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The Judge Institute of Management
University of Cambridge
Trumpington Street
Cambridge CB2 1AG, UK
www.jims.cam.ac.uk

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Santiago Arbeleche
Centre for Financial Research
Judge Institute of Management
University of Cambridge
Tel: +44 (0) 1223 339651
Fax: +44 (0) 1223 339652
Email: cfr@jims.cam.ac.uk

Professor M A H Dempster
Centre for Financial Research
Judge Institute of Management
University of Cambridge
Tel: +44 (0) 1223 339651
Fax: +44 (0) 1223 339652
Email: m.dempster@jims.cam.ac.uk

Please address enquiries about the series to:

Research Support Manager
Judge Institute of Management
Trumpington Street
Cambridge CB2 1AG, UK
Tel: 01223 760546 Fax: 01223 339701
E-mail: research-support@jims.cam.ac.uk

Econometric Modelling for Global Asset Management

Abstract

This paper focusses on model selection, specification and estimation of a global asset return model within an asset allocation and asset and liability management framework. The development departs from a single currency capital market model with four state variables: stock index, short and long term interest rates and currency exchange rates. The model is then extended to the major currency areas, United States, United Kingdom, European Union and Japan, and to include a US economic model containing GDP, inflation, wages and government borrowing requirements affecting the US capital market variables. In addition, we develop variables representing emerging market stock and bond indices. In the largest extension we treat a four currency capital markets model and US, UK, EU and Japan macroeconomic variables. The system models are estimated with seemingly unrelated regression estimation (SURE) and generalised autoregressive conditional heteroscedasticity (GARCH) techniques. Simulation, impulse response and forecasting performance is discussed in order to analyse the dynamics of the models developed.

Keywords

Global Capital Markets, Strategic Asset Allocation, Asset Return Models, Econometric Estimation, Stochastic Optimisation, Asset Liability Management.

1. Introduction

Asset liability management (ALM) is a framework which requires a statistical model specification appropriate to the asset classes of interest, model estimation, variable simulation and scenario generation. Given an asset allocation model in terms of a given utility or loss function and constraints or restrictions to the investment strategies which express the trade off between risk and return in terms of these scenarios an optimal allocation can then be found, see Wilkie (1986, 1995), Mulvey & Vladimirou (1992), Dert (1995). The reported research has been conducted in a joint project between a university and a fund management firm.

The variables in the basic statistical asset return model considered here are a *stock index*, *short term interest rate, long term interest rate* and *exchange rate*. These constitute a capital market model to be linked in each major currency area to an underlying macroeconomic model which involves the rates of change in the *consumer price index, wages and salaries, gross domestic product* and *public sector borrowing requirement*. The regions for which the full model is to be estimated are the United States, United Kingdom, Japan and the European Union - those which issue the main international currencies. For each region the result will be a system of inter-related equations estimated by the seemingly unrelated regression (SURE) technique, see Zellner (1962) and Theil (1971). Other econometric and time series methods such as vector autoregression (VAR) (Sims 1980) and GARCH models (Bollerslev, 1986) are also employed.

The *capital market model* estimated is a four state variable *Gaussian* model, with drift, volatility and correlation parameters, which is linear in the parameters but nonlinear in the variables. The drifts are modelled as functions of the state variables to explain varying risk premia for the asset classes, while the volatilities (after suitable transformation) and correlations are taken to be constant. The capital markets model includes the four state variables for each of the US, UK, EU and JP. These variables are inter-related in terms of their drift functions as well as through their innovations. The capital market variables from the emerging markets are modelled as univariate processes using ARMA and GARCH methodology. Then the residuals from the emerging market processes are used with the residuals from the specific asset return models to estimate an extended variance-covariance matrix.

In the economic model developed for the US, the growth rates of the state variables are explained in terms of lagged levels of the same variables with innovations contemporaneously correlated. The introduction of the US economic variables potentially allows the US capital market variables to be explained by these economic factors.

The full statistical asset return model is designed to be linked with a statistical model of liabilities to result in a fund management model in which fund wealth at any time is equal to the current value of initial wealth plus total contributions paid in by fund contributors minus total benefits paid out by the fund. Using the estimated coefficients of both models a set of scenarios can be generated with *Monte Carlo simulation* and incorporated in an asset allocation model which is optimized to provide the best initial asset allocation and forward investment strategy in light of liability requirements. The asset return statistical model forms part of a chain of methodologies used to solve this asset liability management problem. This is known in literature generally as *strategic financial planning* (Dempster *et al*., 2003). A brief description of the methodology can be found in Arbeleche *et al*. (2003) and a more complete exposition may be found in Dempster *et al*. (2003).

In a world where information and financial cash flows move so quickly between countries, it is a requirement to have a multi-currency framework when working on current asset management problems. In such a world it is also a must to have multi-asset class systems for diversification. Following these two ideas, a first aim of this paper is to develop a multi-currency, multi-asset class return model which has the capability of predicting the possible future *distributions* of the study variables. A second aim is to use the developed model to try to understand how the world's capital markets and economies currently work. We address this second aim by giving empirical evidence across different sample periods and robust results across global currencies. In this paper we will focus on the specification and estimation of the statistical model's parameters and its predictive capabilities as well as its system dynamics. As for the ALM problem treated in Dempster *et al*. (2003), in this paper state variables are simulated and scenarios generated.

One of our main findings from system estimation is that the world's equity, money and bond markets are linked through currency exchange rates to experience shocks simultaneously. Moreover, volatilities for the model's residuals and actual returns are in a similar range (on a monthly timestep). We introduce here *influence diagrams* showing statistically significant coefficients in the system models. This diagram facilitates the visualisation of inter relationships among variables and across currencies in the models. Interest rate parity is found to be significant in modelling exchange rates as well as in explaining short and long interest rates. Another finding is a relationship between interest rate and stock index returns. There are statistically significant parameters showing a relationship between stock returns and short and long rates across the US, UK, EU and JP. Ang & Bekaert (2003) found similar results with the short rate being a predictive tool for stock excess returns.

In Section 1 of the paper we discuss the basic econometric specification. Section 2 analyses the historical time series and tests for unit roots. In Section 3 we give the estimated system model. Section 4 analyses the residuals of the estimated model and compares the estimated model innovations or disturbances with actual asset returns. Section 6 explains briefly how scenarios can be constructed and measures the system stability and forecasting performance of the model incorporating the US economy. Section 6 extends the model to include the macroeconomic models for the remaining currency areas: UK, EU and Japan. Finally, we outline the major findings and summarise in Section 7.

2. Asset Return and Economic Growth Rate Econometric Model Specification

A preliminary version of the models described in this section, estimated from 1993:12 to 2001:02 can be found in Dempster *et al*. (2003). Emphasis is placed here on covariance stationary models. A stochastic process y_t is *covariance stationary* if the expected value of y_t is independent of t, its variance is finite, a positive constant and independent of t; and the covariance of y_t and y_s is a finite function of t-s, but not of t or s. It is common to assume that the innovations are independently generated from one period to the next, with the following assumptions:

$$
\begin{aligned}
&E[\boldsymbol{e}_t] = 0, \\
&\mathrm{Var}[\boldsymbol{e}_t] = s^2, \\
&\text{and } \mathrm{Cov}[\boldsymbol{e}_t, \boldsymbol{e}_s] = 0 \; for t \neq s.
\end{aligned}
$$

2.1 Capital Market Model

Figure 1 depicts the global structure of the asset return model. There are three investments categories or major asset classes, namely cash, bonds and equity, in the four major currency areas US, UK, EU and JP. The arrows symbolize possible explanatory dependence to be subjected to coefficient hypothesis testing and only the statistically significant relations are kept in the final parsimonious estimated model.

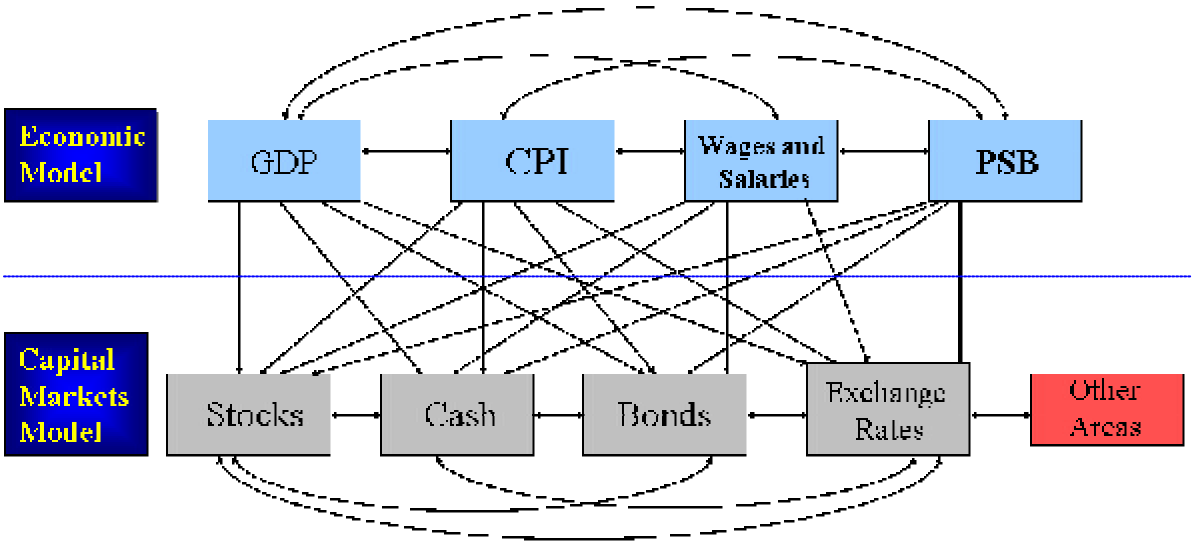

Figure 1: Major Currency Area Detailed Model Structure

The canonical structure of the model shown in Figure 1 can be interpreted as a general unreduced model when the full set of model parameters are estimated.

2.2 Econometric Model for the Capital Markets

The asset return model is in the econometric estimation framework which was developed by Wilkie (1986, 1995) and employed in the work of Dert (1995), Consigli and Dempster (1998) and Boender *et al*. (1998) among others. We state the capital market model first in the familiar continuous time framework. The state variables of our stochastic processes for the model are an equity index (**S** – stocks), short term interest rate (**R** – cash), long term interest rate (**L** – bonds) and exchange rate (**X** – domestic currency/US dollars). These variables are assumed to satisfy the *stochastic differential equations* (SDEs)

$$
\begin{aligned}
\frac{d\boldsymbol{S}}{\boldsymbol{S}} &= \boldsymbol{\mu}_S dt + s_S d\boldsymbol{Z}_S \\
d\boldsymbol{R} &= \boldsymbol{\mu}_R dt + s_R d\boldsymbol{Z}_R \\
d\boldsymbol{L} &= \boldsymbol{\mu}_L dt + s_L d\boldsymbol{Z}_L \\
\frac{d\boldsymbol{X}}{\boldsymbol{X}} &= \boldsymbol{\mu}_X dt + s_X d\boldsymbol{Z}_X ,
\end{aligned}
\qquad (1)
$$

where the $d\mathbf{Z}_i$ (i = **S**, **R**, **L**, **X**) are increments of correlated *Wiener processes*. In the system (1) all left hand side variables are measured in rates, there are two proportional returns (in the case of **S** and **X**) and two changes of rates (for **R** and **L**). Explanatory state variables in the specification are in original levels (**S** and **X**) or rate (**R** and **L**) form in that the *drift* $\boldsymbol{\mu}_i$ and *volatility* s_i parameters are assumed to be functions of the state variables and time t to result in the system

$$
\begin{aligned}
\frac{d\mathbf{S}}{\mathbf{S}} &= m_S(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)dt + s_S(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)d\mathbf{Z}_S \\
d\mathbf{R} &= m_R(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)dt + s_R(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)d\mathbf{Z}_R \\
d\mathbf{L} &= m_L(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)dt + s_L(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)d\mathbf{Z}_L \\
\frac{d\mathbf{X}}{\mathbf{X}} &= m_X(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)dt + s_X(\mathbf{S}, \mathbf{R}, \mathbf{L}, \mathbf{X}, t)d\mathbf{Z}_X .
\end{aligned}
\qquad (2)
$$

For this continuous time system the following functional forms were assumed for the drift and volatility functions for subsequent econometric estimation:

$$
\begin{aligned}
\frac{d\mathbf{S}}{\mathbf{S}} &= \left(a_{S1} + a_{S2}\mathbf{S} + a_{S3}\mathbf{R} + a_{S4}\mathbf{L} + a_{S5}\mathbf{X}\right)dt + s_S d\mathbf{Z}_S \\
d\mathbf{R} &= \left(a_{R1}\mathbf{R} + a_{R2}\mathbf{S} + a_{R3} + a_{R4}\mathbf{L} + a_{R5}\mathbf{X}\right)dt + s_R \mathbf{R} d\mathbf{Z}_R \\
\frac{d\mathbf{L}}{\mathbf{L}} &= \left(a_{L1} + a_{L2}\mathbf{S} + a_{L3}\mathbf{R} + a_{L4}\mathbf{L} + a_{L5}\mathbf{X}\right)dt + s_L d\mathbf{Z}_L \\
d\mathbf{X} &= \left(a_{X1}\mathbf{X} + a_{X2}\mathbf{S} + a_{X3}(\mathbf{R}^F - \mathbf{R}) + a_{X4}(\mathbf{L}^F - \mathbf{L}) + a_{X5}\right)dt + s_X \mathbf{X} d\mathbf{Z}_X ,
\end{aligned}
\qquad (3)
$$

where $\mathbf{R}^F$ and $\mathbf{L}^F$ stand for an appropriate *foreign short term rate* and *foreign long term rate* respectively. Frankel (1995) includes the differential between domestic and foreign money supplies and short and long term rate differentials as well. His specification is: $\mathbf{x} = \beta_0 (m - m^*) + \beta_1(r - r^*) + \beta_2 (l - l^*) + \mathbf{e}$, where * denotes a foreign variable. Note that in (3) the proportional change in the stock price **S** and the long rate **L** have a constant volatility term, while the changes in the exchange rate **X** and the short rate **R** have a volatility proportional to the state variable.

Some discussion of the specification of (3) is in order. First note that the stochastic differential equation specifying the evolution of the stock index **S** is a generalisation of geometric Brownian motion in which the levels of the other state variables and a constant help to explain the proportional return of the index. Similarly the SDE for the long term rate **L** is a generalisation of the Rendleman and Bartter (1980) interest rate model found useful by Dempster & Thorlacius (1998) in the Falcon Asset model. Note that the evolution of the mean function of these processes, while nonlinear, is neither exponential nor even monotone (see Dempster *et al.* (2003a)). Indeed, the non-constant terms in the drift of each stochastic differential equation of (3) may be considered to be a specification of the *market price of risk* of the corresponding asset class which varies over time with the levels of the full set of capital market variables. The SDE's for the short rate **R** and the exchange **X** are generalisations of Cox’s (1975) *constant elasticity of variance* (CEV) specification with volatility state parameter 1 (as opposed to the ½ of the Cox, Ingersoll and Ross (1985) model) as found useful previously in the Falcon Asset model.

In order to have constant volatility Wiener increments for all four state variables we transform this continuous time Markov diffusion system, which is linear in coefficients and non-linear in independent variables, to obtain

$$
\begin{aligned}
\frac{d\mathbf{S}}{\mathbf{S}} &= \left(a_{S1} + a_{S2}\mathbf{S} + a_{S3}\mathbf{R} + a_{S4}\mathbf{L} + a_{S5}\mathbf{X}\right)dt + s_s d\mathbf{Z}_s \\
\frac{d\mathbf{R}}{\mathbf{R}} &= \left(a_{r1} + a_{r2}\frac{\mathbf{S}}{\mathbf{R}} + \frac{a_{r3}}{\mathbf{R}} + a_{r4}\frac{\mathbf{L}}{\mathbf{R}} + a_{r5}\frac{\mathbf{X}}{\mathbf{R}}\right)dt + s_r d\mathbf{Z}_r \\
\frac{d\mathbf{L}}{\mathbf{L}} &= \left(a_{l1} + a_{l2}\mathbf{S} + a_{l3}\mathbf{R} + a_{l4}\mathbf{L} + a_{l5}\mathbf{X}\right)dt + s_l d\mathbf{Z}_l \\
\frac{d\mathbf{X}}{\mathbf{X}} &= \left(a_{x1} + a_{x2}\frac{\mathbf{S}}{\mathbf{X}} + a_{x3}\frac{(\mathbf{R}^F - \mathbf{R})}{\mathbf{X}} + a_{x4}\frac{(\mathbf{L}^F - \mathbf{L})}{\mathbf{X}} + \frac{a_{x5}}{\mathbf{X}}\right)dt + s_x d\mathbf{Z}_x.
\end{aligned}
\tag{4}
$$

To obtain a statistically estimable form of the continuous time system (4) with discretely sampled data we approximate differentials with differences (? t for dt) to obtain

$$
\begin{aligned}
\frac{\Delta S_t}{S_t} &= \left(a_{S1} + a_{S2}S_t + a_{S3}R_t + a_{S4}L_t + a_{S5}X_t\right)\Delta t + s_s\sqrt{\Delta t}e_t^S \\
\frac{\Delta R_t}{R_t} &= \left(a_{R1} + a_{R2}\left(\frac{S_t}{R_t}\right) + a_{R3}\left(\frac{1}{R_t}\right) + a_{R4}\left(\frac{L_t}{R_t}\right) + a_{R5}\left(\frac{X_t}{R_t}\right)\right)\Delta t + s_R\sqrt{\Delta t}e_t^R \\
\frac{\Delta L_t}{L_t} &= \left(a_{L1} + a_{L2}S_t + a_{L3}R_t + a_{L4}L_t + a_{L5}X_t\right)\Delta t + s_L\sqrt{\Delta t}e_t^L \\
\frac{\Delta X_t}{X_t} &= \left(a_{X1} + a_{X2}\left(\frac{S_t}{X_t}\right) + a_{X3}\left(\frac{R_t^F - R_t}{X_t}\right) + a_{X4}\left(\frac{L_t^F - L_t}{X_t}\right) + a_{X5}\left(\frac{1}{X_t}\right)\right)\Delta t + s_X\sqrt{\Delta t}e_t^X,
\end{aligned}
\tag{5}
$$

where the e's are realisations of correlated standard normal variates. Finally, if we set the time scale to be one month (? t := 1 month) the discretised model to be estimated is given by

$$
\begin{aligned}
\frac{S_{t+1} - S_t}{S_t} &= a_{S1} + a_{S2}S_t + a_{S3}R_t + a_{S4}L_t + a_{S5}X_t + s_s e_t^S \\
\frac{R_{t+1} - R_t}{R_t} &= a_{R1} + a_{R2}\left(\frac{S_t}{R_t}\right) + a_{R3}\left(\frac{1}{R_t}\right) + a_{R4}\left(\frac{L_t}{R_t}\right) + a_{R5}\left(\frac{X_t}{R_t}\right) + s_R e_t^R \\
\frac{L_{t+1} - L_t}{L_t} &= a_{L1} + a_{L2}S_t + a_{L3}R_t + a_{L4}L_t + a_{L5}X_t + s_L e_t^L \\
\frac{X_{t+1} - X_t}{X_t} &= a_{X1} + a_{X2}\left(\frac{S_t}{X_t}\right) + a_{X3}\left(\frac{R_t^F - R_t}{X_t}\right) + a_{X4}\left(\frac{L_t^F - L_t}{X_t}\right) + a_{X5}\left(\frac{1}{X_t}\right) + s_X e_t^X.
\end{aligned}
\tag{6}
$$

The final model specification for the capital markets in the US (with a monthly time step) including the US macroeconomic variables as independent variables with two lags is given by

$$\frac{\boldsymbol{S}_{t+1}^{US} - S_t^{US}}{S_t^{US}} = \begin{pmatrix} a_{S1}^{US} + a_{S2}^{US} S_t^{US} + a_{S3}^{US} R_t^{US} + a_{S4}^{US} L_t^{US} + a_{S5}^{US} X_t^{UK} + \\ b_{S2}^{US} S_{t-1}^{US} + b_{S3}^{US} R_{t-1}^{US} + b_{S4}^{US} L_{t-1}^{US} + b_{S5}^{US} X_{t-1}^{UK} + \\ c_{S2}^{US} CPI_t^{US} + c_{S3}^{US} WS_t^{US} + c_{S4}^{US} GDP_t^{US} + c_{S5}^{US} PSB_t^{US} + \\ d_{S2}^{US} CPI_{t-1}^{US} + d_{S3}^{US} WS_{t-1}^{US} + d_{S4}^{US} GDP_{t-1}^{US} + d_{S5}^{US} PSB_{t-1}^{US} \end{pmatrix} + s_S^{US} \boldsymbol{e}_{St}^{US} \tag{7}$$

$$\frac{\boldsymbol{R}_{t+1}^{US} - R_t^{US}}{R_t^{US}} = \begin{pmatrix} a_{R1}^{US} + a_{R2}^{US}\left(\frac{S_t^{US}}{R_t^{US}}\right) + a_{R3}^{US}\left(\frac{1}{R_t^{US}}\right) + a_{R4}^{US}\left(\frac{L_t^{US}}{R_t^{US}}\right) + a_{R5}^{US}\left(\frac{X_t^{UK}}{R_t^{US}}\right) + \\ b_{R2}^{US}\left(\frac{S_{t-1}^{US}}{R_t^{US}}\right) + b_{R3}^{US}\left(\frac{1}{R_t^{US}}\right) + b_{R4}^{US}\left(\frac{L_{t-1}^{US}}{R_t^{US}}\right) + b_{R5}^{US}\left(\frac{X_{t-1}^{UK}}{R_t^{US}}\right) + \\ c_{R2}^{US} CPI_t^{US} + c_{R3}^{US} WS_t^{US} + c_{R4}^{US} GDP_t^{US} + c_{R5}^{US} PSB_t^{US} + \\ d_{R2}^{US} CPI_{t-1}^{US} + d_{R3}^{US} WS_{t-1}^{US} + d_{R4}^{US} GDP_{t-1}^{US} + d_{R5}^{US} PSB_{t-1}^{US} \end{pmatrix} + s_R^{US} \boldsymbol{e}_{Rt}^{US} \tag{8}$$

$$\frac{\boldsymbol{L}_{t+1}^{US} - L_t^{US}}{L_t^{US}} = \begin{pmatrix} a_{L1}^{US} + a_{L2}^{US} S_t^{US} + a_{L3}^{US} R_t^{US} + a_{L4}^{US} L_t^{US} + a_{L5}^{US} X_t^{UK} + \\ b_{L2}^{US} S_{t-1}^{US} + b_{L3}^{US} R_{t-1}^{US} + b_{L4}^{US} L_{t-1}^{US} + b_{L5}^{US} X_{t-1}^{UK} + \\ c_{L2}^{US} CPI_t^{US} + c_{L3}^{US} WS_t^{US} + c_{L4}^{US} GDP_t^{US} + c_{L5}^{US} PSB_t^{US} + \\ d_{L2}^{US} CPI_{t-1}^{US} + d_{L3}^{US} WS_{t-1}^{US} + d_{L4}^{US} GDP_{t-1}^{US} + d_{L5}^{US} PSB_{t-1}^{US} \end{pmatrix} + s_L^{US} \boldsymbol{e}_{Lt}^{US}. \tag{9}$$

In this formulation the US economic variables (**CPI**, **WS**, **GDP** and **PSB**) form part of the explanatory variables for the US capital markets (**S**, **R** and **L**). The e terms are correlated standard normal or standardised student t random variables (for simulation purposes). The parameters to be estimated are the a, b, c, d and s terms. The a and b terms are parameters corresponding to the state capital markets variables at time t and one period lagged respectively; while the c and d terms are the parameters for the economic state variables with similar lag structure, time t for parameters c, and time $t-1$ for parameters d.

In the case of the US, there is no exchange rate equation because the base currency of the whole model is the US dollar. Nevertheless, the model can be easily expressed another currency, such as the pound, euro or yen, as depends on the home currency of an investor.

For the remaining currency areas, the final model specification for the capital markets is of the form

$$\begin{aligned} \frac{\boldsymbol{S}_{t+1}^{i} - S_t^{i}}{S_t^{i}} = {} & a_{S1}^{i} + a_{S2}^{i} S_t^{i} + a_{S3}^{i} R_t^{i} + a_{S4}^{i} L_t^{i} + a_{S5}^{i} X_t^{i} + \\ & b_{S2}^{i} S_{t-1}^{i} + b_{S3}^{i} R_{t-1}^{i} + b_{S4}^{i} L_{t-1}^{i} + b_{S5}^{i} X_{t-1}^{i} + s_S^{i} \boldsymbol{e}_{St}^{i} \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\boldsymbol{R}_{t+1}^{i} - R_t^{i}}{R_t^{i}} = {} & a_{R1}^{i} + a_{R2}^{i}\left(\frac{S_t^{i}}{R_t^{i}}\right) + a_{R3}^{i}\left(\frac{1}{R_t^{i}}\right) + a_{R4}^{i}\left(\frac{L_t^{i}}{R_t^{i}}\right) + a_{R5}^{i}\left(\frac{X_t^{i}}{R_t^{i}}\right) + \\ & b_{R2}^{i}\left(\frac{S_{t-1}^{i}}{R_t^{i}}\right) + b_{R3}^{i}\left(\frac{1}{R_t^{i}}\right) + b_{R4}^{i}\left(\frac{L_{t-1}^{i}}{R_t^{i}}\right) + b_{R5}^{i}\left(\frac{X_{t-1}^{i}}{R_t^{i}}\right) + s_R^{i} \boldsymbol{e}_{Rt}^{i} \end{aligned} \tag{11}$$

$$\frac{\boldsymbol{L}_{t+1}^{i} - L_t^i}{L_t^i} = a_{L1}^i + a_{L2}^i S_t^i + a_{L3}^i R_t^i + a_{L4}^i L_t^i + a_{L5}^i X_t^i + \\ b_{L2}^i S_{t-1}^i + b_{L3}^i R_{t-1}^i + b_{L4}^i L_{t-1}^i + b_{L5}^i X_{t-1}^i + s_L^i \boldsymbol{e}_{Lt}^i \tag{12}$$

$$\frac{\boldsymbol{X}_{t+1}^{i} - X_t^i}{X_t^i} = a_{X1}^i + a_{X2}^i\left(\frac{S_t^i}{X_t^i}\right) + a_{X3}^i\left(\frac{R_t^{US} - R_t^i}{X_t^i}\right) + a_{X4}^j\left(\frac{L_t^{US} - L_t^i}{X_t^i}\right) + a_{X5}^i\left(\frac{1}{X_t^i}\right) + \\ b_{X2}^i\left(\frac{S_{t-1}^i}{X_t^i}\right) + b_{X3}^i\left(\frac{R_{t-1}^{US} - R_{t-1}^i}{X_t^i}\right) + b_{X4}^i\left(\frac{L_{t-1}^{US} - L_{t-1}^i}{X_t^i}\right) + b_{X5}^i\left(\frac{1}{X_t^i}\right) + s_X^i \boldsymbol{e}_{Xt}^i. \tag{13}$$

where i = UK, EU and JP. The e terms are correlated standard normal or standardised student t random variables. The a and b terms are parameters of the state capital market variables at time t and one period lagged respectively. As in the US case, we have non-linear drifts, a lag structure and constant volatilities.

The *interest rate parity* (IRP) theorem is represented in the exchange rate equation by means of the home and foreign short and long term interest rate differentials. The first is through the differential of the short rate R between the base currency (US) and the home currency i. So effectively we are modelling the return on the exchange rate $\left(\frac{X_{t+1}^i - X_t^i}{X_t^i}\right)$ with the differential of rates as a proportion of the exchange rate level $\left(\frac{R_t^{US} - R_t^i}{X_t^i}\right)$ as an independent or explanatory variable. The second representation of IRP is included in a similar fashion but with respect to long term interest rates as $\left(\frac{L_t^{US} - L_t^i}{X_t^i}\right)$.

2.3 Economic Model

In order to achieve more economic reality in the explanation of asset returns and exchange rates, we first introduce US economic variables. These economic variables are assumed to influence only US financial variables directly (see equations (7) to (9)). We specify their relationships as

$$\frac{\boldsymbol{CPI}_{t+1} - CPI_t}{CPI_t} = a_{CPI_1} + a_{CPI_2} CPI_t + a_{CPI_3} WS_t + a_{CPI_4} GDP_t + a_{CPI_5} PSB_t + \\ b_{CPI_2} CPI_{t-1} + b_{CPI_3} WS_{t-1} + b_{CPI_4} GDP_{t-1} + b_{CPI_5} PSB_{t-1} + s_{CPI} \boldsymbol{e}_t^{CPI} \tag{14}$$

$$\frac{\boldsymbol{WS}_{t+1}-WS_t}{WS_t}=a_{WS_1}+a_{WS_2}CPI_t+a_{WS_3}WS_t+a_{WS_4}GDP_t+a_{WS_5}PSB_t+ \\ b_{WS_2}CPI_{t-1}+b_{WS_3}WS_{t-1}+b_{WS_4}GDP_{t-1}+b_{WS_5}PSB_{t-1}+s_{WS}\boldsymbol{e}_t^{WS} \tag{15}$$

$$\frac{\boldsymbol{GDP}_{t+1}-GDP_t}{GDP_t}=a_{GDP_1}+a_{GDP_2}CPI_t+a_{GDP_3}WS_t+a_{GDP_4}GDP_t+a_{GDP_5}PSB_t+ \\ b_{GDP_2}CPI_{t-1}+b_{GDP_3}WS_{t-1}+b_{GDP_4}GDP_{t-1}+b_{GDP_5}PSB_{t-1}+s_{GDP}\boldsymbol{e}_t^{GDP} \tag{16}$$

$$\boldsymbol{PSB}_{t+1}-PSB_t=a_{PSB_1}+a_{PSB_2}CPI_t+a_{PSB_3}WS_t+a_{PSB_4}GDP_t+a_{PSB_5}PSB_t \\ b_{PSB_2}CPI_{t-1}+b_{PSB_3}WS_{t-1}+b_{PSB_4}GDP_{t-1}+b_{PSB_5}PSB_{t-1}+s_{PSB}\boldsymbol{e}_t^{PSB}. \tag{17}$$

In line with the capital markets model, we explain the left hand side of the economic model (forward return from *t+1* to *t*) with levels at time *t* and time *t-1*. As before, the innovations e are contemporaneously correlated although not serially correlated and the step time is monthly (i.e. $\Delta t := 1$).

One problem encountered in this research is the restriction of some macroeconomic variables, for example GDP, to quarterly figures. The time scale of the model is however monthly, so some transformation is needed. We take the cube root of the change from quarter to quarter as a proxy of the monthly percentage change. Another possibility is to take one third of the absolute change from quarter to quarter as a proxy of the monthly absolute change. As an example, the value at month 6 is the value at the end of Q2 while the end of Q3 is the value at month 9; so that value at month 7 is the previous (month 6) plus a third of the Q3 – Q2 differential and so on. We chose the latter approach because we are working with returns, and the second method produces slightly different proportional changes between months as opposed to a constant proportional change for each three month period.

2.4 Emerging Market Models

For the *emerging market* stock ($\mathbf{S}^{\mathrm{EM}}$) and bond ($\mathbf{B}^{\mathrm{EM}}$) index processes we specify the following model:

$$\frac{\boldsymbol{S}_{t+1}^{EM}-S_t^{EM}}{S_t^{EM}}=a_S^{EM}+a_{S1}^{EM}\left(\frac{S_t^{EM}-S_{t-1}^{EM}}{S_{t-1}^{EM}}\right)-a_{S2}^{EM}\sqrt{H_{t-1}^S}e_{t-1}^S+\sqrt{H_t^S}e_t^S \\ \boldsymbol{H}_t^S=b_S+p_SH_{t-1}^S-q_SH_{t-1}^S\left(e_{t-1}^S\right)^2 \tag{18}$$

$$\frac{\boldsymbol{B}_{t+1}^{EM}-B_t^{EM}}{B_t^{EM}}=a_B^{EM}+s_{\boldsymbol{B}^{EM}}e_t^B\ . \tag{19}$$

The model specification for the equity index process is thus effectively an ARMA (1,1) model with a GARCH (1,1) error structure. Let y_t be the proportional return on the emerging markets stock index process and let $\boldsymbol{u}_t := \sqrt{H_t}e_t$. Then the ARMA/GARCH specification can then be written as

$$\boldsymbol{y}_t=a_0+a_1\boldsymbol{y}_{t-1}-b_1u_{t-1}+\boldsymbol{u}_t \\ H_t=g+pH_{t-1}-qu_{t-1}^2,\qquad \boldsymbol{u}_t:=\sqrt{H_t}e_t \tag{20}$$

which has the same structure as the original GARCH specification (Bollerslev, 1986). H_t is the *conditional variance* of $\boldsymbol{u}_t$ and its *unconditional variance* is given by $s_u^2 = \frac{g}{1-p-q}$. Under GARCH specifications, the conditional variance is changing whereas the unconditional variance is constant so long as $p + q \leq 1$. For $p + q \geq 1$, the unconditional variance is not defined, which is known as non-stationarity in variance. If $p + q = 1$, then this is termed a unit root in variance situation, also called integrated GARCH (IGARCH).

The main difference between the emerging markets and the developed economies framework is the structural form of the model. Whereas in the developed economies we use economic theory and system estimation techniques (econometric modelling), in the case of the emerging markets we simply estimate univariate models with dynamic volatility for the stock and bond indices in the emerging markets.

2.5 System Form

The previous detailed specification (equations (7) to (17)) can be stated in vector form. Setting Δ equal to the forward difference, then

$$\Delta \mathbf{x} = diag\left(\mathbf{x}\right)\left[m\left(\mathbf{x}\right) + \sqrt{\mathbf{S}}\,\boldsymbol{e}\right], \tag{21}$$

where diag(.) is the operator which creates a diagonal matrix from a vector, m is a nonlinear first order autoregressive filter, $\sqrt{\mathbf{S}}$ is the Cholesky factor of the innovation correlation matrix $\mathbf{S}$ and the vector $\boldsymbol{e}$ has uncorrelated standardised Gaussian coordinates. This allows a contemporaneously correlated but serially uncorrelated innovation (disturbance) structure. This system is linear in parameters to be estimated but nonlinear in variables so that system stability must be tested using impulse response techniques (see Section 5.2 below).

3. Historical Time Series Analysis

We begin the estimation process with a graphical analysis of the asset class data. The data used as proxies for the variables in the system are shown in Table 1 and their graphical descriptions in Figure 2, which contains four panels showing the equity index, short and long interest rates and exchange rates over the 420 month period from March 1968 to February 2003. The first three panels show variables for the 4 currency areas (US, UK, EU and JP), whereas the fourth panel shows only 2 of the 3 exchange rates with the US dollar for presentation purposes.

In the econometric specification we fit the *returns* of the variable rather than the level; the resulting fitting represents the mean *return* at each time point. As the graphical analysis suggests, it is possible that some series contain a unit root; we will discuss this point in Section 3.2.

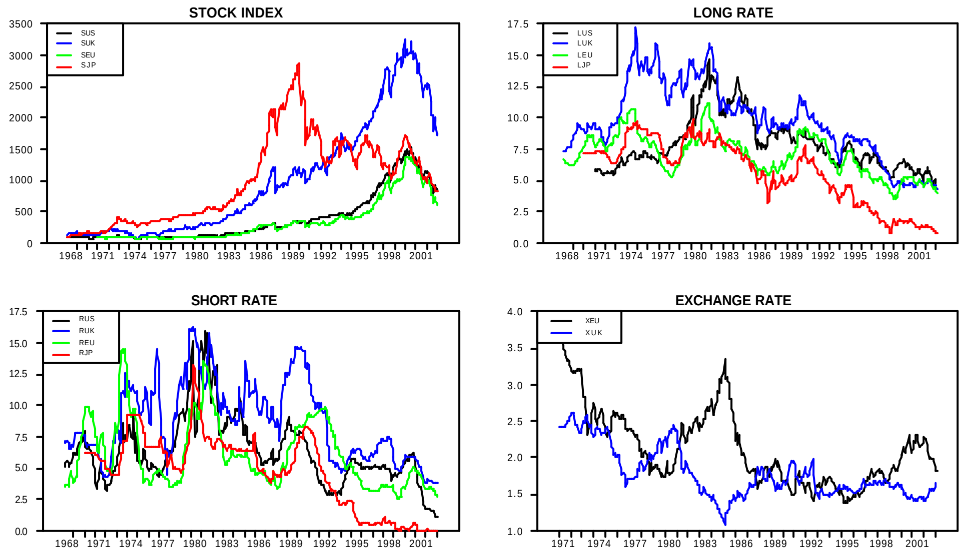

Figure 2: US, UK, EU and JP Variables in Levels

3.1 Data Analysis

The summary statistics for the original time series are presented in Table 2. From this data we can construct several models and the time horizon of the shortest time series will set the sample size for the complete system estimation. A four currency model for the US, UK, EU and JP can use up to 384 observations but the inclusion of the emerging markets limits the beginning of the estimation period to the end of 1993 date when the bond index for emerging markets started to be published and reduces the sample size.

In this paper we will present two of the models examined in our research. First, we set out a 19 equation model with 4 capital markets plus the US economy and emerging markets equity and bond indices estimated with a sample size of 108 observations. Second, we extend the economic models to the four currency areas and exclude emerging markets. This allows us to estimate a 31 equation model (4 capital markets plus 4 economies) over the sample period 1971:01 to 2002:12 to result in a sample size of 384 observations.

Table 3 presents the results of the data transformation to returns on the assets. Over the whole sample period, the highest average return was given by UK equity with a monthly average return of 0.79%, US had a 0.64% average monthly return. The largest fall in one month corresponds to the UK in October 1987, with 26.6% lost in that month when the US dropped 21.8%.

3.2 Unit Root Analysis

A unit root process is also called *difference stationary* or *integrated of order one* - I(1) - because its first difference is a stationary process. Many economic series can be characterised as being I(1), but also their linear combinations may appear to be stationary. Such variables are *cointegrated* and the weights in the estimated linear combination are a *cointegration vector*.

An $I(0)$ series is a stationary series, while an $I(1)$ series contains 1 unit root and it is nonstationary. An $I(2)$ series contains 2 unit roots, and so on. So the number of unit roots corresponds to the number of times we must transform the data by (i.e. first differences, differences in logs or proportional returns) in order to induce stationarity. Tests examining the existence of a unit root underlying a time series y are based on the null hypothesis $H_0: \ f = 1$ for $y_t = f y_{t-1} + u_t$ versus the alternative hypothesis $H_1: \ f < 1$. The null hypothesis states that the process underlying the series contains a unit root versus the alternative that this process is stationary.

We test the whole set of time series individually for underlying unit root processes. In the time series literature this is done using suitable test statistics. The procedures for unit root testing, among others, are the Dickey Fuller test (Fuller 1976; Dickey & Fuller 1979, and Fuller 1996), the Phillip Perron test and graphical analysis using the autocorrelation (ACF) and partial autocorrelation (PACF) functions. An alternative is a test based on the Bayesian odds ratio proposed by Sims (1988). For more techniques for analysing unit roots, such as employing Lyapunov exponents, see Dechert & Gencay (1993).

There are several ways to set up the basic unit root test methodology based on auto regression, with or without a time trend, with or without drift, for the significant difference of the estimated autoregression coefficient from 1, i.e. the unit root, in a left tail one sided test of the null hypothesis using a *t*-test. We tried several variations, but applied a *t*-test for a unit root incorporating an intercept and a time trend given the paths shown in Figure 2 (Fuller, 1996, see also Hayashi, 2000). With a sample size N > 250, the probability that the t- statistic $\boldsymbol{t}$ is less than –3.42 is 0.05, i.e. $P(\boldsymbol{t} < -3.42)$, $P(\boldsymbol{t} < -3.69)$ is 0.025 and $P(\boldsymbol{t} < -3.98)$ is 0.01. When N> 100, then $P(\boldsymbol{t} < -3.45) = 0.05$, $P(\boldsymbol{t} < -3.73) = 0.025$ and $P(\boldsymbol{t} < -4.05) = 0.01$.

The sample size used for the unit root tests each time series was the maximum number of observations available (see Table 2) and the analysis was done on a univariate basis for each time series applying both the Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests. The results are summarised in Table 4. For level variables the existence of a unit root could not be rejected at the 5% level, while for all returns existence can be rejected at the 0.1% level justifying the I(0) nature of the system specification in returns. The time series in this paper were for specification purposes transformed to make them stationary (see Section 1.2). There are different types of suitable transformations such as first differences, differences in logs and (proportional) returns to mention a few, but we chose the return transformation because it offers the actual return on the invested asset class. It is also easier to interpret than logarithmic transformation and unlike first differences has the advantage of being unit free.

When the system models were specified and before estimation, we ran the DF and ADF tests again for the corresponding sample period (either 108 or 384 observations) with similar results.

4. General Model Estimation

The initial model specification or general unreduced model contains 188 coefficients. In order to arrive at a final parsimonious specification, non-significant coefficients with respect to individual t-tests for zero value on OLS estimates were deleted sequentially. The final parsimonious model contains 84 coefficients with most of the coefficients are significant at at least 5% significance level. The process of eliminating insignificant coefficients can be automated if the model is linear; in our nonlinear model coefficient elimination had to be done manually.

System estimation in econometrics started with the work of Zellner (1962) and Theil (1971). This framework is still being used and more recent descriptions can be found in Hamilton (1994) and Hayashi (2000). System estimation such as *seemingly unrelated equations* (SURE), is found to gain efficiency in parameter estimation over ordinary least squares (OLS) if the residuals are correlated among equations and the regressors are not the same in each equation (as opposed to the situation in a VAR framework). The nonlinear SURE specification employed here can be interpreted as a near-VAR model or alternatively as a structural econometric system, due to a specification which attempts to include economic relations between the financial and economic variables such as interest rate and purchasing power parity. In SURE regressions none of the variables or parameters in the *N* equations need be related; the connection between the equations can lie solely in the disturbance terms which are correlated across different equations. The SURE estimation technique allows contemporaneous - but not serial - correlation of the disturbances (innovations). Thus the disturbance vector **e** has contemporaneously correlated components but is serially uncorrelated, i.e. $V(\mathbf{e}_t) := (s_{ij})$ and $E(\mathbf{e}_{it}\ \mathbf{e}_{jt}) = 0$ for $t \neq t$. The contemporaneous covariances are estimated from the data by the SURE technique.

In summary the advantages of SURE are: 1) disturbances in a particular equation are contemporaneously correlated with the disturbances in other equations; 2) the right hand side does not need to be the same across equations as in a VAR; 3) models may be formulated with constraints across parameters in different equations; 4) SURE offers a full fixed variance-covariance matrix of disturbance terms.

4.1 Econometric Results for the Capital Market and US Economic Model 1993-2002

The parsimonious model estimated with SURE contains 84 significant coefficients out of the 188 originally present in the unreduced model. Computations were carried out with the econometric regression analysis of time series (RATS) software version 5.04. The regression results are presented in Table 5, which shows the variable, the coefficient value and its statistical significance level.

For the emerging market indices, we tried different ARMA/GARCH specifications for the two processes underlying the data. We found ARMA(1,0)/GARCH(1,1) the most appropriate model for the emerging markets equity index process (S^{EM}). For the emerging markets bond index process $\mathbf{B}^{EM}$, the best specification is a discretization of geometric Brownian motion with drift.

Specifically, the results for the ARMA/GARCH fitting over the sample period for the stock process S^{EM} are:

$$\frac{\boldsymbol{S}_{t+1}^{EM} - S_t^{EM}}{S_t^{EM}} = 0.1870\left(\frac{S_t^{EM} - S_{t-1}^{EM}}{S_{t-1}^{EM}}\right) + \boldsymbol{u}_t$$

$$\boldsymbol{H}_t^S = 0.0003 + 0.8140\left(H_{t-1}^S\right) + 0.1315\left(u_{t-1}^2\right), \qquad \boldsymbol{u}_t := \sqrt{H_t}e_t.$$

Recall that in GARCH specification H_t is the *conditional variance* of $\boldsymbol{u}_t$ and the *unconditional variance* is given by $s_{\boldsymbol{u}}^2 = \dfrac{g}{1-p-q}$. For the stock index process the unconditional variance is $s_{S^{EM}}^2 = \dfrac{.0003}{1-.8140-.1315}$ and the result is 0.55% per month with standard deviation 7.42% per month. The annualised volatility is (.0742) $\sqrt{12}$ =.2570 or 25.7%.

The specific results for the emerging market bond process $\mathbf{B}^{EM}$ are:

$$\frac{B_{t+1}^{EM} - B_t^{EM}}{B_t^{EM}} = 0.0093 + 0.0524 e_t$$

This is a discretised geometric Brownian motion for the level with an annualised drift of (.0093)(12)=.1116 or 11.2%. The volatility estimate for the emerging markets bond index process is 5.24% per month with an annualised volatility of (.0524) $\sqrt{12}$ =.1815.

4.2 Influence Diagrams for the Capital Markets and US Economic Model 1993-2002

In order to interpret the different relationships among the whole set of variables in the system, we developed *influence diagrams* (see Figure 3). They provide the same information as detailed econometric results (see Table 5), but using them it is visually easier to capture the different relationships among variables in the model and across currency areas.

The arrows represent only statistically significant relationships in the sense of a statistical significant coefficient in the final parsimonious model, as in Table 5 (with constants excluded). A solid arrow represents an influence on a dependent *return* variable of an explanatory *level* variable at time *t* while a dotted arrow represents an influence at time *t-1*. The significance levels are the same as in Table 5. The rectangles represent macroeconomic variables while the ellipses represent capital market variables. The % shown inside an ellipse or rectangle represents the explanatory power or $\bar{R}^2$ (adjusted R squared). The system model presented in Figure 3 was estimated for the period 93:12 to 02:12. Influence diagrams for the period 1993:12 to 2001:02 were first provided for the benefit of Dempster *et al.* 2003a (Section 3.6).

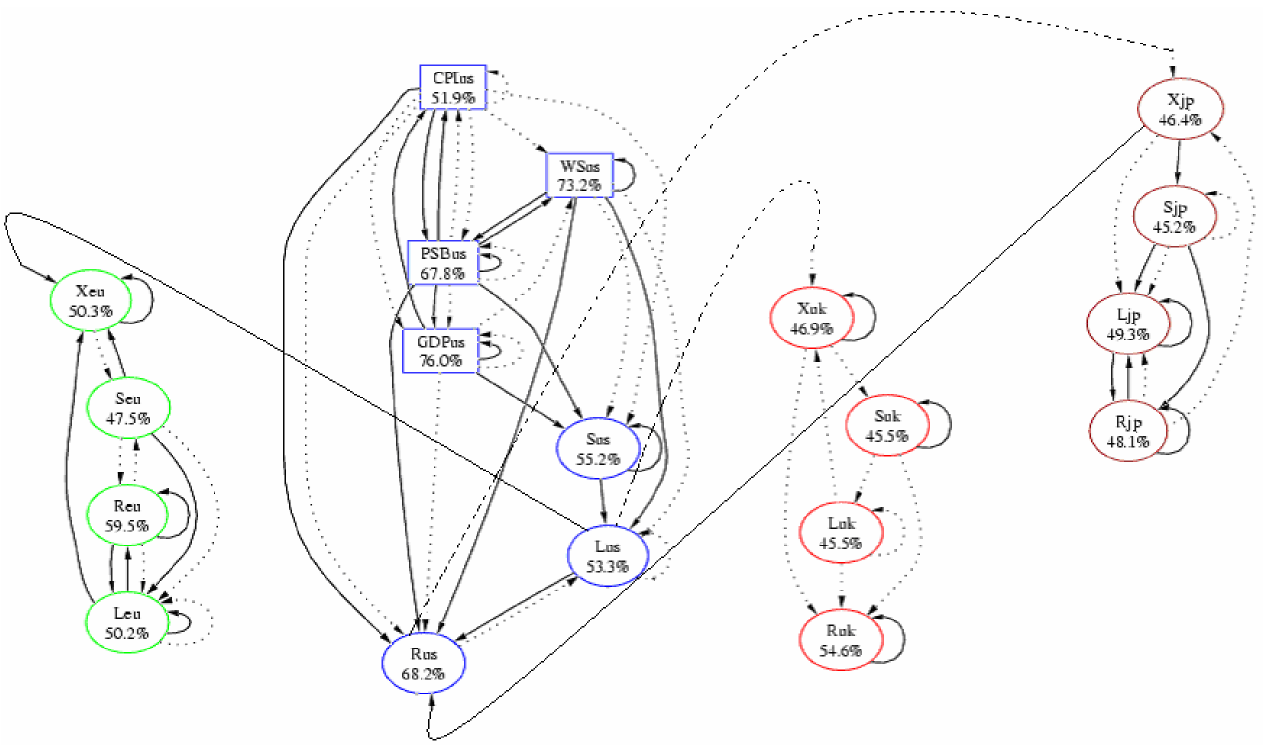

Figure 3: Influence Diagram for CMM and US Economic Model 1993-2002

In the estimated model, the return on US stocks (S^{US}) from time *t* to *t+1* is affected by the stock index itself and the economic variables GDP^{US} and PSB^{US} at time *t* and CPI^{US} and WS^{US} at time *t-1*. This results in a coefficient of determination of 55.2%. The *t*-statistic value of these coefficients is higher than 2, meaning that they all are significant at at least the 5% level (except for R^{EU}_{t-1} in S^{EU}, S^{EU}_{t-1} in L^{EU} and $1/R^{JP}$ in R^{JP} which are significant at 10%, see Table 5). If we compare the fitting for the US equity returns with those of other currency areas, we find a better fit (S^{US} is 55.2% compared to S^{UK} 45.5%, S^{EU} 47.5% and S^{JP} 45.2%).

It is also helpful to appreciate that the effect of the long interest rate in the US (L^{US}) in explaining the exchange rate in the UK (X^{UK}) can be interpreted as some empirical evidence of the interest rate parity theorem (IRP). Interest rate parity is also found significant in the dollar exchange rate for EU; the bond yields in the EU and the US help to explain the behaviour of the exchange rate process. For Japan, IRP also holds, but in this case through the short term interest rates in the US and JP which help to model the Yen/US dollar exchange rate.

5. Quasi-maximum Likelihood Estimation of the Contemporaneous Covariance Matrix of the Innovations

In order to create a system in which the developed economies are linked to the emerging markets, we re-estimate a variance/covariance matrix using the residuals from the systems for both types of economies estimated separately. The procedure is to obtain the residuals from the SURE estimation of the developed markets and the normalised residuals from the ARMA/GARCH estimation of the emerging markets and re-estimate the full variance/covariance matrix. Table 7 contains the resulting matrix correlation of the residuals with their variances shown on the diagonal. Table 8 shows the corresponding standard deviations.

5.1 Variance Correlation Matrix of Actual Returns and Model Residuals

We may compare the behaviour of the historical *actual returns* with the estimates of the corresponding *innovations* from the *residuals* from the model estimation. Tables 6 and 7 refer to the same time span for both sets of variables. Whereas the actual US stock returns over the period 1993:12 to 2002:12 show a correlation of 79.1% with S^{UK}, 78.6% with S^{EU}, 38.6% with S^{JP} and 67.1% with S^{EM} (see Table 6), the corresponding model disturbance term for the US equity shows a correlation of 76.2% with S^{UK}, 76.6% with S^{EU}, 46.6% with S^{JP}, and 70.5% with the emerging markets equity (see Table 7). The historical short term interest rate return in the US (R^{US}) over the same period has a correlation of 27.8%, 32.3% and 17.3% with UK, EU, and JP respectively (see Table 6). Analogously the residuals from R^{US} in the model show 1.2%, 18.2% and 16.8% for UK, EU and JP (see Table 7). In the case of the long term interest rate for the US (L^{US}) the correlations with respect to UK, EU, JP and emerging markets bonds (B^{EM}) are 50.3%, 53.6%, 13.0%, and –7.8%. The correlation values for the model residuals are 46.0%, 53.1%, 1.8% and 3.1% correspondingly.

Historical equity returns in the United Kingdom are correlated with S^{EU}, S^{JP} and S^{EM} at 84.3%, 38.7%, and 65.8% respectively (see Table 6). The disturbance terms derived from the model for S^{UK} had 83.7%, 39.4% and 67.0% respectively (see Table 7). The return on the short term interest rate in the UK (R^{UK}) is correlated with R^{EU} and R^{JP} with 27.4% and 10.0%. Correspondingly, the residuals have 15.1% and 20.7% correlations. The long term interest rate in the UK (L^{UK}) is correlated over the same time period with L^{EU}, L^{JP} and B^{EM} at 70.7%, -4.1%, and -17.6% respectively. The residual counterparts are 72.3%, -8.6% and -11.1%.

There are significant similar correlations between the stock and (bond) returns in the emerging markets and the world equity, cash and bond markets. The fact that this relation exists between the nonlinear autoregressive dynamic model's residuals and the actual returns suggests that these markets are contemporaneously linked by shocks transmitted mainly thorough the foreign exchange markets.

The volatilities of the model residuals seem also in line with the actual realised volatility of the assets returns. The diagonals of both matrices (see Tables 6 and 7) look similar. Furthermore, the standard deviation of the residuals is comparable with the standard deviation for the assets found in Table 3 over a larger sample period.

6. System Dynamics

In this section we study the dynamics of the estimated system by simulation, impulse response analysis and forecasting performance.

6.1 Bootstrapping Simulation

In Dempster *et al.* (2003) simulations were performed with Monte Carlo techniques using a preliminary version of the model developed here in Sections 2 to 4. Two major pieces of information are required to generate scenarios, namely the model's coefficient values, which will generate the drift vector of the stochastic process; and the variance covariance matrix, which will generate the correlated innovation terms from independent standard normal pseudo random numbers. In this section we present an alternative type of scenario generation; rather than drawing from pseudo random normal distributions we draw at random from the residuals derived from the model. Hence the name *bootstrapping*. One advantage of bootstrapping is that it allows inference without imposing strong statistical distribution assumptions, since the empirical distribution is employed. Thus we do not impose a normal distribution on the innovations' behaviour, but instead let the innovations take a value at random from the corresponding residuals. Given a sample size T the probability of selecting a particular value from the computed residuals is $1/T$. Bootstrapping thus draws from the sample data points themselves and this is performed here by repeated sampling with replacement from the vector of residuals from the SURE model.

An example of scenario generation for the UK can be seen in Figure 4, for the stock index, short and long term interest rates, and the US$/£ exchange rate. The graphs show 5 year out of sample scenarios.

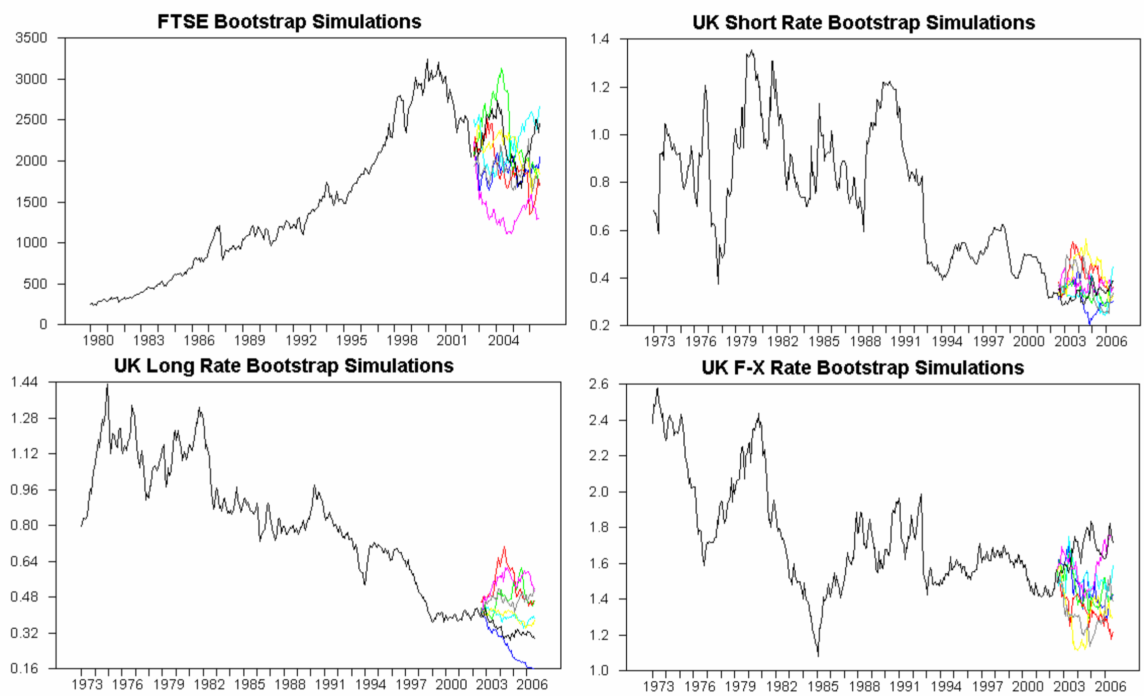

Figure 4: Simulated Paths for Stock Index, Short, Long and Exchange Rates for the UK

Given a reasonable number of scenarios or plausible realisations of the variables under study, one can get an idea of the probability distribution of the different states by computing quantiles of the out-of-sample scenarios. This quantile analysis is performed for the Monte Carlo simulations in Dempster *et al*. 2003. In this paper we focus instead on the impulse responses of the variables in the system and their forecast performance.

6.2 Impulse Response Dynamics

We first constructed three subsystems of the full system to analyse the impulse responses of the model, US-UK, US-EU and US-JP. All these models contain the following order: S^{US}, R^{US}, L^{US}, S^i, R^i, L^i, X^i, for i = UK, EU, JP. The estimation of these 7 variable systems was performed with the system equation specification and the sample period of Table 5. All three nonlinear autoregressive subsystems appear to be *stable* since the impulse responses converge to zero in a few steps – such system stability overall is critical for scenario generation of asset returns over long horizons.

Full system stability evaluation in the form of orthogonal impulse response analysis was also performed on the full 19 variable system. Again the responses of all equations to shocks to each other equation residuals converged to zero after a few steps, confirming that the full nonlinear autoregressive system appears stable.

6.3 Forecasting Performance

Next we discuss the forecasting performance of the estimated system over short horizons. In evaluating the out-of-sample forecasting capability of an econometric model it is necessary to first define appropriate measures of forecasting error and their various statistical properties. The concept of a forecast error is very simple: it is the difference between the forecast value and the actual historical value of the variable under study. So we define the forecast *error* as: $e_{it} := y_{it} - \hat{y}_{it}$, where $\hat{y}_{it}$ is the forecast at step t from the ith variable in the system and y_{it} is the corresponding actual value of the dependent variable (return).

The *sum of forecast errors* is defined as

$$SFE_t := \sum_{i=1}^{N_t} e_{it} \tag{22}$$

where N_t is the number of times that a t-step forecast has been computed (t =1 for one-step-ahead forecasts, t =2 for two-step-ahead forecasts, etc.).

The *mean error* is given by

$$ME_t := \frac{SFE_t}{N_t} = \frac{\sum_{i=1}^{N_t} e_{it}}{N_t}. \tag{23}$$

The *sum of absolute errors* is defined as

$$SAE_t := \sum_{i=1}^{N_t} |e_{it}|, \tag{24}$$

so that the *mean absolute error* is given by

$$MAE_t := \frac{SAE_t}{N_t} = \frac{\sum_{i=1}^{N_t} |e_{it}|}{N_t}. \tag{25}$$

The *sum of squared errors* is given by

$$SSE_t := \sum_{i=1}^{N_t} e_{it}^2. \tag{26}$$

Consequently the *root mean square (RMS) error* is defined as

$$RMS_t := \sqrt{\frac{SSE_t}{N_t}} = \sqrt{\frac{\sum_{i=1}^{N_t} e_{it}^2}{N_t}}. \tag{27}$$

RMS error is the classical measure of forecasting performance and is widely used.

We will also use another statistic called *Theil's U statistic* which is a ratio of the RMS error to the RMS error of a naïve forecast of no change in the dependent variable, see e.g. Brooks (2002). So we define the SSE of a naïve model as

$$SSEnf_t := \sum_{i=1}^{N_t} \left(y_{it} - \breve{y}_{i0} \right)^2, \tag{28}$$

where $\breve{y}_{i0}$ is the naïve or flat forecast, i.e. no change in the dependent variable from the previous period. It follows that the RMS of the naïve model of a no change forecast is

$$RMSnf_t := \sqrt{\frac{SSEnf_t}{N_t}} = \sqrt{\frac{\sum_{i=1}^{N_t}\left(y_{it} - \breve{y}_{i0}\right)^2}{N_t}}. \tag{29}$$

The *Theil-U statistic* is the ratio of root mean square errors from the two models

$$U_t := \frac{RMS_t}{RMSnf_t} = \frac{\sqrt{\frac{SSE_t}{N_t}}}{\sqrt{\frac{SSEnf_t}{N_t}}} = \frac{\sqrt{\frac{\sum_{i=1}^{N_t}\left(y_{it} - \hat{y}_{it}\right)^2}{N_t}}}{\sqrt{\frac{\sum_{i=1}^{N_t}\left(y_{it} - \breve{y}_{i0}\right)^2}{N_t}}}. \tag{30}$$

(31)

A value higher than one means the model did worse than the naïve method. However, a value of less than one should not necessarily be interpreted as a major success, as there are simple procedures that can produce such a value for series with a strong trend, which is of course not so applicable to returns forecasting. For more on forecast evaluation criteria see Clements and Hendry (2000).

Table 9 shows the results of a recursive 1, 2 and 3 month ahead evaluation of forecast performance of the 19 equation nonlinear system, which includes the 4 currency area capital markets and the US macroeconomy (Dempster *et al*. (2003) refers to BMSIM 4 regions plus US economy). Recall that the 19 equation model was estimated over the period December 1993 to December 2002 to have 84 significant coefficients as presented in Table 5. Note that according to the Theil statistics the system's forecasting performance is better at 3 months than at one.

7. Extension to the Global Macroeconomy

In this section we extend the macroeconomic modelling to the remainder of the currency areas: UK, EU and Japan, but exclude emerging markets. The resulting model is a 31 equation system and it is estimated over the period 1971-2002.

The influence diagrams facilitate the examinations of the inter relationships among different variables and across currency areas. The estimate model presented is the (parsimonious) model where only statistically significant coefficients remain in the final estimation; the sample period is 1971-2002, see Figure 5 in the Appendix.

Interest rate parity can be assumed to apply when we see the relationships between the short and long rates and the exchange rates. Recall that the system's domestic currency is US and in the exchange rate equation (13) the foreign long and short interest rates used to compute the interest rates differentials for the different currencies are those of the US.

The exchange rate in UK is affected by the long rate in the UK and the US, as well as the short rate in both regions. The EU exchange rate is affected only by the long rate in the EU and the US. Finally the exchange rate for Japan is affected by short and long interest rates in both currency areas, JP and US.

The matrices depicted in Tables 6 and 7 (19 equation model, i.e. 4 capital markets plus US economy) can be compared to the 31 equation model (4 capital markets and 4 economies) of this section with Tables 10 to 13 from the Appendix. Besides the interest rates and the exchange rates relations, we notice three explanatory relations from the economic model to the capital markets variables. The first being wages and salaries WS as an explanatory variable for the short term interest rate R for the US, UK and EU. Relation two was identified as CPI explaining long term interest rate L for the US, UK and EU. The last relation is GDP as explanatory variable for the short rate R in the US, EU and JP. These three relations support standard economic theory.

8. Conclusions

An asset return model is developed which includes economic variables to help model the financial markets for developed economies and emerging markets. We found the correlations of the model's residuals to be similar to those of the actual returns which supports our main econometric finding that the world's equity, money and bond markets are linked simultaneously through currency exchange rates in reacting to exogenous shocks. Volatilities for the model residuals and actual returns are also consistent.

Parameter values and *influence diagrams* showing statistically significant coefficients for system estimates are presented. With this type of diagram it is easier to understand the inter relationships among variables and across currencies in the estimated model. Interest rate parity is found to be significant in modelling exchange rates as well as explaining short and long rates in the system model across currencies in the different versions analysed. Interest rate parity is found to be significant in the versions where the macroeconomies are included and some relations suggested by economic theory have been identified from the macroeconomy to the capital markets.

Another finding concerns the relationship between interest rate and stock returns. We find statistically significant links between stock returns and short and long rate, across the US, UK, EU and JP. Recent research shows similar results in the sense of the short rate as a predictive tool for stock excess returns (Ang & Bekaert, 2003).

Econometric modelling is an important tool in using the estimated coefficient and residual structures to simulate possible paths of the state variables for optimal asset allocation models. The dynamics of the estimated system can be analysed to understand relationships among variables and to check for stability and test economic theory within a statistical framework. More results related to those presented here may be found in Arbeleche (2004) and Dempster *et al.* (2003), which describes in detail the applications of this research to asset liability management.

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Tables

Variable	Corresponding Proxy	Data Source	Frequency
S^{US}	S&P 500 index	DataStream	Monthly
R^{US}	US 3 month T-bill rate	DataStream	Monthly
L^{US}	US 30 year T-yield	DataStream	Monthly
S^{UK}	FTSE all share index	DataStream	Monthly
R^{UK}	UK 3 month T-bill rate	DataStream	Monthly
L^{UK}	UK 20 year Gilt rate	DataStream	Monthly
X^{UK}	US$/£ exchange rate	Bloomberg	Monthly
S^{EU}	MSCI Europe w/o UK index	Morgan Stanley	Monthly
R^{EU}	German 3 month FIBOR rate	DataStream	Monthly
L^{EU}	German 10 year bond yield	DataStream	Monthly
X^{EU}	DM/US$ exchange rate	Bloomberg	Monthly
S^{JP}	TOPIX index	DataStream	Monthly
R^{JP}	Japan 3 month CD rate	DataStream	Monthly
L^{JP}	Japan 10 year bond yield	DataStream	Monthly
X^{JP}	Yen/US$ exchange rate	Bloomberg	Monthly
CPI^{US}	US consumer price index	DataStream	Monthly
WS^{US}	US wages and salaries	DataStream	Monthly
GDP^{US}	US gross domestic product	DataStream	Quarterly
PSB^{US}	US public sector requirements	DataStream	Quarterly
S^{EM}	MSCI Emerging Markets index	Morgan Stanley	Monthly
B^{EM}	EMBI+ index	J.P. Morgan	Monthly

Table 1: Data Proxies for Model Variables

Series	Obs	Mean	Std Error	Min	Max
SUS	420	384.02	389.11	63.54	1517.68
RUS	420	6.33	2.67	1.14	16.01
LUS	384	7.91	2.11	4.63	14.68
SUK	420	986.60	909.65	66.66	3242.06
RUK	420	8.79	3.14	3.78	16.26
LUK	420	9.75	2.99	4.37	17.18
XUK	385	1.78	0.35	1.08	2.62
SEU	386	352.76	341.44	71.71	1382.76
REU	420	6.15	2.69	2.58	14.57
LEU	420	7.05	1.65	3.53	11.20
XEU	386	2.13	0.52	1.37	3.64
SJP	420	996.31	679.48	105.02	2881.37
RJP	397	4.76	2.97	0.03	13.19
LJP	398	5.65	2.55	0.78	9.97
XJP	385	188.32	74.57	84.33	357.72
EMBI	108	147.99	44.27	72.09	215.71
MSCIEM	183	344.52	122.63	100.00	577.29

Table 2: Statistics for the Original Time Series

Series	Obs	Mean	Std Error	Min	Max
SUS	419	0.637	4.520	-21.76	16.31
RUS	419	-0.056	7.464	-36.52	31.35
LUS	383	0.009	3.688	-11.44	15.16
SUK	419	0.790	6.087	-26.59	52.68
RUK	419	0.106	7.397	-17.85	56.67
LUK	419	-0.063	3.426	-11.41	10.49
XUK	384	-0.056	2.981	-12.31	14.55
SEU	385	0.601	4.547	-21.78	13.12
REU	419	0.122	6.230	-21.72	29.32
LEU	419	-0.073	3.246	-07.69	10.59
XEU	385	-0.128	3.253	-10.17	11.24
SJP	419	0.624	5.148	-20.42	18.15
RJP	396	0.232	18.00	-70.59	54.55
LJP	397	-0.300	7.371	-40.27	72.93
XJP	384	-0.228	3.332	-15.01	10.92
EMBI	104	0.854	5.302	-28.74	10.70
MSCIEM	182	0.812	6.882	-29.29	18.11

(In percentage)

Table 3: Statistics for the Return on Assets

Variable	Lags	DF Levels	Returns	Lags	ADF Levels	Returns
S^{US}	0	-1.64	-19.48	2	-1.56	-11.16
R^{US}	0	-2.37	-16.87	2	-2.72	-10.43
L^{US}	0	-1.69	-16.72	2	-1.83	-11.70
S^{UK}	0	-1.93	-17.09	2	-1.90	-10.56
R^{UK}	0	-2.45	-17.73	2	-2.79	-11.12
L^{UK}	0	-2.60	-14.69	2	-3.06	-10.93
X^{UK}	0	-2.14	-18.13	2	-2.42	-10.80
S^{EU}	0	-1.67	-16.62	2	-1.91	-9.76
R^{EU}	0	-1.64	-12.28	2	-2.73	-8.86
L^{EU}	0	-1.90	-12.71	2	-2.65	-8.91
X^{EU}	0	-2.34	-18.55	2	-2.60	-10.50
S^{JP}	0	-0.86	-18.37	2	-1.07	-10.56
R^{JP}	0	-1.77	-12.31	2	-2.48	-9.75
L^{JP}	0	-2.63	-17.45	2	-2.81	-13.83
X^{JP}	0	-2.08	-18.59	2	-2.35	-9.92
CPI^{US}	0	-0.74	-11.14	2	-1.19	-7.02
WS^{US}	0	-1.28	-25.25	2	-1.08	-13.77
GDP^{US}	0	-2.34	-7.83	2	-1.75	-9.12
PSB^{US}	0	-0.62	-9.29	2	-1.37	-12.66
S^{EM}	0	-1.38	-11.35	2	-1.83	-7.71
B^{EM}	0	-2.86	-9.72	2	-2.45	-7.03

Table 4: Dickey Fuller (DF) and ADF Test Statistics for Unit Roots

Variable	Coeff	Signif
Dependent Variable SUS		
R Bar **2	**0.552**	
1. Constant	0.506	0.024
2. SUS	-2.9E-04	0.000
3. CPIUS{1}	-0.008	0.006
4. WSUS{1}	-1.7E-04	0.005
5. GDPUS	2.0E-04	0.000
6. PSBUS	3.5E-04	0.000
Dependent Variable RUS		
R Bar **2	**0.682**	
7. Constant	0.937	0.015
8. LRUS	0.090	0.000
9. CPIUS	0.029	0.014
10. CPIUS{1}	-0.036	0.001
11. WSUS	-3.3E-04	0.006
12. GDPUS{1}	1.7E-04	0.032
13. PSBUS	4.7E-04	0.000
Dependent Variable LUS		
R Bar **2	**0.533**	
14. Constant	0.300	0.000
15. SUS	0.000	0.013
16. RUS{1}	-0.007	0.039
17. LUS{1}	-0.019	0.001
18. WSUS	-2.4E-04	0.006
19. WSUS{1}	1.9E-04	0.027
Dependent Variable SUK		
R Bar **2	**0.455**	
20. SUK	-1.2E-05	0.017
21. XUK{1}	0.019	0.015
Dependent Variable RUK		
R Bar **2	**0.546**	
22. Constant	-0.122	0.000
23. INVRUK	-1.746	0.000
24. SR1UK	2.3E-04	0.000
25. LR1UK	0.121	0.000
26. XR1UK	0.723	0.001
Dependent Variable LUK		
R Bar **2	**0.455**	
27. Constant	0.066	0.046
28. SUK{1}	-1.5E-05	0.059
29. LUK{1}	-0.005	0.044

Variable	Coeff	Signif
Dependent Variable XUK		
R Bar **2	**0.469**	
30. Constant	-0.107	0.000
31. INVXUK	0.169	0.000
32. LFLX1UK	-0.007	0.004
Dependent Variable SEU		
R Bar **2	**0.475**	
33. Constant	0.082	0.001
34. REU{1}	-0.005	0.107
35. XEU{1}	-0.032	0.015
Dependent Variable REU		
R Bar **2	**0.595**	
36. Constant	-0.128	0.000
37. INVREU	-0.537	0.000
38. LREU	0.142	0.000
39. SR1EU	0.000	0.000
Dependent Variable LEU		
R Bar **2	**0.502**	
40. SEU	7.5E-05	0.050
41. SEU{1}	-6.2E-05	0.105
42. REU	-0.042	0.002
43. REU{1}	0.035	0.008
44. LEU	0.066	0.000
45. LEU{1}	-0.063	0.000
Dependent Variable XEU		
R Bar **2	**0.503**	
46. Constant	-0.087	0.001
47. SXEU	7.6E-05	0.000
48. LFLXEU	0.015	0.016
49. INVXEU	0.083	0.012
Dependent Variable SJP		
R Bar **2	**0.452**	
50. Constant	0.185	0.005
51. SJP{1}	-5.4E-05	0.015
52. XJP	-0.001	0.008
Dependent Variable RJP		
R Bar **2	**0.481**	
53. Constant	-0.142	0.001
54. SRJP	-3.6E-05	0.016
55. INVRJP	-0.025	0.096
56. LRJP	0.061	0.000

Variable	Coeff	Signif
Dependent Variable LJP		
R Bar **2	**0.493**	
57. Constant	0.469	0.010
58. SJP	3.7E-04	0.007
59. SJP{1}	-3.1E-04	0.028
60. RJP	0.238	0.008
61. RJP{1}	-0.153	0.071
62. LJP	-0.096	0.000
63. XJP{1}	-0.003	0.002
Dependent Variable XJP		
R Bar **2	**0.464**	
64. Constant	-0.020	0.019
65. RFRX1JP	0.638	0.007
Dependent Variable CPIUS		
R Bar **2	**0.519**	
66. Constant	0.063	0.000
67. CPIUS{1}	-0.001	0.000
68. GDPUS	5.7E-06	0.000
69. PSBUS	-2.3E-05	0.020
70. PSBUS{1}	2.2E-05	0.027
Dependent Variable WSUS		
R Bar **2	**0.732**	
71. CPIUS{1}	-0.001	0.000
72. WSUS	-1.5E-04	0.000
73. GDPUS{1}	8.3E-05	0.000
74. PSBUS	4.8E-05	0.000
Dependent Variable GDPUS		
R Bar **2	**0.760**	
75. CPIUS{1}	5.3E-05	0.000
76. GDPUS	7.2E-05	0.000
77. GDPUS{1}	-7.3E-05	0.000
78. PSBUS	1.1E-05	0.034
79. PSBUS{1}	-1.2E-05	0.038
Dependent Variable PSBUS		
R Bar **2	**0.678**	
80. CPIUS{1}	-8.349	0.018
81. CPIUS	8.561	0.014
82. WSUS	-0.009	0.075
83. PSBUS	0.604	0.000
84. PSBUS{1}	-0.602	0.000

Linear Systems Estimation by Seemingly Unrelated Regressions Monthly Data From 1993:12 To 2002:12 Usable Observations 108

Table 5: SURE Regression Results CMM plus US Economic Model 1993:12 to 2002:12

	S^{US}	R^{US}	L^{US}	S^{UK}	R^{UK}	L^{UK}	X^{UK}	S^{EU}	R^{EU}	L^{EU}	X^{EU}	S^{JP}	R^{JP}	L^{JP}	X^{JP}	CPI^{US}	WS^{US}	GDP^{US}	PSB^{US}	S^{EM}	B^{EM}
S^{US}	0.002	0.021	0.086	**0.791**	0.031	-0.148	-0.134	**0.786**	-0.036	-0.126	0.209	**0.386**	0.073	0.057	-0.085	0.091	0.086	0.209	0.266	**0.671**	**0.550**
R^{US}		0.004	0.102	0.096	**0.278**	0.106	-0.142	0.140	**0.323**	0.240	0.102	0.053	**0.173**	0.124	0.050	-0.048	0.154	0.039	0.218	0.004	-0.072
L^{US}			0.001	0.174	-0.048	**0.503**	-0.140	0.179	0.020	**0.536**	0.050	0.136	-0.023	**0.130**	-0.030	0.021	0.104	0.003	0.032	**0.239**	**-0.078**
S^{UK}				0.002	0.006	-0.177	**-0.318**	**0.843**	0.039	-0.112	**0.232**	**0.387**	0.147	0.090	**-0.064**	0.015	-0.011	0.109	0.221	**0.658**	**0.519**
R^{UK}					0.001	0.133	0.245	0.150	**0.274**	0.153	-0.066	0.077	**0.100**	-0.175	0.172	-0.010	0.134	0.024	0.093	-0.048	-0.020
L^{UK}						0.001	-0.009	-0.073	-0.027	**0.707**	0.019	0.105	-0.112	**-0.041**	0.153	0.101	0.029	-0.108	-0.100	-0.009	**-0.176**
X^{UK}							4.1E-04	-0.288	-0.048	-0.016	**-0.605**	-0.120	-0.077	-0.059	**-0.281**	0.023	0.056	0.035	-0.050	-0.190	-0.193
S^{EU}								0.003	0.017	-0.032	0.383	**0.434**	0.128	0.117	0.106	0.049	0.101	0.193	0.290	**0.635**	**0.490**
R^{EU}									0.002	0.348	-0.021	-0.052	**0.316**	-0.051	-0.086	-0.079	0.019	-0.026	0.028	-0.123	-0.069
L^{EU}										0.001	-0.029	0.108	0.042	**0.048**	-0.001	0.036	0.080	-0.011	-0.025	0.011	-0.161
X^{EU}											0.001	0.270	0.034	0.091	**0.440**	0.085	0.037	0.084	0.163	0.166	0.219
S^{JP}												0.003	-0.043	0.156	0.064	0.007	-0.158	0.023	0.181	**0.470**	**0.343**
R^{JP}													0.111	0.098	-0.058	-0.064	0.025	0.096	-0.127	0.008	0.056
L^{JP}														0.013	-0.152	-0.166	-0.105	0.019	-0.037	-0.010	-0.154
X^{JP}															0.001	-0.086	0.119	-0.009	0.065	-0.033	0.104
CPI																8.5E-06	0.331	0.653	-0.084	-0.013	0.178
WS																	6.5E-05	0.520	0.188	-0.065	0.061
GDP																		2.0E-05	0.074	0.007	0.174
PSB																			373.3	0.143	0.042
S^{EM}																				0.005	**0.706**
B^{EM}																					0.003

Table 6: Variance/Correlation Matrix of Actual Returns 1993:12 to 2002:12

	S^{US}	R^{US}	L^{US}	S^{UK}	R^{UK}	L^{UK}	X^{UK}	S^{EU}	R^{EU}	L^{EU}	X^{EU}	S^{JP}	R^{JP}	L^{JP}	X^{JP}	CPI	WS	GDP	PSB	S^{EM}	B^{EM}
S^{US}	0.002	-0.019	0.103	**0.762**	-0.054	-0.110	-0.104	**0.766**	0.006	-0.082	0.177	**0.466**	0.088	0.120	-0.120	-0.052	-0.215	0.040	0.170	**0.705**	**0.498**
R^{US}		0.002	0.004	0.050	**0.012**	-0.073	-0.180	0.081	**0.182**	0.012	0.131	-0.091	**0.168**	0.024	0.149	-0.094	0.177	0.044	0.010	-0.006	0.017
L^{US}			0.001	0.207	0.024	**0.460**	-0.178	0.203	0.145	**0.531**	0.076	0.074	0.003	**0.018**	-0.021	0.112	0.173	0.083	-0.103	**0.177**	**0.014**
S^{UK}				0.002	-0.017	-0.165	**-0.293**	**0.837**	0.101	-0.102	**0.198**	**0.394**	0.209	0.084	**-0.112**	-0.046	-0.226	0.031	0.119	**0.670**	**0.517**
R^{UK}					0.001	0.192	0.237	0.137	**0.151**	0.090	-0.059	0.068	**0.207**	-0.249	0.194	-0.029	0.072	-0.039	-0.110	-0.079	-0.070
L^{UK}						0.001	-0.076	-0.029	0.071	**0.723**	0.061	0.098	-0.096	**-0.086**	0.197	0.156	0.232	-0.034	-0.119	-0.055	**-0.165**
X^{UK}							3.8E-04	-0.259	0.014	-0.052	**-0.614**	-0.096	-0.075	-0.063	**-0.286**	-0.043	0.033	0.000	0.102	-0.199	-0.194
S^{EU}								0.003	0.046	-0.028	0.338	**0.438**	0.189	0.098	0.065	-0.053	-0.111	0.045	0.149	**0.670**	**0.469**
R^{EU}									0.001	0.427	-0.079	0.058	**0.388**	-0.009	-0.107	-0.129	-0.176	0.210	-0.156	-0.094	-0.109
L^{EU}										0.001	-0.048	0.067	0.006	**-0.045**	0.038	-0.031	0.059	-0.007	-0.116	-0.077	-0.154
X^{EU}											0.001	0.200	0.025	0.055	**0.443**	0.114	-0.014	0.058	-0.017	0.173	0.206
S^{JP}												0.003	-0.006	0.162	0.025	0.013	-0.130	0.221	0.199	**0.428**	**0.328**
R^{JP}													0.101	0.120	-0.073	-0.182	-0.080	0.148	-0.135	0.002	0.019
L^{JP}														0.011	-0.157	-0.159	-0.085	0.158	-0.127	0.002	-0.102
X^{JP}															0.001	-0.110	0.110	-0.120	0.004	-0.022	0.059
CPI																3.7E-06	0.015	-0.085	-0.177	0.001	0.119
WS																	2.3E-05	0.069	0.123	-0.075	-0.086
GDP																		1.1E-06	-0.104	0.065	-0.029
PSB																			209.8	0.158	0.066
S^{EM}																				0.986	**0.707**
B^{EM}																					0.003

Table 7: Variance/Correlation Matrix of CMM plus US Economy plus EM Residuals 1993:12 to 2002:12

	SUS	RUS	LUS	SUK	RUK	LUK	XUK	SEU	REU	LEU	XEU
S. D.	0.041	0.044	0.034	0.041	0.031	0.032	0.020	0.053	0.036	0.034	0.025
	SJP	**RJP**	**LJP**	**XJP**	**CPI**	**WS**	**GDP**	**PSB**	**SEM**	**BEM**	
S. D.	0.050	0.318	0.106	0.036	0.002	0.005	0.001	14.48	0.993	0.995	

Table 8: Standard Deviations of Model Residuals

Forecast Statistics for Series DSSUS Step ME MAE RMS TheilU N.Obs 1 0.0015 0.034 0.044 0.610 85 2 0.0012 0.034 0.044 0.603 84 3 0.0013 0.035 0.044 0.653 83	**Forecast Statistics for Series DXXEU** Step ME MAE RMS TheilU N.Obs 1 0.0001 0.020 0.025 0.728 85 2 -0.0003 0.020 0.025 0.677 84 3 -0.0002 0.020 0.025 0.659 83
Forecast Statistics for Series DRRUS 1 0.0002 0.034 0.046 0.662 85 2 -0.0002 0.034 0.046 0.610 84 3 0.0000 0.034 0.047 0.659 83	**Forecast Statistics for Series DSSJP** 1 -0.0002 0.041 0.049 0.718 85 2 -0.0004 0.041 0.049 0.747 84 3 -0.0001 0.042 0.049 0.698 83
Forecast Statistics for Series DLLUS 1 0.0000 0.027 0.035 0.686 84 2 0.0000 0.028 0.035 0.639 83 3 -0.0008 0.027 0.035 0.705 82	**Forecast Statistics for Series DRRJP** 1 0.0034 0.239 0.354 0.802 85 2 0.0032 0.242 0.356 0.688 84 3 0.0028 0.245 0.358 0.647 83
Forecast Statistics for Series DSSUK 1 0.0008 0.033 0.043 0.723 85 2 0.0006 0.033 0.043 0.697 84 3 0.0007 0.034 0.043 0.704 83	**Forecast Statistics for Series DLLJP** 1 -0.0019 0.080 0.114 0.726 85 2 -0.0024 0.081 0.114 0.679 84 3 -0.0035 0.081 0.115 0.564 83
Forecast Statistics for Series DRRUK 1 -0.0013 0.023 0.030 0.839 85 2 -0.0010 0.023 0.030 0.789 84 3 -0.0007 0.023 0.030 0.758 83	**Forecast Statistics for Series DXXJP** 1 0.0006 0.026 0.035 0.668 85 2 0.0003 0.026 0.035 0.731 84 3 0.0006 0.026 0.035 0.717 83
Forecast Statistics for Series DLLUK 1 -0.0030 0.025 0.032 0.747 85 2 -0.0032 0.025 0.032 0.665 84 3 -0.0037 0.025 0.032 0.799 83	**Forecast Statistics for Series DCPIUS** 1 0.0001 0.001 0.002 0.731 85 2 0.0001 0.001 0.002 0.612 84 3 0.0000 0.001 0.002 0.621 83
Forecast Statistics for Series DXXUK 1 0.0012 0.016 0.019 0.650 85 2 0.0016 0.015 0.019 0.647 84 3 0.0016 0.016 0.019 0.737 83	**Forecast Statistics for Series DWSUS** 1 -0.0004 0.002 0.003 0.725 85 2 -0.0004 0.002 0.003 0.700 84 3 -0.0005 0.002 0.003 0.767 83
Forecast Statistics for Series DSSEU 1 0.0013 0.044 0.058 0.714 85 2 0.0009 0.044 0.058 0.720 84 3 0.0010 0.045 0.058 0.708 83	**Forecast Statistics for Series DGDPUS** 1 0.0000 0.0007 0.001 0.901 84 2 0.0000 0.0007 0.001 0.637 83 3 0.0000 0.0008 0.001 0.520 82
Forecast Statistics for Series DRREU 1 -0.0009 0.025 0.037 0.772 85 2 -0.0003 0.025 0.037 0.626 84 3 0.0002 0.025 0.037 0.624 83	**Forecast Statistics for Series DPSBUS** 1 0.820 9.0 15.95 0.864 84 2 0.872 9.0 16.04 0.611 83 3 0.981 9.0 16.11 0.498 82
Forecast Statistics for Series DLLEU 1 -0.002 0.027 0.033 0.769 85 2 -0.002 0.027 0.033 0.644 84 3 -0.003 0.027 0.033 0.690 83	

Table 9: Forecasting Performance Results Capital Markets plus US Economy

Appendix – Influence Diagrams and Covariance Matrices for the Global Capital Markets Global Macroeconomy

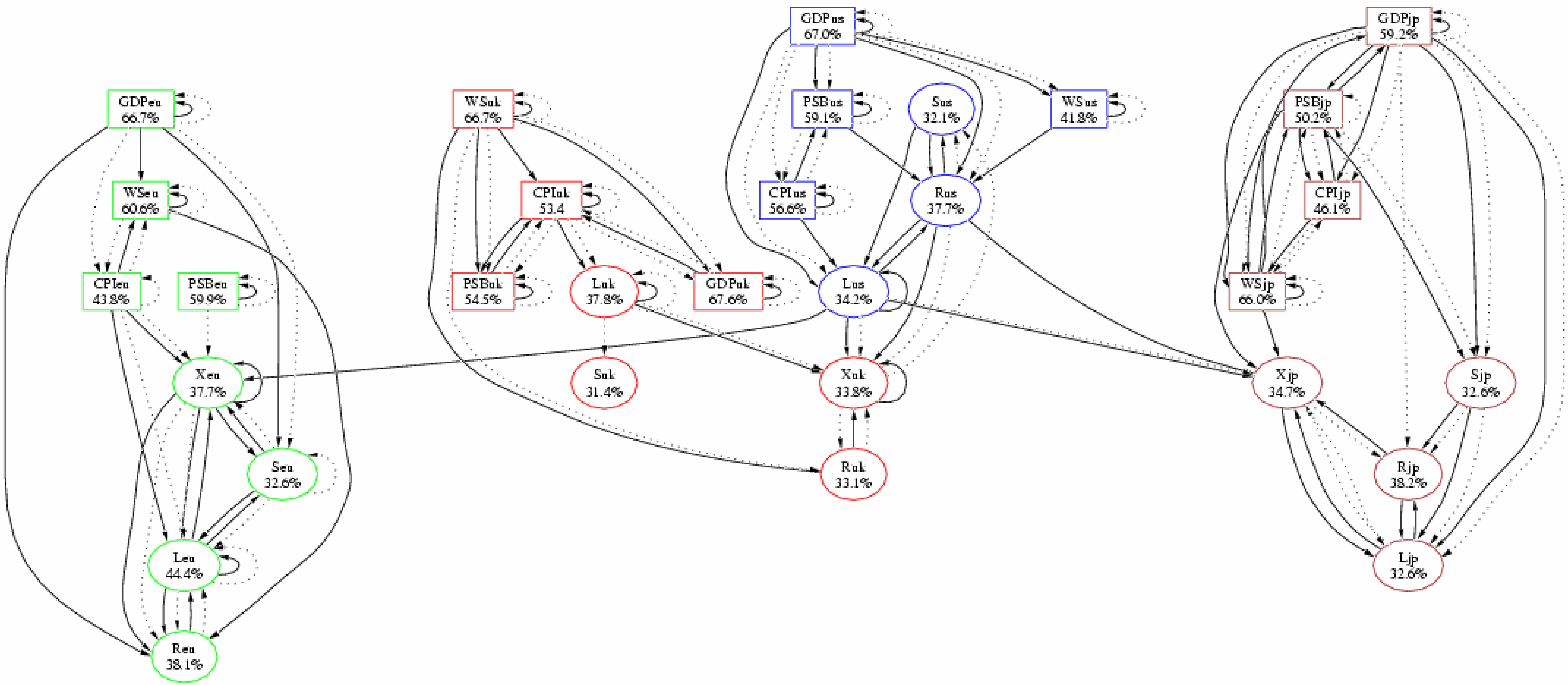

Figure 5: Influence Diagrams for the Global Economy and Capital Markets Model 1971-2002

	SUS	RUS	LUS	SUK	RUK	LUK	XUK	SEU	REU	LEU	XEU	SJP	RJP	LJP	XJP
SUS	**0.002**	-0.060	-0.240	0.593	-0.011	-0.158	-0.017	0.637	-0.093	-0.159	0.006	0.360	-0.010	-0.015	-0.041
RUS		**0.006**	0.394	-0.059	0.153	0.157	-0.158	0.032	0.111	0.245	0.174	0.017	0.101	0.147	0.138
LUS			**0.001**	-0.107	0.064	0.300	-0.122	-0.093	0.034	0.382	0.145	-0.040	-0.024	0.198	0.098
SUK				**0.004**	-0.241	-0.336	-0.029	0.609	-0.181	-0.198	0.070	0.336	0.004	-0.014	-0.030
RUK					**0.006**	0.417	-0.162	-0.075	0.191	0.172	0.014	0.015	0.030	0.028	0.124
LUK						**0.001**	-0.163	-0.190	0.133	0.430	0.015	-0.089	-0.023	0.088	0.130
XUK							**0.001**	-0.094	-0.003	-0.030	-0.647	0.038	-0.018	-0.089	-0.455
SEU								**0.002**	-0.109	-0.193	0.179	0.451	0.037	0.027	0.046
REU									**0.004**	0.450	-0.038	-0.078	0.175	0.047	0.038
LEU										**0.001**	0.071	-0.047	0.074	0.180	0.104
XEU											**0.001**	0.005	0.007	0.107	0.549
SJP												**0.003**	-0.061	-0.056	-0.085
RJP													**0.032**	0.106	-0.016
LJP														**0.006**	0.091
XJP															**0.001**

	CPIUS	WSUS	GDPUS	PSBUS	CPIUK	WSUK	GDPUK	PSBUK	CPIEU	WSEU	GDPEU	PSBEU	CPIJP	WSJP	GDPJP	PSBJP
SUS	0.010	0.080	0.113	0.087	0.102	0.097	0.121	0.085	0.092	0.083	0.084	0.035	0.020	0.078	0.005	0.125
RUS	0.089	0.096	0.113	0.155	-0.008	0.033	0.034	-0.033	0.071	0.008	0.012	0.063	-0.026	-0.027	-0.005	-0.022
LUS	0.139	0.088	0.115	0.093	0.068	0.045	0.054	0.035	0.116	0.023	0.058	0.093	0.030	-0.061	-0.110	0.007
SUK	0.014	0.082	0.122	0.079	0.179	0.164	0.149	0.031	0.118	0.063	0.077	0.014	0.052	-0.004	-0.096	0.030
RUK	0.066	0.104	0.068	0.060	0.008	0.080	0.066	-0.025	0.027	0.022	0.045	-0.028	-0.019	0.069	0.152	0.022
LUK	0.111	0.007	0.028	-0.037	0.012	-0.016	0.009	0.056	0.071	0.030	0.027	0.060	0.058	0.060	0.103	0.069
XUK	-0.018	0.039	-0.039	-0.028	0.008	0.002	-0.017	-0.012	-0.070	-0.054	-0.045	0.006	-0.033	0.096	0.050	0.025
SEU	0.001	0.095	0.128	0.177	0.070	0.106	0.114	0.100	0.087	0.094	0.125	0.036	0.009	0.051	-0.082	0.114
REU	0.117	0.055	0.074	0.048	0.000	0.021	0.010	-0.026	-0.039	0.047	0.065	0.014	-0.010	0.108	0.179	0.071
LEU	0.140	0.051	0.064	0.044	-0.016	-0.007	-0.038	0.012	0.067	0.007	0.023	0.097	-0.019	-0.016	-0.047	0.031
XEU	0.025	-0.014	0.000	0.119	-0.061	-0.048	-0.040	-0.013	0.033	0.004	0.004	0.029	-0.048	-0.130	-0.088	0.058
SJP	0.041	0.063	0.133	0.094	0.114	0.100	0.141	0.045	0.141	0.121	0.109	0.027	0.033	0.052	-0.059	0.059
RJP	0.010	-0.001	0.015	-0.109	-0.027	0.038	0.012	-0.103	0.016	-0.004	-0.000	0.018	-0.016	0.005	0.014	-0.013
LJP	0.014	-0.047	0.008	-0.040	-0.042	0.012	-0.005	0.025	0.007	0.007	-0.010	0.132	-0.002	0.072	0.036	0.091
XJP	-0.044	-0.040	-0.077	0.038	-0.051	-0.046	-0.054	-0.020	0.030	-0.046	-0.042	-0.092	-0.021	-0.080	-0.044	-0.149
CPIUS	**0.000**	0.493	0.779	-0.067	0.702	0.785	0.756	-0.022	0.635	0.564	0.552	0.021	0.498	0.068	0.063	-0.030
WSUS		**0.000**	0.644	0.133	0.452	0.537	0.546	-0.090	0.287	0.388	0.404	-0.107	0.260	0.279	0.202	0.107
GDPUS			**0.000**	0.064	0.660	0.800	0.837	-0.044	0.632	0.595	0.631	-0.014	0.408	0.195	0.154	0.040
PSBUS				**139.9**	-0.086	-0.029	-0.038	0.089	-0.059	-0.009	0.036	0.075	-0.026	0.039	0.024	0.075
CPIUK					**0.000**	0.710	0.735	0.079	0.607	0.467	0.477	0.038	0.584	0.076	0.114	0.040
WSUK						**0.000**	0.904	-0.019	0.609	0.574	0.588	0.004	0.475	0.163	0.144	-0.018
GDPUK							**0.000**	0.015	0.611	0.572	0.624	-0.000	0.434	0.202	0.225	0.023
PSBUK								**3E+06**	0.080	-0.008	-0.049	0.208	0.045	0.135	0.133	0.092
CPIEU									**0.000**	0.507	0.502	0.044	0.430	0.118	-0.033	-0.025
WSEU										**0.000**	0.892	-0.044	0.377	0.085	0.032	0.021
GDPEU											**0.000**	-0.088	0.336	0.143	0.113	0.044
PSBEU												**8E+00**	0.034	-0.100	-0.210	-0.030
CPIJP													**0.000**	-0.048	0.028	-0.007
WSJP														**0.364**	0.418	0.257
GDPJP															**0.001**	0.294
PSBJP																**4E+06**

Table 10: Variance\ Correlation Matrix of Returns 1971-2002

	SUS	RUS	LUS	SUK	RUK	LUK	XUK	SEU	REU	LEU	XEU	SJP	RJP	LJP	XJP
SUS	**0.002**	-0.101	-0.257	0.580	-0.003	-0.127	-0.039	0.629	-0.074	-0.126	-0.010	0.364	0.029	0.003	-0.031
RUS		**0.005**	0.398	-0.113	0.128	0.149	-0.172	-0.013	0.095	0.231	0.169	-0.036	0.100	0.143	0.170
LUS			**0.001**	-0.131	0.051	0.320	-0.124	-0.136	0.020	0.381	0.128	-0.079	-0.027	0.190	0.093
SUK				**0.004**	-0.259	-0.317	-0.051	0.602	-0.159	-0.150	0.068	0.337	0.024	-0.008	-0.030
RUK					**0.006**	0.404	-0.150	-0.085	0.163	0.136	0.007	-0.004	0.027	0.004	0.127
LUK						**0.001**	-0.172	-0.174	0.078	0.375	0.010	-0.117	-0.028	0.066	0.135
XUK							**0.001**	-0.109	-0.012	-0.067	-0.662	0.038	-0.025	-0.087	-0.469
SEU								**0.002**	-0.111	-0.213	0.168	0.433	0.073	0.040	0.056
REU									**0.003**	0.391	-0.051	-0.079	0.163	-0.004	0.031
LEU										**0.001**	0.066	-0.070	0.037	0.105	0.098
XEU											**0.001**	0.001	0.014	0.084	0.544
SJP												**0.003**	-0.065	-0.064	-0.084
RJP													**0.029**	0.130	-0.015
LJP														**0.005**	0.071
XJP															**0.001**
CPIUS															
WSUS															
GDPUS															
PSBUS															
CPIUK															
WSUK															
GDPUK															
PSBUK															
CPIEU															
WSEU															
GDPEU															
PSBEU															
CPIJP															
WSJP															
GDPJP															
PSBJP															

	CPIUS	WSUS	GDPUS	PSBUS	CPIUK	WSUK	GDPUK	PSBUK	CPIEU	WSEU	GDPEU	PSBEU	CPIJP	WSJP	GDPJP	PSBJP
SUS	-0.152	-0.016	-0.073	0.044	0.040	-0.118	0.009	0.080	0.025	-0.042	-0.063	0.040	-0.025	0.066	-0.074	0.072
RUS	0.125	0.019	0.107	0.061	-0.076	-0.016	-0.069	-0.035	0.080	0.035	0.009	0.007	-0.057	-0.020	0.007	-0.013
LUS	0.182	0.059	0.194	-0.009	0.032	0.019	0.009	0.017	0.118	-0.002	0.034	0.075	-0.029	-0.061	-0.046	0.065
SUK	-0.201	-0.020	-0.067	0.053	0.113	0.033	0.043	0.062	0.033	-0.087	-0.081	0.033	-0.020	-0.002	-0.123	0.064
RUK	0.067	0.078	0.025	0.016	-0.027	-0.009	-0.003	-0.024	0.027	0.061	0.042	-0.021	-0.051	0.049	0.124	0.009
LUK	0.185	-0.010	0.045	-0.061	0.024	-0.066	0.031	0.035	0.115	0.030	0.017	0.044	0.043	0.122	0.127	0.076
XUK	0.021	0.094	-0.003	-0.004	0.025	0.057	-0.059	-0.007	-0.067	0.051	0.043	-0.003	-0.034	0.051	0.081	-0.014
SEU	-0.119	-0.009	-0.076	0.109	0.037	-0.050	0.011	0.120	0.055	-0.007	-0.006	0.043	0.006	0.049	-0.111	0.084
REU	0.092	0.039	0.098	-0.051	-0.082	-0.044	-0.062	-0.060	-0.059	-0.004	0.046	0.028	-0.056	0.035	0.192	0.056
LEU	0.149	0.037	0.077	-0.027	-0.059	0.004	-0.119	-0.030	0.119	0.014	0.026	0.063	-0.016	-0.018	0.027	0.089
XEU	0.076	0.014	0.096	0.066	-0.045	-0.012	0.046	-0.014	0.077	-0.017	-0.002	-0.018	0.018	-0.101	-0.004	0.084
SJP	-0.116	-0.033	-0.050	0.126	-0.028	-0.095	0.002	0.048	0.063	0.038	0.014	0.022	-0.070	0.034	-0.081	0.051
RJP	-0.032	-0.001	-0.056	-0.142	-0.050	-0.004	-0.107	-0.108	0.064	0.005	-0.034	0.045	0.001	0.057	-0.032	0.021
LJP	0.056	-0.040	0.007	-0.131	0.009	0.018	-0.006	0.033	0.025	-0.044	-0.055	0.089	0.050	0.097	0.010	0.112
XJP	-0.024	0.028	-0.024	0.012	0.016	0.014	-0.053	-0.045	0.084	-0.037	-0.030	-0.072	0.048	-0.055	-0.021	-0.169
CPIUS	**0.000**	-0.055	0.100	-0.086	0.126	0.175	-0.008	-0.033	0.158	0.099	0.026	0.064	0.176	0.108	-0.095	0.018
WSUS		**0.000**	0.224	0.176	-0.002	0.004	-0.030	-0.169	-0.223	0.022	0.017	-0.194	-0.040	0.031	0.057	0.091
GDPUS			**0.000**	0.088	-0.029	0.053	0.128	-0.085	0.006	0.090	0.125	-0.080	-0.058	-0.034	-0.008	0.068
PSBUS				**81.6**	-0.063	0.022	0.022	0.182	-0.070	0.104	0.179	0.064	-0.020	-0.036	-0.030	-0.015
CPIUK					**0.000**	0.070	0.150	0.191	0.228	-0.095	-0.092	0.004	0.279	0.021	0.166	-0.021
WSUK						**0.000**	0.460	-0.006	0.008	-0.004	0.011	0.028	0.107	-0.038	-0.047	-0.040
GDPUK							**0.000**	0.088	-0.032	0.026	0.126	-0.014	-0.040	0.010	0.150	-0.011
PSBUK								**2E+06**	0.044	-0.068	-0.126	0.138	0.067	0.047	0.186	-0.048
CPIEU									**0.000**	0.099	0.019	0.053	0.186	0.164	-0.242	0.029
WSEU										**0.000**	0.769	-0.044	0.107	-0.033	-0.121	0.080
GDPEU											**0.000**	-0.127	0.059	-0.021	-0.029	0.068
PSBEU												**5E+00**	0.023	0.025	-0.037	0.072
CPIJP													**0.000**	-0.088	-0.051	-0.023
WSJP														**0.169**	-0.016	0.129
GDPJP															**0.001**	0.137
PSBJP																**3E+06**

Table 11: Variance\ Correlation Matrix of Residuals Parsimonious Model 1971-2002

	SUS	RUS	LUS	SUK	RUK	LUK	XUK	SEU	REU	LEU	XEU	SJP	RJP	LJP	XJP
SUS	**0.00**	0.06	0.03	0.78	0.03	-0.18	-0.11	0.77	-0.04	-0.15	0.20	0.40	0.05	0.07	-0.10
RUS		**0.00**	0.13	0.13	0.31	0.13	-0.15	0.18	0.30	0.26	0.09	0.08	0.27	0.14	0.05
LUS			**0.00**	0.13	-0.05	0.50	-0.11	0.12	0.01	0.53	0.02	0.14	-0.05	0.15	-0.05
SUK				**0.00**	0.00	-0.20	-0.31	0.83	0.04	-0.14	0.22	0.41	0.13	0.11	-0.08
RUK					**0.00**	0.13	0.24	0.16	0.28	0.16	-0.06	0.08	0.10	-0.17	0.17
LUK						**0.00**	-0.01	-0.10	-0.02	0.71	0.02	0.12	-0.12	-0.03	0.15
XUK							**0.00**	-0.27	-0.03	0.00	-0.59	-0.11	-0.07	-0.05	-0.27
SEU								**0.00**	0.01	-0.06	0.38	0.46	0.11	0.14	0.09
REU									**0.00**	0.35	-0.05	-0.07	0.34	-0.07	-0.10
LEU										**0.00**	-0.05	0.11	0.04	0.05	-0.01
XEU											**0.00**	0.26	0.03	0.08	0.43
SJP												**0.00**	-0.06	0.14	0.06
RJP													**0.11**	0.09	-0.06
LJP														**0.01**	-0.16
XJP															**0.00**

	CPIUS	WSUS	GDPUS	PSBUS	CPIUK	WSUK	GDPUK	PSBUK	CPIEU	WSEU	GDPEU	PSBEU	CPIJP	WSJP	GDPJP	PSBJP	SEM	BEM
SUS	0.08	0.08	0.21	0.29	-0.00	0.17	0.17	0.13	0.09	0.09	0.13	0.12	0.15	0.10	0.12	0.19	0.67	0.55
RUS	-0.06	0.19	0.07	0.21	-0.03	-0.10	-0.03	-0.04	0.05	-0.01	0.05	0.11	-0.09	-0.08	0.01	-0.11	0.04	-0.05
LUS	-0.00	0.10	-0.00	0.04	0.04	-0.13	-0.01	0.14	0.09	-0.14	0.04	0.14	-0.03	0.03	-0.18	0.04	0.22	-0.10
SUK	-0.00	-0.02	0.11	0.24	-0.04	0.09	0.06	0.12	0.09	0.01	0.07	0.02	0.07	0.03	0.10	0.16	0.65	0.52
RUK	-0.01	0.13	0.02	0.10	0.05	-0.01	0.01	-0.12	0.05	0.02	0.10	-0.09	0.08	0.10	0.09	-0.12	-0.05	-0.03
LUK	0.10	0.03	-0.12	-0.10	0.02	-0.19	-0.10	0.05	0.15	-0.11	-0.01	0.18	0.08	-0.08	-0.29	-0.04	-0.02	-0.19
XUK	0.04	0.05	0.03	-0.04	0.08	-0.01	-0.01	0.04	0.03	-0.11	0.03	-0.05	0.12	0.27	0.10	-0.01	-0.18	-0.20
SEU	0.03	0.10	0.19	0.31	-0.02	0.14	0.15	0.15	0.15	0.04	0.13	0.05	-0.00	0.13	0.10	0.21	0.63	0.49
REU	-0.09	0.03	-0.01	0.01	0.03	-0.00	-0.07	-0.09	-0.18	0.00	0.01	0.06	0.03	0.04	0.17	0.04	-0.12	-0.05
LEU	0.03	0.08	-0.01	-0.03	0.00	-0.11	-0.00	0.04	0.12	-0.07	0.07	0.10	0.03	-0.02	-0.16	0.02	0.00	-0.17
XEU	0.07	0.04	0.10	0.15	-0.03	0.11	0.09	-0.12	0.04	0.07	0.10	0.04	-0.10	-0.20	-0.05	0.05	0.16	0.24
SJP	0.01	-0.16	0.03	0.18	-0.05	-0.12	-0.01	0.08	0.00	-0.06	0.05	0.06	0.03	-0.01	-0.03	0.14	0.48	0.37
RJP	-0.07	0.02	0.09	-0.12	-0.16	0.07	0.07	-0.11	0.07	-0.02	0.02	0.01	-0.16	0.00	0.07	-0.01	-0.01	0.05
LJP	-0.17	-0.10	0.03	-0.05	-0.20	-0.06	-0.05	0.01	0.05	-0.03	-0.09	0.14	-0.21	0.20	0.14	0.14	-0.00	-0.13
XJP	-0.09	0.12	-0.01	0.06	-0.08	0.06	0.04	-0.04	0.14	0.05	0.03	-0.17	-0.17	-0.02	0.02	-0.31	-0.04	0.11
CPIUS	**0.00**	0.33	0.66	-0.09	0.49	0.64	0.64	-0.17	0.31	0.56	0.55	0.04	0.13	-0.15	-0.37	-0.06	-0.02	0.18
WSUS		**0.00**	0.52	0.19	0.35	0.50	0.48	-0.15	0.10	0.37	0.30	-0.05	-0.03	0.17	0.08	0.08	-0.07	0.05
GDPUS			**0.00**	0.08	0.48	0.87	0.93	-0.12	0.40	0.59	0.76	0.01	0.04	0.18	-0.01	0.08	-0.00	0.16
PSBUS				**4E+02**	0.00	0.03	0.09	0.19	0.01	0.03	0.13	0.12	0.04	0.11	0.17	0.06	0.15	0.06
CPIUK					**0.00**	0.44	0.50	0.06	0.23	0.34	0.37	0.17	0.39	0.17	-0.00	0.05	-0.10	0.14
WSUK						**0.00**	0.90	-0.09	0.38	0.72	0.72	0.03	0.02	0.17	0.04	-0.02	-0.08	0.18
GDPUK							**0.00**	0.01	0.44	0.66	0.77	0.08	0.05	0.20	0.03	0.03	-0.03	0.13
PSBUK								**9E+06**	0.19	-0.13	-0.13	0.21	0.06	0.23	0.27	0.05	0.11	-0.07
CPIEU									**0.00**	0.27	0.34	0.05	-0.25	0.27	-0.15	-0.11	0.08	0.09
WSEU										**0.00**	0.56	0.01	-0.07	0.12	-0.01	-0.03	-0.08	0.08
GDPEU											**0.00**	-0.02	0.03	0.13	-0.06	-0.01	0.01	0.11
PSBEU												**24.83**	0.10	0.02	-0.01	0.04	0.02	-0.05
CPIJP													**0.00**	-0.16	0.03	0.20	0.08	0.18
WSJP														**0.30**	0.48	0.22	0.07	0.06
GDPJP															**0.00**	0.37	0.04	0.13
PSBJP																**1E+07**	0.15	0.20
SEM																	**0.00**	0.71
BEM																		**0.00**

Table 12: Variance\ Correlation Matrix of Returns 1993-2002

	SUS	RUS	LUS	SUK	RUK	LUK	XUK	SEU	REU	LEU	XEU	SJP	RJP	LJP	XJP
SUS	**0.00**	-0.09	-0.06	0.77	0.04	-0.14	-0.12	0.76	-0.04	-0.15	0.13	0.42	0.09	0.08	-0.11
RUS		**0.00**	0.07	0.02	0.12	0.08	-0.17	0.03	0.14	0.08	0.05	-0.02	0.30	0.12	0.15
LUS			**0.00**	0.04	-0.09	0.50	-0.10	-0.00	-0.00	0.51	-0.03	0.07	-0.04	0.12	-0.03
SUK				**0.00**	-0.05	-0.18	-0.33	0.83	-0.00	-0.16	0.19	0.42	0.17	0.12	-0.12
RUK					**0.00**	0.18	0.28	0.12	0.07	0.12	-0.17	0.07	0.10	-0.18	0.18
LUK						**0.00**	-0.03	-0.08	-0.02	0.69	-0.03	0.09	-0.11	-0.02	0.16
XUK							**0.00**	-0.31	-0.03	-0.01	-0.60	-0.12	-0.06	-0.04	-0.26
SEU								**0.00**	-0.03	-0.12	0.31	0.45	0.14	0.13	0.05
REU									**0.00**	0.35	-0.04	-0.15	0.33	-0.10	-0.09
LEU										**0.00**	-0.12	0.01	0.00	-0.04	-0.01
XEU											**0.00**	0.22	0.06	0.03	0.40
SJP												**0.00**	-0.08	0.12	0.03
RJP													**0.10**	0.14	-0.06
LJP														**0.01**	-0.18
XJP															**0.00**

	CPIUS	WSUS	GDPUS	PSBUS	CPIUK	WSUK	GDPUK	PSBUK	CPIEU	WSEU	GDPEU	PSBEU	CPIJP	WSJP	GDPJP	PSBJP	SEM	BEM
SUS	-0.06	-0.06	-0.16	0.14	-0.04	-0.21	0.02	0.16	0.02	-0.08	-0.06	0.11	0.20	0.04	-0.08	0.05	0.70	0.52
RUS	-0.20	0.10	0.06	-0.03	0.06	0.03	0.01	-0.02	0.01	-0.03	-0.13	-0.08	-0.10	-0.15	0.08	-0.17	-0.03	-0.00
LUS	0.07	0.05	0.22	-0.12	0.08	0.03	-0.04	0.10	0.08	-0.08	-0.04	0.08	-0.13	-0.03	0.01	0.12	0.13	-0.06
SUK	-0.09	-0.07	-0.09	0.15	-0.02	-0.13	0.07	0.17	0.08	-0.12	-0.03	0.07	0.12	-0.00	0.07	0.11	0.67	0.51
RUK	-0.03	0.08	-0.21	-0.09	0.09	0.00	-0.10	-0.05	0.07	0.05	-0.04	-0.09	-0.02	0.12	0.10	-0.16	-0.02	-0.03
LUK	0.10	0.01	-0.05	-0.08	0.06	-0.10	-0.10	0.04	0.15	-0.06	-0.16	0.10	-0.03	0.10	-0.01	0.01	-0.04	-0.11
XUK	0.04	0.07	-0.03	0.03	0.11	0.03	-0.01	0.05	-0.02	0.06	0.03	-0.06	-0.02	0.18	0.04	-0.08	-0.21	-0.21
SEU	-0.11	0.03	-0.14	0.20	-0.04	-0.19	0.04	0.17	0.12	-0.18	-0.03	0.07	0.10	0.04	-0.03	0.11	0.65	0.48
REU	-0.20	0.03	0.08	-0.14	-0.11	-0.04	-0.26	-0.10	-0.11	-0.09	-0.02	0.14	-0.03	-0.13	0.15	0.09	-0.13	-0.12
LEU	-0.06	0.05	-0.01	-0.07	-0.06	0.02	-0.20	0.00	0.09	-0.05	-0.08	0.05	-0.00	-0.06	0.00	0.10	-0.09	-0.12
XEU	-0.07	0.08	0.17	0.04	-0.17	-0.07	0.03	-0.07	-0.02	-0.04	0.11	-0.06	-0.01	-0.26	0.07	0.14	0.17	0.26
SJP	0.05	-0.16	-0.03	0.21	-0.05	-0.16	0.07	0.13	-0.02	-0.12	0.16	0.08	0.08	-0.04	-0.06	0.08	0.47	0.40
RJP	-0.17	0.03	0.03	-0.16	-0.16	0.03	-0.18	-0.11	0.12	0.04	-0.11	0.05	-0.10	0.10	0.01	0.02	-0.02	-0.02
LJP	-0.11	-0.09	0.02	-0.11	-0.09	-0.03	-0.06	0.02	0.02	-0.05	-0.01	0.09	-0.05	0.25	0.01	0.17	-0.02	-0.12
XJP	-0.17	0.15	-0.01	0.03	-0.04	-0.01	0.06	-0.06	0.12	0.06	-0.03	-0.11	-0.02	-0.10	-0.09	-0.36	-0.03	0.09
CPIUS	**0.00**	-0.14	-0.08	-0.19	-0.04	0.10	-0.05	-0.22	0.00	0.06	-0.03	0.04	0.04	0.12	-0.15	0.07	0.06	0.12
WSUS		**0.00**	0.22	0.17	0.09	-0.10	-0.11	-0.28	-0.10	-0.13	-0.17	-0.16	-0.14	-0.08	0.07	0.09	-0.10	-0.05
GDPUS			**0.00**	-0.16	0.01	0.15	0.23	-0.18	-0.09	-0.24	0.27	-0.23	-0.08	-0.14	0.02	0.16	-0.05	-0.04
PSBUS				**2E+02**	0.02	-0.17	0.22	0.30	-0.06	-0.03	0.16	0.11	0.01	-0.04	0.06	-0.05	0.14	0.06
CPIUK					**0.00**	-0.07	0.26	0.16	0.10	-0.03	-0.05	0.10	0.15	0.10	0.21	-0.09	0.03	0.14
WSUK						**0.00**	0.31	-0.23	0.02	0.33	0.27	0.04	-0.02	-0.13	0.04	0.06	-0.01	0.10
GDPUK							**0.00**	0.10	-0.16	0.19	0.32	0.07	0.07	-0.06	0.08	-0.06	0.15	0.17
PSBUK								**6E+06**	0.12	-0.09	-0.16	0.18	0.09	0.01	0.21	-0.08	0.10	-0.00
CPIEU									**0.00**	-0.03	-0.19	0.00	-0.12	0.37	-0.16	-0.05	0.09	0.05
WSEU										**0.00**	-0.02	0.03	0.09	-0.07	0.06	0.00	0.05	0.05
GDPEU											**0.00**	-0.15	0.10	-0.10	-0.09	0.04	0.04	0.03
PSBEU												**1E+01**	0.13	0.07	0.17	0.08	0.07	0.04
CPIJP													**0.00**	-0.04	0.04	0.06	0.13	0.28
WSJP														**0.12**	-0.16	0.08	0.06	-0.07
GDPJP															**0.00**	0.21	-0.03	-0.04
PSBJP																**8E+06**	0.09	0.10
SEM																	**0.99**	0.71
BEM																		**0.99**

Table 13: Variance\ Correlation Matrix of Model Residuals 1993-2002