Order Book Modeling and Financial Stability

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Outline of presentation

- I- Intro and motivation
- II- Model description
- a) Design
- b) Stylized facts compliance
- III- Policy testing

Introduction Model Operation

I - Intro and Motivation

The majority of financial markets adopts an order book mechanism for orders matching (Rosu 2009). Two convergent perspectives:

- A- The macroeconomic framework of financial markets: bubbles and crises are often described as emergent aggregate behaviors resulting from individual independent actions, unintendently leading to synchronization. It might give support to:
 - a) understanding authentic ingredients of market dynamics,
 - b) explaining price variability,
 - c) addressing policies that can dampen dangerous fluctuations.

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 - a) understanding authentic ingredients of market dynamics,
 - b) explaining price variability,
 - c) addressing policies that can dampen dangerous fluctuations.
- B- The microstructure of order book dynamics:

It allows the analysis of mechanisms determining stock prices from an "inner" point of view: trading strategies, behavioral heterogeneity, composition of the population of investors, stylized facts.

It might give support to:

- a) trading decisions, specially referring to optimal investment strategies,
- b) market and price impact evaluations,
- c) design computer-based trading algorithms.

In previous studies, some of the macroeconomic sources of financial avalanches has been discussed (see AEB, Pluchino, Rapisarda, Helbing, 2013a, 2013b; AEB, Pluchino, Rapisarda, 2013, 2014, 2015, 2016).

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Existing Models Model description

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The main motivation of this research is, then, to highlight the most evident microstructural causes of markets instability, in order to discuss actual chances to control them by means of policies.

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Many of them have inspired parts of the proposed model.

Chakravarty and Holden (1995), where a market maker sets "the official quote" and, after this, traders decide whether to submit limit or market orders by comparing personal evaluation of the asset and the official quote;

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Hollifield *et al.* (2006), who showed that, in a perfectly liquid market, the market's design may provide incentives to traders either to buy, or to sell, or to abstain from trading, according to their private values;

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Rosu (2010), who showed how informed traders may choose between market and limit orders, concluding that the price impact of each trader's decisions is not sufficiently strong to be seen by others;

Bak et al. (1997), who modeled a zero intelligence order book model, where buy and sell orders are two different types of particles on a one-dimensional space whose points represent prices and each particle moves along the price line by following a random walk. Such an approach has been followed, among others, by: Bouchaud *et al.* (2009), Farmer and Foley (2009), Farmer et al. (2005), and Smith et al. (2003);

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Cont *et al.* **(2010)**, who modeled the evolution of the limit order book by focusing on the conditional probability of certain events and assuming that relative prices of orders are drawn from a power-law distribution;

A median approach: ABM (I)

The **agent-based** approach gives the chance to simulate a global environment where interactions among heterogeneous traders give rise to emergent phenomena at the aggregate level. This may also provide analysts with tools for policy assessment.

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Consiglio *et al.* (2005) who modeled heterogenous traders in a multiple assets market where compliance to empirical features of financial data is shown to be related to the interplay of investors' financial constraints and to the frictions generated by the trading mechanism itself;

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Existing Models

Model description

Introduction

Model Operation

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Tedeschi *et al.* (2009) and (2012), who showed that imitative behaviors let some "market guru" have a role in explaining the distinctive features of empirical data.

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Such a simplified classification may appear too naive (Bouchaud *et al.*, 2009). As other classifications proposed (e.g.: informed/uninformed, liquidity hunters/liquidity providers, rational investors/speculators, etc), it allows to consider two opposite market approaches.

II - Model description Fundamentalists

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where Θ_t is a bounded random variable, drawn with uniform distribution within the interval $[-\sigma_{\Theta}, \sigma_{\Theta}]$ and Θ_0 is set at the beginning of simulations.

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where Φ_t is a bounded random variable, drawn with uniform distribution within the interval $[-\sigma_F, \sigma_F]$.

Existing Models Model description

Chartists are trend followers. Therefore, the individual price forecast of a chartist is built upon the inspection of past prices. Similarly to Alfi *et al.*(2008), each chartist chooses a time window of a certain length and looks back to that portion of the past market prices series in order to define her personal reference value:

$$RV_t = \sum_{j=(t-\tau)}^t p_j / \tau$$
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$${}_{c}\boldsymbol{\rho}_{t}^{exp} = \boldsymbol{\rho}_{t} + \frac{\boldsymbol{\rho}_{t} - \boldsymbol{R}\boldsymbol{V}_{t}}{\tau - 1} \pm \Lambda_{t}$$
(4)

where Λ_t is a random variable with uniform distribution drawn from the interval $[-\sigma_C, \sigma_C]$ so that two traders with same time-window length may still have different expectations.



No matter which group an investor belongs to, the trading strategy is quite simple:

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At the beginning of simulations, all traders are endowed with a given amount of cash money, m_t , and a given quantity of asset, a_t .

Existing Models Model description

Each trader sets the price for her order, according to the decided strategy, as follows.

Similarly to the approach of Lux and Marchesi (1999) and (2000), traders perceive the market pressure: adjustments are allowed in the price settings procedure, because of demand/supply mismatching.

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- For bid orders:

$${}^{i}_{B}\boldsymbol{p}_{t} = {}^{i}_{B} \boldsymbol{w}_{t} + \mu [\boldsymbol{z}_{1} \Delta \boldsymbol{n} + \boldsymbol{z}_{2} \Delta \boldsymbol{p}]$$

$$(5)$$

where: μ , z_1 , $z_2 \in [0, 1]$ are individual variables computed at each iteration for each agent and measure the influential weight of the market environment; $_B^i w_t$ is the individual willingness to pay of the bidder, measured as the trader's price expectation, net of her personal estimate of askers' willingness to accept, i.e. $(^i p_t^{exp} - _A^i w_t^{exp})$; Δn is a function of n_B and n_A , which are, respectively, the number of bidders and the number of askers; and, finally, Δp is the price pressure of either excess demand or excess supply.

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II- For ask orders:

$${}^{i}_{A}\boldsymbol{p}_{t} = {}^{i}_{A}\boldsymbol{w}_{t} + \boldsymbol{\mu}[\boldsymbol{z}_{1}\boldsymbol{\Delta}\boldsymbol{n} + \boldsymbol{z}_{2}\boldsymbol{\Delta}\boldsymbol{p}] \tag{6}$$

where ${}^{i}_{A}w_{t}$ is the individual willingness to accept of the bidder, measured as the trader's price expectation, added of her personal estimate of bidders' willingness to pay, i.e. $({}^{i}p_{t}^{exp} + {}^{i}_{B}w_{t}^{exp})$ and all other variables are defined as before.

The quantity to be traded in each order, is decided in two alternative ways:

a- Randomly

Each investor trades a quantity drawn randomly from a feasible interval. Respectively for bid and ask orders, quantities will be set as:

$${}^{i}q_{t}^{B}=\omega$$
 and ${}^{i}q_{t}^{A}=\eta$ (7)

where ω and η are uniformly distributed random variables with values drawn, respectively, from the intervals $[1, m_t/p_t]$ and $[1, a_t]$.

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b- Rationally

Each investor decides the order quantity as a function of her expectations, according to her optimism/pessimism. Respectively for bid and ask orders, quantities will be set as:

$${}^{i}q_{t}^{B} = \beta(m_{t}/p_{t}) \quad and \quad {}^{i}q_{t}^{A} = \alpha a_{t}$$
 (8)

where, similarly to De Long *et al.* (1990), $\beta = 1 - (p_t/p_t^{exp})$ and $\alpha = 1 - (p_t^{exp}/p_t)$ measure, respectively, optimism and pessimism.

Orders are registered in two sections of the order book, ranked with respect to price:

- bid orders decreasingly,

in such a way that the highest bid price, i.e. the *best bid* ($_{B}p_{t}^{best}$), is the first of the list, and the trader who posted it, i.e. the *best bidder*, with the highest willingness to pay, has the priority;

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- ask orders increasingly, in such a way that the lowest ask price, i.e. *best ask* $({}_{A}p_{t}^{best})$, is the first of the list, and the trader who posted it, i.e. *best asker*, with the lowest willingness to accept, has the priority.

Existing Models Model description

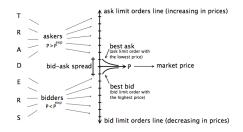
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The "regularity" of orders is also checked: bidders must have sufficient capital to buy the ordered quantity, and askers must have enough stocks to sell the ordered quantity. Irregular orders are deleted. Regular orders have a finite time validity (governed by a parameter): after that period, they are cancelled. Agents cannot have simultaneous active orders.



Existing Models Model description

II - Model description 4) Order Book Setting: Market and Limit Orders

Following the same approach used in Chiarella *et al.* (2009), the taxonomy of orders is restricted to just market and limit orders. The distinction between such types of orders is simple: it derives from the comparison between their prices ($_Ap_t$ and $_Bp_t$) and the current counter-side prices.

Thus, a bid order with price $_Bp_t$ will be:

- a limit order if $_{B}p_{t} <_{A} p_{t}^{best}$
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It is worth noticing that such a taxonomy of orders is partially inaccurate: many other types of orders exist and market and limit orders work in a more complicated way than the one depicted here. For simplicity, the same approach usually adopted in literature is used here. Next releases of the model, will account for a more detailed reproduction of order types (market, limit, stop, stop loss, etc.) and for other aspects of the order book mechanism, such as, among others, the minimum lot, the depth analysis and the bid-ask spread due to commissions/transaction costs.

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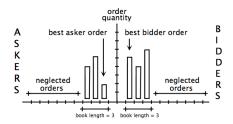
The successive trader becomes the new "best" trader (with new price and quantity). The previously unsatisfied trader continues the interrupted transaction with such a new counterpart, at the new best price.

The update process is possible only for one side of the book and can be repeated until either the unmatched quantity has been entirely negotiated, or the allowed order book length (regulated by a parameter) is reached.

Existing Models Model description

II - Model description Market Orders: The Order Book Length

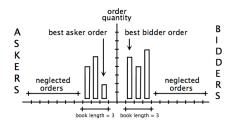
In the figure, the role of the order book length parameter is shown: only the first 3 orders per side will be considered. The best asker is offering 2 shares and the best bidder is demanding 5 shares. The first transaction will be executed for 2 units at the best ask. Then, the best asker is erased. The update process finds the second asker.



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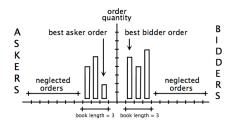


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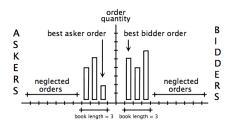
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Notice, finally, that no more than 3 counterparts would have been matched in any case.

When a limit order enters the book, it is ranked accordingly to its price as above explained, and remains active according to the validity settings. When a limit order enters the book, it is ranked accordingly to its price as above explained, and remains active according to the validity settings.

A distinctive feature of this model is that, during each simulative run, no fictitious data is inserted in the price series (and consequently in the returns series). Other order book models, e.g. Chiarella and Iori (2002) and Chiarella *et al.* (2009) add a mid price, defined as the average between the best bid and the best ask, when transactions do not occur. In the model here presented, when best quotes do not match, if no market orders are executed, agents continue to update their forecast, generate their prices, and post their orders until some transaction *truly* happens.

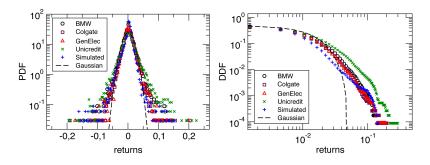
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Moreover, limit orders standing in the book, indirectly express a signal about the "market sentiment". This induces the dynamic adjustment of the price settings rules, as above explained, by means of Δn and Δp components.

Stylized facts compliance

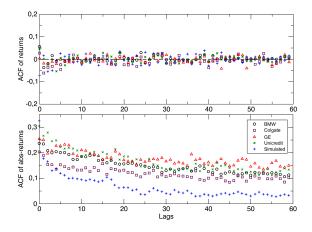
Stylized facts compliance Fat Tails of Returns PDF



In the left panel of Fig. 1, density functions of empirical returns of five financial time series (daily closings ranging from 2/1/1973 to 30/6/2016) are reported in comparison with simulated returns.

The right panel highlights the same result, by showing the decumulative distribution functions of returns, defined as the probability to find a return greater than a certain value, for all series plotted in the left one.

Stylized facts compliance ACF of Returns Distribution and Volatility Clustering



In Fig. 2, the autocorrelation functions of returns of five financial series and simulated returns are reported. In the top panel of Fig. 2, the ACF of plotted returns shows the absence of autocorrelation. In the bottom panel, the ACF of absolute values of returns shows a positive and decaying autocorrelation (*volatility clustering*, Mandelbrot 1963).

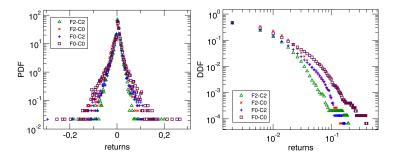
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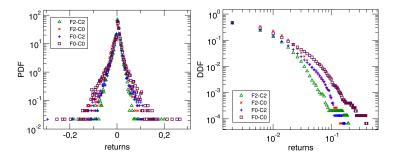
Policy Testing Heterogeneity of Traders



Part of literature (e.g. Chiarella-Iori, 2002, Chiarella *et al.*, 2009) suggests that fat tails are generated by the presence of chartism. Fig. 3 shows that, in the present model, fat tails are not generated by the presence/absence of one of the population components, but by the lack of heterogeneity among investors.

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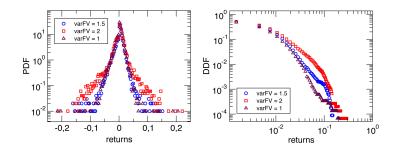


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In other contributions, as for example in Consiglio *et al.* (2005), it is suggested that the homogeneity of the population, may be the cause of more extreme price variations. Indeed, the model confirms such a vision: population may be divided in just two groups, but the heterogeneity of traders within each one, dampens returns fluctuations.

Stylized facts compliance

Policy Testing Variability of Fundamental Variable

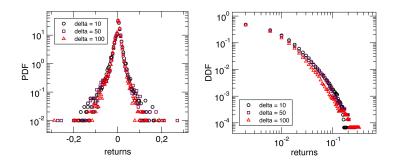


Kirchler and Huber (2007), highlighted that the variability of the fundamental value is the cause of the emergence of fat tails.

Such a conclusion is consistent with results obtained by the model and shown in Fig. 4. Nonetheless, it is not the unique cause. It should be considered in combination with the temporal horizon observed by chartists.

Stylized facts compliance

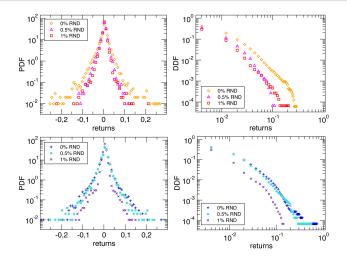
Policy Testing Variability of Chartists' Information



Each chartist sets the length of her observational time window as the sum of two components: one is fixed for everyone, the second is an individual choice. Results plotted in both panels of the Fig. 5, have been obtained by fixing the chartist-window value to $200 + \kappa$, where κ is a bounded random variable, different for each trader, uniformly drawn from the interval $[0, \delta]$.

Stylized facts compliance

Policy Testing Random Traders



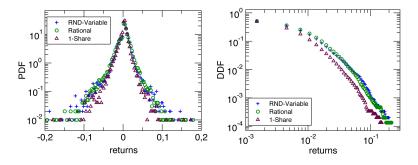
This model confirms that the presence of random investors plays a role in reducing fat tails of returns (top panels 2000 traders, bottom panels 4000).

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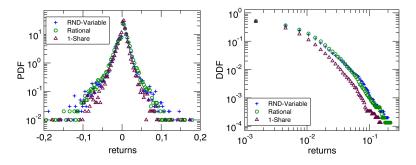
Policy Testing Quantity Setting: Randomness Still Surprises!!



As said above, the presence of random traders may surprisingly help market stabilization. Each random trader decides whether to buy or to hold or to sell just by tossing a coin, and sets her prices at the current market price value. In other words, random traders post market orders at a random pace.

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Further, if we look at how non-random investors decide the quantity of shares to buy/sell, the random quantity selection performs (in terms of market stability) very similarly to the rational one (i.e. the order quantity is proportional to the optimism/pessimism of the trader). Fig. 8 shows this evidence and also the typical 1-share case.

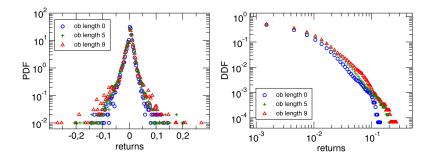
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Order Book Modeling and Financial Stability

Stylized facts compliance

Policy Testing Order Book length



As shown in the above Fig. 9, the order book length parameter plays a crucial role in explaining returns variability. This appears quite natural, since it regulates the number of counterparts that any market order can match progressively, in order to negotiate its desired quantity. In other words, it indirectly shows the price impact of orders. In true markets, true investors have many incentives to minimize the price impact of orders.

A policy on this might obtain results even stronger than showed ones.

Stylized facts compliance

Policy Testing Orders Validity

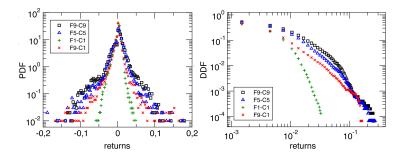


Fig. 10, shows that the validity of orders is the other key variable that can be tuned to dampen fluctuations. Longer lived limit orders remain standing in the book and create a mass of possible transactions if the market price triggers them.

Policy interventions addressed to restrict the validity of orders should consider the individual strategies adopted by traders in order to manage the price/market impact of their investments. These strategies count on different order types and accurate operational timing. As previously anticipated, next releases of the model will care of some of these more advanced issues.

Alessio Emanuele Biondo

Conclusive Remarks

Presented results highlighted several aspects that can be referred to two relevant addresses of policy interventions:

- information diffusion:

the informative problem matters from both a microeconomic and a macroeconomic point of view; in fact, the heterogeneity of traders simulates the degree of the convergence of opinions; and the variability of external variables reflects the credibility of market signals; these aspects represent crucial fields of action for monetary and financial policies;

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Refinements of the model will cover both these topics: 1) the information dynamics among traders will be modeled by means of a dedicated layer where behaviors are observable according to different topologies; 2) portfolio analysis will be added, in such a way that the model may compare different trading strategies of both population groups in terms of profitability; 3) conflicts among micro- and macro-targets will be described, and the strategic use of different orders types will be described.

Thank you for your attention.

Order Book Modeling and Financial Stability

Alessio Emanuele Biondo



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