Filtering for Risk Assessment of Interbank Network

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Background: Overnight Market

- By regulations, banks in the United States are required to maintain reserves either as cash or as fed funds.
- The day-to-day banking activities are unlikely to leave banks with the desired level of reserve.
- To meet shortfalls, regulated banks can trade fed funds in the interbank market.
- Other sources of overnight liquidity include Repos and discount window.

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- Other sources of overnight liquidity include Repos and discount window.

Overall, the overnight market serves as:

1. the most immediate source of liquidity
2. an important indicator of system functionality
3. a crucial role for implementation of monetary policy

Motivation: Interbank Functionality

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- Theory proposes two possible channels that lead to disruptions:

1. Liquidity Hoarding
   - Each bank's uncertainty about its own funding needs skyrocketed, Brunnermeier (2009)
2. Counterparty Risk
   - Asymmetric information during crisis, Heider et al (2015)

Motivation: Interconnectedness

- Different line of research looks at the interconnectedness of financial institutions and its impact on systemic risk.
- Acharya's (2009) Theory of Systemic Risk.
- Adrian and Brunnermeier (2011): policies that focus on bank's individual-level fail to respond to systemic risk
- Uncertainty about network structure and fire sales, Caballero and Simsek (2013)

Motivation: Network Architecture

- There is mixed evidence about the interconnectedness and network resilience
- Allen and Gale (2000) interconnectedness is beneficial from risk-diversification
- However, there are limits for diversification benefits,
  - Allen et al (2012), Battiston et al (2012)

Research Question

- Can we reverse engineer signals from the interbank market and identify its network structure?
- If yes, how can we assess its resilience to mitigate liquidity shocks?
- Moreover, what role does the network structure (especially interconnectedness) play in the interbank market functionality?

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- The interbank market allows to answer the above mainly due to
  1. the overnight transactions
  2. the pairwise lender-to-borrower relationship

Contribution

- We propose a framework that
  1. Deduces and evaluates information from the interbank market
  2. Identifies the interbank network structure along with its interconnectedness
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  3. Serves as early warning system to detect distress

- Using simulation studies, we find that interconnectedness is risk mitigating to some degree, beyond which systemic risk increases exponentially.
  - Contribute to the debate e.g. Stiglitz (2010)

Model: Assumptions

- There are N banks interacting during T periods via overnight borrowing and lending.
- Interest rate is zero and banks hold zero cash reserve.
- Net cash flows, $x_{i,t} = x_{i,t}^{A} - x_{i,t}^{D}$, is the only source of stochasticity, $\forall i = 1, .., N$ and $t = 0, 1, ..., T$.

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- Banks with positive net cash flows always lend, if there is demand.
- If no lending takes place, positive net cash flows are invested in assets.
  - This, hence, induces non-stationarity in net cash flows over time.

Model: Assumptions II

- For all $i = 1, ..., N$

$$x_{i,t} \sim N\left(\mu_{i,t}, \sigma_{i,t}\right), \tag{1}$$

- For all i, j and t

$$\rho_{ij,t} = \rho_{ij} \tag{2}$$

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- There are two states: $\mu_{i,t} \in \left\{\mu_i^{(1)}, \mu_i^{(2)}\right\}$ and $\sigma_{i,t} \in \left\{\sigma_i^{(1)}, \sigma_i^{(2)}\right\}$, such that

$$\frac{\mu_i^{(1)}}{\sigma_i^{(1)}} > \frac{\mu_i^{(2)}}{\sigma_i^{(2)}} \tag{3}$$

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- The transition between the two states follows a Markov chain process of order 1,

$t \setminus t+1$	1	2
1	p_i	$1 - p_i$
2	$1 - q_i$	q_i

Model: Dynamics

- At $t = 0$, if $x_{i,0} < 0$, bank i borrows $y_{.i,0} = -x_{i,0}$ in the interbank market.
- At $t = 1$, bank i repays $y_{.i,0}$, such that its net cash flows become $x_{i,1} - y_{.i,0}$.
- If $x_{i,1} - y_{.i,0} < 0$, then i borrows again at $t = 1$:

$$y_{.i,1} = -(x_{i,1} - y_{.i,0}) \tag{4}$$

- Otherwise, $y_{.i,1} = 0$

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- Otherwise, $y_{.i,1} = 0$
- To generalize, we have

$$y_{.i,t} = \begin{cases} -\left[x_{i,t} - (y_{.i,t-1} - y_{i.,t-1})\right] & B_{i,t} = 1 \\ 0 & B_{i,t} = 0 \end{cases}, \tag{5}$$

where $B_{i,t}$ returns 1 if bank i borrows at time t and 0 otherwise.

Filtering System: Estimation I

- There are two main challenges: truncation and non-stationarity
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- For a given state s, the conditional likelihood for bank i at time t is

$$\ell_{i,t}^{(s)} = \begin{cases} \phi\left(\left[y_{\cdot i,t} - (c_{i,t-1} - \mu_i^{(s)})\right]/\sigma_i^{(s)}\right)(1/\sigma_i^{(s)}) & B_{i,t} = 1 \\ 1 - \Phi\left([c_{i,t-1} - \mu_i^{(s)}]/\sigma_i^{(s)}\right) & B_{i,t} = 0 \end{cases} . \quad (6)$$

where $c_{i,t-1} = y_{\cdot i,t-1} - y_{i\cdot,t-1}$.

Filtering System: Estimation II

- Given two states, i.e. $s \in \{1, 2\}$, the conditional likelihood at time t for bank i is

$$\begin{aligned}\ell_{i,t} = \\ &= \xi_{i,t-1}^{(1)} \left[p_i \cdot \ell_{i,t}^{(1)} + (1 - p_i) \cdot \ell_{i,t}^{(2)}\right] \\ &+ \xi_{i,t-1}^{(2)} \left[(1 - q_i) \cdot \ell_{i,t}^{(1)} + q_i \cdot \ell_{i,t}^{(2)}\right]\end{aligned} \tag{7}$$

Filtering System: Estimation II

- Given two states, i.e. $s \in \{1, 2\}$, the conditional likelihood at time t for bank i is

$$\begin{aligned} \ell_{i,t} = & \\ & = \xi^{(1)}_{i,t-1}\left[p_i \cdot \ell^{(1)}_{i,t} + (1 - p_i) \cdot \ell^{(2)}_{i,t}\right] \\ & + \xi^{(2)}_{i,t-1}\left[(1 - q_i) \cdot \ell^{(1)}_{i,t} + q_i \cdot \ell^{(2)}_{i,t}\right] \end{aligned} \tag{7}$$

- Given information up till time t, the probability of i being in either state is

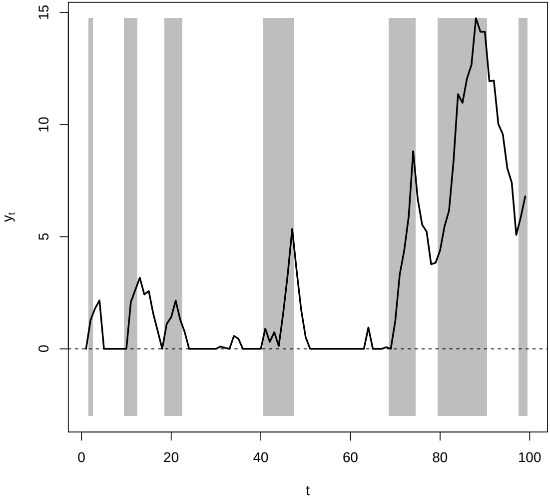
$$\xi^{(1)}_{i,t} = \frac{\left[p_i \cdot \xi^{(1)}_{i,t-1} + (1 - q_i) \cdot \xi^{(2)}_{i,t-1}\right] \ell^{(1)}_{i,t}}{\ell_{i,t}}, \tag{8}$$

and

$$\xi^{(2)}_{i,t} = 1 - \xi^{(1)}_{i,t}. \tag{9}$$

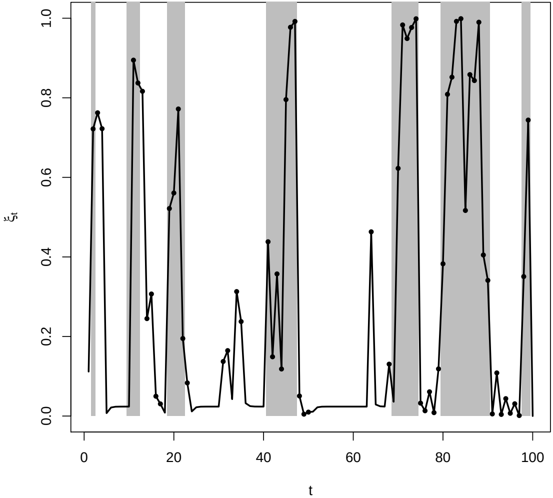
Filtering System: Illustration I

Figure: Total Borrowings by Bank *i* over Time



Filtering System: Illustration II

Figure: State 2 Filter for Bank *i* over Time



Filtering System: Interconnectedness I

- We proxy interconnectedness between two banks by the correlation coefficient between their net cash flows, i.e. ρ_{ij}.
- To do so, we derive the conditional joint likelihood between banks i and j, which is given by:

$$\ell_{ij,t} = \left[P_i \otimes P_j \vec{\ell}_{ij,t} \right]^T \vec{\xi}_{ij,t-1}, \tag{10}$$

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- where

$$\vec{\ell}_{ij,t} = \left[\begin{array}{cccc} \ell_{ij,t}^{(11)} & \ell_{ij,t}^{(12)} & \ell_{ij,t}^{(21)} & \ell_{ij,t}^{(22)} \end{array} \right]^T \tag{11}$$

and

$$\ell_{ij,t}^{(s_1 s_2)} = \begin{cases} f(y_{.i,t}, y_{.j,t} \mid \Omega_{t-1}) & B_{i,t} = 1, B_{j,t} = 1 \\ f(y_{.i,t}, y_{.j,t} = 0 \mid \Omega_{t-1}) & B_{i,t} = 1, B_{j,t} = 0 \\ f(y_{.i,t} = 0, y_{.j,t} \mid \Omega_{t-1}) & B_{i,t} = 0, B_{j,t} = 1 \\ f(y_{.i,t} = 0, y_{.j,t} = 0 \mid \Omega_{t-1}) & B_{i,t} = 0, B_{j,t} = 0 \end{cases} . \tag{12}$$

Filtering System: Interconnectedness II

- Let Θ_i denote bank's i specific parameters.
- What determines the interbank market interconnectedness is the correlation matrix among all agents' net cash flows.
- Specifically, let **R** denote the correlation matrix of the network.
- The ith row and jth column element of **R** denotes the correlation in net cash flows between bank i and j, ρ_{ij} for all $i \neq j$.

Filtering System: Interconnectedness II

- Let Θ_i denote bank's i specific parameters.
- What determines the interbank market interconnectedness is the correlation matrix among all agents' net cash flows.
- Specifically, let $\mathbf{R}$ denote the correlation matrix of the network.
- The ith row and jth column element of $\mathbf{R}$ denotes the correlation in net cash flows between bank i and j, ρ_{ij} for all $i \neq j$.
- Hence, given real transaction, the proposed filtering system identifies the interbank network by estimating $\mathbf{R}$ and $\Theta_i, \forall i = 1, ..., N$.

Simulation Study: Structure

- We look at $N = 20$ banks that interact over $T = 30$ periods.
- For each bank i we identify Θ_i.
- We impose that banks fall into two clusters:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_{12} \\ \mathbf{R}'_{12} & \mathbf{R}_2 \end{bmatrix}, \tag{13}$$

  - The **within** correlation in R_1 and R_2 is uniform and equals to ρ_1 and ρ_2, respectively.
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Special Cases

1. **Quasi-Autarky**: $\rho_1 = \rho_2 = 0.5$ while $\rho_{12} = 0$
2. **Full Integration**: $\rho_1 = \rho_2 = \rho_{12} = 0.25$.

Simulation Study: Risk Assessment I

- Let $\mathcal{S}$ denote the network's structure.
- For each structure $\mathcal{S}$, we simulate the network $\mathcal{R} = 1000$ times.
- We assess systemic risk using the following two metrics:

$$\bar{F}_{\mathcal{S}} = \frac{\sum_{r=1}^{\mathcal{R}} F_{\mathcal{S}}^{r}}{\mathcal{R}} \tag{14}$$

and

$$Q_{\mathcal{S},0.99} = \inf \left\{ Q \mid \frac{1}{\mathcal{R}} \sum_{r=1}^{\mathcal{R}} I \left\{ F_{\mathcal{S}}^{r} \leq Q \right\} = 99\% \right\}, \tag{15}$$

Where $F_{\mathcal{S}}^{r}$ is the number of failed banks in the network in run r and network structure $\mathcal{S}$.

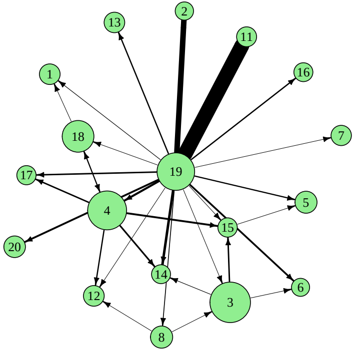
Simulation Study: Risk Assessment II

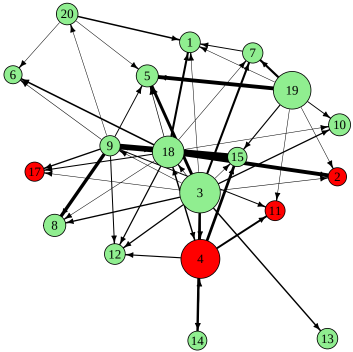
- We define a shock of magnitude $k > 1$ on bank i at time t in the following manner:

$$x_{i,t} \to x_{i,t} - k \cdot |x_{i,t}|. \tag{16}$$

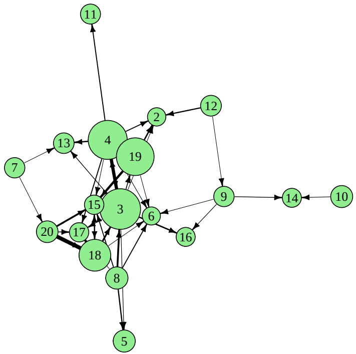
- Specifically, we set $k = 10$ and $t = 10$, while looking at the largest four banks in the network.

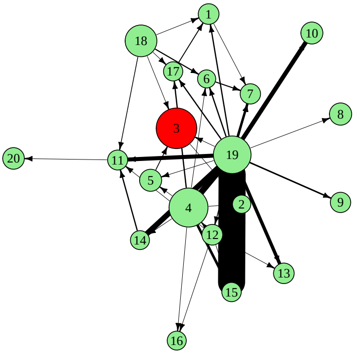
Simulation Study: Quasi-Autarky





Simulation Study: Full Integration

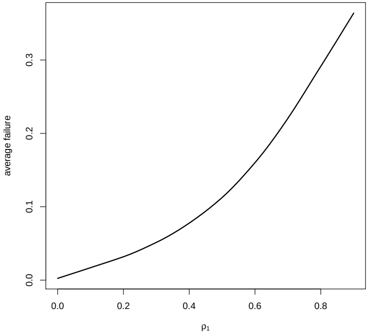


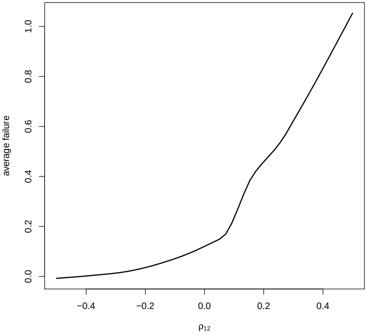


Simulation Study: Quasi-Autarky versus Full Integration

	$k = 0$	1 bank when $k = 10$	2 banks when $k = 10$	4 banks when $k = 10$
		Panel (a)	Quasi Autarky	
$\bar{F}_{\mathcal{S}}$	0.18	0.64	1.47	3.47
$Q_{\mathcal{S},0.99}$	4.01	5.00	5.00	7.00
		Panel (b)	Full Integration	
$\bar{F}_{\mathcal{S}}$	0.13	0.56	1.53	3.52
$Q_{\mathcal{S},0.99}$	4.00	4.00	8.00	8.00

Simulation Study: Sensitivity to Correlation I





Simulation Study: Sensitivity to Correlation II

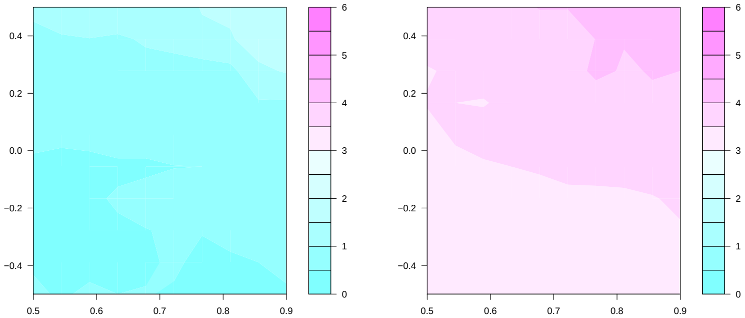
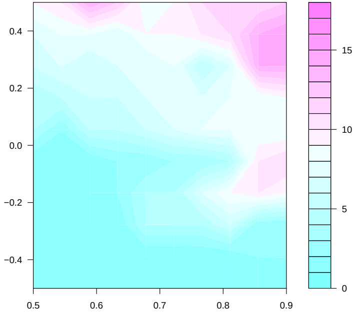
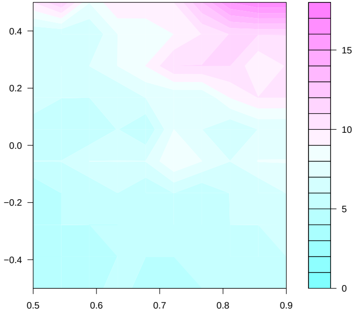


Figure: Average Failure

Simulation Study: Sensitivity to Correlation III

Figure: 99^{th} Percentile Failure





Simulation Study: Regression I

	Dependent variable: average failure							
	no shock		1 bank with $k = 10$		2 banks with $k = 10$		4 banks with $k = 10$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\rho_1 \cdot \rho_{12}$	0.876^{***}	0.876^{***}	1.293^{***}	1.293^{***}	1.154^{***}	1.154^{***}	1.126^{***}	1.126^{***}
	(0.064)	(0.045)	(0.091)	(0.067)	(0.069)	(0.059)	(0.061)	(0.053)
$(\rho_1 \cdot \rho_{12})^2$		2.001^{***}		2.698^{***}		1.629^{***}		1.345^{***}
		(0.198)		(0.293)		(0.260)		(0.233)
Constant	0.204^{***}	0.101^{***}	0.773^{***}	0.634^{***}	1.611^{***}	1.527^{***}	3.574^{***}	3.505^{***}
	(0.015)	(0.014)	(0.021)	(0.021)	(0.016)	(0.019)	(0.014)	(0.017)
Observations	100	100	100	100	100	100	100	100
Adjusted R^2	0.652	0.829	0.672	0.823	0.735	0.810	0.775	0.775

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Simulation Study: Regression II

	Dependent variable: 99^{th} percentile							
	no shock		1 bank with $k = 10$		2 banks with $k = 10$		4 banks with $k = 10$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\rho_1 \cdot \rho_{12}$	9.997***	9.997***	14.047***	14.047***	7.735***	7.735***	9.350***	9.350***
	(0.725)	(0.666)	(1.045)	(1.028)	(0.667)	(0.670)	(0.754)	(0.607)
$(\rho_1 \cdot \rho_{12})^2$		12.888***		9.355**		1.620		19.714***
		(2.935)		(4.528)		(2.951)		(2.673)
Constant	3.132***	2.468***	5.498***	5.015***	5.226***	5.142***	7.108***	6.091***
	(0.165)	(0.214)	(0.237)	(0.330)	(0.152)	(0.215)	(0.171)	(0.195)
Observations	100	100	100	100	100	100	100	100
Adjusted R^2	0.656	0.710	0.645	0.656	0.574	0.571	0.607	0.745
Note:						$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$		

Summary

- We identify the interbank network using a statistical learning procedure that reverse engineers transactions in the overnight interbank market.
- Given the structure of the network, we conduct a series of simulation studies for risk assessment.
- Integration appears to be optimal when systemic risk is absent.
- However, this evidence is undermined when systemically important institutions suffer liquidity shocks.

Thanks!

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