Cambridge Centre for Risk Studies

Research Showcase 21 June 2015

CAMBRIDGE BANKING MODEL

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Centre for **Risk Studies**

UNIVERSITY OF CAMBRIDGE
Judge Business School

CJBS 25

Cambridge Banking Model

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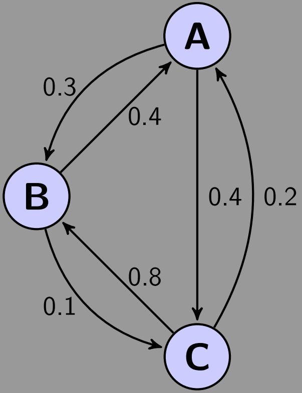
Overview

Financial Network Model

n institutions (banks)

Banks can invest in $n - 1$ institutions



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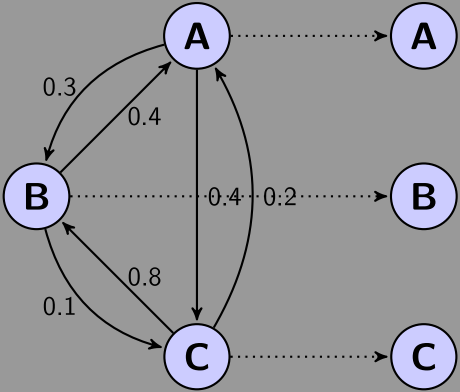
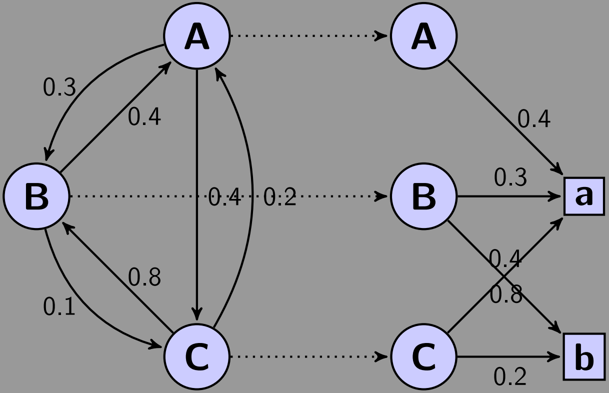


Financial Network Model

n institutions (banks)

m external assets

Banks can invest in $n-1$ institutions or m assets.



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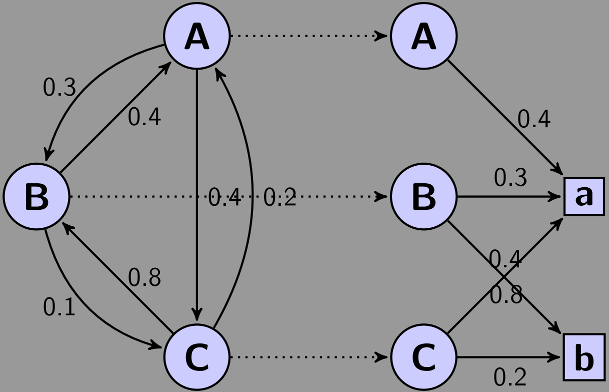


Assets (liabilities) can be external or inter-bank, with totals as

$$A_i^e = \sum_{k=1}^{m} A_{ik}^e \text{ and } A_i^b = \sum_{j=1}^{n} A_{ij}^b$$

Balance Sheets

State Variables

- $E_i(t)$ equity of institution i at time t
- $A_i(t)$ total assets of institution i at time t
- $D_i(t)$ total liabilities of institution i at time t
- A^b_{ij} amount institution i lends to institution j
- A^e_{ik} amount institution i invests in asset k
- $l_i(t)$ leverage of institution i at time t

Assets	**Liabilities**
$A^e = 0.4$	$D^b = 0.6$
$A^b = 0.7$	E

Table: Balance Sheet of Bank A

The balance sheet is defined as

$$A^e_i(t) + A^b_i(t) = A_i(t) = D_i(t) + E_i(t)$$

Leverage of a bank is the ratio of assets and equity

$$l_i(t) = \frac{A_i(t)}{E_i(t)}$$

.

Balance Sheets (cont.)

Financial System

- $l_i(t)$ leverage of institution i at time t
- $l^e_{ik}(t)$ external leverage of institution i with respect to asset k at time t
- $l^b_{ij}(t)$ inter-bank leverage of institution i towards institution j at time t
- $l^e_i(t)$ total external leverage of institution i at time t
- $l^b_i(t)$ total inter-bank leverage of institution i at time t

Leverage (disaggregated) of a bank is the sum of it's external and inter-bank leverage.

$$
\begin{aligned}
l_i(t) &= \frac{A^e_i(t)}{E_i(t)} + \frac{A^b_i(t)}{E_i(t)} \\
&= l^e_{ik}(t) + l^b_{ij}(t)
\end{aligned}
$$

l^e_{ik} can be seen as elements of the adjacency matrix of an bi-partite external leverage network and l^b_{ij} of a mono-partite interbank leverage network. The totals would be the sum along the columns:

$$l^e_i = \sum_{k=1}^{m} l^e_{ik} \text{ and } l^b_i = \sum_{j=1}^{n} l^b_{ij}$$

Loss in Equity Suffered

Distress or Vulnerability

$h_i(t)$ cumulative relative equity loss of institution i at time t

$H(t)$ cumulative relative equity loss of of the financial system at time t

losses of banks relative to it's equity and with respect to a baseline at $t = 0$:

$$h_i(t) = \min\{1, \frac{E_i(0) - E_i(t)}{E_i(0)}\}$$

with bank under distress for $h_i(t) \in (0,1] \forall t$ and default for $h_i(t) = 1$.

losses of the financial system relative to total equity and with respect to a baseline at $t = 0$ is the weighted average cumulative relative equity loss of each bank:

$$H(t) = \sum_{i=1}^{n} w_i h_i = \sum_{i=1}^{n} \frac{E_i(0)}{\sum_{j=1}^{n} E_j(0)} h_i$$

Loss in Equity Induced to the System

Impact

DR_i global relative equity loss induced by the default of institution i

DebtRank DR_i is the impact induced by the default of each bank individually on the system:

$$DR_k(t) = \sum_{i=1}^{n} h_i(T) E_i(0)$$

.

This is the exact solution for systemic risk as defined in BCBS [2013]

Distress Propagation Circle

Asset Losses

negative shock on the value of assets causes losses in banks, which is absorbed by equity.

Inter-Bank Losses

Inter-Bank Losses: distress from asset losses puts inter bank obligations under pressure. Those losses are again absorbed by equity.

Fire Sale

banks need to adjust their leverage to meet regulatory requirements by selling assets. The price impact leads to further pressure on asset prices. This closes the virtuous circle.

Asset Losses

Price Shock

$p_k(t)$ unit price of asset k at time t

$r_k(t)$ relative price (shock) of asset k at time t

a shock

$$r_k(1) = \frac{p_k(0) - p_k(1)}{p_k(1)} < 0$$

on the value of asset k reduces the value of the investment in external assets in bank i by

$$\sum_k r_k(1) A_{ik} = \sum_k r_k(1) l_{ik} E_i = E_i \sum_k r_k(1) l_{ik}$$

the loss needs to be compensated by reduction in equity

$$A^e_{ik}(0) - A^e_{ik}(1) = \sum_k r_k(1) A^e_{ik}(0) = E_i(0) - E_i(1)$$

individual and global relative equity loss at time $t = 1$ are:

$$h_i(1) = \min\{1, \sum_k l_{ik} r_k(1)\} \text{ and } H(1) = \sum_{i=1}^{n} w_i h_i(1)$$

Inter-Bank Losses

Distress Propagation

$V_t(A_{ij})$ market to market value of A_{ij}

The distress that propagates from j into each of the lenders i is the relative loss with respect to the original face value

$$\frac{A_{ij} - V_t(A_{ij})}{A_{ij}} = f(h_j(t-1)).$$

individual relative loss in equity:

$$h_i(t) = \frac{E_i(t) - E_i(0)}{E_i(0)} = \min\left\{1, \sum_{i \in S_A(t)} l_{ij} f(h_i(t-1))\right\}$$
$$= \left(l_i^e + \sum_j l_{ij}^b l_j^e\right) r(1)$$

where $S_A(t)$ is the set of active[1] nodes.

[1] nodes that transmit distress at time t, as in Battiston et al. [2012]

Fire Sale

Price Impact

- Q_i quantity of assets of bank i
- $\hat{p}$ shock price
- η price impact factor

Banks try to sell external assets in order to repay obligations to move to the original leverage:

$$l_i(0) = l_i(t) = \frac{A_i^e(t) + A_i^b(t)}{E_i(t)}$$
$$= \frac{(Q_i(0) + \Delta Q)\hat{p} + A_i^b(t)}{E_i(t)}$$

price impact is linear (proportional to relative change in demand):

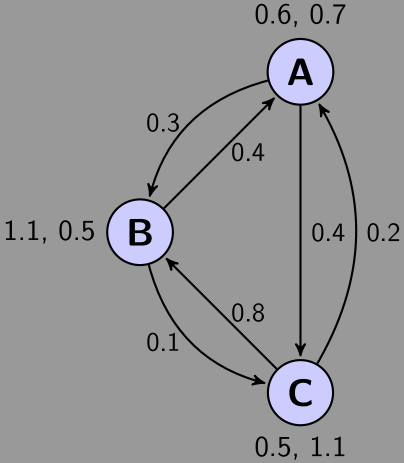
$$r(t) = \eta \frac{\Delta Q_i}{Q_i(0)} = \eta \frac{D_i(0)}{Q_i(0)\hat{p}} (l_i^e)^2 r(1)$$

relative loss in equity:

$$h_i(t) = \frac{E_i(t) - E_i(0)}{E_i(0)} = \left(l_i^e + \sum_j l_{ij}^b l_j^e \right) r(1) + \eta \frac{D_i(0)}{Q_i(0)\hat{p}} (l_i^e)^2 r(1)$$

Network reconstruction

Inter-Bank Network



$$\begin{pmatrix} 0.0 & 0.3 & 0.4 \\ 0.4 & 0.0 & 0.1 \\ 0.2 & 0.8 & 0.0 \end{pmatrix} \quad \begin{pmatrix} 0.7 \\ 0.5 \\ 1.0 \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & 1.1 & 0.5 \end{pmatrix}$$

Network reconstruction

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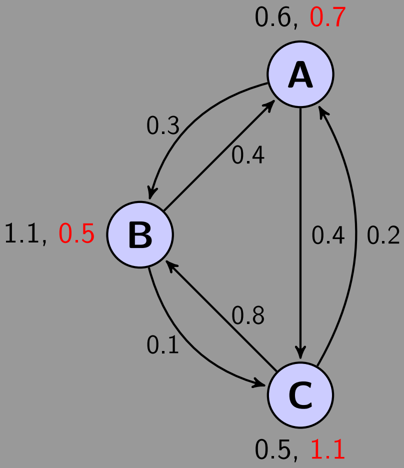

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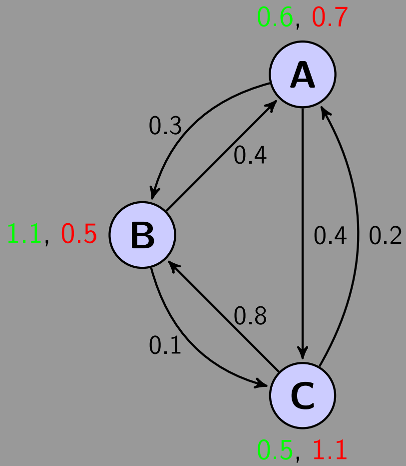

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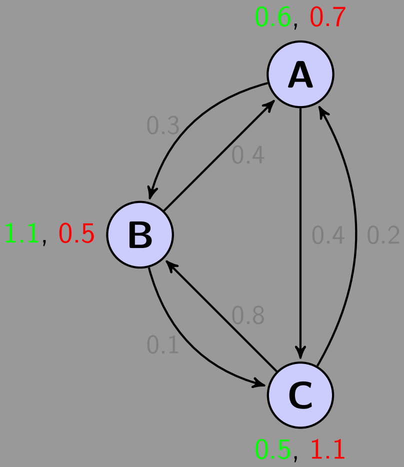

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Quiz

Why are this two matrices similar?

$$\begin{pmatrix} 0.0 & 0.2 & 0.5 \\ 0.5 & 0.0 & 0.0 \\ 0.1 & 0.9 & 0.0 \end{pmatrix} \qquad \begin{pmatrix} 0.0 & 0.3 & 0.4 \\ 0.4 & 0.0 & 0.1 \\ 0.2 & 0.8 & 0.0 \end{pmatrix}$$

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$$\begin{pmatrix} 0.6 & 1.1 & 0.5 \end{pmatrix} \qquad \begin{pmatrix} 0.6 & 1.1 & 0.5 \end{pmatrix}$$

Both matrices have the same sum over rows and columns

- no unique mapping between marginals and exposure
- possible networks range from maximum entropy to minimum density (e.g. diversification vs. costs for relationships)

Network reconstruction

Fitness Model

x_i^{in} lending propensity
x_i^{out} borrowing propensity
p_{ij} exposure probability

Lending and borrowing propensity is the relative exposure

$$x_i^{in} = \frac{A_i}{\sum_j A_j} \text{ and } x_i^{out} = \frac{L_i}{\sum_j L_j}$$

Fitness model applied to interbank network we assume x_i to be the fitness level.

The probability that bank i lends to bank j is :

$$p_{ij} = \frac{z x_i^{in} x_j^{out}}{1 + z x_i^{in} x_j^{out}},$$

where z is a free parameter. The total number of links is equal to the expected value $\sum_i \sum_{j \neq i} p_{ij}$

[De Masi et al., 2006]

Network reconstruction (cont.)

Exposure Volume Allocation

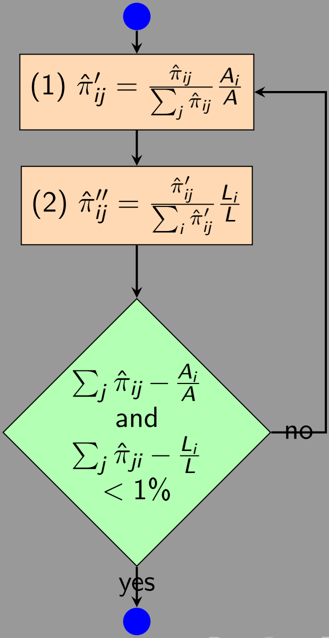
π_{ij} average relative exposure

$$\pi_{ij} = \frac{1}{2}\left(x_{ij}^{in} + x_{ij}^{out}\right)$$

Constraint: sum of exposures equal total assets of bank i

$$1 = \sum_j \pi_{ij}$$

Interactive prop. fitting algorithm: estimate the relative exposure π_{ij} iterating (1) and (2).



Stress Test Scenarios

Trigger by Asset Shock

Shock on assets causes losses in banks; losses propagate to the inter-bank market, spread across the network causes further losses. Feedsback on asset prices.
Historic examples:

- The Tulip and Bulb Craze (1637)
- South Sea Bubble (1720)
- Subprime Mortgage Crisis (2008)

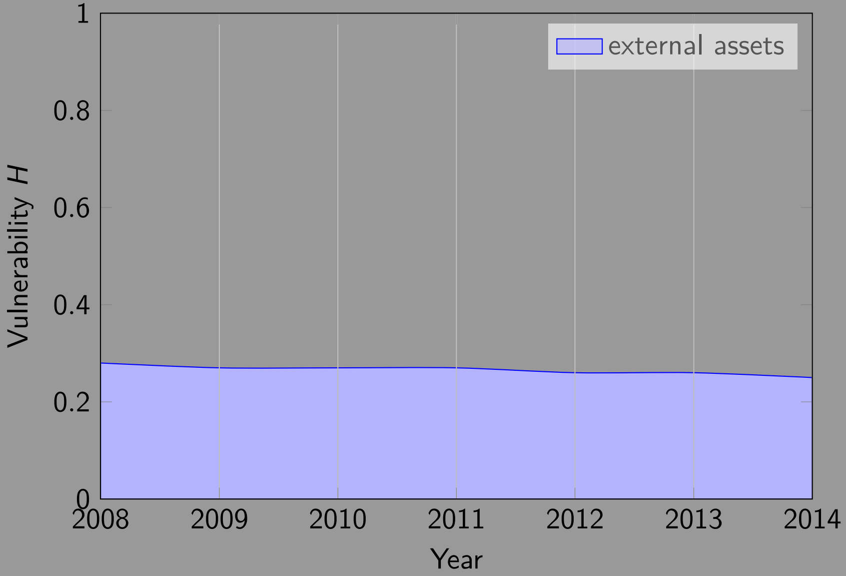
Trigger by Bank Default

Banks fail and default on their obligations. Losses propagate via inter-bank and common asset holdings. Feedback on prises.

Historic examples:

- Jay Cooke & Company crisis (1873)
- Banker's Panic (1907)
- Great Depression (1929)

Stress Test Results



[2]

[2] this data is for illustration only, Tufte [2001]

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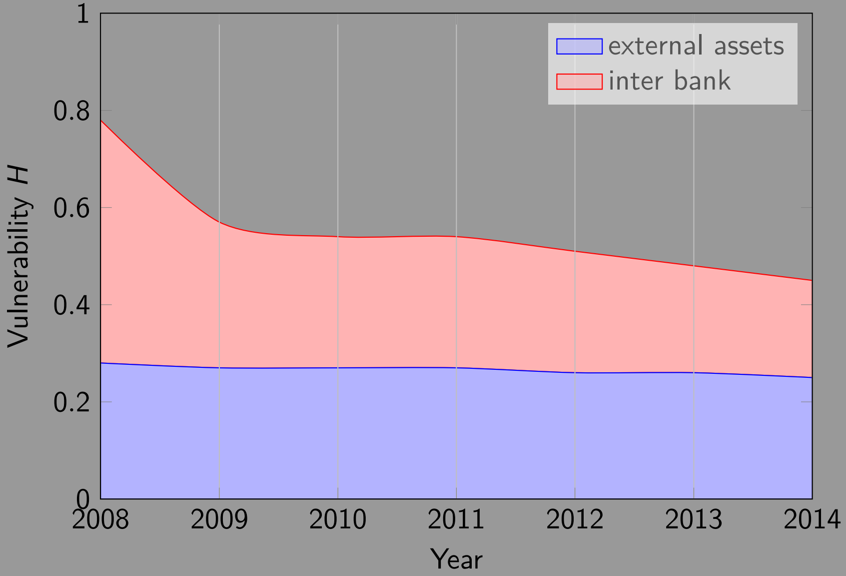

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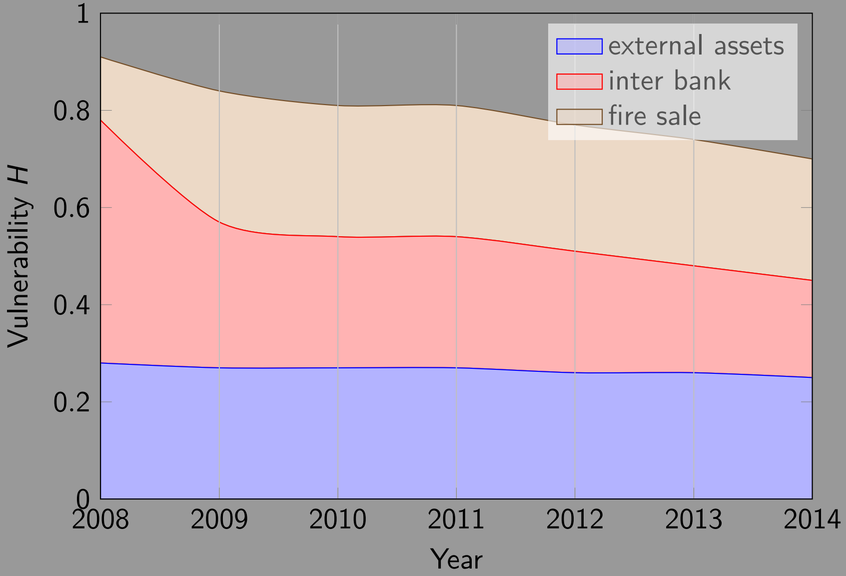

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