Reconsidering long-term risk quantification methods when routine VaR models fail to reflect economic cost of risk.

Executive Summary

The essay discusses issues and challenges of long-term risk measurement from the context of Economic Capital quantification for market risk. Economic Capital reflects counter -cyclical long-term view on risk with very high confidence level implied by target credit rating and measured with well-known and popular Value-at-Risk (VaR) concept. The framework is widely used by financial institutions (FI) for internal capital adequacy purposes (to reflect economic, not regulatory cost of risk), efficient capital allocation and risk adjusted performance measurement of business lines.

Typically, market risk is assumed to be liquid and therefore short-term VaR methods used to reflect current market volatility and tactically manage positions within risk limits. While it is suitable for market risk steering this approach is inappropriate for Economic Capital quantification. But instead of rethinking VaR modelling approach FI often adjust existing set of short-term VaR models for Economic Capital specifics (e.g. long-term horizon and high confidence level). Such adjustments could be appropriate only under strict set of assumptions that often does no hold and hence lead to misleading results.

As one way to address the risk measurement issues arising mainly from popular "square root of time" VaR scaling procedure[1], the historical bootstrapping technique was described and compared to most popular range of practises for market risk modelling.

The long-term risk measurement is highly relevant for banks and other financial institutions (e.g. insurance, asset management companies) willing or obliged to accept long-term market risk due to specifics of underlying business model. Therefore accurate and sound modelling approaches are vital to ensure confidence in risk measurement and empower efficient application of risk models to ongoing business of FI.

I. Introduction

In a broad sense, Economic capital (ECap) is defined by the Basel Committee as the methods or practices that allow banks to consistently assess risk and attribute capital to cover the economic effects of risk-taking activities[2]. The key goal of organisation-wise aggregated Economic Capital is to inform shareholders and management on the level of risk in the business and whether it is in line with risk appetite, i.e. ability and willingness to take risk.

Risk appetite and thus ECap should be defined on business planning and budgeting time horizon that is typically 1 year.

The focus of the essay is challenges of Ecap quantification for particular risk type – market risk. While the long time horizon for risk quantification naturally not a big issue for other, mostly non-tradable, Pillar I risks[3] (credit risk and operational risk), one could find it difficult to address long-term horizon in case of market risk.

Market risk is used to be considered liquid and often calculated on 1 day time horizon (10 days for regulatory reporting purposes). Though after financial crisis Basel Committee commented that "While individual banks might judge that they can all promptly exit or hedge their risk exposures without

---

[1] Multiplication of 1 day VaR on √T, where T – number of business days in required time horizon

[2] Basel Committee on Banking Supervision, 2009. "Range of practices and issues in economic capital frameworks", available on www.bis.org. [Accessed 25 February 2016]

[3] On Pillar I risks definition see Basel Committee on Banking Supervision, 2006. "International Convergence of Capital Measurement and Capital Standards", available on www.bis.org. [Accessed 25 February 2016]

affecting market prices, the market is likely to turn rapidly illiquid in times of banking system stress if the banking system as a whole holds similar exposures" and introduced Fundamental Review of the Trading Book standards[4] that extends time horizon for market risk estimates. But liquidity is not the only one reason that could lead to higher time horizon for market risk – time horizon also should reflect willingness and ability of FI to reduce or hedge market risk, and this is highly business and exposure specific.

Despite fundamental differences in market risk steering (cyclical short-term view) and Ecap goals (counter-cyclical long-term considering specifics of risk positions) application of popular and well-recognized methods for Daily VaR calculation are often applied for Ecap measurement through the number of adjustments (e.g. for time horizon and confidence level). But such method could be justified and applied under specific assumptions that rarely hold[5].

The discussion of an issue is developed from the authors' project experience of Economic Capital models development for large national development bank[6] which required advanced understanding of risk taken and relevant capitalization requirements from government. Not regulated by central bank, bank planned to develop economically justifiable methods to measure risk taking into account specifics of the exposures.

In 2008 crisis the development bank was requested to slow down the free fall of domestic equity market (which was considered as crisis panic while companies' fundamentals have not changed that dramatically) by investing in largest companies of the index which financial health was vital for the economy. Thus, probably not typical for development banks, the bank ended up with massive concentrated equity risk. Hedging of the risk was expensive and not feasible.

Activities on exposure management required discussion with government, thus extending potential time horizon for risk measurement. The bank had set of daily VaR models for market risk measurement and common practise to adjust for ECap purposes led to risk estimate almost equal entire exposure level thus questioning appropriateness of such adjustments.

The reminder of the essay is organized as follows: in Section 2 Economic Capital framework development and application is reviewed. Section 3 describes key conceptual differences of ECap and market risk steering models and provides overview of market practise to tackle the issues. Section 4 elaborates on VaR reparametrization approach for ECap measurements and corresponding flaws and Section 5 introduce historical bootstrapping for ECap modelling and compares results of different model specification from Section 4. Finally, Section 6 concludes and lists important issues for further consideration.

II. Overview of Economic Capital framework development

More technically as risk metric Economic Capital is defined as amount of capital required to absorb unexpected losses from variety of risk types at defined time horizon at defined confidence level and

---

[4] Basel Committee on Banking Supervision, 2016. "Minimum capital requirements for market risk", available on www.bis.org. [Accessed 25 February 2016]

[5] See Diebold, F.X. Hickman, A., Inoue, A. and Schuermann, T., 1998. "Converting 1-Day Volatility to h-Day Volatility: Scaling by Root-h is Worse than You Think,". Wharton Financial Institutions Center, Working Paper 97-34.

[6] National development banks have different business model and goals from investment and commercial banks. Such banks typically are not regulated be central banks and serve governments as instrument to invest in specific projects (e.g. projects not attractive from risk/return profile to profit-oriented banks), ensure and protect economic growth. National development banks capitalised by governments and their creditworthiness usually considered to be of sovereign grade.

measured trough VaR approach[7]. In banking area Economic Capital is used for internal risk measurement and management processes and subject to supervisory review under Pillar II (supervisory review process) of the Basel II Framework.

Figure 1 presents high-level description of Economic Capital framework building process.

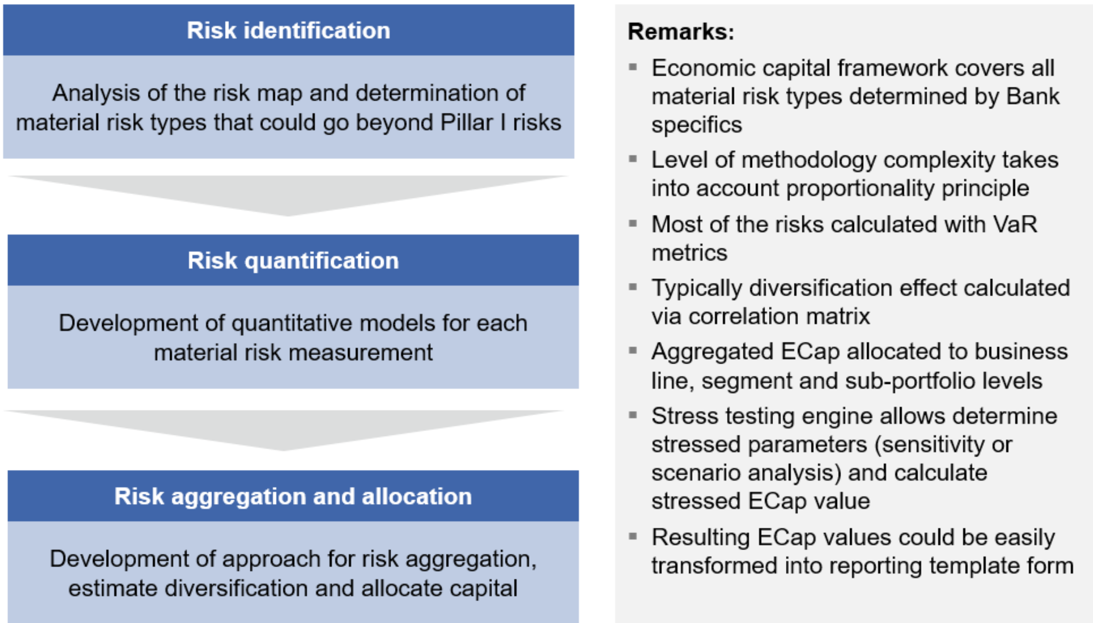


Figure 1. Economic Capital framework development

On the aggregate level Economic Capital is compared to Risk Taking Capacity. For all risk taken FI should ensure that it has enough financial resources to cover unexpected losses from business it is willing to run (Figure 2).

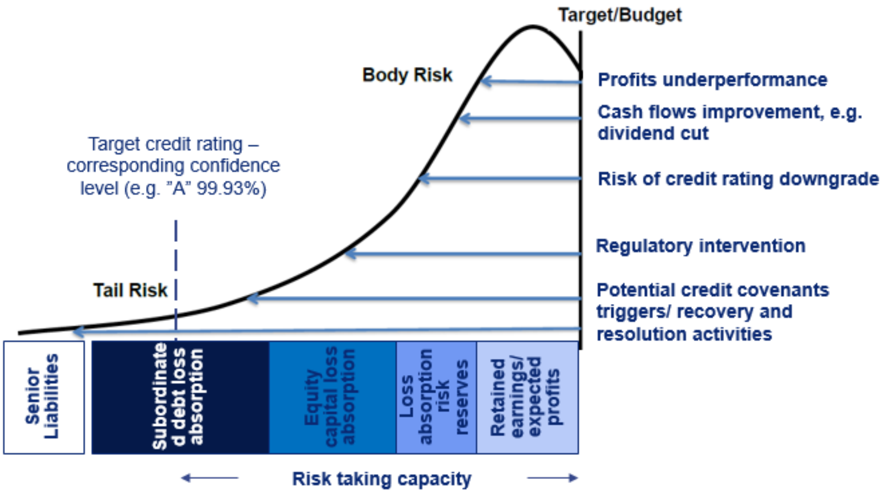


Figure 2. Illustrative P/L distribution and Risk Taking Capacity

Economic Capital is also important input for Risk Adjusted Return on Capital (RAROC) metric. RAROC is defined as expected profit divided by Economic Capital and widely used to compare risk-adjusted profitability of different business lines and inform capital allocation decisions. As example

[7] For formal definition of VaR and its application see Jorion, Philippe (2006). "Value at Risk: The New Benchmark for Managing Financial Risk (3rd ed.)". McGraw-Hill. ISBN 978-0-07-146495-6.

high profits associated with high risk could reflect same utility comparing to low profits associated with low risk from risk-adjusted point of view.

Successful incorporation of ECap framework into business activities requires full understanding and acceptance of the models by their final users, not developers. Hence target approach should correctly reflect underlying risk factors movements and at the same time be transparent, easy to implement and to manage. Moreover, ECap models should reflect exposure specifics and ability to manage that exposure.

III. Key conceptual differences between VaR steering goals and Economic Capital

As mentioned earlier market risk steering and ECap have different goals and hence measurement focus and that should motivate different underlying modelling approach. Key differences are summarised in the Table 1.

Table 1. Overview of market practise for Economic Capital measurement methodology and parametrization

Topic	Market Risk steering (Daily VaR)	Economic capital for market risk
Cyclicity	Daily market risk management typically use the most relevant market information for efficient current market risk steering (e.g. limit setting) and use the most precise short-term forecasts of market behavior	Economic Capital framework should consider a high severity of market environment incorporating the history of significant financial stresses, where behavior of market risk factors could differ greatly from current short-term forecasts
Underlying data	1-3 years of the latest data on market risk factors. Could include 1 year of significant period of stress for regulatory purposes (stressed VaR)	The large data sample of latest data that that covers at least 12 months of significant financial stress (e.g., 8-10 years) or market risk model calibrated to the period of financial stress
Confidence level	Typically 95/99%. 99% for regulatory purposes	Typically confidence level is chosen based on target credit rating of financial institution (e.g. 99.99% for AAA)
Time horizon	1 day. Scaling using "square root of time" rule to 10 days for regulatory purposes	Typically 1 year, as for Credit Risk and Operational Risk. But taking into account that financial instruments are much more liquid, time horizon could be shorten, but still no less than defined conservative floor (e.g. 3 months). Time horizon also could take into account specifics of positions, e.g. how quickly the decision to reduce/hedge the position could be agreed within financial institution.
Methodology	Common to put more emphasis to the latest short/medium term market risk information by using corresponding volatility weighting (e.g. EWMA)	Weights are not used because historical extreme events should remain in the tail of the distribution

Based on authors' project experience and publically available reports, most of financial institutions that develop ECap framework recognize conceptual differences but respond with leveraging existing daily VaR modelling capabilities to adjust. Table 2 contain high-level market practise overview of ECap modelling for market risk.

Table 2. Overview of market practise.

Topic	Summary for Large European Banks	Large German commercial bank	Large European development bank
Measurement methodology	Often banks use Pillar 1 market risk model methodology for ECap quantification and do not have separate market risk model for ECap purposes. Most popular approach is Historical simulation	Scaling Stressed VaR, that generated by Monte Carlo simulation of various distributions depending on risk class	Historical simulation with "square root of time" scaling to higher time horizon
Confidence Level	Typically corresponds to target credit rating (AAA ~ 99.99%, AA+ ~ 99.98% etc.)	99.98%	99.99%
Time horizon	Either 1 year for all asset or various liquidity horizon with conservative floor and additional penalties depending on trading desks	Various liquidity horizons with a floor of 3 months for all asset classes	▪ 2 months for IRR for liquid instruments and FX for liquid currencies ▪ 12 months for credit spread, illiquid IRR and FX
Underlying historical data	Typically Banks include stress data to reduce pro-cyclicality	Stress window of financial crisis (June 2008-June 2009)	Large window of through-the-cycle data history, which includes a stress period

Source: Pillar III and annual reports of European banks, project experience

IV. How FI respond to the ECap specifics with daily VaR models adjustments?

So the main specifics of Economic Capital are very high confidence level, long-term horizon and emphasis on counter-cyclicity. Table 3 below illustrates parametrization issues based on very popular historical VaR model as starting point.

Table 3. Short-term model parametrization

Topic	Possible parametrization solution	Potential issues for ECap
Underlying data and pro-cyclicity	▪ Analogues to post-crisis Basel 2.5 framework VaR could be calibrated to the period of significant financial stress ▪ The volatility weighting is not appropriate	▪ Given high confidence level for ECap 252 returns observation from 1 year of stress is not enough to derive accurate high quantile from underlying historical distribution
High confidence level	▪ Given data constrains it is not always possible to derive accurate explicit quantile from historical distribution, though we could approximate higher quantile by multiplying VaR on the ratio of standard normal distribution quantiles with different confidence levels or add more data to the sample	▪ Underestimation of tail risk from quantile scaling based on standard normal distribution ▪ Increasing of daily returns data sample could dilute desired calibration of the model to stress period
Long-term time horizon	▪ The most popular technique is to apply "square root of time" rule to scale 1-day VaR to whatever time horizon ▪ Could use larger windows to calculate historical returns (e.g. 2 weeks, 1 month)	▪ "Square root of time" rule tend to overestimate VaR when applied to long time horizons and valid approach only under strict assumptions on data generation process

		▪ Increasing of returns window could not be feasible as reduce amount of data points for analysis in case of non-overlapping observations, and leads to autocorrelation effects in case of moving window, i.e. overlapping observations

Figure 3 below illustrates the effects of listed adjustments for initial 10% VaR of returns.

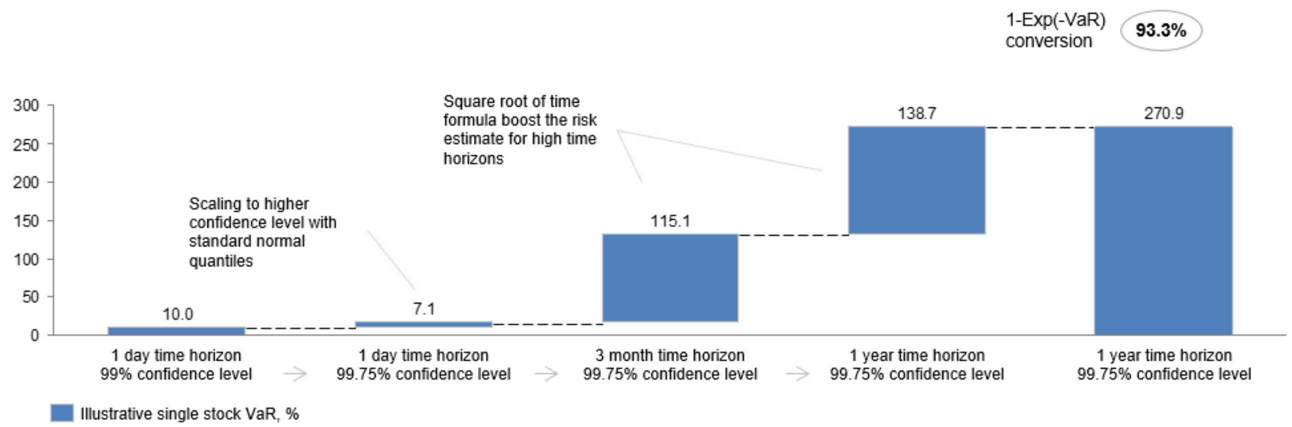


Figure 3. Effects of reparametrization of VaR for ECap purposes

As could be seen from the graph, "square root of time" scaling rule has a greatest effect on VaR metric. If we start with initial VaR of 10% (that could be easily seen on emerging markets) 1-year VaR will well exceed 100% and direct multiplying to position value does not make sense as we can not lose more than the entire position value, at least from market risk perspective. Taking into account log-returns of asset prices the VaR equals 93.3% which could be treated as bankruptcy scenario. In that case one could find it difficult to argue that past volatility of share price led us to conclusion that company could go bust and we should hold market risk capital equal to entire position value, and in case of market index almost impossible.

Clearly such reparametrization provides very conservative estimate of market risk. With "square root of time" rule for VaR scaling we essentially scale high quantile of loses, assuming there could be adverse loses every day on chosen time horizon. To overcome the issues it is important to consider alternative methods to build distributions for long-term without applying "square root of time" rule.

V. Overcoming modeling issues and business acceptance challenges

Issues of "square root of time" rule application to convert Daily VaR to long-term horizons is not new and have been discussed by a number of academics and practitioners[8]. Diebold and Inoue[9] argued that the rule is misleading and inappropriate for empirical returns data and overestimates risk on long horizon. The research by Kaufmann[10] examinees the number of different returns generation processes to test "square root of time" rule and concludes that results are highly model dependant and for low confidence levels (<99%) and relatively short period scaling (e.g. 10 days) the rule is a good

[8] See p. 3, p. 26. Basel Committee on Banking Supervision, 2009. "Range of practices and issues in economic capital frameworks", available on www.bis.org. [Accessed 25 February 2016]

[9] Diebold, F.X. Hickman, A., Inoue, A. and Schuermann, T., 1998. "Converting 1-Day Volatility to h-Day Volatility: Scaling by Root-h is Worse than You Think". Wharton Financial Institutions Center, Working Paper 97-34

[10] Kaufmann, R. and Patie, P., 2003. "Strategic Long-Term Financial Risks: The One Dimensional Case". RiskLab Report, ETH Zurich.

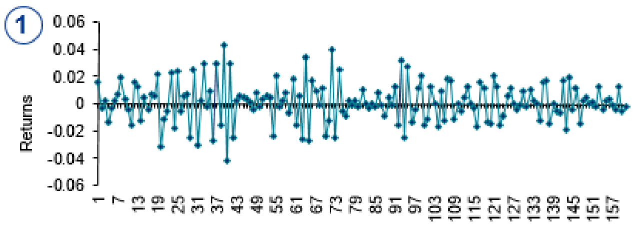
approximation. Though Kaufmann mentioned importance of trend absence in the data and additionally refers to Brummelhuis[11] who showed that the rule is strongly violated as confidence level get closer to one, which is Economic Capital specification case.

A number of analytical adjustments to the rule were proposed (e.g. Drost and Nijman[12]) but they are highly model-dependent and require careful and non-trivial investigation of return generation process to be applied, such kind of approaches could be hard to communicate within financial institution. Thus on average "square root of time" rule could be safe to apply for general case and tendency of the rule to overestimate risk is not critical from solvency point of view, though will face high resistance from business acceptance perspective.

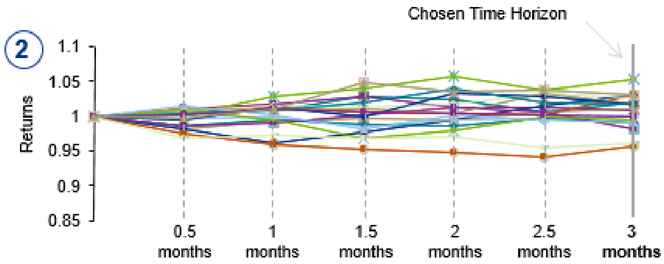
One of the possible method to overcome limitations of short-term models reparametrization and fully take in account ECap specifics is to apply historical bootstrapping to simulate distributions on desired time horizon from historical returns.

Historical bootstrapping algorithm for VaR estimate consists of three key steps:

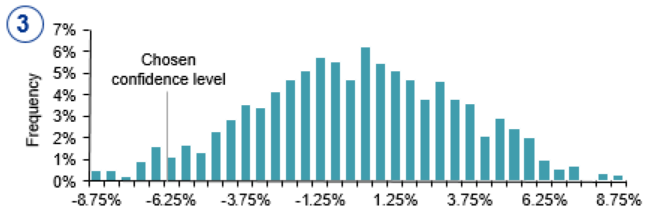
1. Construct the dataset of risk factors changes based on the chosen time interval (e.g., 2-weeks returns on the stock – **ri,** i=1,…n). The approach assumes that all historical risk factor changes are independent and could occur in future with equal probability.



2. For chosen time horizon calculate the amount of risk factor from the sample required to build the path for desired time horizon, e.g. for 3 –months time horizon we need 3 month / 2 weeks = 6 risk factor samples. Build risk factors path – m historical risk factors changes (m = 6 for our example) should be randomly selected from the sample and multiplied (1+r1)(1+r2)….(1+rm). Then build risk factors path N times (e.g. 500 000).



3. Based on the calculated risk factor changes build the corresponding distribution and derive the quantile for desired confidence level.



[11] R. Brummelhuis and R. Kaufmann, 2003. "Estimating 10-day value-at-risk," Working Paper, ETH Zurich

[12] F.C. Drost and T.E. Nijman, 1993 "Temporal aggregation of GARCH processes," Econometrica, vol. 61, pp. 909–927.

The advantages of historical bootstrapping approach are:

1. Transparent, easy to understand algorithm without underlying distribution assumptions, hence could be applied in general case and easily communicated to Economic Capital model users;
2. Could efficiently combine 1 year of stress period and 1 year of recent market data to introduce counter-cyclicity taking recent market volatility into account;
3. The sample could be created from 2-weeks non-overlapping returns mitigating autocorrelation effects in time series;
4. Explicit high quantile calculation from simulated distribution. Square root of time scaling and/or confidence level scaling are not required.

Example of bootstrapping model specification could be found in Appendix 1. Figure 4 illustrates comparison of different market risk models with parametrization relevant to Economic Capital purposes. On Figure 4 “1-day stress data” refers to the daily return during 2008 crisis period, “all data” refers to 6 years of returns including 2008 crisis period. Varying liquidity horizons were used for all models (3 months for liquid instruments, 1 year for illiquid instruments, see Appendix 1).

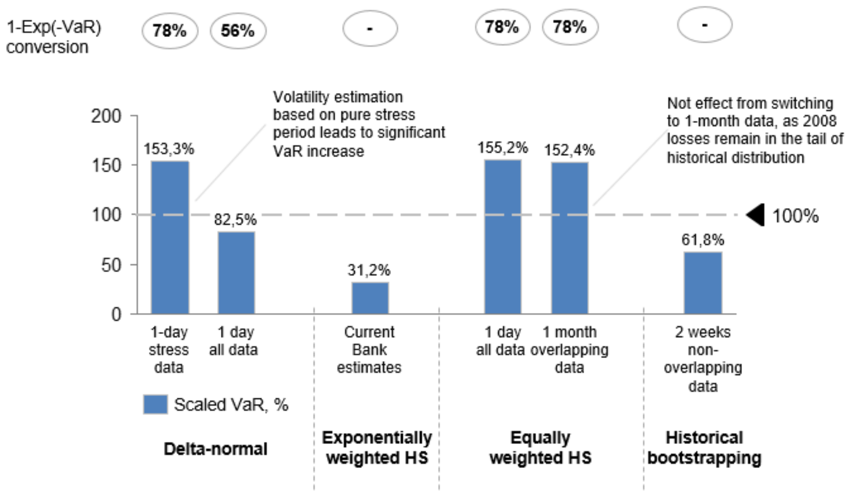


Figure 4. Comparison of different VaR models with elements of ECap parametrization

The flaws of Daily VaR adjustment for ECap purposes could be easily seen:

- Delta-Normal model is well known for tail risk underestimation as assumes normal distributions of returns. Flaw of the approach could be seen from inconsistency of results for two different data sets;
- Calibrated to calm market period on the date of estimation exponentially weighted historical simulation model underestimates tail risk as no stress data reflected in distribution;
- Historical VaR without data weighting tends to overestimate risk even when 1 year time horizon reduced to 3 month for liquid instruments. Also it is well-known that daily returns and 1-month overlapping returns autocorrelation violate i.i.d. assumption critical to “square root of time” rule application.

Historical bootstrapping provides adequate results without any distribution and scaling assumptions, flexible in taking into account specifics of the underlying exposures and could be applied in general case without complicated examinations of data generation process.

VI. Furtherer long-term risk measurement developments and issues to address

Economic Capital is a vital element for promoting advanced risk management in FI, e.g risk appetite framework within organization, risk adjusted performance measurement and optimal capital allocation. The value of the concepts is recognized and put on a target operating models for many financial institutions. Moreover, consideration of better long-term risk measurement is vital for asset portfolios where the value is subject to movements in market risk factors over entire holding period.

In the essay above some major problems of long-term market risk measurement were discussed and relatively simple algorithm was proposed to tackle them and encourage better business acceptance and understanding of the framework. The robust parametric representation is obvious way to move forward. Below are other important points to be considered for further development:

Addressing portfolio composition fluctuations. Potential loses in market risk portfolio depends not only of risk factors volatility but also on the position vector that describes the portfolio. Fixed portfolio composition often assumed in risk modelling. And while over a short period of time it could be a safe assumption, over the long time it is unlikely the case. FI management, traders and portfolio managers could react to particular price path with rebalancing/reducing the positions over the time horizon and it’s not reflected in the risk metric. Supplementing VaR algorithm with position vector model that could reflect stop loss policies and other tactical and strategic considerations will not just provide better risk understanding, but also would allow to align time horizons for all instruments and mitigate risk aggregation issues for market risk and for wider aggregation with other risk types.

Stress testing and scenario analysis. While ECap/Long-term VaR could provide base long-term indication of risk, scenario analysis could be seen as instrument to analyse any market risk concerns on more frequent basis and inform decisions to create additional capital buffers for specific events. Top-down and macro stress scenarios were pushed forward by regulators in post-crisis period and generally FI encouraged to develop stress testing expertise. Such techniques should be also regularly used for unique specifics of individual portfolios and work in tandem with long-term risk metrics.

Regulatory incentive to invest in internal risk projects. The overall post crisis risk compliance vector is going towards more granular rules based regulation as opposite to principle based one. The number of recent initiatives that could be summarized as Basel 4 put more weight on less risk sensitive conservative standardised approaches and make internal based approaches more linked to standardised floors. The risk compliance costs are so high that it potentially could leave FI with no resources to invest in internal projects. And such projects essentially address FI unique expertise and business model and vital for better risk management within organization.

Appendix 1. Possible ECap model specification. Equity risk example.

1. Measurement granularity and data requirements

The Economic Capital is calculated on portfolio, subportfolio and financial instruments level. Subportfolio could be defined from business perspective needs for risk analysis. All financial instruments should be included for ECap calculation purposes.
For each instrument time series of closed prices are required both for1 year period of financial stress and 1 year of recent data. In case there is not enough data for financial instrument reference instrument could be used instead. Reference instrument should broadly reflect the same level of market risk.
2-weeks non-overlapping returns are used as input data for historical bootstrapping algorithm. Starting day of calculating 2-weeks returns should be derived from closest to median volatility of 10 different options (window moved by 0 days, 1 day, 2 day etc.).

2. Model parametrization

Confidence level for ECap estimate should imply target credit rating of the institution. Time horizon for measurement defined as:

- 3 months for financial instruments included in the main equity index of the country (liquid positions);
- 1 year for financial instruments not included in the index (illiquid positions).

Number of iterations for historical bootstrapping algorithm should be no less than 500000.

3. Measurement algorithm

1. Based on built data sample elements are chosen randomly to build financial asset price path. Draws are independent and number of draws corresponds to desired time horizon.

$$R_i = (1 + r_{j_1}^1) * (1 + r_{j_2}^2) * ... * (1 + r_{j_m}^m) - 1, \quad i = 1, ..., N$$, where

R_i – risk factor value on chosen time horizon; m – required number of sample elements to build the path; $r_{j_1}^1, ... , r_{j_m}^m$ – randomly chosen sample elements; N - number of iterations.

2. Required distribution is derived based on N financial instrument price paths from previous step. VaR estimates is then derived form the distribution.

$$(\%)VaR_{CL}^i(TH^i) = -\min(0, K_{CL}^{TH_i})$$
$$VaR_{CL}^i(TH^i) = (\%)VaR_{CL}^i(TH^i) * Exp^i, \quad \mathrm{i} = 1, ..., \mathrm{N}$$

$(\%)VaR_{CL}^i(TH^i)$ – relative VaR of financial instrument; $VaR_{CL}^i(TH^i)$ – absolute VaR of financial instrument; N – number of financial instruments included in ECap calculation; CL– confidence level; TH– time horizon; $K_{CL}^{TH_i}$ – empirical quantile derived from historical bootstrapping procedure; Exp^i – current market value of financial instrument.