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Abstract

This paper presents a model featuring both endogenous growth and endogenous cycles. It explains how entrepreneurial choice regarding innovation within an existing leading technology or the search for a new leading technology, contributes to the existence of long waves. A major contribution of the model is that with time, the periodicity of long waves will, on average, become shorter. The intuition behind the shortening of the long wave is simple; if an endogenous growth approach is adopted, and economic growth results in the accumulation of knowledge which increases the productivity of resources in the R&D sector, then a faster pace of innovation is an implied result of the higher growth outcome. As the pace of innovation rises, the exploitation of successive leading technologies becomes more rapid, causing a shortening of the long wave.

INTRODUCTION

This paper develops a model of endogenous growth and endogenous cycles to present a formalisation of the phenomenon of long waves in economic growth.¹ The major contribution of the paper is that sufficient conditions are set so that not only does a long wave pattern emerge but also it is identified with time the periodicity of each long wave will, on average, become progressively shorter.² The time varying pace of innovation distinguishes this paper from a number of other endogenous growth models such as Lucas (1988), Romer (1990) and Grossman and Helpman (1991) where the pace of innovation is assumed to be constant.³

Using a typical knowledge diffusion mechanism as popularised by Griliches (1960) in which the pace of innovation varies in proportion with the saturation of some bounded maximum number of innovation opportunities, it is established that long waves are the result of rational maximising behaviour on behalf of entrepreneurs in their choice between innovation within an existing leading technology or the search for a new leading technology.⁴ In this respect, the paper resembles a similar study by Bresnahan and Trajtenberg (1994) where waves of innovation in 'General Purpose Technologies' (GPTs) generate cycles in the rate of economic growth. GPTs are characterised by their pervasive influence throughout the economy; advances in other technologies may take place while GPTs are being exploited, but most entrepreneurial behaviour is centred on the current GPT. In short, a GPT has the endearing quality of being able to 'plug into' many existing avenues for technological exploitation.⁵ Therefore, GPTs play a crucial

¹ Analysis of long waves is generally recognised to have begun with the Russian economist Nikolai Kondratieff who used earlier work by Jevons (1884) and a number of Dutch economists to establish long-run fluctuations in price growth as an indicator of economic activity. For a survey of the literature examining the long wave phenomenon pre - Kondratieff, refer to Kleinknecht (1987).

² Jovanovic and Rob (1990) explored the possibility of long wave periodicity variation but not at the aggregate economy level.

³ Barro and Sala-i-Martin (1997) allowed for a time variant form of innovation in their study of convergence using an endogenous growth model.

⁴ The implications of the bunching of innovations for long waves has also been explored by Mensch (1979), Perez (1983), Shleifer (1986) and Stein (1994).

⁵ This 'granular' nature of technology was explored by Baser and Weil (1998) in their explanation as to the differing pace of diffusion for different technologies – a concept explored more fully in this paper.

role as the 'engines of growth'. An additional feature of the model developed in this paper in contrast to earlier studies is that knowledge accumulation causes the average periodicity of each long cycle/wave to decline over time as the time it takes to exploit each GPT becomes sequentially shorter.

The intuition behind the shortening of the long wave is as follows: 1) using an endogenous growth approach, knowledge accumulation generates increasing productivity in the R&D sector; 2) since productivity in the R&D sector is measured in terms of output of innovations per unit of time, higher productivity implies a faster pace of innovation and a more rapid exploitation of any finite number of financially viable innovation opportunities in a GPT; 3) since the pace of innovation for each GPT is modelled using a knowledge diffusion process which results in a wave-like pattern emerging for the rate of innovation exploitation per unit of time, the rise in productivity in the R&D sector causes a shortening of this long wave effect.⁶ The crucial relationship in the model is the assumption that knowledge can be transmitted between successive GPTs with improvements in productivity in the R&D sector being cumulative. Intuitively this seems a plausible result as quite often the benefits of knowledge accumulation are not always specific to only one technology.

Modelling a time varying periodicity of the long wave may prove useful for the empirical validation of the proposed long wave phenomenon. Since Kondratieff's hypothesis was translated into English in 1935 (he first published in German in 1926), the actual empirical validation of the existence of long waves in economic activity has been an area of considerable controversy.⁷ Kuznets (1940), Rostow (1975) and Solomou (1987) have all found little or no empirical verification for the existence of long waves. In fact, Kuznets (1958) argued that the 55-70 year cycles in economic

⁶ If the amount of financially viable innovations per GPT is declining, then the improving rate of innovation output implies the long wave is compressing at a rate even faster than the constant number of innovations scenario. It is only in the case of financially viable innovation opportunities for each new GPT is rising, that there arises the possibility of the periodicity of the long wave remaining constant. This is a limiting case scenario because it requires financially viable innovation opportunities to be growing at a rate equal to the cumulative knowledge effect upon the growth of productivity in the R&D sector.

⁷ See Kondratieff, 'The Long Waves in Economic Life', *Review of Economics and Statistics*, 1935.

activity that Kondratieff attempted to justify were more consistent with shorter 22 year cycles. Alternately, studies by Clark et al. (1981), van Ewijk (1982) and Metz (1992) have argued in favour of a 65-70 year economic cycle; not to mention the seminal contribution in Schumpeter's *Business Cycles* in 1939.

A number of reasons can be expounded for the differences in the empirical validation of long waves. Firstly, the paucity of reliable economic data time-series over such a long timeframe has a tendency to bias results and generate inconsistencies between studies based on different data sets. Secondly, the fact that there have been large structural changes in economies since the industrial revolution makes cross-century comparisons difficult and has made econometric results reliant upon dubious trend filtering devices. Finally, it is possible that the average periodicity of long waves is non-constant causing the various econometric modelling outcomes to be dependent upon the length of the data set used and the assumptions made regarding the timeliness of long wave fluctuations. It is this area of long wave periodicity that this paper intends to make a contribution.

Since the model developed in this paper is required to capture the varying contributive effects of both genesis and post-genesis innovations in GPTs to economic growth, it is necessary to frame the model around the utility function of a representative agent. Section *I(a)* of the paper provides the foundations for linking entrepreneurial innovation to intertemporal consumer utility. Sections *I(b)* and *I(c)* examine the production and R&D sectors (respectively). Section *I(d)* formalises the technology search / technology exploitation choice faced by entrepreneurial agents, while sections *I(e)* and *I(f)* provide the necessary solutions for general equilibrium by examining the financing of innovation and the labour market (respectively). The characteristics of the general equilibrium result are examined in section *II* which includes an investigation of innovation/growth waves and wave compression.

I. THE MODEL

(a) Intertemporal Consumption

The model is an adapted version of Grossman and Helpman's (1991) endogenous growth model linking innovation to value-added economic growth. To start with, the standard assumptions are made regarding an infinitely lived representative agent model. That is, assume a fixed population, (Z), consumers share identical preferences and each individual maximises utility over an infinite horizon. The representative agent's utility maximand is as follows

$$(1) \quad U_t = \int_t^{\infty} e^{-\rho(\tau-t)} \log D(\tau) d\tau$$

where $D(\tau)$ represents an index of consumption at time τ , and ρ is the subjective discount rate. The natural logarithm of the consumption index measures the instantaneous utility of the representative agent at each moment in time. Utility is directly related to product variety with consumer preferences extending over an infinite range of products indexed by $j \in [0, \infty)$. If products in the range $[0, N]$ are available, then the consumer utility index is specified as follows

$$(2) \quad D = \left[\int_0^N x(j)^\alpha dj \right]^{1/\alpha} \quad 0 < \alpha < 1$$

where $x(j)$ denotes consumption of brand j and α is the elasticity of demand for this particular good.⁸ Since consumer preferences extend over an infinite range of products $j \in [0, \infty)$ and consumers always prefer more product variety, then there is always an

⁸ Dixit and Stiglitz (1977) have highlighted a number of useful properties of this form of utility function. In particular, cross-preference stability is ensured by a diminishing marginal utility from consumption of more units of a particular good ($0 < \alpha < 1$).

incentive for entrepreneurs to undertake innovative activity. This is because the elasticity of substitution between any two products, ε , is constant, and greater than one

$$(3) \quad \varepsilon = \frac{1}{(1-\alpha)} > 1.$$

indicating an expanding range of market opportunities for entrepreneurial agents.

As is well known (see Grossman and Helpman 1991 [Ch 3]), solving for instantaneous utility maximisation using the budget constraint of total income at each particular point in time, $E = \int_0^N p(j)x(j)dj$, gives the instantaneous demand function for the

representative consumer who maximises consumption over the entire range of goods as

$$(4) \quad x(j) = \frac{Ep(j)^{-\varepsilon}}{\int_0^N p(j')^{1-\varepsilon} dj'}.$$

A necessary condition for intertemporal equilibrium is for *real* aggregate supply to equate to *real* aggregate demand. Since the budget constraint is expressed in nominal terms, then a necessary condition for intertemporal equilibrium is

$$(5) \quad D = \frac{E}{p_D}$$

where p_D represents an 'ideal' price index of equilibrium prices for the basket of goods consumed in instantaneous equilibrium at each point in time and is approximated by

$p_D = \left[\int_0^N p(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$. Innovation which expands product diversity, lowers the price

per unit of utility for consumers and thus lowers p_D . To take into account this effect of innovation upon intertemporal consumption, it is necessary to substitute equation (5) into equation (1) assuming that indirect utility is weakly separable in the level of

spending and the ideal price index. When this is done, it is possible to rewrite the intertemporal consumption function as

$$(6) \quad U_t = \int_t^{\infty} e^{-\rho(\tau-t)} [\log E(\tau) - \log p_D(\tau)] d\tau .$$

Equation (6) highlights that intertemporal utility evolves according to the difference between expenditure growth and growth in the innovation determined ideal price index. Assuming that households can borrow or lend freely at the instantaneous interest rate, then a necessary condition for dynamic equilibrium is that intertemporal expenditure changes in proportion to the difference between the interest rate r and the subjective discount rate ρ

$$(7) \quad \frac{\dot{E}}{E} = r - \rho .$$

Proof of this result can be obtained in Grossman and Helpman (1991). Choosing a numeraire $E(t) = I$ so that the evolution of nominal spending with respect to time is normalised so that at every moment nominal spending remains constant, the intertemporal consumption equilibrium condition which was identified in equation (7) can now be interpreted as

$$(8) \quad r(t) = \rho \quad \text{for all } t .$$

This provides the intertemporal solution for utility maximisation of the representative consumer in response to entrepreneurial induced innovation. How this rate of entrepreneurial innovation varies over time is discussed in the following section.

(b) *The Production Sector*

It is assumed for simplicity that production is determined only by labour input, L . Under conditions of general equilibrium α represents both the marginal utility and the marginal product of a particular good. Within the production sector there is constant returns and the production function is of the form

$$(9) \quad Y = L\alpha.$$

Using this production function, entrepreneurs manufacture a series of differentiated products. It is assumed that each differentiated product is manufactured by a single, atomistic firm controlled by a single entrepreneur. Entrepreneurs capture markets via the process of product innovation and gain (by assumption) the right to be the only producer of a particular product variety. Monopolistic competition prevails and competitors do not try to undertake the production of existing goods because it is assumed that imitation is costly and incumbent producers engage in Bertrand Oligopolistic restrictive pricing behaviour or 'limit pricing'. Given these conditions, entrepreneurs are either manufacturing products that they have previously developed or they are managing the allocation of labour to the R&D sector with the aim of developing a genesis or post-genesis innovation. This up-front cost of R&D in the production process is regarded as a fixed cost.

Profits, π , are maximised for the successful entrepreneur under Bertrand Oligopolistic conditions by charging a price for a innovative product variety, $p(j)$, which takes into account both the marginal cost of production and the utility derived from a particular product variety. Since it is assumed that labour is the only input into the production process and there are constant returns to scale in the production sector, the marginal cost of production is simply the wage rate, w . This wage rate is deflated by the respective utility derived from the innovative product to set an equilibrium price condition for the producer of

$$(10) \quad p(j) = \frac{w}{\alpha}.$$

(c) *The Research and Development Sector*

Within the R&D sector increasing returns apply since it is assumed knowledge spillovers contribute to the stock of public knowledge (K_N) which in turn raises the output of innovations (both genesis and post-genesis) per unit of labour input (η_i) in the R&D sector.⁹ To quantify the impact that K_N has upon η_i , it is helpful to make the simplification assumption that the factor proportion relationship between an additional unit of public knowledge accumulation and an additional product innovation is one to one¹⁰

$$(11) \quad K_N = N.$$

A distinction is made between knowledge regarding a particular GPT (K_i) and knowledge regarding other GPTs (K_o) when formalising the relationship between K_N and η_i . The reason for this is that it can be argued that knowledge regarding the same GPT is of greater value in developing new product varieties within that GPT than is knowledge about other GPTs. Subsequently, K_i and K_o are assigned the values π and ζ (respectively) where for any particular GPT i , $\pi > \zeta$.¹¹ The relationship between knowledge accumulation and the productivity of labour in the R&D sector is therefore

$$(12) \quad \eta_i = \pi(K_i) + \zeta(K_o) \quad \text{where } K_N = K_i + K_o = N.$$

⁹ The subscript i denotes the particular generation of GPT to which η applies; where $0 < i < \infty$.

¹⁰ For earlier work on the implications of the growth of knowledge, refer to Nelson (1982) and Telser (1982). For a more recent study refer to Porter and Stern (2000).

¹¹ The introduction of a distinction between knowledge accumulated from other GPTs also provides a convenient formalisation for identifying the degree to which 'lock-in' of technology specific plays an important role in shaping future innovations; see for instance Arthur (1989).

Even though knowledge spillovers assist prospective entrepreneurs with the pursuit of new product varieties by lowering the effective cost of innovation (η_i), this rising pace of entrepreneurial innovation for any one GPT cannot continue indefinitely. It is assumed that the number of financially viable innovations (excluding the genesis innovation) within any given GPT i is universally bounded and finite (\bar{n}_i). Furthermore, as the number of innovations within a given GPT approaches its limit and discernibly different new product varieties within this GPT become scarcer, it is assumed that the pace of innovation begins to fall. A number of reasons have been espoused for the initial rise, then fall in the rate of innovation diffusion. These range from risk aversion in adopting a technology about which little is known (see Mansfield 1961), to production function discontinuity (see Mansfield 1977).¹²

Using $L_{R\&D}$ to denote the amount of labour devoted to the R&D sector, and using the relationship between K_N and η_i , it is possible to denote the development of new products per unit of time (\dot{N}) within a particular GPT as

$$(13) \quad \dot{N} = L_{R\&D} \eta_i \varpi_i.$$

where $\varpi_i = \left[\frac{(\bar{n}_i - n_i)}{\bar{n}_i} \right]$ and represents a weighting reflecting the number of financially viable innovation opportunities remaining within a particular GPT. This weighting only applies to \bar{n}_i and consequently does not apply to entrepreneurs seeking a genesis innovation (GPT search). Rather, the outcome of GPT search is stochastic but $L_{R\&D} \eta_i$ is directly proportional to the expected probability of success.¹³

(d) *The GPT Search / GPT Exploitation Choice*

¹² The microeconomic behaviour of firms in the competitive diffusion of knowledge about a particular technology has been explained in an excellent paper by Jovanovic and MacDonald (1994). In fact, studies as early as Schumpeter (1939) have described a swarming type process in the pace of adoption of a new technology along with its implications for entrepreneurial profit.

¹³ It is possible that certain 'star scientists' can significantly improve the odds of success. This heterogenous ability characteristic will not be covered explicitly here but we refer the reader to Darby and Zucker (2002) for a discussion of the implications arising out of such an assumption.

Profit maximising entrepreneurs face the option of undertaking either GPT search (pursuing a genesis innovation) or GPT exploitation (pursuing a post-genesis innovation).

At this point it is perhaps useful to define 'GPT search' and 'GPT exploitation', given that the entrepreneurial decision between these two pursuits is crucial to the formation of long waves. GPT search is defined as the sequential development of a particularly radical innovation that embodies a new leading technology. Tylecote (1992) has identified six major technologies (GPTs) that have been developed in modern economic history, these are; water power, steam power, steel-making, electricity, microelectronics and biotechnology.¹⁴ These technologies were at first announced with a radical innovation which for the purposes of this paper are classified as genesis innovations. For example, Trevithick's steam locomotive in 1805 acted as a harbinger to the development of steam power; Bessemer's process of removing impurities from pig iron in 1854 eventually led to the development of cheap steel. Innovations which follow genesis innovations are basically concerned with the refinement of GPTs into an array of recognisably different product varieties. This is the GPT exploitation phase of the innovative cycle. Post-genesis innovations take place in varying degrees; significant post-genesis innovations constitute the development of an entirely new product line, such as Bell's telephone in 1876 which was a major innovative adaptation of Faraday's earlier work on electromagnetic induction in 1831; less significant post-genesis innovations are represented by minor refinements to an existing product line such as the numerous changes that have been made to the physical housing of telephone electronics in attempt to target various segments of the consumer market.

Proposition 1. Depending upon the relative profit returns from both activities, a majority of entrepreneurs will opt for the pursuit of innovation within an existing GPT

¹⁴ See Arthur (2002) for a recent discussion of the comparability between these major technologies.

(GPT exploitation) in preference to GPT search until the number of financially viable product innovations within the existing GPT is nearly exhausted.

Proof: Using the demand function given in equation (4) it is possible to demonstrate that entrepreneurs who successfully develop a post-genesis innovation have an operating profit function consistent with total revenue minus total cost

$$(14) \quad P_{PG}(j) = p(j)x(j) - wx(j) - R(j)$$

where it is assumed for simplicity that there is a one to one relationship between labour input and product output, and that R represents a royalty paid to the individual entrepreneur who developed a GPT by other innovators within that GPT. That is, once a genesis innovation has taken place it is assumed that other entrepreneurs are able to learn about the new GPT via a reverse engineering process but that the genesis innovator is able to enforce some property rights over this knowledge.¹⁵

The operating profit function for developing a genesis innovation is different in the respect that it is stochastic and that the royalty no longer represents a cost of production but instead constitutes a revenue. Furthermore, consistent with Young (1993a and 1993b) an extra cost (C) is introduced for developing a new GPT because of the lack of accumulated knowledge. Thus the operating profit function for developing a genesis innovation is

$$(15) \quad P_G(j) = \beta \left[p(j)x(j) + R \bar{n}_i - wx(j) - C \right]$$

where β represents the aggregate probability of success for GPT search; which is assumed to be constant.

¹⁵ This raises policy related issues concerning the optimum reward for genesis innovators to sustain high rates of innovation and minimise periods of GPT search.

If $P_G < P_{PG}$, profit maximising entrepreneurs will consistently choose the pursuit of GPT exploitation rather than undertaking the relatively high risk, low return GPT search.

Assuming $P_G > P_{PG}$ and that all entrepreneurs are of equal ability, there is the incentive for all entrepreneurs to engage in GPT search.¹⁶ However, this result is bounded by the constant stochastic outcome of GPT search. Following Romer (1990) and assuming excludability of information between entrepreneurial agents undertaking technology search, it is possible to argue that as the number of entrepreneurs undertaking GPT search increases, the probability of success for any individual entrepreneur declines. Consequently, if $P_G > P_{PG}$ it is only profitable for a relatively small proportion of entrepreneurs to enter into GPT search.

Proposition 2. When the number of financially viable product innovations in the existing GPT (at the current level of η_i) is reached, all entrepreneurs switch their energies toward GPT search if the next leading technology has not already emerged; alternately, these entrepreneurs will turn their attention toward the exploitation of the next leading technology if this GPT has already been developed.

Proof: If $P_G < P_{PG}$ entrepreneurs are eventually confronted with no financially viable innovation opportunities remaining within the existing GPT (at the prevailing level of η_i). It is at this point that they switch their focus toward GPT search.¹⁷ The evolution of GPTs is demonstrated to be continuous by appealing to the law of large numbers as described by Feller (1968) and identifying that there is an increasing probability of a

¹⁶ Jovanovic and Rob (1989) explore the possibility of heterogeneity in entrepreneurial agents and its implications for the pace of innovation diffusion.

¹⁷ This does not imply that all financially viable post-genesis innovations in a particular GPT (η_i) will necessarily be exploited in the first attempt. The constraint imposed by the current level of η_i may necessitate some small number of the universally bounded finite number of financially viable innovation opportunities in a GPT to go undeveloped until productivity in the R&D sector rises by a sufficient amount. When this is the case, entrepreneurs will continuously backtrack to exploit old GPTs as η_i rises through time. Intuitively this stands to reason as the knowledge garnered from latter generations of technology makes possible the pursuit of previously unobtainable innovations.

new leading technology emerging out of GPT search as the time spent on this activity extends. That is, assuming all entrepreneurs and the labour they employ in GPT search are of equal ability, each entrepreneur chooses a different technology and the aggregate probability of success in each time period (β) is constant. Using ξ_i to denote the development of a genesis innovation and repeating the number of trials over the finite time period κ , the contribution made to the development of the next leading technology over this time period is dependent upon κ itself, the innovative productivity of resources in the R&D sector (η_i) and β

$$(16) \quad \xi = f[\beta(\xi; \eta_i, \kappa)].$$

As time approaches infinity and the number of trials effectively becomes infinite, the outcome of GPT search is no longer stochastic but is deterministic. As a consequence, when $P_G < P_{PG}$ the pace of innovation will be characterised by alternating phases of GPT exploitation and GPT search.

If $P_G > P_{PG}$ the alternating pattern of GPT exploitation / GPT search is fundamentally similar however if one individual engaged in GPT search happens to be successful and a new GPT appears before the existing GPT has been fully exploited, it is important to identify that entrepreneurs will still focus their attention on exploiting the superseded technology. This is because more knowledge has been accumulated about the superseded GPT than the latest GPT which implies that η_i is higher for the older GPT. Intuitively this is a plausible result, as given the same profit function for an innovation in both technologies, it is more likely that entrepreneurs will innovate in a GPT in which a great deal is known, rather than in a GPT about which little is known. This appears to demonstrate a historical phenomenon described in Helpman and Trajtenberg (1994) where there appears to be a significant lag between the development of a genesis innovation and the subsequent post-genesis exploitation of a new GPT. Rather, it is not until innovations in the older GPT become scarce that entrepreneurs are enticed to

investigate the new GPT. This is evidenced by $\eta_i \varpi_i$ for the superseded GPT gradually falling to parity with the $\eta_i \varpi_i$ for the latest GPT; at this point entrepreneurs switch *en-masse* to the exploitation of the new GPT.

(e) *Financing GPT Exploitation and GPT Search*

Assuming symmetric demand conditions, there is equilibrium in the distribution of entrepreneurial ability and given that $E=1$, competition ensures an average profit return for each innovation (both genesis and post-genesis) of

$$(17) \quad P_A = \frac{1 - \alpha - R}{N}.$$

The household-based capital market uses this profit return to calculate the distribution of savings funds between bonds and equities to facilitate consumption smoothing over their infinite life span. Loans from the household-based capital market are crucial in converting innovation opportunities into a growing diversification of products. The fixed pool of household savings constitutes the entire financial market; it is assumed that there is no savings undertaken by firms. Stocks and bonds are assumed to represent perfect financial substitutes within households portfolio's. Equilibrium requires that the return from holding entrepreneurial equity is the same as the return from holding bonds. The return on bond investment is simply the nominal interest rate (r) multiplied by the value of bonds held, v . The return on entrepreneurial equity is the average profit return (P_A), assuming all profits are paid in the form of dividends, plus any expected capital gains or losses, $\left(\dot{v}\right)$. Capital gains and losses are calculated as the discounted present value stream of future profit. This present value stream of future profit will decline as new innovations are brought into production and the market share of existing producers is diluted.¹⁸ Consumers continue to purchase products from older GPTs at but these

¹⁸ For an empirical study of the link between the stock market and the pace of innovation, refer to Greenwood and Jovanovic (1999).

GPTs comprise an ever declining share of their total expenditure.¹⁹ The functional form for equilibrium in the household-based capital market is

$$(18) \quad P_A + \dot{v} = r v.$$

From equation (13), an entrepreneur who devotes l units of labour to the pursuit of post-genesis innovation acquires the ability to produce $dN = \varpi_i l \eta_i$ new varieties of product. Alternately, for the entrepreneurs who decide to pursue a genesis innovation, $dN = \beta l \eta_i$ represents the expected outcome of their combined GPT search. Combining these results, capital markets place valuation on the labour choice for innovation equivalent to $v[\beta \varpi_i l \eta_i]$ where $\varpi = 1$ for GPT search, $\beta = 1$ for GPT exploitation. Value maximisation by entrepreneurs requires that l will be set as large as possible whenever the market valuation, $v[\beta \varpi_i l \eta_i]$ is greater than the cost of devoting labour to developing an innovation, (wl) . If this minimum market valuation condition holds, the labour demand by entrepreneurs for innovation activities is unbounded. Stability requires

$$(19) \quad v = \frac{w}{\beta \varpi_i \eta_i}.$$

(f) *The Labour Market*

In ensuring stability for the financing of GPT exploitation and GPT search, wages are the crucial variable. From equation (13) labour demand in the R&D sector is given by

$$(20) \quad L_{R\&D} = \left[\frac{\dot{N}}{\eta_i \varpi_i} \right].$$

¹⁹ This co-existence of different generations of technology contrasts Schumpeter's hypothesis of 'creative destruction' where the most recent technology supplants that of the earlier generation (Schumpeter, 1942).

The labour demand in the production sector of the economy is derived by obtaining the number of products sold. Since aggregate spending is arbitrarily set at the constant $E=I$ and given an input-output coefficient of one for labour input relative to output in the production sector, then the labour demand in this sector of the economy is

$$(21) \quad L_p = \frac{1}{p}.$$

Using L_s to denote the fixed labour supply the constant population supplies at every moment of time the labour market clearing condition is

$$(22) \quad \frac{\dot{N}}{\eta_i \varpi_i} + \frac{1}{p} = L_s.$$

A necessary condition placed upon this labour market condition is that employment in the production sector must be non-negative. Given this, the equilibrium price of an innovative product must satisfy $p \geq \frac{1}{L_s}$.

The higher the amount of GPT search and GPT exploitation for a given η_i implies a higher rate of labour demand in the R&D sector. Rising wages are therefore the only factor placing an upper bound on the rate of innovation at any particular point in time.²⁰ This ensures the stability condition identified in equation (19) is satisfied.

II. EQUILIBRIA

(a) *General Equilibrium: Linking Innovation to Economic Growth*

Innovation (both genesis and post-genesis) will only take place when the financial market valuation placed upon innovative activity, v , is greater than the valuation placed

²⁰ In accordance with equation (10), as wages rise with the rate of innovation, so will the rate of inflation.

on the production of the existing range of products, \bar{v} . Combining equations (10), (19) and (21) gives this relative valuation boundary condition as

$$(23) \quad \bar{v} = \frac{\alpha}{\beta \varpi_i \eta_i L}.$$

Assuming that the relative valuation condition is met, GPT search and/or GPT exploitation does not continue unbounded; the labour market resource constraint and rising wages sets an upper limit on the amount of innovative activity undertaken in any one period of time. The rate of product development is determined by the pricing equation (10), the equilibrium market valuation condition in equation (19) and the labour market equilibrium condition in equation (22)

$$(24) \quad \dot{N} = L_S \eta_i \varpi_i - \frac{\alpha \beta}{v}.$$

Provided the relative valuation boundary condition is met, equation (24) represents the potential rate of innovation per unit of time. For this rate of innovation to actually take place, it must be financed. The dynamic financial market valuation placed upon innovative activity is obtained by combining the formulae for the intertemporal consumption maximisation (8) and the operating profit condition (17) into the no arbitrage condition (18)

$$(25) \quad \dot{v} = \rho v - \frac{1 - \alpha - R}{N}.$$

The system of equations is now formalised into the dual differential equations (24) and (25). Consistent with Grossman and Helpman (1991), economic growth is directly proportional to the rate of innovation per unit of time with real GDP defined as the sum of value-added manufacturing plus R&D

$$(26) \quad G \equiv p_D D + v \dot{N}.$$

Growth in real GDP is equivalent to the weighted average growth rates in the index of manufactured output and research output

$$(27) \quad g_G = g \left[\frac{A(1-\alpha)}{\alpha} + (1-A) \right]$$

where A is the weighting applied to the respective indices.

(b) Innovation/Growth Waves

Case 1: $P_G < P_{PG}$

Assuming there is no incentive to undertake GPT search simultaneously with GPT exploitation and imputing values for \bar{n}_i , π , α , R , ρ and L_s , it is possible to solve the differential equation system for the rate of innovation per unit of time to establish its wave like pattern.²¹ Before doing this however, it is first necessary to verify the existence of a steady-state equilibrium for the imputed variables.

Proposition 3. For the imputed values $\bar{n}_i = 1000$, $\pi = 0.1$, $\alpha = 0.5$, $R = 0.1$, $\rho = 0.04$ and $L_s = 1000$ there exists a steady-state equilibrium value.

Proof: See Appendix.

Having established the existence of a steady-state equilibrium for the imputed variables, the rate of innovation is simply equation (24). To ascertain the rate of innovation per unit of time it is necessary to anchor the rate of innovation relative to some variable.

²¹ The endogenous cycles displayed in this model differs from earlier models - such as Stiglitz (1993) - in the sense that endogenous cycles are the result of the exhaustion of innovation opportunities not the misallocation of resources.

Doing this for N relative to \bar{n}_i gives the following wave like pattern for the rate of innovation over time.

Figure 1: An Innovation/Growth Wave

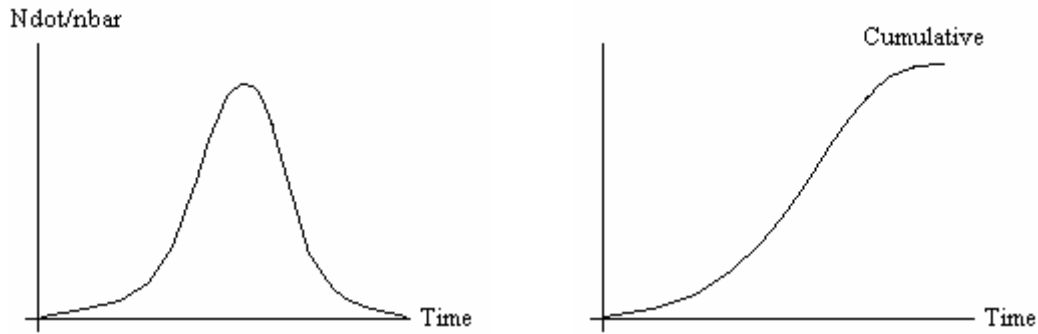


Figure 1 illustrates that as knowledge regarding a new GPT begins to spread, the pace of innovation per unit of time begins to rise at a faster rate. However, as innovation opportunities become more scarce, this pace of innovation falls. The cumulative relationship resembles the typical innovation diffusion pattern as identified in Griliches (1960). Given the direct relationship between the rate of innovation and the rate of economic growth identified in equation (26), economic growth will follow a similar pattern. For the imputed values, the periodicity of the innovation/growth wave is 64 years. The main determinants of this periodicity are the rate at which η_i increases for a unitary increase in N , and the value of \bar{n}_i . The wave like pattern of innovation exploitation per unit of time is a generalised result with the exception of extremely low values of $L_{R\&D}$ or η_i which generate negative values for \dot{N} as innovation opportunities become more scarce.

Case 2: $P_G > P_{PG}$

Assuming it is profitable for a small proportion of entrepreneurs to undertake GPT search, it is still possible to identify a wave like pattern in the rate of innovation per unit of time. Using the same imputed values as in Case 1, the only difference in the pattern

of innovation is that it takes a longer period of time to reach the full exploitation of \bar{n}_i . This is because it is now profitable for some small portion of L_S to be devoted to GPT search which results in less labour being engaged in the exploitation of the current GPT. Consistent with Proposition 2, if one individual has been successful in their GPT search, it will not be profitable for entrepreneurs to shift to the new GPT until the superseded GPT has been fully exploited (at the prevailing level of η_i).

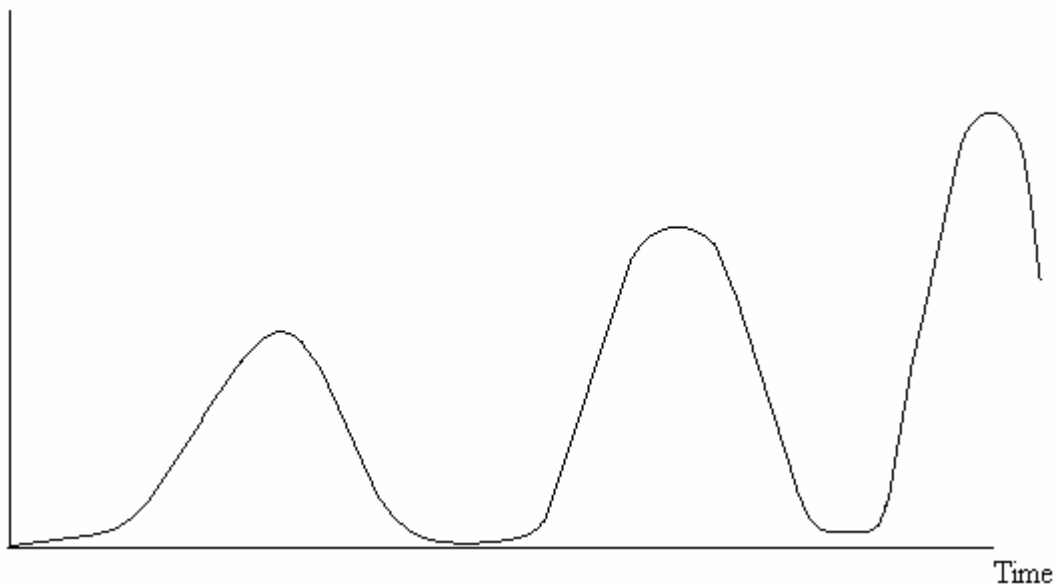
(c) *Long Wave Compression*

Case 1: $P_G < P_{PG}$

Consistent with equation (13), the cumulative nature of knowledge implies that η_i will be continuously rising which has implications for the pace of exploration of sequential GPTs. Linking together waves of innovation requires some assumption regarding the influence of information learnt in older GPTs upon η_i .²² Assuming $\zeta = 0.01$ generates the following result for long wave compression when there are alternating phases of GPT search / GPT exploitation.

Figure 2: Long Wave Compression - $P_G < P_{PG}$

\dot{N} / \bar{n}



²² The solution procedure remains basically the same with the exception of defining the steady-state position for the very last GPT.

For the very last GPT the final value of \bar{n}_i is never actually reached. Intertemporal solution requires the assumption of a significantly large number of GPTs and then undertaking a recursive backward solving solution for each GPT using the same approach as identified in Proposition 2 but also taking into account the declining influence of ζ upon η_i . It was found that in excess of 10 GPTs was sufficient to enable this procedure to be adopted while minimising any present value potential error for the established time path that may emerge since \bar{n}_i for the last GPT never actually reaches ∞ .

As can be seen in figure 2, the cumulative effect of knowledge upon η_i results in the periodicity of the long wave shortening for each new GPT. The speed of this compression is directly dependent upon the assumed value of ζ . Inclusive at the beginning of each wave is a finite period of GPT search in which there is a high probability of a new leading technology emerging. This finite period of GPT search at the start of each long wave will also decline as η_i rises. As this period of GPT search declines so too does the duration and severity of each low growth period in which the transition between leading technologies takes place (as evidenced in figure 2 by a higher base value and shorter intermission period between the second and third cycle).

For the imputed values, the speed of compression resulted in the duration of the long wave approximately halving for each new GPT. The speed of compression for the second wave for different values of ζ is illustrated in the following figures.

Figures 3(i) and 3(ii): The Speed of Long Wave Compression

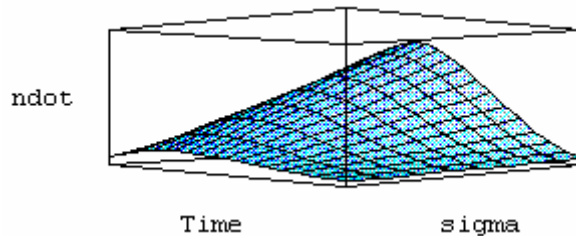
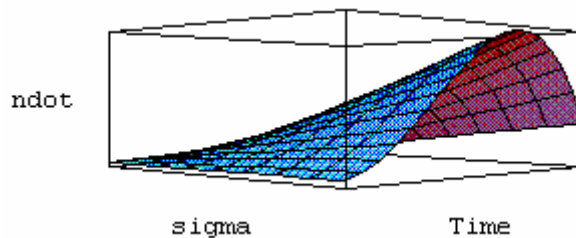


Figure 3(i) above illustrates the elongated long wave for a low value of ζ . Figure (ii) below takes the opposite side view and illustrates the much shorter secondary wave for a higher value of ζ .



What is a reasonable approximation for the value of ζ will only be acquired by a close examination of the empirical data.

Proposition 4. Although it was initially assumed that there are an equal finite number of financially viable innovation opportunities for each GPT, the compression of the long wave still holds when \bar{n}_i is allowed to vary; provided $\frac{d\bar{n}_i}{dt} < \zeta$.

Proof: Initially it was assumed that \bar{n}_i was constant, thus a positive value for ζ implies more rapid rate of exploitation of innovation opportunities with the periodicity of each sequential wave (τ_i) declining as $t \rightarrow \infty$. If \bar{n}_i is, on average, declining then $\frac{d\tau_i}{dt}$ will fall at a faster rate than in the case of a constant \bar{n}_i . If \bar{n}_i is, on average, rising then $\frac{d\tau_i}{dt}$ will still fall as $t \rightarrow \infty$, provided $\frac{d\bar{n}_i}{dt} < \zeta$. It is only in the limiting case where the influence of earlier GPTs upon the productivity of labour in the R&D sector is rising at a rate less than (or equal) to the average growth of \bar{n}_i , that the periodicity of the long wave will be rising (or constant).

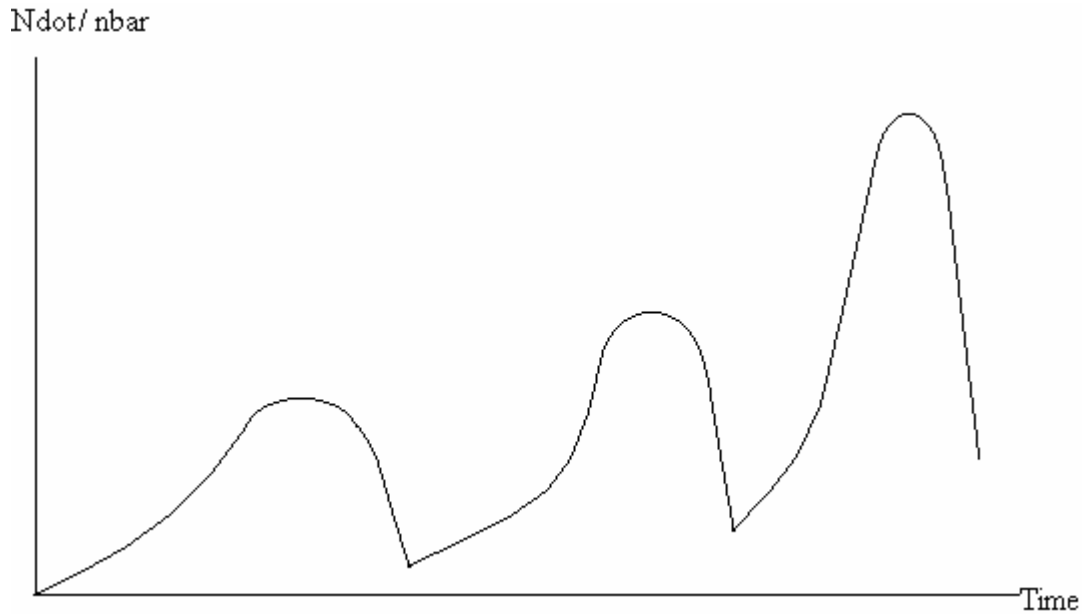
Case 2: $P_G > P_{PG}$

When it is profitable for a small proportion of entrepreneurs to constantly undertake GPT search there will be a similar pattern of wave compression as was exhibited in Case 1. Assuming that the small minority of entrepreneurs are successful in their endeavours within the duration of the exploitation of the current leading technology (τ_i), there will be an instantaneous transition to the next leading technology at the end of each long wave. This implies that there will be no period of protracted economic growth as entrepreneurs search for the next GPT. Rather, entrepreneurs switch *en-masse* to whatever new GPT that has emerged once the relative return in the superseded GPT falls below the crucial threshold identified by $\varpi_{i-1}\eta_{i-1} < \varpi_i\eta_i$.²³ Given this

²³ If more than one possible leading technology has emerged over the time period τ_i , the technology that is initially the most popular choice will become the next GPT. This is because the technology that is the most popular choice will also accumulate the

behaviour, the pattern that emerges in the rate of compression of the long wave for an assumed value of $\zeta = 0.01$ is identified in the following figure.

Figure 4: Long Wave Compression - $\underline{P_G > P_{PG}}$



III. CONCLUSION

The model developed in this paper can be characterised as exhibiting both endogenous growth and endogenous cycles. Entrepreneurial choice between the pursuit of GPT search and GPT exploitation generates the existence of long waves in economic activity which, because of knowledge spillovers between GPTs have a tendency to compress over time. Knowledge spillovers also have the effect of raising the rate of economic growth by raising the rate of innovation output per unit of time. Related to this issue, accumulated knowledge results in the smoother transition between declining and new GPTs so that there is a greater potential for shorter and less severe slumps in economic activity during these transition periods. This is because entrepreneurial agents become

most technology specific knowledge and will therefore quickly establish a comparative advantage over other competing leading technologies in terms of η_i .

more adept at developing new leading technologies to provide the engine for future economic growth.

The model suffers from the limitation of an inability to describe the simultaneous existence of different GPTs. This is flagged as an area of future research. The main conclusions of the model are centred on the assumption that what is learnt in earlier GPTs has a pervasive influence upon the productivity of exploiting new GPTs. The proposed existence of such an effect, along with its implications for economic growth, will only be verified by an examination of the data.

APPENDIX

Proof of Proposition 3: A Steady-State Solution for Imputed Values

Using the time-elimination technique developed by Mulligan and Sala-i-Martin (1993) and solving for the steady-state condition for the differential equations (24) and (25) by setting $\dot{N} = 0$ and $\dot{v} = 0$ (respectively), gives

$$(A1) \quad v = \frac{\alpha}{L_s \eta_i \varpi_i} \quad \text{for } \dot{N} = 0 \text{ in equation (24);}$$

$$(A2) \quad v = \frac{1 - \alpha - R}{\rho N} \quad \text{for } \dot{v} = 0 \text{ in equation (25).}$$

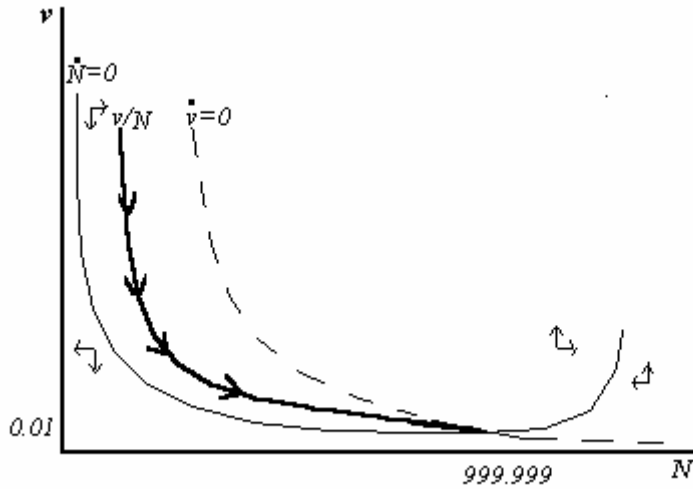
Assuming transversality conditions are satisfied, then it is possible to take the ratio of equation (24) and (25) and use L' Hopitals rule to give the slope of the optimal path at the steady-state equilibrium value

$$(A3) \quad \left. \frac{dv}{dN} \right|_{N=N^*} = \frac{\dot{v}}{\dot{N}} = \lim_{N \rightarrow N^*} \frac{\rho v^2 N - v(1 - \alpha - R)}{L_s \eta_i v N \varpi_i - \alpha N};$$

using L' Hopitals rule

$$(A4) \quad \lim_{N \rightarrow N^*} \frac{dv'}{dN'} \Big|_{N=N^*} = \frac{\rho v^2 - 2\rho N v v' - v'(1 - \alpha - R)}{L_s \eta_i v \varpi_i - L_s \eta_i N v + L_s \eta_i v' N \varpi_i + L_s \eta_i' v \varpi_i - \alpha}$$

Using an interpolation function technique, it is possible to use computer packages such as *Mathematica* and MATLAB to solve for the optimal path given starting values obtained by simultaneously solving equations (A1) and (A2), and using equation (A4) to give the slope of \dot{v}/\dot{N} at this point. An additional boundary condition is required for ζ to incorporate the impact of knowledge accumulation over time. Solving for the imputed variables gives the following steady-state solution.



Full exploitation of each and every \bar{n}_i only occurs if $L_{R\&D} \geq \bar{n}_i$ or, to begin with, η_i is sufficiently high for the given values of $L_{R\&D}$ and \bar{n}_i . If for earlier GPTs \bar{n}_i is not fully exploited, then there arises the possibility of the dual exploitation of current and past GPTs as η_i rises through time and previously non-viable innovations become commercially feasible. If this were the case, adjustments would need to be made in regard to the number of innovation opportunities available per unit of time. This issue

of backtracking for previously non-obtainable values of \bar{n}_i becomes less of a problem as $t \rightarrow \infty$ and η_i rises.

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