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Structured Products for Pension Funds

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Abstract. This paper introduces the use of dynamic stochastic optimisation for the design of structured products for pension funds. The design of such products involves econometric modelling, economic scenario generation, generic methods of solving optimization problems and modelling of required risk tolerances. In nearly all the historical backtests using data over roughly the past decade the system described (with transactions costs taken into account) outperformed the benchmark S&P500.

1 Introduction

Defined benefit pension plans and most state schemes are becoming inadequate to cover the gap between the contributions of people while working and their pensions once retired. A long-term minimum guarantee return plan with a variable time-horizon and with the possibility of making variable contributions during the lifetime of the product in addition to the initial contribution is a new investment instrument aimed at attracting investors who are worried about the volatility of financial markets. Although potentially highly profitable for the provider, the design of such instruments is not a trivial task, as it encompasses the need to do long-term forecasting for investment classes and handling a number of stochastic factors together with providing a guarantee. This paper shows that dynamic stochastic optimisation methodology is an ideal technique to solve these kinds of problems.

This paper describes the approach and outcomes of a joint project with a leading firm operating in the European fund management industry to develop a state-of-the-art dynamic asset liability management (ALM) system for pension fund management. The liabilities considered in this paper take the form of guaranteed returns, which we refer to as *quasi-liabilities*.

2 Critical Issues for Pension Fund Management

Asset liability management concerns optimal strategic planning for management of financial resources in stochastic environments, with market, economic and actuarial risks all playing an important role. The task of a pension fund, in particular, is to attempt to guarantee benefit payments to retiring clients by investing part of their current wealth in the financial markets. The responsibility of the pension fund is to hedge the client's risks, while meeting the solvency standards in force in such a way that all guarantees are met.

Below we list below some of the most important issues a pension fund manager has to face in the determination of the optimal asset allocations over the time to product maturity.

Stochastic nature of asset returns
The future asset returns over the life of the product are unknown. It is critical that the portfolio decisions are based on a realistic representation of these returns.

Long investment horizons
The typical investment horizon is very long (30 years). This means that the fund portfolio will have to be rebalanced many times and can make "buy&hold" Markowitz-style portfolio optimisation inefficient. Multi-period techniques are needed to take explicitly into account the on-going rebalancing of the asset-mix.

Risk of under-funding
There is a very important requirement to monitor and manage the probability of under-funding for both individual clients and the fund, that is the probability that the pension fund will not be able to meet its targets without resort to its parent guarantor.

Management constraints
The management of a pension fund is also dictated by a number of solvency requirements which are put in place by the appropriate regulating authorities. These constraints greatly affect the suggested allocation and must always be considered. Moreover, since the fund's portfolio must be actively managed, the markets' bid-ask spreads, taxes and other frictions must also be modelled.

3 Pension Fund Management Through Stochastic Optimization

Most firms currently use *static* portfolio optimisation, such as the Markowitz mean-variance allocation [15], which is short-sighted and when rolled forward can lead to radical portfolio rebalancing unless constrained by the portfolio manager. Although such models have been extended to take account of liabilities in terms of expected solvency (surplus) levels (see e.g. [18]), many difficulties with static models remain. In practice fund allocations are likely to be wealth dependent and face time-varying investment opportunities, path-dependent returns due to cash flows and transactions costs. Hence *all* conditions necessary for a sequence of myopic static model allocations to be dynamically optimal are likely to be violated [20, pp 9-11].

By contrast, the dynamic stochastic programming models incorporated in the system described below automatically hedge current portfolio allocations against future uncertainties over a longer horizon, leading to more robust decisions and previews of possible future problems and benefits. It is this feature and its ability to incorporate different attitudes to risk that make dynamic stochastic optimisation the most natural framework for the effective solution of pension fund ALM problems.

Strategic ALM requires the dynamic formulation of portfolio rebalancing decisions together with appropriate risk management in terms of a *dynamic stochastic optimisation problem*. Decisions under uncertainty require a complex process of future prediction or projection and the simultaneous consideration of a number of alternatives, some of which must be optimal with respect to a given objective or utility function. The problem is that these decisions are only *known* to be optimal or otherwise *after* the realisation of all random factors involved in the decision process. In dynamic stochastic optimisation the unfolding uncertain future is represented by a large number of future scenarios (see e.g [14] and the references therein) and contingent decisions are made in stages according to tree representations of future data and decision processes. The initial – *implementable stage* – decisions are made with respect to all possible variations of the future (in so far as it is possible to predict and generate this future) and are thus hedged within the constraints against all undesirable outcomes. Each particular optimisation problem is formulated for a specific application combining the goals and the constraints reflecting risk/return relationships. The dynamic nature of stochastic optimisation: decisions – observed output – next decisions – etc … allows a choice of strategy which is the best suited for the stated objectives. For example, for pension funds the objective may be a guaranteed return with a low unexpected risk and decisions reviewed every year.

Figure 1 summarizes the processes, models, data and other inputs required to construct a strategic system for dynamic asset liability management with periodic portfolio rebalancing. It should be noted that knowledge of several independent highly technical disciplines is required in addition to professional domain knowledge.

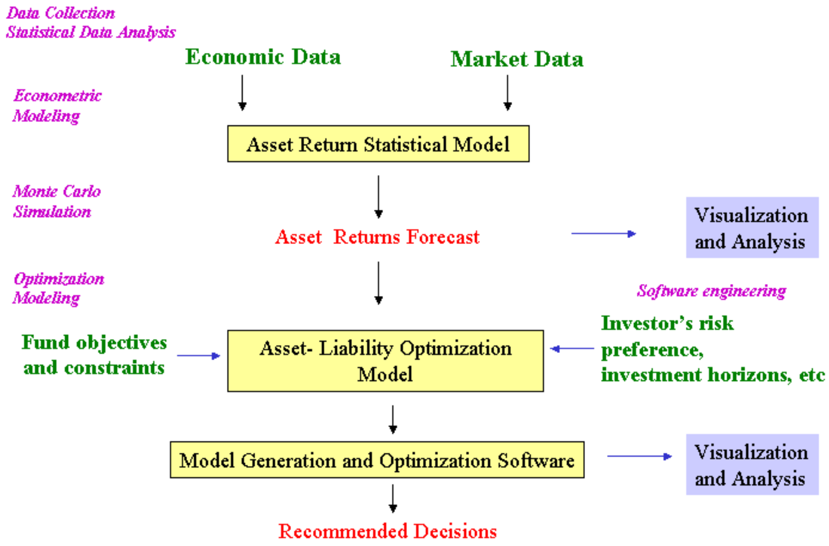

Fig. 1. Strategic financial planning data requirements, models and objectives

Implementing the dynamic stochastic optimisation methodology can be highly non-trivial. The investment fund will naturally want its set of potential investments to be as large as possible. Thus, it will want the option to invest in global assets ranging from relatively low risk, such as cash, to relatively high risk, such as emerging markets equity. The inclusion of such assets greatly increases the complexity and the amount of uncertainty in the problem since it necessitates the modelling to some degree of not only the asset returns, but also of exchange rates and correlations. Further sources of complexity arise from the multi-period nature of the problem and frictions such as market transaction costs and taxes. However, the advance of computing technology and the development of effective algorithms have made dynamic stochastic optimisation problems significantly more tractable. We have applied the *Stochastics*™ optimisation system [8] to solve pension fund management problems with global investments and have shown that it outperforms alternatives by analysing the returns of its recommended portfolio decisions in terms of the appropriate Sharpe ratio [10].

Dynamic stochastic optimization model

We focus here on *strategic asset allocation* which is concerned with allocation across broad asset classes such as equity and bonds of a given country. The canonical pension fund problem is as follows:

> *Given a set of assets, a fixed planning horizon and a set of rebalance dates, find the trading strategy that maximizes utility subject to the constraints.*

Different pension plan instruments are given by alternative utility functions (fund risk tolerances) and the specification of risk management objectives through the constraints.

We consider a discrete time and space setting. It is assumed that the fund operates from the view point of one currency which we call the *home currency*. There are T+1 times (the first T are decision points) indexed by t=1,...,T+1, where T+1 corresponds to the planning *horizon* at which no decisions are made. Uncertainty is represented by a finite set of time evolutions of states of the world, or *scenarios,* denoted by $\omega \in \Omega$. Assets take the form of *equity*, *bonds* and *cash*. Let I denote the set of all assets. Let $x_{it}(\omega)$ denote the amount *held* of asset i between times t and t+1 in state ω, and let $x_{it}^{+}(\omega)/x_{it}^{-}(\omega)$ denote the amount *bought/sold* of asset i at time t in state ω. Let $v_{it}(\omega)$ denote the *return* on asset i between times t-1 and t in state ω, and let $p_{it}(\omega)$ denote the *exchange rate* of asset i at time t in state ω. A *trading strategy* results in a *wealth before rebalancing* of $w_t(\omega)$ for t=2,...,T+1 and $\omega \in \Omega$, and a *wealth after rebalancing* of $W_t(\omega)$ for t=1,…,T and $\omega \in \Omega$.

Subject to the constraint structure, the fund acts by choosing the trading strategy which maximizes the (von Neumann-Morgenstern) *expected utility* of the wealth process which is assumed to take the form

$$E[U(\mathbf{w}_2,...,\mathbf{w}_{T+1})] = \sum_{\omega\in\Omega} p(\omega)\sum_{t=2}^{T+1} u_t(w_t(\omega)), \quad p(\omega) := 1/|\Omega|, \tag{4}$$

where p(ω) is the probability of state ω. *Utility functions* are used in our system to represent the general attitude to risk of the fund's participants over a specified fund horizon. For example, short horizon funds which are likely to attract relatively risk averse participants would typically use a different utility than very long horizon funds whose long term participants can afford to tolerate more risk in the short run. In principle different attitudes to risk may be imposed at *each* decision point with the *additively separable* utility U through a sum of different *period utility functions* u_t, t=2,…,T+1, or through a sum of period utility functions with a common form and different period-specific values of its parameters. In practice a common specification of period utility is usually used and adjustment of the period-specific parameter values allows the shaping of the fund wealth distribution across scenarios at a decision point. We consider the following period utility functions:

Exponential (CARA): $u(w) = -e^{-aw}$, $a > 0$ (5)

Downside-quadratic: $u(w) = (1-a)w - a(w-\tilde{w})_-^2$, $0 \le a \le 1, 0 \le \tilde{w} \le \infty$. (7)

Note that the downside-quadratic utility function reduces to the linear utility function (u(w)=w) for a=0. (Other period utility functions are discussed in [7].)

The basic constraints of the optimization model are those of the dynamic CALM model (*cf.* [4]):

- *Cash balance constraints.* These ensure that the net flow of cash at each time and state is zero and take the form

$$\sum_{i\in I} p_{it}(\omega)(gx_{it}^-(\omega) - fx_{it}^+(\omega)) = 0,$$

  where f and g are proportional transaction costs.
- *Inventory balance constraints*. These give the amount invested in each asset at each time and state and take the form

$$x_{it}(\omega) = x_{it-1}(\omega)(1+v_{it}(\omega)) + x_{it}^+(\omega) - x_{it}^-(\omega).$$

  This approach, due to Bradley & Crane [2], allows tax and business modelling structures to be incorporated in constraints (see e.g.[3]).
- *Wealth constraints*. These define the before and after rebalancing wealths at each time and state and take the form

$$\sum_{i\in I} p_{it}(\omega)(1+v_{it}(\omega))x_{it-1}(\omega) = w_t(\omega)$$
$$W_t(\omega) = \sum_{j\in I} p_{jt}(\omega)x_{jt}(\omega).$$

Besides these basic constraints, the fund may face the following portfolio restrictions:

- *Solvency constraints*. These constrain the fund wealth at each time to be non-negative and take the form

$$w_t(\omega) \ge 0$$

- *Cash borrowing/short limits*. These limit the amount borrowed/shorted of an asset and take the form

$$p_{it}(\omega)x_{it}(\omega) \ge \overline{x}_i$$

- *Position limits*. These limit the amount invested in an asset to be less than some proportion of the fund wealth and take the form

$$p_{it}(\omega)x_{it}(\omega) \le \phi_i W_t \ (\omega)$$

- *Turnover (liquidity) constraints*. These limit the change in the amount invested in an asset from one period to the next and take the form

$$| \ p_{it}(\omega)x_{it}(\omega) - p_{it-1}(\omega)x_{it-1}(\omega) \ | \le \alpha_i W_t \ (\omega) .$$

All the above constraints are piecewise linear convex.

For backtesting purposes we specify the following three types of constraint structures. T1 constraints have no position limits or turnover constraints, T2 constraints have position limits and no turnover constraints and T3 constraints contain both position limits and turnover constraints. Short selling and borrowing are not allowed in any of these constraint structures. Assuming that the simulated price processes are non-negative, this automatically enforces solvency constraints.

- *Guaranteed return constraints*. The *return guarantee* to an individual investor is absolute given the solvency of the guarantor. In the situation of a banking group such as the fund manager and its parent guarantor this necessitates strategies both to implement the absolute guarantee for individuals and to manage the investment strategy of the fund so as to ensure meeting the guarantee for *all* participants of the fund with a high probability.

Mathematically, this latter goal can be met by imposing a *probabilistic* constraint of the *value at risk* type on the wealth process at specific trading dates, computing expected shortfall across scenarios which fail to meet the fund guarantee and adding the corresponding penalty terms to period objective functions. For example, at the horizon T+1 or any intermediate date t' this would take the form $P\left(\mathbf{w}_{t'} \ge w_{t'}^{*}\right) \ge 1-\alpha,$ where α: = 0.01 or 0.05, corresponding to respectively 99% or 95% confidence, and $w_{t'}^{*}$ is calculated from the initial wealth and the guaranteed period rate *r* as $w_0\left(1+r\right)^{t'}$. However, such scenario-based probabilistic constraints are extremely difficult to implement in that they convert the convex (deterministic equivalent) large scale optimisation problem to a *nonconvex* one. For practical purposes we have developed the *capital guaranteed products algorithm* implemented for a pension fund using parametric nested optimization techniques [7].

Asset return statistical models

Our main *asset return model* (BMSIM) used to generate scenarios for the ALM problem is based on a set of continuous time *stochastic differential equations* for the financial and economic dynamics of interest. We then discretise time to obtain the corresponding system of stochastic *difference* equations, *estimate* them econometrically (in the econometric estimation tradition initiated by Wilkie [23,24]) and *calibrate* the output of their simulation with history by various *ad hoc* or semi-formal methods of parameter adjustment. (See, for example, [19] and [6].)

The global structure of this model involves investments in the three major asset classes – cash, bonds and equities – in the four major currency areas – US, UK, EU and Japan (JP) – together with emerging markets (EM) equities and bonds. Each currency area is linked to the others *directly* via an exchange rate equation and *indirectly* through correlated innovations (disturbance or error terms). Figure 2 shows the structure of the canonical model of a major currency area. The home currency for these models is assumed to be the US dollar, but scenarios can be generated in any of the four major currencies since cross rates are forecast and any other currency (e.g. the Euro) can be taken as the home currency for the statistical estimation. Detailed specifications of this model are given in [1] and [7].

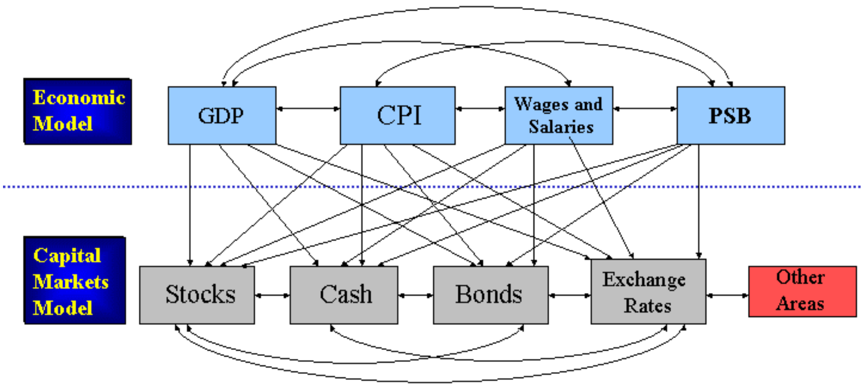

Figure 2. Major currency area detailed model structure

The formulation of the US capital market model with the UK/US exchange rate is given below where S denotes the equity level, R denotes the short term (money market) interest rate, L denotes the long term interest rate, X denotes the exchange rate, C denotes the consumer price index, W denotes the wages level, G denotes GDP and P denotes public sector borrowing.

$$\frac{\boldsymbol{S^{US}_{t+1}} - S^{US}_t}{S^{US}_t} = \begin{pmatrix} a^{US}_S + a^{US}_{SS} S^{US}_t + a^{US}_{SR} R^{US}_t + a^{US}_{SL} L^{US}_t + a^{US}_{SX} X^{US}_t + \\ b^{US}_{SS} S^{US}_{t-1} + b^{US}_{SR} R^{US}_{t-1} + b^{US}_{SL} L^{US}_{t-1} + b^{US}_{SX} X^{US}_{t-1} + \\ c^{US}_{SC} C^{US}_t + c^{US}_{SW} W^{US}_t + c^{US}_{SG} G^{US}_t + c^{US}_{SP} P^{US}_t + \\ d^{US}_{SC} C^{US}_{t-1} + d^{US}_{SW} W^{US}_{t-1} + d^{US}_{SG} G^{US}_{t-1} + d^{US}_{SP} P^{US}_{t-1} \end{pmatrix} + \sigma^{US}_S \boldsymbol{\varepsilon^{US}_{St}}$$

$$\frac{\boldsymbol{R^{US}_{t+1}} - R^{US}_t}{R^{US}_t} = \begin{pmatrix} a^{US}_R + a^{US}_{RS}\left(\frac{S^{US}_t}{R^{US}_t}\right) + a^{US}_{RR}\left(\frac{1}{R^{US}_t}\right) + a^{US}_{RL}\left(\frac{L^{US}_t}{R^{US}_t}\right) + a^{US}_{RX}\left(\frac{X^{US}_t}{R^{US}_t}\right) + \\ b^{US}_{RS}\left(\frac{S^{US}_{t-1}}{R^{US}_t}\right) + b^{US}_{RR}\left(\frac{1}{R^{US}_t}\right) + b^{US}_{RL}\left(\frac{L^{US}_{t-1}}{R^{US}_t}\right) + b^{US}_{RX}\left(\frac{X^{US}_{t-1}}{R^{US}_t}\right) + \\ c^{US}_{RC} C^{US}_t + c^{US}_{RW} W^{US}_t + c^{US}_{RG} G^{US}_t + c^{US}_{RP} P^{US}_t + \\ d^{US}_{RC} C^{US}_{t-1} + d^{US}_{RW} W^{US}_{t-1} + d^{US}_{RG} G^{US}_{t-1} + d^{US}_{RP} P^{US}_{t-1} \end{pmatrix} + \sigma^{US}_R \boldsymbol{\varepsilon^{US}_{Rt}}$$

$$\frac{\boldsymbol{L^{US}_{t+1}} - L^{US}_t}{L^{US}_t} = \begin{pmatrix} a^{US}_L + a^{US}_{LS} S^{US}_t + a^{US}_{LR} R^{US}_t + a^{US}_{LL} L^{US}_t + a^{US}_{LX} X^{US}_t + \\ b^{US}_{LS} S^{US}_{t-1} + b^{US}_{LR} R^{US}_{t-1} + b^{US}_{LL} L^{US}_{t-1} + b^{US}_{LX} X^{US}_{t-1} + \\ c^{US}_{LC} C^{US}_t + c^{US}_{LW} W^{US}_t + c^{US}_{LG} G^{US}_t + c^{US}_{LP} P^{US}_t + \\ d^{US}_{LC} C^{US}_{t-1} + d^{US}_{LW} W^{US}_{t-1} + d^{US}_{LG} G^{US}_{t-1} + d^{US}_{LP} P^{US}_{t-1} \end{pmatrix} + \sigma^{US}_L \boldsymbol{\varepsilon^{US}_{Lt}} .$$

$$\frac{\boldsymbol{X}_{t+1}^{c}-X_t^c}{X_t^c}=\left(\begin{array}{l} a_X^c+a_{XS}^c\left(\dfrac{S_t^c}{X_t^c}\right)+a_{XR}^c\left(\dfrac{R_t^{US}-R_t^c}{X_t^c}\right)+a_{XL}^j\left(\dfrac{L_t^{US}-L_t^c}{X_t^c}\right)+a_{XX}^c\left(\dfrac{1}{X_t^c}\right)+\\ b_{XS}^c\left(\dfrac{S_{t-1}^c}{X_t^c}\right)+b_{XR}^c\left(\dfrac{R_{t-1}^{US}-R_{t-1}^c}{X_t^c}\right)+b_{XL}^c\left(\dfrac{L_{t-1}^{US}-L_{t-1}^c}{X_t^c}\right)+b_{XX}^c\left(\dfrac{1}{X_t^c}\right)\end{array}\right)+\sigma_X^c\boldsymbol{\varepsilon}_{Xt}^{c}.$$

Here and below the a,b,c,d and σ terms are parameters of the model and the ε terms are correlated standard normal random variables. All dependent variables in this specification are in terms of returns, while the explanatory variables are in original level or rate form. Although linear in the parameters, this model is second order autoregressive and nonlinear in the state variables, making its long run dynamics difficult to analyse and potentially unstable. Due to its linearity in the parameters this model may be estimated using the *seemingly unrelated regression* (SUR) technique, see e.g. [9, Chapter 11], recursively until a parsimonious estimate is obtained in which all non-zero parameters are statistically significant. Since the work of Theil in the 1970's, it has been demonstrated that there is a gain in efficiency in using SUR vs. OLS if the residuals are correlated across equations and/or the regressors are not the same in each equation.

The formulation of the US economic model is given next.

$$\frac{\boldsymbol{C}_{t+1}-C_t}{C_t}=\left(\begin{array}{l} a_{cpi_1}+a_{cpi_2}C_t+a_{cpi_3}W_t+a_{cpi_4}G_t+a_{cpi_5}P_t\\ +b_{cpi_2}C_{t-1}+b_{cpi_3}+W_{t-1}+b_{cpi_4}G_{t-1}+b_{cpi_5}P_{t-1}\end{array}\right)+\sigma_C\boldsymbol{\varepsilon}_t^C$$

$$\frac{\boldsymbol{W}_{t+1}-W_t}{W_t}=\left(\begin{array}{l} a_{ws_1}+a_{ws_2}C_t+a_{ws_3}W_t+a_{ws_4}G_t+a_{ws_5}P_t\\ +b_{ws_2}C_{t-1}+b_{ws_3}+W_{t-1}+b_{ws_4}G_{t-1}+b_{ws_5}P_{t-1}\end{array}\right)+\sigma_W\boldsymbol{\varepsilon}_t^W$$

$$\frac{\boldsymbol{P}_{t+1}-P_t}{P_t}=\left(\begin{array}{l} a_{psb_1}+a_{psb_2}C_t+a_{psb_3}W_t+a_{psb_4}G_t+a_{psb_5}P_t\\ +b_{psb_2}C_{t-1}+b_{psb_3}+W_{t-1}+b_{psb_4}G_{t-1}+b_{psb_5}P_{t-1}\end{array}\right)+\sigma_P\boldsymbol{\varepsilon}_t^P$$

$$\frac{\boldsymbol{G}_{t+1}-G_t}{G_t}=\left(\begin{array}{l} a_{gdp_1}+a_{gdp_2}C_t+a_{gdp_3}W_t+a_{gdp_4}G_t+a_{gdp_5}P_t\\ +b_{gdp_2}C_{t-1}+b_{gdp_3}+W_{t-1}+b_{gdp_4}G_{t-1}+b_{gdp_5}P_{t-1}\end{array}\right)+\sigma_G\boldsymbol{\varepsilon}_t^G.$$

This model captures the interactions of the capital markets with the economy in each major currency area. For stability the specification is in terms of returns similar to the capital markets. This is again a second order autoregressive model in the state variables which is *linear* in parameters and *nonlinear* in variables. It may be estimated using the techniques mentioned above.

The emerging markets equity and bond index returns are modelled with the following AR(1) / GARCH(1,1) processes in a similar fashion as the original work on generalized autoregressive conditional heteroscedasticity models done by Bollerslev (1986) as

$$\begin{aligned}\boldsymbol{y}_t&=a_0+a_1y_{t\text{-}1}-a_2u_{t\text{-}1}+\boldsymbol{u}_t\\ \boldsymbol{u}_t&=\sqrt{H_t}\boldsymbol{\varepsilon}_t\\ H_t&=b_0+b_1H_{t\text{-}1}-b_2u_{t-1}^2,\end{aligned}$$

where y_t denotes the index return. These models are estimated using maximum likelihood and the residuals are used to estimate the correlations of their innovations with those of the other state variables.

Asset classes are given by data on selected stock indices, government bonds and treasure bills: S&P 500 stock index, FTSE stock index and MSCI Europe stock index; US 3 month T-bill rate and US 30 year T-bond with semi-annual compounding; UK 3 month T-bill rate and UK 20 year GILT rate with semi-annual compounding, and so on. Sources were Data Stream and Bloomberg at monthly frequency from 1977 except for economic variables available only quarterly. Monthly levels were computed for the latter by taking the cube root of the actual quarterly return and finding the corresponding monthly levels between announcements. A consistent database of model data is currently being maintained and updated monthly by the fund manager.

Various subsystems of this model have been estimated using the SURE model maximum likelihood estimation procedures of RATS 4.0. For each model the full set of model parameters was first estimated and insignificant (at the 5% level) variables sequentially removed to obtain a parsimonious final model with all statistically significant coefficients. The seemingly unrelated regression nature of the model is obvious as each currency area is directly related only through exchange rates and indirectly related through shocks. In light of Meese & Rogoff's [16, 17] classical view on the inefficacy of macroeconomic explanations of exchange rates even at monthly frequency, after considerable single equation and subsystem analysis we have found that interest rate parity expressed as inter-area short and long rate differences – *together* with other local capital market variables – has significant explanatory power, while purchasing power parity expressed various ways has less (*cf.* [11]).

We found model residuals correlated across equations and a similar level of correlation was also found in actual returns for the same variables and periods of time. Our main econometric finding is that the world's equity markets are linked simultaneously through shocks [1] and [7].

Montel Carlo simulation

Given initial values of its state variables, asset return scenarios may be simulated without stochastic innovations as a discrete time deterministic dynamical system defining the mean paths of the state variables. The nonlinear dynamics of this deterministic system may be exceedingly complex and the system may rapidly explode or die to zero values of some variables for certain configurations of the (significant) estimated parameters. Graphical emulation of the central tendencies of the historical paths by this deterministic system is a necessary condition for the generation of realistic scenarios by Monte Carlo simulation of the stochastic dynamical system. Intuitions can be developed to make the achievement of reasonably accurate calibrations tractable and we have developed a prototype graphical interface tool *stochgen 3.0* [8] to aid the process graphically.

A second asset return model we have used is is a *vector autoregressive* (VAR) system, VARSIM, which is *stable* in the state variable returns, so that the deterministic system converges to steady state returns and shocks to the corresponding stochastic system are non-persistent. Finally, a third asset return model we have used is a historical bootstrap model, HSIM. This model treats the process generating the historical data as *stationary* with independent increments, and conducts historical simulation by resampling from the empirical distributions of state variable returns constructed from the historical paths over the in-sample period. All these models have been evaluated and we report dynamic stochastic optimisation backtest results for all three approaches to scenario generation for our dynamic ALM problem in our joint project [7] in the next section.

Due to a finite sample of scenarios, there will always be sampling error in the generation of scenario return distributions relative to the calibrated estimated system model. This can lead to serious errors and spurious arbitrages in subsequent portfolio optimisation. However these may be suppressed by ensuring that the sample return distributions corresponding to all generated scenarios at a specific point in time have two moments matched to those of the statistical model underlying the simulations [12, 13, 21].

In order to mirror reality the alternative unfolding future scenarios in the model must be organized in a *tree* form. Each path from the root to a leaf node in the tree *represents* a scenario and the nodes represent decision points – forward portfolio rebalances. The root node represents the initial *implemented* decision that is an initial portfolio balance. Note that the Monte Carlo simulation of scenarios corresponding to a given scheme is a nontrivial matter requiring generic software to handle a complex simulator. We have used the generic *stochgen 2.3* software of the *Stochastics*™ toolchain for dynamic stochastic optimisation [8]. The *stochgen* software must handle at each node multiple conditional stochastic simulations of versions of the asset return model initialised by the data at the node and two previous timesteps (months) along the scenario path. Notice that the simulation time step (a month) is much shorter than the decision point frequency (for forward portfolio rebalancing: quarterly, semi annually or annually). In the backtests reported below balanced scenario trees with high initial branching were used.

4 System Historical Backtests

A number of historical backtests have been run on variants of the global model, see [21] for complete details. The aims of these tests were several. First, we wished to evaluate how well the system would have performed had it been implemented in practice relative to a benchmark. Since each backtest is performed from the point of view of a US dollar-based firm, the benchmark used is the S&P 500. Second, we wished to understand the impact of alternative utility functions on optimal portfolio decisions. Thirdly, we were interested in what effects imposing the practical diversification and liquidity (turnover) constraints would have on backtest returns. A comparison of backtests for single and multi period formulations of ALM problems can be found in [21]. All portfolio rebalances are subject to a 1% value tax on transactions which of course does not apply to the benchmark index. Monthly data were available from July 1977 to August 2002.

Table 1 shows the results in terms of annualised returns of a typical backtest with a 2 year telescoping horizon and semi-annual rebalancing from February 1999 to February 2001 using a model with 8192 scenarios, a 128.16.2.2 branching structure and a terminal wealth criterion. During this period the S&P500 returned 0 percent. With no position limits the model tends to pick the best asset(s) and so in this case a high annual historical return to the chosen low diversification portfolios is an indication of the predictive merits of the tuned econometric model used to generate the scenarios. When more realistic constraints are imposed in this test however portfolios become well diversified and there is little to choose from in the results corresponding to the various attitudes to risk. However, performance is improved by the use of the emerging market asset returns even though they were actually not used in the optimal portfolios. Corresponding results for the addition of the US economic model to the system are mixed. When this backtest was extended one period to August 2001 – when the S&P500 annualised return over the 2.5 year period was –2.3% – similar results were obtained with the best position limited result being 6.8% per annum for the downside-quadratic utility and target wealth a 61% increase over the period.

Table 1. Asset allocation backtests: Annualised returns from February 1999 – 2001

	Capital Markets		Capital Markets + Emerging Markets		Capital Markets + Emerging Markets + US Economic Model	
Utility Function	**No Limits**	**20% Limits**	**No Limits**	**20% Limits**	**No Limits**	**20% Limits**
Linear	91%	9%	92%	10%	31%	11%
Downside-quadratic	54%	9%	70%	11%	29%	9%
Exponential	72%	9%	92%	10%	51%	11%

A summary of backtest results using the downside-quadratic utility function is given in Table 2. Taken altogether this utility function appears to be most effective and limited tests with other utility functions confirm the results in Table 2.

Table 2. Summary of historical backtests[1]

Initial Estimation Period	Out-of-sample Period	Length	Asset Return Model	Simulator	Number of Scenarios k	Rebalance Frequency	Risk Management Criterion	Horizon	Constraint Annualised Return % (see Section 5.4)			S&P 500 Benchmark Annualised Return %
									T1	T2	T3	
1972-1990	1990-1995	5 years	3 areas (ex Japan)	BMSIM	4	annual	terminal	telescoping	10.33	9.34	-	7.41
1992-1996	1996-2001	5 years	4 areas	BMSIM	4	annual	terminal	telescoping	13.36	7.13	-	14.12
1992-1996	1996-2001	5 years	4 areas	VARSIM	4	annual	terminal	telescoping	1.51	8.30	-	14.12
1992-1999	1999-2001	2.5 years	4 areas	BMSIM	8.2	semi-annual	terminal	telescoping	27.89	6.48	2.69	-2.30
1992-1999	1999-2001	2.5 years	above + emerging markets	BMSIM	8.2	semi-annual	terminal	telescoping	16.98	5.72	3.38	-2.30
1992-1999	1999-2001	2.5 years	above + US economy	BMSIM	8.2	semi-annual	terminal	telescoping	19.16	4.64	-0.38	-2.30
1992-1999	1999-2001	2.5 years	4 areas	VARSIM	8.2	semi-annual	terminal	telescoping	-6.40	-	-3.92	-2.30
1990-1996	1996-2001	5 years	4 areas	BMSIM	8.2	annual	all periods	telescoping	8.54	-	8.37	14.12
1990-1996	1996-2001	5 years	4 areas	VARSIM	8.2	annual	all periods	telescoping	5.78	9.99	**9.37**	14.12
1990-1996	1996-2001	5 years	4 areas	HSIM	8.2	annual	all periods	telescoping	4.95	-	6.04	14.12
1972-1991	1991-2001	10 years	4 areas	VARSIM	8.2	annual	all periods	5-year rolling	3.56	-	9.98	12.72

[1] "-" entries not calculated.

Note here that imposing the practical liquidity and turnover (T3) constraints, which could be expected generally to reduce returns, sometimes led to significantly increased returns. Overall, we found that the imposition of the T3 constraints in the model forced its decisions to take full advantage of the information in future scenarios and optimal forward rebalances to result in well diversified portfolios and significant improvement in historical backtest performance.

5 Conclusions

This paper has described the use of dynamic stochastic optimisation methodology for structured products for pension fund management. Practical solutions to the design of guaranteed return investment products for pension funds have been outlined. In nearly all the historical backtests using data over roughly the past decade the global asset allocation system outperformed the S&P500 when transactions costs are taken into account. Nearly all system returns for the nonlinear statistical model were positive – even through the recent high tech crash. Thus the dynamic stochastic optimisation approach to pension fund management is a practical reality today.

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