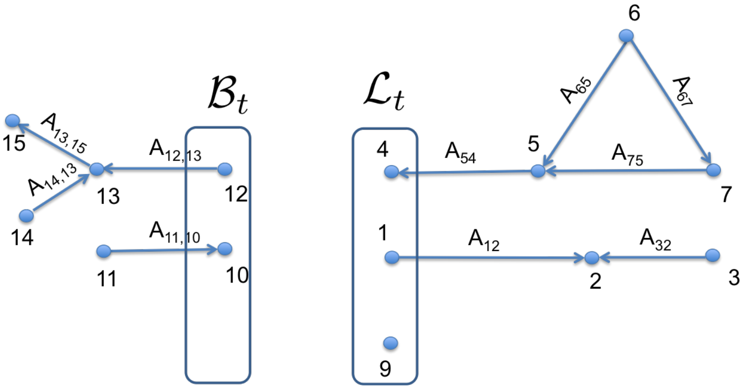

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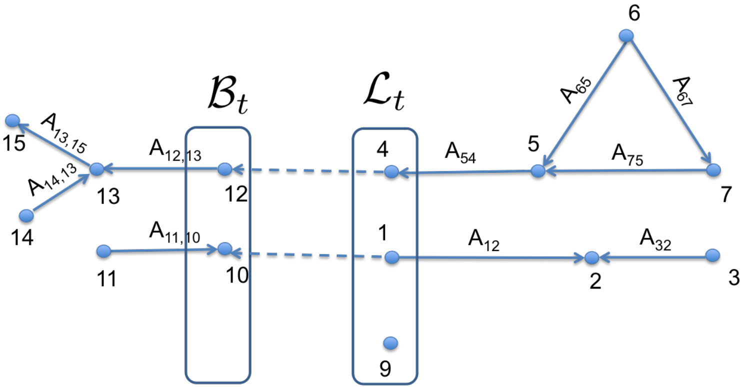

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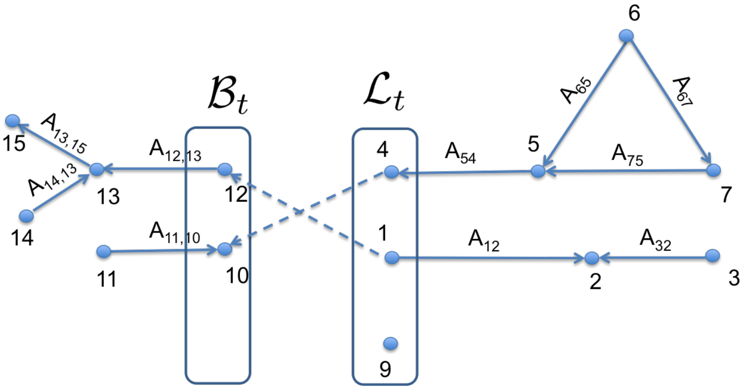

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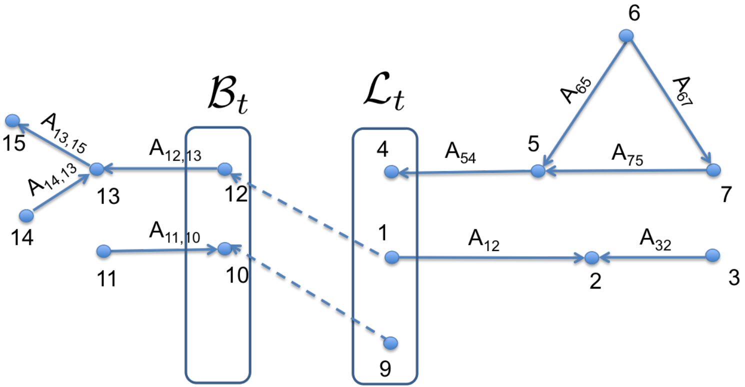

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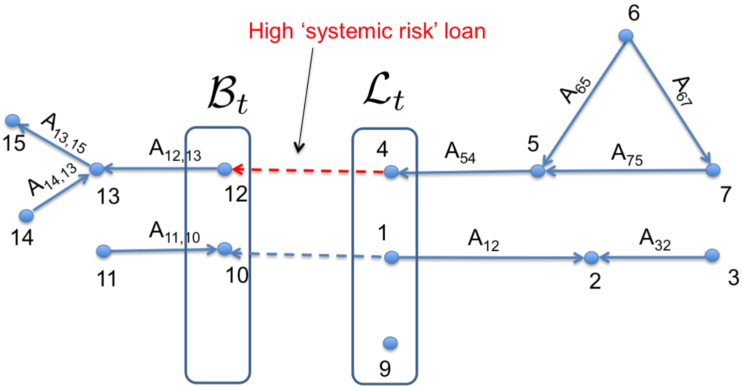

High 'systemic risk' loan: Bank 12 inherits the high systemic impact $SI^4(\bar{A}_t, \vec{E}_t)$ of bank 4

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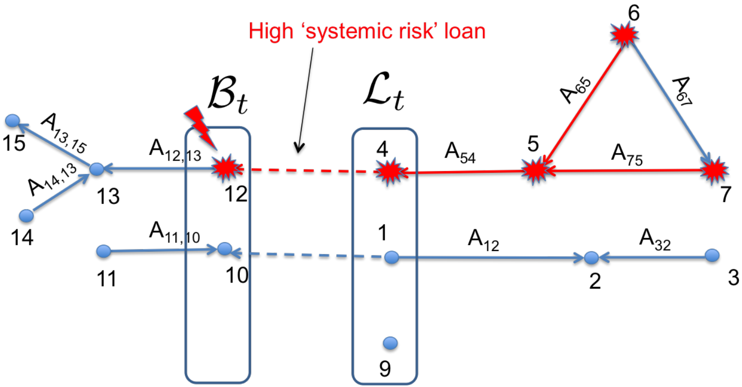

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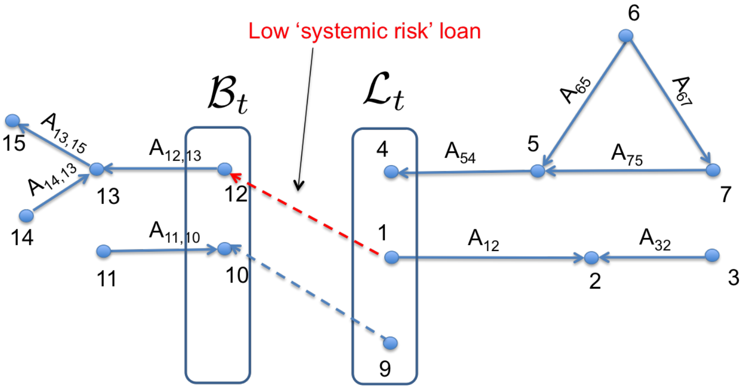

Low 'systemic risk' loan: Bank 12 does not inherit any systemic systemic impact of bank 1 (here $SI^1(\bar{A}_t, \vec{E}_t) = 0$)

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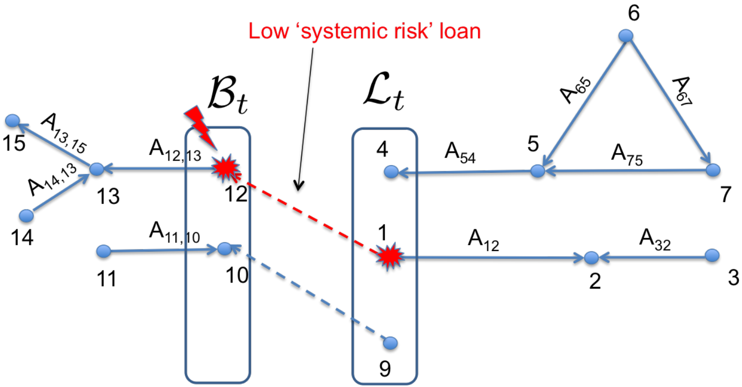

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Matching Markets

Matching markets: Designed to resolve a range of complex economic problems

- Example 1: Students to Schools (Roth, 1984, 1999)
- Example 2: Kidney donors to receivers (Roth et al., 2003)
- Example 3: Online matching platforms (e.g. Airbnb, Uber)

→ We need an equilibrium concept based on stable matchings (Gale & Shapley (1962))

Simple model of a credit system

At each discrete time $t \in \{0, 1, 2, ...\lfloor T \rfloor\}$, each bank $i \in \mathcal{N}$ receives a liquidity shock ϵ_t^i

$$\epsilon_t^i = \begin{cases} +1 & \text{with prob. } y/2 \quad \text{(bank } i \text{ in supply of liquidity)} \\ -1 & \text{with prob. } y/2 \quad \text{(bank } i \text{ in demand of liquidity)} \\ 0 & \text{with prob. } 1-y \end{cases}$$

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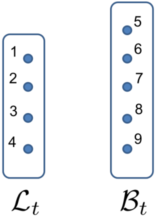
Induces a set of lenders and a set of borrowers:



Bilateral Contracts and Preference Lists

Simple bilateral contracts

- Each borrower j has an exogenous failure probability ρ_j and a reservation rate $\bar{r}_j$.
- Each lenders has an exogenous baseline lending rate r_i and adds a (fair) risk premium $h_i(\rho_j)$:

$$r_{ij} = r_i + h_i(\rho_j)$$

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- Any borrower j prefers borrowing from lenders with lower rates (up to a maximal rate)
  If $r_{1j} < r_{2j} < r_{3j} < \bar{r}_j < r_{4j}$
  $\rightarrow$ Preference list $P_\beta^j = 1, 2, 3.$

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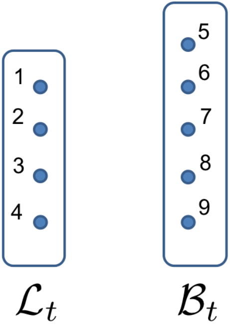
  $\rightarrow$ Preference list $P^j_\beta = 1, 2, 3.$
- Risk premium makes lenders indifferent as to who they lend to

Two-sided matching

Let $\mathbf{P} = \{P_\beta^5, P_\beta^6, P_\beta^7, ...\}$ be the set of all preference lists

We call the triplet $(\mathcal{L}_t, \mathcal{B}_t, \mathbf{P})$ a market for liquidity at time t.



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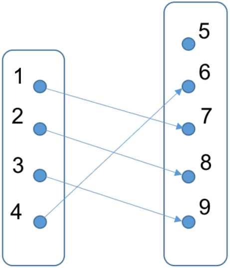
Outcome at each t, is a matching μ_t:



$\mathcal{L}_t$ $\mathcal{B}_t$

$\mu_t(1) = 7$, $\mu_t(2) = 8$, etc...

Equilibrium Concept: Stable Matching

Definition (Stable Matching)

A matching μ_t^* is stable if :

- (I) No set of borrowers $\vec{b} \in \mathcal{B}_t$ could agree to swap their counter-parties.
- (II) The lending rates are below the borrowers' reservation rates (i.e. $r_{ij} \leq \bar{r}_j$)

$\rightarrow$ **In words**: No bank could benefit from behaving differently

Systemic Risk Created by a Stable Matching

Lemma (Equilibrium Multiplicity under Bilateral Contracting)

Any matching μ_t such that the lending rates are below the borrowers' reservation rates (i.e. $r_{ij} < \bar{r}_j$) is stable.

- Many networks can emerge in equilibrium !
  $\rightarrow$ Results from borrowers having *homogenous* preferences (the all prefer the lender i with lowest baseline rate r_i)
- How to compare the different equilibrium matchings?
  $\rightarrow$ Need a notion of efficiency.

Systemic Risk-Efficient Equilibrium

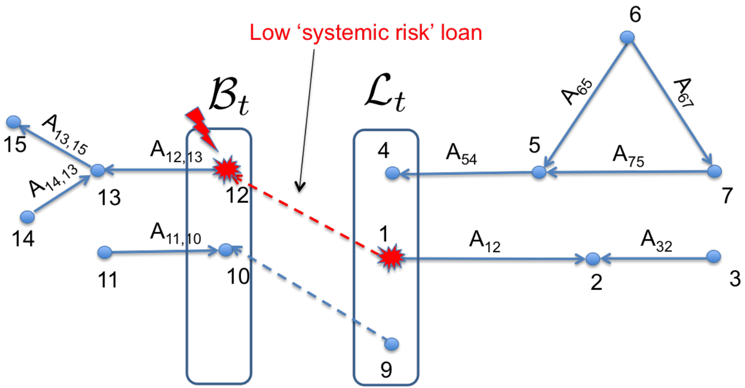
Definition (Systemic Risk-Efficient Equilibrium)

An equilibrium matching $\mu_t^{*,eff}$ is systemic risk-efficient if it minimizes systemic risk given a certain transaction volume v:

$$\mu_t^{*,eff} \in \underset{\mu_t^*: Vol(\mu_t^*)=\nu}{\text{argmin}} \; ESL(\bar{A}_t^*, \vec{E}_t).$$

Revisiting the toy example

Systemic Risk Efficient Equilibrium



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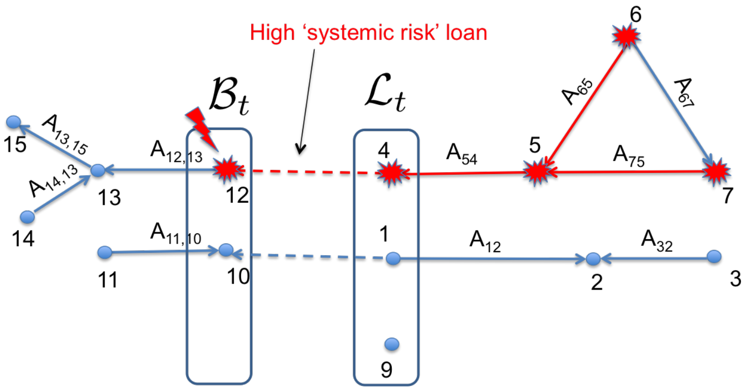
Systemic Risk *Inefficient* Equilibrium



Transaction-Specific Tax

Question: Can we select a systemic risk-efficient matching that is sustained as a *unique* equilibrium ?

Answer: Yes, by creating *heterogeneous* preferences by means of a transaction-specific tax.

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- $\mathcal{T} = \{\tau_{ij}\}$: a matrix of transaction-specific taxes, $i \in \mathcal{L}_t$ and $j \in \mathcal{B}_t$
- $\tau_{ij} \geq 0$ is a mark-up applied to the interest rate paid by bank j when it borrows from bank i:

$$r_{ij}^{\mathcal{T}} = r_i + h_i(\rho_j) + \tau_{ij}$$

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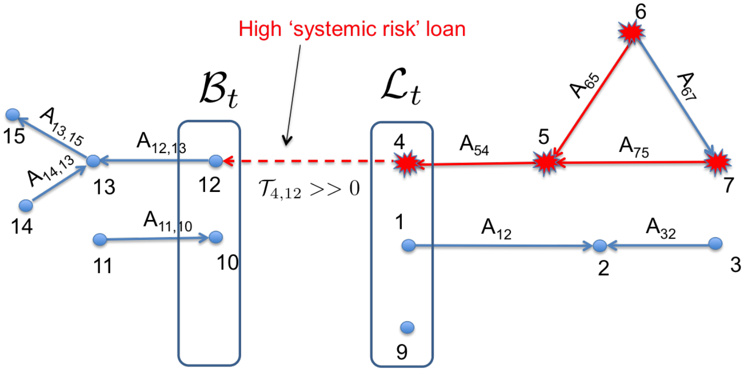
$\rightarrow$ Idea introduced in Poledna & Thurner (2016)

- Each borrower can now prefer a different lender

Equilibrium Selection and Uniqueness

Idea: leave desired matches untaxed and tax the undesired matches



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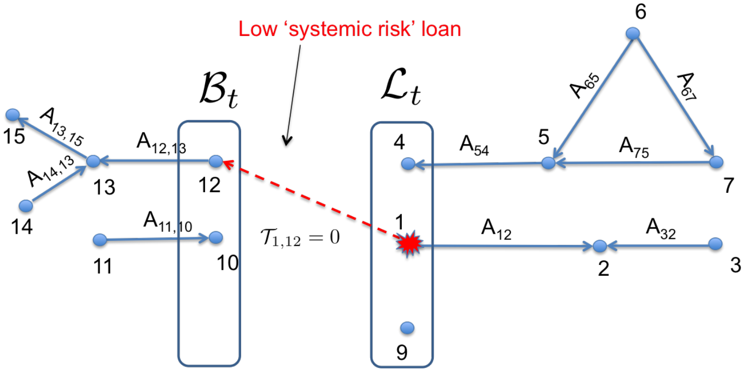

Systemic Risk under SRT

- $\mathcal{T}$ re-orders the preferences of each borrower over the set of lenders

  $\rightarrow$ allows a regulator to create *heterogeneous* preferences, i.e. each borrower j can now have a *different* preference list P^j_β with optimal match on top.

Proposition (Systemic Risk under Systemic Risk Tax)

For some desired volume ν, there exists $\mathcal{T}$ such that μ^*_t is unique and systemic risk efficient. We call this $\mathcal{T}$ a Systemic Risk Tax (SRT).

SRT versus Tobin-like tax

A Tobin-like tax is a particular case of the SRT $\mathcal{T}$

- Borrowing rate under SRT $\mathcal{T}$:

$$r_{ij}^{\mathcal{T}} = r_i + h_i(\rho_j) + \tau_{ij}$$

where $\tau_{ij} = 0$ for desired matches and $\tau_{ij} > 0$ for undesired ones

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$\rightarrow$ Makes all transactions more costly, without re-ordering the borrowers' preference lists.
$\rightarrow$ Cannot select a particular systemic risk efficient equilibrium.

SRT versus Tobin-like Tax

Proposition (Tobin-like tax versus Systemic Risk Tax)

There always exists $\mathcal{T}$ *such that* $ESL(\bar{A}_t^{*,\mathcal{T}}, \vec{E}_t) \leq ESL(\bar{A}_t^{*,\kappa}, \vec{E}_t)$ *and* $Vol(\mu_t^{*,\mathcal{T}}) \geq Vol(\mu_t^{*,\kappa})$

In words: SRT can achieve higher trading volume *and* lower systemic risk

Regulator's Optimization Problem

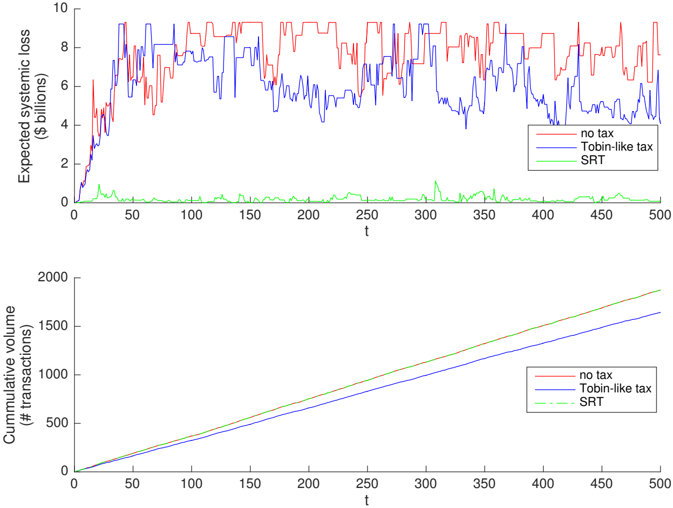
Solve this problem on a dynamically evolving complex network:

- Banks receive liquidity shocks and trade that liquidity in the form of interbank loans
- At each t, regulator solve following one-period-ahead optimization problem

$$\hat{\mathcal{T}} \in \underset{\mathcal{T}:Vol(\mu_t^{*,\mathcal{T}})=\nu}{\text{argmin}} \; ESL(\bar{A}_t^{*,\mathcal{T}}, \vec{E}_t)$$

$\rightarrow$ Optimize matching of lenders and borrowers, given a certain transaction volume

Regulator's Optimization Problem



Papers

- *Incentivizing Resilience in Financial Networks*. Leduc, M.V. and S. Thurner. (2016)

Companion papers:

- *Elimination of systemic risk in financial networks by means of a systemic risk transaction tax*. Poledna, S. and Thurner, S. (2016). Quantitative Finance,

- *Systemic risk management in financial networks with credit default swaps*. Leduc, M. V., Poledna, S., and Thurner, S. (2016).

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