Preparatory Work for Quantitative Techniques for Management

1 Introduction

Quantitative Techniques for Management assumes knowledge of very basic mathematics, including standard functions and notation, probability and graphical presentations of data. You are also assumed to be familiar with basic MicroSoft Excel skills including the ability to calculate basic statistics in MicroSoft Excel. *The goal of this document is to test and improve your familiarity and fluency with these techniques because we do not have time to cover them in class.*

See Section 1.1 to test or improve your Excel skills, and Section 1.2 for basic maths & stats skills.

1.1 MicroSoft Excel

If you have the required Excel skills, then it will only take you 5 minutes to verify this by scanning the tutorial at

`http://best-excel-tutorial.com/54-basics`

If not, you will need to work through the tutorial prior to the course.

- Opening, saving and printing Excel files
- Moving around a spreadsheet and selecting ranges
- Inserting and deleting worksheets, rows and columns
- Entering, editing and deleting text, numerical data and formulas
- Formatting text, numbers and formulas
- Performing basic arithmetic calculations $(+, -, *, /)$
- Applying simple functions such as MAX, AVERAGE, COUNT, IF
- Using relative and absolute references to cells in formulas
- Creating simple tables
- Plotting pie, bar, column and line charts

1.2 Maths, probability and statistics

To begin, please attempt the self-test questions in Section 2. If these questions are easy for you and you are confident with your maths training and abilities, you need not read further.

If some of the self-test questions are challenging and you take quite a while to finish, then you should invest in the following to improve your skills. Identify the relevant exercises in Section 3–6 of this document and try to work through them. If you are unconfident about your approach or find the exercises difficult, we recommend some revision materials below.

Note: Some of you may think that the topics covered in Section 3–6 or the website or textbooks suggested below are very easy, while others may find them quite challenging. There is no single best way of approaching this; be prepared to try out the resources we suggest, or others, in order to find what suits you.

S-cool **website**

`http://www.s-cool.co.uk/a-level/maths`

This covers:

- `Statistics - The Basics` (sampling, mode, median, mean, quartiles, standard deviation, and variance)
- `Representation of Data` (frequency tables, frequency diagrams, line graphs, cumulative frequency, cumulative frequency diagrams, bar charts, pie charts, and scatter diagrams)
- `Probability` (permutations, combinations, probability, mutually exclusive events, independent events, tree diagrams, the 'and' rule, and conditional probability)
- `Probability Distributions` (probability density functions, cumulative distribution functions, expectation, variance, and standard deviation). There is no need to read binomial distributions and geometric distributions.

For each topic covered in *S-Cool*, the reader is guided through three steps: (1) Revise It, (2) Test It, and (3) Remember It.

Reference books

If you want to review maths & stats using a book, we suggest books of Morris [1, 2], see detailed references at the end of this document. Note that book [2] will be one of the main textbooks for the course. If using edition 5 of book [2], for example, then review Chapter 1 (basic numeracy), pages 133-141 of Chapter 8 (elementary probability), pages 56-77 of Chapter 5 (data presentation).

2 The Self-test Questions and Answers

This section provides some example questions you can try to test yourself on the preparatory topics.

1. Find:

(a) $\sum_{i=1}^{4} i$, (b) $\sum_{i=1}^{4} i^2$, (c) $\sum_{i=1}^{4} i!$, (d) $\sum_{i=1}^{4} ix$, (e) $x^2(x^3x^4)^2$, where x is some unknown value.

2. For $x_1 = 3.1$, $x_2 = 1.2$, $x_3 = 3.7$, $x_4 = 0.6$, $x_5 = 5.3$, find: (a) $\bar{x} = \frac{\sum_{i=1}^{5} x_i}{5}$, (b) $\sum_{i=1}^{5}(x_i/5)$, (c) $\sum_{i=1}^{5}(x_i - \bar{x})$, (d) $\sum_{i=1}^{5}(x_i - \bar{x})^2$, (e) $\sqrt{\frac{\sum_{i=1}^{5}(x_i-\bar{x})^2}{4}}$.

3. For the functions $f(x) = 35x - 10$ and $g(x) = 5x + 20$, find: (a) $f(10)$, (b) $g(20)$, (c) The value of x, where $f(x) = g(x)$.

4. If there is an equal probability of a child being born a boy or a girl, what is the probability that a couple who have three children have two children of the same sex and one of the opposite sex?

5. How many ways are there of choosing four students from a group of six students?

6. What is the probability, after rolling a pair of dice, that the numbers add up to 7? How much more likely is rolling 7 than rolling 4?

7. In a certain firm 40% of the workforce are women. Only 25% of the female workforce is management grade, whereas for male workers the figure is 30%. What is the probability that a worker selected at random is both female and management grade?

8. You toss a coin twice and observe your "score" as follows: two tails score 1; a head and a tail, or vice versa, score 3; two heads score 9. What is your average score?

9. Table 1 shows the average number of miles travelled by each sales representative per "effective call". An effective call is one that results in a certain level of sales.

Average no. of miles per effective call	No. of sales representatives
At least 10, less than 20	4
At least 20, less than 30	7
At least 30, less than 40	18
At least 40, less than 50	8
At least 50, less than 60	3

Table 1: The frequency table for the number of sales representatives.

(a) Give a table and graph showing the (cumulative) number of sales representatives with less than a given number of miles per effective call.

(b) What is the percentage of sales reps who require less than 40 miles per effective call?

(c) What is the likelihood of a randomly chosen sales rep averaging at least 20 miles but less than 40 miles per effective call?

10. A college admissions section has collected the following distribution of ages, shown in Table 2, for last year's applicants who are up to 21 years old: Assume that the average 17

Age	No. of applicants
17	3
18	74
19	47
20	18
21	11

Table 2: The frequency table for the number of applicants.

year old is 17.5 years old, and so on.

(a) What is the average age of up-to-21-year-old students, to two decimal places?

(b) What is the standard deviation of age?

(c) What is the median age group (i.e. the median "bin")?

Answers

1. (a) 10, (b) 30, (c) 33, (d) $10x$, (e) x^{16}.

2. (a) 2.78, (b) 2.78, (c) zero, (d) 14.5, (e) 1.91. Note that a different answer is obtained for $\sqrt{\frac{\sum_{i=1}^{5}(x_i-\bar{x})^2}{5}}$.

3. (a) 340, (b) 120, (c) $x = 1$.

4. 3/4.

5. 15 ways.

6. 1/6. Rolling 7 is twice as likely as rolling 4.

7. 1/10.

8. The mean is 4.

9. (a) See Table 3 and Figure 1.

Average no. of miles per effective call	Cumul. No. of representatives
less than 10	0
less than 20	4
less than 30	11
less than 40	29
less than 50	37
less than 60	40

Table 3: The cumulative frequency table for the number of representatives.

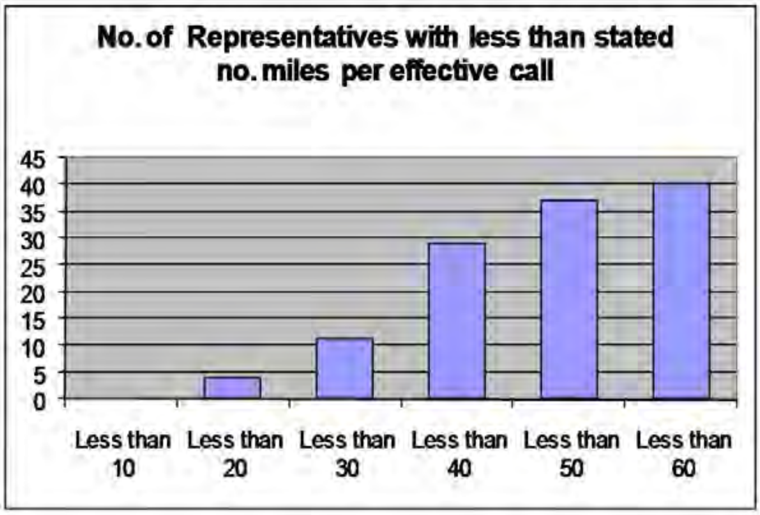

Figure 1: The cumulative distribution graph for the number of representatives.

(b) 72.5% or 73%.

(c) 0.625 or 0.63.

10.

(a) Mean age is 19.24 years.

(b) Standard deviation for the population is 0.948, to three decimal places. If you use the standard deviation formula for samples, i.e. multiply the population standard deviation by $\sqrt{(n/(n-1))}$ where n is the sample size - here $n = 153$ - then the result is 0.951.

(c) The median age group is 18 (median student is no. 76.5 or 77).

3 Mathematics

3.1 Percentages, fractions, decimals, and absolute values

Fractions, decimals and percentages are three different ways of writing the same number. Five examples are shown in Table 4. For example, $\frac{1}{2} = 0.5 = 50\%$.

Fraction	Decimal	Percentage
1	1	100%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.2	20%
$\frac{2}{3}$	0.67	66.67%

Table 4: Fraction and decimal numbers and percentages.

Question 3.1

(a) *What are the decimal and percentage values for $\frac{3}{6}$?*

(b) *What are the percentage and fraction value for 0.8?*

(c) *What are the fraction and decimal value for 40%?*

(d) *Linda is given a 20% discount on a laptop costing £1000. How much discount would Linda get in £?*

(e) *Adam paid £200 for an air ticket, for which the full price is £300. How much discount did Adam get in percentage?*

The absolute value of a real number is its numerical value without regard to its sign. For example, 10 is the absolute value of both 10 and -10. The absolute value of a real number x is denoted by $|x|$. The absolute value is used in hypothesis testing and linear regression. (We will not provide questions to test this simple concept.)

3.2 Squares, square roots, and powers

The square of a number x is denoted by x^2 and is found by multiplying the number by itself. Hence the square of 8 is 64.

The square root of a number x is denoted by $\sqrt{x}$ or $x^{1/2}$ or $x^{\frac{1}{2}}$, and is equal to the number which, when squared, gives x. Hence the square root of 64 is 8.

Powers (also known as exponents) are an extension to the idea of squares. Instead of writing, for example, $x \times x \times x \times x \times x$, we would write x^5. The basic format of numbers involving power is $M \times 10^n$, where M is a real number between 1 and 10 and n is a whole number.

$$\begin{aligned}
&10^0 = 1, \\
&2 \times 10^2 = 2 \times 10 \times 10 = 200, \\
&3 \times 10^{-2} = 3/(10^2) = 0.03.
\end{aligned}$$

Rules for the power operator are:

$$\begin{array}{ll}
x^m x^n = x^{m+n}. & \text{Example :} 5^2 \times 5^3 = 5^5 = 3125. \\
(x^n)^m = x^{nm}. & \text{Example :} (3^2)^3 = 3^6 = 729. \\
(xy)^m = x^m y^m. & \text{Example :} (3 \times 2)^4 = 3^4 \times 2^4 = 1296. \\
(\frac{x}{y})^m = \frac{x^m}{y^m}. & \text{Example :} (\frac{1}{2})^3 = \frac{1^3}{2^3} = 1/8. \\
\frac{x^m}{x^n} = x^{m-n} \text{ if } x \neq 0. & \text{Example :} \frac{5^3}{5^2} = 5^1 = 5.
\end{array}$$

Squares and square roots are often used when calculating variance and standard deviation in statistics. Powers are often used in calculating present and future values in finance.

Question 3.2

(a) *Calculate the square and square root of 81.*

(b) *Let* $m = 2$, $n = 3$, $k = 2$, $x = 10$, $y = 2$ *and* $z = 5$. *Calculate* $(yz)^m \times (\frac{x}{z})^n \times \frac{x^n}{x^k}$.

(c) *Let* $x = 2$, $y = 6$, $z = 20$, $n = 2$, $m = 3$, *and* $k = 4$. *Calculate* $\frac{(x^n+y)^m}{z^k}$.

(d) *Suppose the annual interest rate for fixed term deposits is* $r = 10\%$. *Jerry just deposits* $PV =$ *£1,000 in the bank. Five years later, the future value using compound interest that Jerry will receive is* $FV = PV(1+r)^n$, *where* $n = 5$. *Help Jerry to calculate* FV.

(e) *Jane wants to start a new business in four years. Her analysis shows that she needs a fixed amount of* $FV =$ *£*$2,000,000$ *to get her business started. If she saves her money in the bank, the amount of money that she needs to invest now is* $PV = FV/((1+r)^n)$, *where* $n = 4$ *and* $r = 10\%$ *is the annual interest rate. Help Jane to calculate* PV.

3.3 Subscript notation, indices and sums

If we have a collection of n values of a quantity x, we often denote them using a subscript: x_1, x_2, x_3, x_4, . . . , x_n. For example, we collect some data on interest rates in four different bank accounts. Here, $n = 4$, and if we number the banks from one to four, we might have:

i :	1	2	3	4
x_i :	2.1	6.5	4.7	3.4

The subscript is often referred to as the index. The term x_i is read "x-eye". In the above table, we may write $x_1 = 2.1$, or $x_i = 2.1$ when $i = 1$.

We frequently write sums of values, and there exists a useful shorthand notation (known as summation notation) for doing this. For example, if we wanted to denote the sum $(x_1 + x_2 + x_3 + x_4)$ of the values in the above table, we would write:

$$\sum_{i=1}^{4} x_i = 2.1 + 6.5 + 4.7 + 3.4 = 16.7.$$

The summation notation is frequently used in calculating probabilities, central tendency and variation statistics.

Question 3.3

(a) *Suppose the inflation rates in the past five years are 2.2, 3.1, -0.2, 1.5, and 3.4, and they are denoted by quantity* x. *What is* x_i *when* $i = 3$? *What is the value for* x_4?

(b) *In (a), what is the average inflation rate in the past five years?*

(c) *If quantity* x *has five values 2, 3, 4, 2, and 5, then calculate the square of* x_i *for each* i *and calculate* $\sum_{i=1}^{5}(x_i)^2$. *Note the summation notation is written slightly differently here.*

(d) *In the last 4 days a car yard has sold 4, 3, 8 and 2 new cars and earned daily revenues of £40,000, £38,000, £37,000 and £42,000, respectively. Let* x_i *denote the number of cars sold on day* i *and* y_i *the average price per car sold on day* i. *What are values for* x_3 *and* y_3?

(e) *In (d), calculate* $\sum_{i=1}^{4} x_i y_i$.

3.4 Functions

When the values of two variables are related in some way, we often say that one variable is a function of the other. For example, if we have two variables, x and y, and the value of y can be calculated if we know the value of x, we would say that y is a function of x. We write this as:

$$y = f(x)$$

which is read "y equals f of x". In particular, $y = a + bx$ is a linear function, where x and y are variables and a and b are parameters.

The exponential function with a format of $f(x) = e^{ax}$ is very useful in modelling variable service times in a queueing system such as call centres and supermarket tills. Here x is the variable, a is a parameter, and $e = 2.71828$ is the Euler constant (approximately).

If $f(x)$ and $g(x)$ are two functions, then $\min(f(x), g(x))$ and $\max(f(x), g(x))$ are called the minimum and maximum functions of $f(x)$ and $g(x)$, respectively. Note that $\min(a, b)$ is equal to the minimum (the smallest value) between a and b, and $\max(a, b)$ is equal to the maximum (the largest value) between a and b.

Question 3.4

(a) *Consider the function $y = f(x) = 6x - 3$. If $x = 3$, what is the value of y?*

(b) *Consider the function $y = f(x) = 2x^2 - 3x + 15$. If $x = 2$, what is the value of y?*

(c) *Consider functions $y = f(x) = 2 + 4x$ and $z = g(x) = 6 - 2x$. Define new functions $u = \min(f(x), g(x))$ and $v = \max(f(x), g(x))$. If $x = 1.5$, what are values of y, z, u and v?*

(d) *Alex receives a fixed amount of monthly commission of £50 from a commercial bank. In addition, Alex receives a commission of £200 per home loan. Let x and y be the number of home loans per month that Alex sells and the total amount of commission per month that Alex gets from the bank. Establish a linear function between x and y.*

(e) *The price of a taxi journey is made up of two elements: a fixed charge of £2.50, plus a charge of 80p per mile traveled. Write down a mathematical expression to show the price of a journey as a function of miles traveled.*

3.5 Solving one equation with one unknown

You need to be able to solve equations with one unknown. Remember that the way to do this is to carry out the same set of operations to both sides of the equation simultaneously. Given values for parameters a, b and y, here is a standard procedure for solving $y = ax + b$ for x,

$y - b = ax$ (subtracting b on both sides)

So ... x = (y-b)/a (dividing both sides by a).

Question 3.5

(a) *Solve the equation* $4x + 3 = 2x - 6$ *and equation* $12(x - 1) = 15 + 6x$.

(b) *Find the value for t to satisfy the following equation* $30 + t\frac{\sigma}{\sqrt{n}} = 36$, *where* $\sigma = 18$ *and* $n = 81$.

(c) *Find the value for n to satisfy the following equation* $1950 = 2000 - t\frac{\sigma}{\sqrt{n}}$, *where* $t = 2$ *and* $\sigma = 250$.

(d) *Suppose that the daily demand (the number of units sold each day) and price relationship for a mobile phone shop is represented by: demand = 20 - (3 × price). What price should the shop charge mobile phone customers if they want to sell 5 phones per day? Derive a formula to calculate price based on demand.*

(e) *A cafeteria shop plans to order a number of fresh sandwiches from a nearby bakery every day. The bakery charges the cafeteria a fixed price of £10 for delivery and a unit price of £1.50 per sandwich. Establish the relationship between the total amount of money the cafeteria needs to pay to the bakery and the number of sandwiches the cafeteria orders.*

3.6 Solving two equations with two unknowns

This topic is optional.

Sometimes, it may be necessary to solve two equations with two unknowns simultaneously. In the following system, suppose a, b, c, d, e, and f are parameters, and x and y are unknowns:

$$ax + by = c, \quad dx + ey = f.$$

A typical procedure for solving two equations with two unknowns is: (1) Express one unknown as a linear function of another unknown based on the first equation: $y = (c - ax)/b$, (2) Substitute y by $(c - ax)/b$ in the second equation and obtain a new equation with unknown x, (3) Find the value for x by solving the new equation, (4) Obtain the value for y using expression $y = (c - ax)/b$.

Suppose x and y are unknowns and a and b are parameters. Then equation $y = a + bx$ can be completely represented by a straight line in a two-dimensional plane in which x is represented by the horizontal axis and y by the vertical axis. Here a is often called the y-intercept and b the slope. Two straight lines represented by $y = 1 + 3x$ and $y = 6 - 2x$ are depicted in Figure 2. Note that the coordinates of the intersection of these two lines give the solution of these two equations.

Question 3.6

(a) *Solve the following pair of simultaneous equations:* $x - 2y = 10$, $2x + 3y = 20$.

(b) *Solve the following pair of simultaneous equations:* $x + 4y = 15$, $4x + 2y = 30$.

(c) *Solve the following pair of simultaneous equations:* $3a - 4b + 5 = 0$, $-10a + 6b = 12$.

(d) *Solve the following pair of simultaneous equations:* $t - 2s = 18$, $4s + 5t - 40 = 0$.

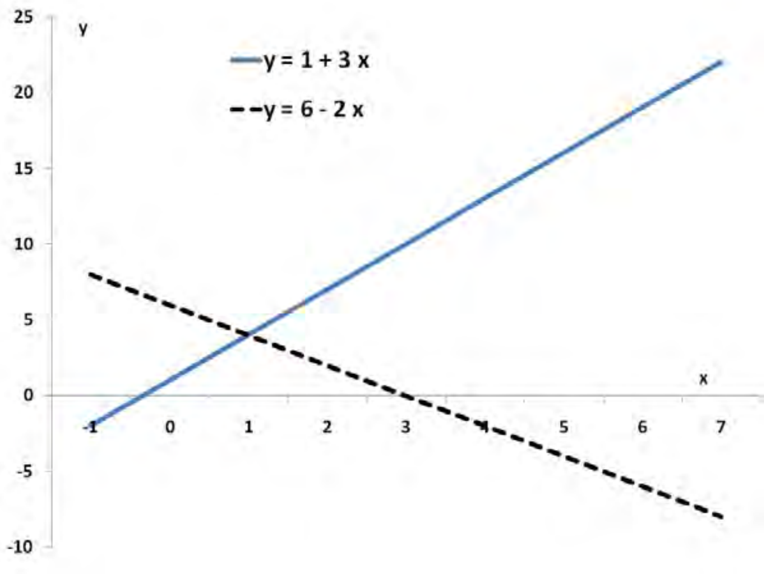

Figure 2: Two straight lines and their intersection.

(e) *Suppose that the demand function for a new luxury entertaining product is* $Q^d = 100 - 10p$ *and the supply function for the same product is* $Q^s = -10 + 12p$*, where* p *is the market price,* Q^d *is the potential demand, and* Q^s *is the potential supply. When* $Q^d = Q^s$ *or the demand is equal to the supply, the market reaches an equilibrium. The corresponding price is called the market clearing price, which can be found by solving two equations:* $d = 100 - 10p$ *and* $d = -10 + 12p$*. Find the market clearing price and its corresponding demand for the product.*

3.7 Inequalities

In many business problems, not every thing can be expressed by equations, but sometimes by the expressions of one thing less than or more than the other. We introduce some notation:

$>$ is read "greater than"

$<$ is read "less than"

$\geq$ is read "greater than or equal to"

$\leq$ is read "less than or equal to"

For example, business managers often want to assess the probability that the business cannot make a profit, i.e., the probability that the profit is less than or equal to zero. If we have the inequality $-3x \leq 12$, then the solution (i.e., equivalent expression with only x "on the left-hand side") is $x \geq -4$.

Question 3.7

(a) *Solve the inequality* $4(x - 1) \geq 10$.

(b) Solve the inequality $-8(3x+4) \leq 2x-5$.

(c) Check if the following two inequalities equivalent:

$$x \leq 120,$$

$$\frac{(x-100)}{\sqrt{100}} \leq 2.$$

(d) Check if the following two inequality expressions are equivalent:

$$70 \leq x \leq 130,$$

$$-2 \leq \frac{(x-100)}{\sqrt{100}} \leq 3.$$

(e) In Question 3.5(e), if the cafeteria has a daily budget of £40 for sandwiches, what is the maximum number of sandwiches the cafeteria can order each day?

4 Probability

4.1 What is probability

Probability is about the chance of something happening. When we talk about how probable something is we say things like 'unlikely', 'probably', 'even chance', 'almost definitely' and '100% certain'. All these are ways of talking about probability. We want to use numbers to describe probabilities of certain events occuring. Examples are:

(a) Winning a Noble prize is possible, but not very likely. We say that the probability of winning a Noble prize is very small for each individual.

(b) The share price for blue-chip company X will move up or down tomorrow, but it is not certain. We say that the probability of having a different share price tomorrow for blue-chip company X is close to one or 100%.

(c) When you toss a perfectly designed die of six sides, the probability of getting a 2 is $\frac{1}{6}$.

Question 4.1

(a) What is the probability of tossing a coin and getting 'heads'?

(b) If an event has a probability of 0.75, is it likely or unlikely to happen?

(c) How many possible outcomes are there if you pick up one card from a standard pack of cards?

(d) If the probability of event A occuring is 0.6 and the chance of event B occuring is 60%, which event has a higher probability of occuring?

(e) Which of the following two events has a lower chance of occuring in any day? (1) It rains in Cambridge, (2) The Dow Jones index rises by 20%.

4.2 The "and" rule

Two uncertain numbers (or 'random variables') are independent if the outcome of one is irrelevant to the outcome of the other, and vice versa. The *multiplication rule* or the *and rule* states that the probability of two independent events both occurring is the product of their individual probabilities: for two independent events A and B, we have

$$\text{Pr(A and B)} = \text{Pr(A)} \times \text{Pr(B)},$$

where Pr(A) denotes the probability that event A occurs. Coin tossing is a natural example of independent events since the likelihood of tossing a head on a second or third (or any other) toss is unaffected by the outcome of the first toss. Applying the and rule, the probability of getting a head on the first toss, and a head on the second, and a head on the third is just $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Question 4.2

(a) *Suppose events A, B and C are independent, Pr(A) = 0.5, Pr(B) = 0.8, and Pr(C) = 0.2. What is Pr(A and B and C)?*

(b) *If you roll two dice, what is the chance that you get two 6s?*

(c) *If you toss a coin twice, what is the chance that you get a head the first time and a tail second time?*

(d) *Suppose that the chance that the interest rate goes up next month is 75% and the chance that the house price goes up next month is 0.6. Assume that these two events are independent. What is the chance that both the interest rate and the house price go up next month?*

(e) *Suppose that the probabilities that the share price for a large oil company goes up, goes down, and stays the same on a typical day are 0.5, 0.4, and 0.1, respectively. Suppose further that the the change in share price on any day is independent of the change on the previous day. What is the probability that the the share price for this company goes down in four consecutive days?*

4.3 The "or" rule

Two events are mutually exclusive if both cannot happen at the same time. The *addition rule* or the *or rule* states that the probability of either of two mutually exclusive events occurring is the sum of their individual probabilities: for two mutually exclusive events A and B, we have

$$\text{Pr(A or B)} = \text{Pr(A)} + \text{Pr(B)}.$$

Clearly, event A and event "not A" are mutually exclusive. Hence Pr(not A) = 1 - Pr(A). When you toss a coin, the chance that you get a head or a tail is 100%.

Question 4.3

(a) *What is the probability of getting an odd number when you roll a die?*

(b) *What is more likely to happen: rolling a die of six sides and getting a number less than or equal to 3, and tossing a coin and getting tails?*

(c) *Pick a card randomly from a standard pack of cards. What is the probability of your card being a king?*

(d) *Two dice are rolled and the numbers are added together. What is the probability that the sum is 4?*

(e) *Suppose that the probability for a long or short length of stay (LOS) in hospitals for urban patients is 20% or 80%, respectively, the probability for a long or short length of stay in hospitals for rural patients is 30% or 70%, respectively. If 60% of patients live in urban areas and the rest live in rural areas, what is the probability that a patient has the following demographic feature: a long LOS from either urban or rural areas.*

5 Statistics

5.1 Mean, median and mode

There is more than one type of averages you can have: mean, median and mode. The measure used most often is the mean value, often called the average value. When people talk about the average of something, like average price, average wage or average height, they are usually talking about the mean value. For a given set of data of consisting n values, the mean is equal to the sum of all n values divided by n, the median is the middle value in the set if they are ordered from the smallest to the largest, and the mode is the value which occurs most frequently.

You rolled a die 10 times and you obtained the following values: 4, 3, 6, 1, 1, 2, 4, 3, 5, 2. Then the mean is $(4+3+6+1+1+2+4+3+5+2)/10 = 3.1$, the median is 3 (which is the fifth and sixth smallest numbers), and the mode is 1 or 2 or 3 or 4 (which all occurred twice).

Question 5.1

(a) *What are mean, median and mode of the following 10 values: 21, 20, 21, 22, 23, 21, 20, 22, 23, 22?*

(b) *The number of goals scored by a football team in the last 10 games are: 2, 0, 1, 1, 4, 0, 2, 3, 4, 4. Calculate the mean, median and mode of the number of goals.*

(c) *What are the mean, median and mode wages of eight CEOs: £150,000, £200,000, £250,000, £60,000, £100,000, £100,000, £120,000, £180,000?*

(d) *There are 30 members in a new and youth internet enterprize club. Four members are 17 year old, eight are 18, seven are 19, and eleven are 20. Calculate the mean, median and mode ages.*

(e) *Jim has traveled to his office by bike in the last four consecutive weeks. The travel times from home to office were: 20.5 minutes on four days, 21 on five days, 22 on nine days, 22.5 on two days. Calculate the mean, median and mode travel times from home to office for Jim.*

5.2 Standard deviation and variance

The averages are measures of central tendency for a set of values/observations. Standard deviation and variance are measures for dispersion or spread or variation or risk for a set of observations. The variance is equal to the average of the sum of the square of the deviation of each data point from the mean: $\nu = (\sum_{i=1}^{n}(x_i - \bar{x})^2)/n$, where n is the number of data points, x_i is the value for observation i, and $\bar{x}$ is the mean of all x_i. The standard deviation is the squared root of the variance: $\sigma = \sqrt{\nu}$.

You rolled a die 10 times and you obtained the following values: 4, 3, 6, 1, 1, 2, 4, 3, 5, 2. Then the mean is $\bar{x} = 3.1$, the variance is

$$\begin{aligned}
\nu &= \frac{1}{10}((4-\bar{x})^2 + (3-\bar{x})^2 + (6-\bar{x})^2 + (1-\bar{x})^2 + (1-\bar{x})^2 + \\
&\quad (2-\bar{x})^2 + (4-\bar{x})^2 + (3-\bar{x})^2 + (5-\bar{x})^2 + (2-\bar{x})^2)) \\
&= \frac{2(1-\bar{x})^2 + 2(2-\bar{x})^2 + 2(3-\bar{x})^2 + 2(4-\bar{x})^2 + (5-\bar{x})^2 + (5-\bar{x})^2}{10} \\
&= 2.49,
\end{aligned}$$

and standard deviation is $\sigma = \sqrt{\nu} = 1.578$.

Question 5.2 *It might be quicker to do the following questions in Excel.*

(a) Calculate the standard deviation and variance for Question 5.1(a).

(b) Calculate the standard deviation and variance for Question 5.1(b).

(c) Calculate the standard deviation and variance for Question 5.1(c).

(d) Calculate the standard deviation and variance for Question 5.1(d).

(e) Calculate the standard deviation and variance for Question 5.1(e).

6 Data Handling and Graphs

If you have not had any experience of using Microsoft Excel to generate graphs, check an online Excel learning tool at the following webpage

`http://best-excel-tutorial.com/54-basics`

6.1 Frequency and cumulative frequency tables

The frequency of a particular data value is the number of times the data value occurs. For example, if four FTSE-100 companies have an annual return of 10%, then an annual return of 10% is said to have a frequency of 4. The frequency of a data value is often represented by f. A frequency table is constructed by arranging collected data values in ascending order of magnitude with their corresponding frequencies.

Cumulative frequency corresponding to a particular value is the sum of all the frequencies up to and including that value. A cumulative frequency table is constructed by arranging collected data values in ascending order of magnitude with their corresponding cumulative frequencies.

Here are annual return figures for 30 FTSE-100 companies: a return of -4% for five of them, -2% for six, 2% for five, 5% for ten, and 10% for four. We can calculate cumulative frequencies: a return of no more than -4% for five of them, no more than -2% for eleven, no more than 2% for sixteen, no more than 5% for twenty six, and no more than 10% for thirty. Clearly, we can easily calculate average and dispersion values of annual returns of these companies.

Table 5 shows the frequencies and cumulative frequencies of annual returns.

Return (x)	Frequency (f)		Return (x)	Cumulative frequency
-4%	5		$\leq -4\%$	5
-2%	6		$\leq -2\%$	11
2%	5		$\leq 2\%$	16
5%	10		$\leq 5\%$	26
10%	4		$\leq 10\%$	30

Table 5: The frequency and cumulative frequency tables of annual returns.

Question 6.1

(a) *Construct frequency and cumulative frequency tables for Question 5.1(a).*

(b) *Construct frequency and cumulative frequency tables for Question 5.1(b).*

(c) *The numbers of executive directors in 36 large car manufacturing companies are: each of five companies has 4 executive directors (ed), each of seven companies has 5 eds, each of eleven companies has 6 eds, each of nine companies has 7 eds, and each of four companies has 8 eds. Construct frequency and cumulative frequency tables.*

(d) *A sample of house prices (in£1000) for some recent sales in Cambridge is the following: 215, 170, 204, 195, 345, 260, 170, 220, 275, 294, 167, 190, 170, 375, 145, 195, 242, 250, 255, 230, 135, 205, 190, 156, 173, 220, 210, 168, 180, 266, 190, 200, 219, 110, 138, 146. Construct frequency and cumulative frequency tables using £50,000 buckets/ranges with the first bucket to be less than £150,000, the second to be greater than or equal to £150,000 and less than £200,000, etc., and the last to be greater than or equal to £350,000. Hint: it might be easier and quicker if you re-order these numbers and/or do it in Excel.*

(e) *Construct frequency and cumulative frequency tables for Question 5.1(e).*

6.2 Pie and bar charts

A pie chart is a circular chart divided into sectors, illustrating relative magnitudes or frequencies. A bar chart or bar graph is a chart with rectangular bars with lengths proportional to the values that they represent.

The MBA career manager in a business school informs that among all MBAs graduated last year, 28% of them are now working in the finance sector, 20% are in the consulting sector, 19% are in the manufacturing sector, 15% are in the service sector, 6% are in the non-profit sector, and 12% are in other sectors. For this same example, we can construct both pie and bar charts as shown in Figure 3.

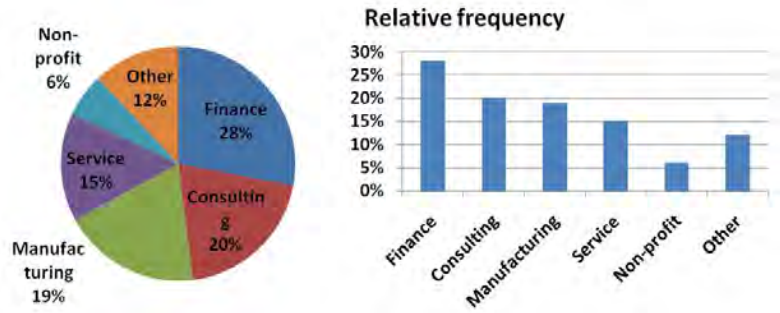

Figure 3: The pie chart on the left and bar chart on the right.

Question 6.2

(a) *Construct both pie and bar charts for Question 5.1(a).*

(b) *Construct a bar chart for Question 5.1(b).*

(c) *Construct a bar chart for Question 6.1(c).*

(d) *You are given the data in Table 6 reflecting the number of people in a study for each investment option. Construct a bar chart to display the data.*

Investment	No. of people
Mutual Fund	357
Saving Account	506
Fixed-term Deposit	158
Individual Stocks	347
Bonds	86
Real Estate	169
Other	41

Table 6: The frequency table for investment options.

(e) *A relative frequency for a given outcome from a set of data points is the fraction or percentage of this outcome occurring and it is equal to the ratio between the frequency for this outcome and the total number of data points in the data set. Construct a relative frequency table as well as a bar chart to display the data in Question 6.2(d). What do you observe from comparing the bar charts for this question and Question 6.2(d)?*

6.3 Line and scatter-plot charts

A line chart is a two-dimensional chart showing time on the horizontal axis and the variable of interest on the vertical axis.

The data in Table 7 represent expenditures on advertising over the period of 1999 to 2009 by an insurance company. The line chart for the above example is shown in Figure 4.

Year	Advertising	Year	Advertising
1999	£21,500	2005	£28,700
2000	24,600	2006	32,500
2001	26,300	2007	34,300
2002	29,800	2008	35,200
2003	33,400	2009	37,500
2004	32,100		

Table 7: The annual advertising expenses.

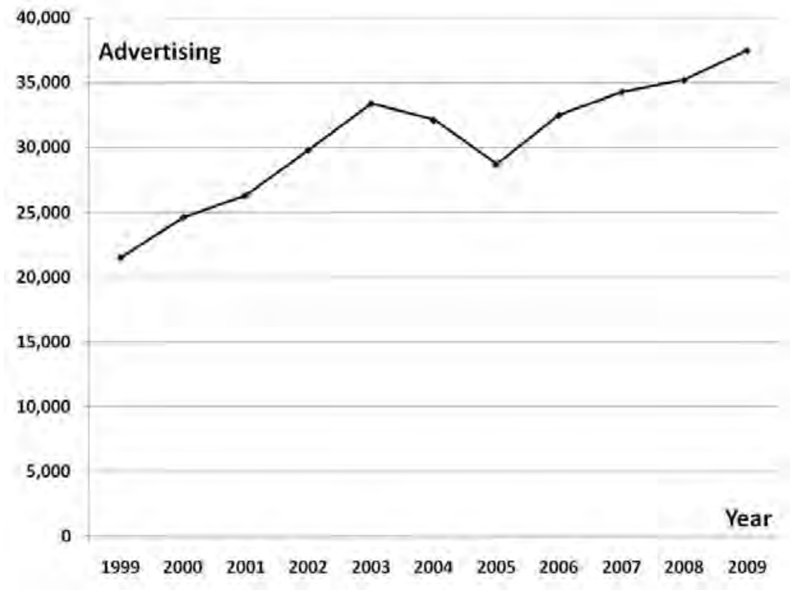

Figure 4: The line chart for annual advertising expenses.

A scatter diagram (or plot or graph) is a two-dimensional graph of plotted points in which the vertical axis represents values of one variable and the horizontal axis represents values of the other. Each plotted point has coordinates whose values are obtained from the respective variables.

An airline company recently selected a sample of 12 IT people. They compared the employees' performance rating, based on a 100-point scale, and the number of overtime hours the employee has worked in the last quarter. The data are shown in Table 8. The scatter plot for the above example is shown in Figure 5.

Employee	Rating	Overtime hours
1	88	51
2	68	32
3	91	99
4	87	94
5	81	72
6	60	24
7	42	26
8	95	84
9	90	79
10	51	34
11	80	66
12	72	48

Table 8: Employees' ratings and overtime hours.

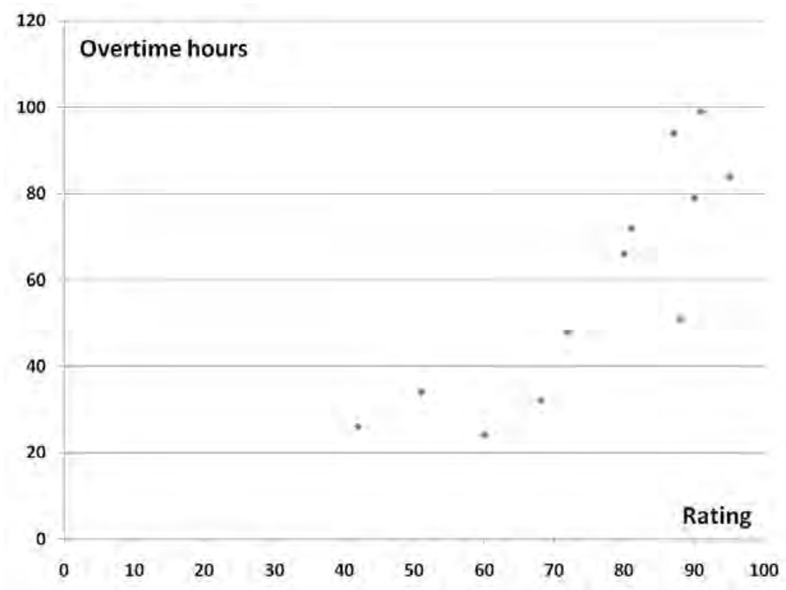

Figure 5: The scatter plot for ratings and overtime hours of employees.

Question 6.3

(a) *Construct a line chart for a time series representing the monthly average temperature in centigrade degree in Cambridge in the last year. The data collected were: 12.2, 10.5, 14.6, 15.7, 16.2, 16.3, 17.4, 18.5, 16.1, 12.2, 11.4, and 10.8.*

(b) *A trucking company carries out major repairs for delivery trucks. The number of repairs recorded in the last 10 years are: 206, 212, 235, 170, 255, 222, 156, 206, 200, and 150. Construct a line chart for the above time series.*

(c) *You are given the data in Table 9 for variables x and y: Plot these variables in scatter plot*

x	y
20	16.4
17	12.5
24	19.5
20	17.6
22	22.3
14	10.6
18	12.1
19	14.6

Table 9: Values for variables x and y.

format.

(d) *The GDP growth in percentage and health spending in percentage of GDP for each of eight countries in year 2006 are presented in Table 10. Construct a scatter plot for the GDP*

Country	GDP growth	Health spending
1	3.40	14.20
2	2.75	10.40
3	3.00	9.80
4	2.60	6.80
5	3.85	8.60
6	2.60	7.25
7	-2.65	7.40
8	1.20	7.70

Table 10: GDP growth in percentage and health spending for eight countries.

growth and the health spending.

(e) *A sales center compares the daily operating cost and the daily sales. A sample of 10 paired observations is listed in Table 11. Construct a scatter plot to show the relationship between the daily sales volume and operating cost.*

7 Answers to Questions

Question 3.1(a): 0.5, 50%.

Day	Sales	Costs
1	£4,220	£300
2	4,200	390
3	4,100	400
4	3,800	350
5	4,300	320
6	3,950	310
7	4,380	405
8	5,200	440
9	6,100	450
10	3,800	360

Table 11: Sales and costs in a sales center.

Question 3.1(b): 80%, $\frac{4}{5}$.

Question 3.1(c): $\frac{2}{5}$, 0.4.

Question 3.1(d): £200.

Question 3.1(e): 33.3%.

Question 3.2(a): 6,561, 9.

Question 3.2(b): 8,000.

Question 3.2(c): 1/160.

Question 3.2(d): £1,610.51.

Question 3.2(e): £1,366,027.

Question 3.3(a): -0.2, 1.5.

Question 3.3(b): 2.0.

Question 3.3(c): 58.

Question 3.3(d): 8, £37,000

Question 3.3(e): £654,000.

Question 3.4(a): 15.

Question 3.4(b): 17.

Question 3.4(c): $y = 8, z = 3, u = 3, v = 8$.

Question 3.4(d): $y = 50 + 200x$.

Question 3.4(e): $y = 2.5 + 0.8x$, where x is the miles traveled and y is the total amount of payment.

Question 3.5(a): -4.5, 4.5.

Question 3.5(b): 3.

Question 3.5(c): 100.

Question 3.5(d): 5, price = (20 - demand)/3.

Question 3.5(e): $y = 10 + 1.5x$, where x is the number of sandwiches to order and y is the total amount of money to pay.

Question 3.6(a): $x = 10$, $y = 0$.

Question 3.6(b): $x = 45/7$, $y = 15/7$.

Question 3.6(c): $a = -27/33$, $b = 7/11$.

Question 3.6(d): $s = -25/7$, $t = 76/7$.

Question 3.6(e): $p = 5$, $d = 50$.

Question 3.7(a): $x \geq 7/2$.

Question 3.7(b): $x \geq -27/26$.

Question 3.7(c): Yes.

Question 3.7(d): No, the latter inequalities is equivalent to $80 \leq x \leq 130$.

Question 3.7(e): 20 (based on equation $y = 10 + 1.5x$).

Question 4.1(a): 1/2.

Question 4.1(b): Likely.

Question 4.1(c): 54.

Question 4.1(d): The same probability.

Question 4.1(e): The Dow Jones index rises by 20%.

Question 4.2(a): 0.08.

Question 4.2(b): 1/36.

Question 4.2(c): 0.25.

Question 4.2(d): 0.45.

Question 4.2(e): 0.0256.

Question 4.3(a): 50%.

Question 4.3(b): The same probability.

Question 4.3(c): 54, 4/54.

Question 4.3(d): $3/36 = 1/12$.

Question 4.3(e): 0.24.

Question 5.1(a): 21.5, 21.5, 21 and 22.

Question 5.1(b): 2.1, 2, 4.

Question 5.1(c): £145,000, £135,000, £100,000.

Question 5.1(d): 18.83333, 19, 20.

Question 5.1(e): 21.5, 22.0, 22.0.

Question 5.2(a): 1.024695, 1.05.

Question 5.2(b): 1.513275, 2.29.

Question 5.2(c): 58.30952, 3400.

Question 5.2(d): 1.067187, 1.138889.

Question 5.2(e): 0.689, 0.475.

Question 6.1(a): See Table 12.

Value	Frequency (f)		Value	Cumulative frequency
20	2		≤ 20	2
21	3		≤ 21	5
22	3		≤ 22	8
23	2		≤ 23	10

Table 12: Frequency and cumulative frequency tables for Question 6.1(a).

Value	Frequency (f)		Value	Cumulative frequency
0	2		≤ 0	2
1	2		≤ 1	4
2	2		≤ 2	6
3	1		≤ 3	7
4	3		≤ 4	10

Table 13: Frequency and cumulative frequency tables for Question 6.1(b).

No. of nonexecutives	Frequency (f)		No. of nonexecutives	Cumulative frequency
4	5		≤ 4	5
5	7		≤ 5	12
6	11		≤ 6	23
7	9		≤ 7	32
8	4		≤ 8	36

Table 14: Frequency and cumulative frequency tables for Question 6.1(c).

Question 6.1(b): See Table 13.

Question 6.1(c): See Table 14.

Question 6.1(d): See Table 15.

Return (x)	Frequency (f)		Return (x)	Cumulative frequency
< 150	5		< 150	5
$\geq 150, < 200$	13		< 200	18
$\geq 200, < 250$	10		< 250	28
$\geq 250, < 300$	6		< 300	34
$\geq 300, < 350$	1		< 350	35
≥ 400	1		≥ 350	36

Table 15: Frequency and cumulative frequency tables for Question 6.1(d).

Question 6.1(e): See Table 16.

Question 6.2(a): See Figure 6.

Question 6.2(b): See Figure 6.

Question 6.2(c): See Figure 6.

Question 6.2(d): See Figure 7.

Question 6.2(e): See the bar chart for relative frequencies in Figure 7 and the relative frequency table in Table 17. Clearly, the bar charts for the frequency and relative frequencies have the same shape.

Time	Frequency (f)		Time	Cumulative frequency
20.5	4		≤ 20.5	4
21.0	5		≤ 21.0	9
22.0	9		≤ 22.0	18
22.5	2		≤ 22.0	20

Table 16: Frequency and cumulative frequency tables for Question 6.1(e).

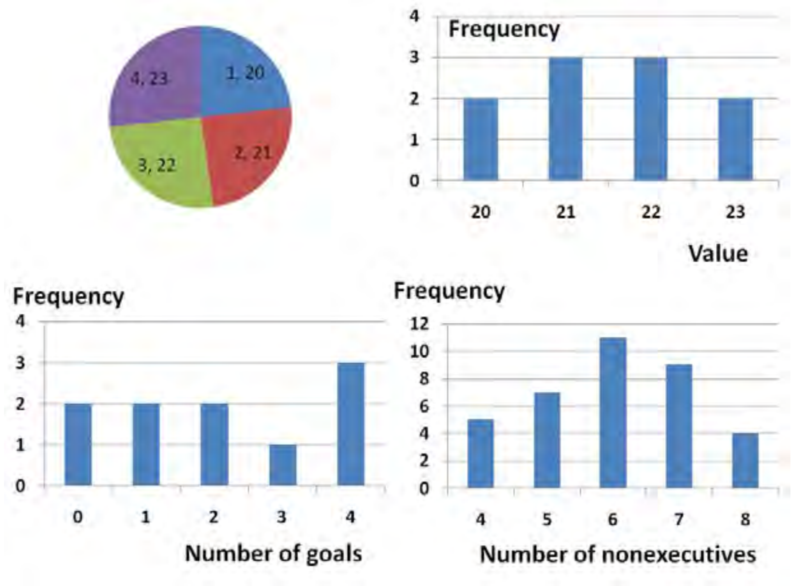

Figure 6: The pie and bar charts: top left and top right for Question 6.2(a), bottom left for Question 6.2(b), and bottom right for Question 6.2(c).

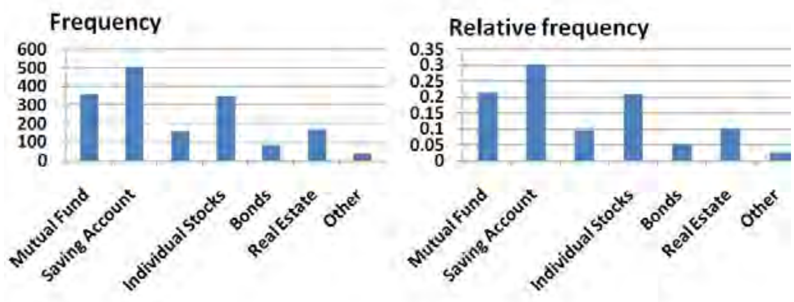

Figure 7: Bar charts for Question 6.2(d) and Question 6.2(e).

Investment	Relative frequency
Mutual Fund	0.214543269
Saving Account	0.304086538
Fixed-term Deposit	0.094951923
Individual Stocks	0.208533654
Bonds	0.051682692
Real Estate	0.1015625
Other	0.024639423

Table 17: The relative frequency table for investment options.

Question 6.3(a): See Figure 8.

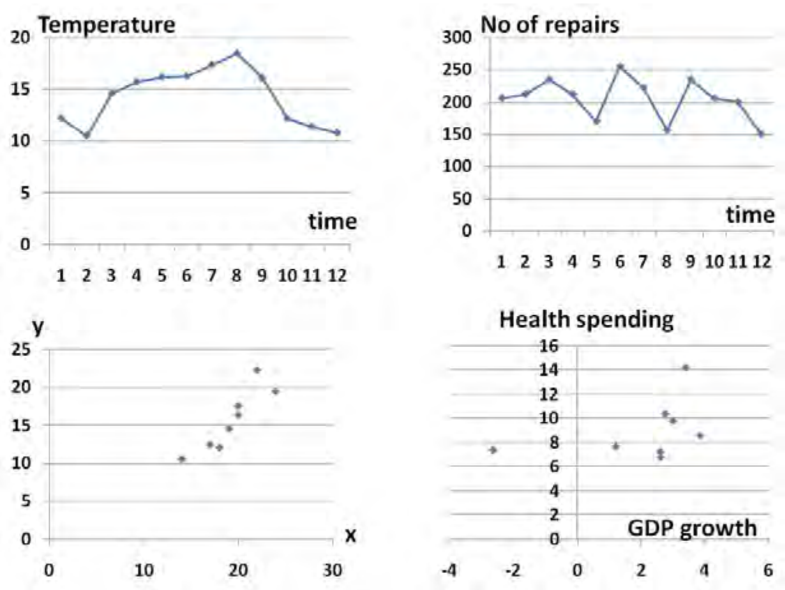

Figure 8: Line charts and scatter plots: top left for Question 6.3(a), top right for Question6.3(b), bottom left for Question 6.3(c), and bottom right for Question 6.3(d).

Question 6.3(b): See Figure 8.

Question 6.3(c): See Figure 8.

Question 6.3(d): See Figure 8.

Question 6.3(e): See Figure 9.

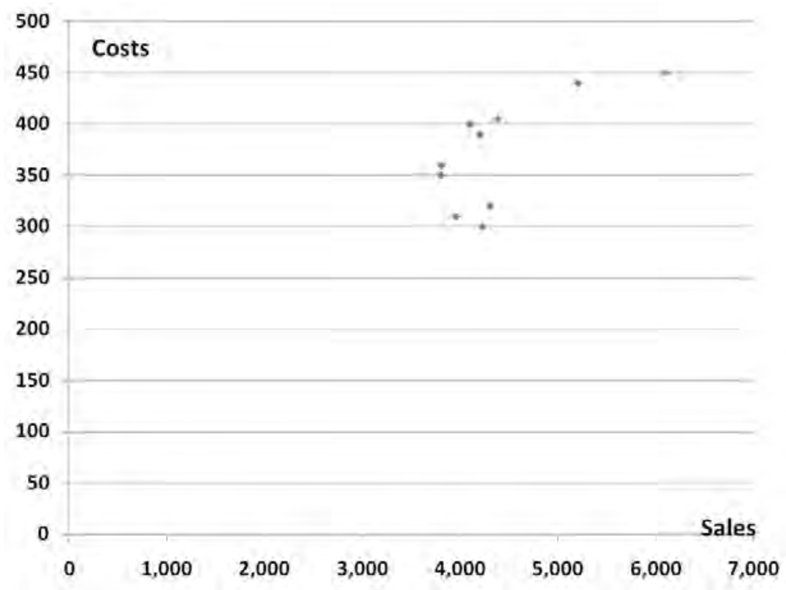

Figure 9: The scatter plot for Question 6.3(e).

References

[1] C. Morris, *Essential Maths for Business and Management*, Palgrave MacMillian, New York, 2007.

[2] C. Morris, *Quantitative Approaches in Business Studies*, inancial Times/Prentice Hall, New York, 2000 or 2003 or 2008. In edition 5 of the book: Review Chapter 1 (basic numeracy), pages 133-141 of Chapter 8 (elementary probability), pages 56-77 of Chapter 5 (data presentation). The libary catelogue of the book at Judge Business School Libary is HD30.215.M67 2000.