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On the Cross-Section of Expected Stock Returns: An Examination of Size Related Anomalies in the German Stock Market*

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Abstract

In recent years the so called anomaly literature provided a vast amount of empirical evidence that the CAPM does not fully explain the cross-sectional variation in expected returns. This paper investigates the cross-section of expected returns in the German stock market with particular focus on their predictability on the basis of size related factors. The findings show that the size anomaly in stock returns is not explainable by systematic risk not captured in the CAPM beta, but rather represents undiversified idiosyncratic risk of small capitalization stocks. The premium on the higher default risk of smaller firms and the illiquidity risk of trading their shares suggests market irrationality in investment portfolio decisions. However, the common problem of asset pricing model misspecifications cannot be rejected.

Keywords: Size-effect, capital market anomaly, capital asset pricing

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I. Introduction

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966) and further enhanced by Black (1972) set the foundation for modern asset pricing theory as it is known and acknowledged today.[1] It postulates that every asset's return is a linear function of its systematic risk, its covariance with the market portfolio's return, and independent of firm specific factors. In efficient capital markets all investors are assumed to diversify their asset portfolios according to Markowitz's (1952) mean-variance based portfolio choice model,[2] thus eliminating their exposure to idiosyncratic risk by holding the market portfolio comprised of all available assets. Despite its preeminent role in the finance profession and in finance theory for over four decades, the CAPM has recently suffered from severe criticism in the financial literature.

The so called anomaly literature provides a vast amount of empirical evidence that the CAPM does not fully explain the cross-sectional variation in expected returns. The simple and intuitive risk-return relationship does not hold empirically. Rather it seems that stock returns are more accurately explained by additional factors not captured in the famous one-factor asset pricing model.[3]

This paper investigates the cross-section of expected returns in the German stock market with particular focus on their predictability on the basis of size related factors. The purpose of this study is to add insights to the economic rationales behind size factors and the sparse literature concerning the unexplained return behaviour of German stocks. Despite the plethora of empirical work for US stocks and some supporting evidence for the size effect in international stock markets,

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1 See W. Sharpe, "Capital Asset Prices," *JF* 19 (1964): 425-442; J. Lintner, "The valuation of risky assets," *RES* 47 (1965): 13-37; J. Mossin, "Equilibrium in a Capital Asset Market," *Econometrica* 1966: 768-783; F. Black, "Capital Market Equilibrium," *JB* 45 (1972): 444-455.

2 See H. Markowitz, "Portfolio Selection," *JF* 7 (1952): 77-91.

3 See e.g. R. Litzenberger and K. Ramaswamy, "The Effect of Personal Taxes and Dividends on Capital Asset Prices," *JFE* 7 (1979): 163-195; S. Basu, "Investment Performance," *JF* 12 (1977): 663-682; R. Banz, "The Relationship Between Return and Market Value of Common Stock," *JFE* 9 (1981): 3-18.

recently, including German stocks, there has not been a study, so far, entirely devoted to the German stock market.[4]

The next section discusses previous studies on the size effect related to this study. Section three presents the methodology, the beta estimation procedure, and regression method, followed by a brief description of the sample. The results are presented in section five and summarized and interpreted in section six.

II. Review of Previous Studies

The explanatory power of a firm's market value for the variation in expected stock returns was first documented by Banz (1981) and subsequently labeled as the size effect.[5] Banz (1981) shows that between 1936 and 1975 stocks of small firms had on average higher risk-adjusted returns than stocks of large firms in the US.[6] Banz grouped his sample into portfolios sorted by market value and estimated betas, and performed a time-series of cross-sectional regressions over his sampling period for an arbitrage portfolio comprised of a long position in the small firm portfolio and a short position in the large firm portfolio.[7] He concludes that the CAPM is misspecified, but fails to give an economic explanation why size is a factor in stock returns or whether it is just a proxy for a risk factor not captured in securities' betas.

An explanation that smaller firms are riskier and therefore deserve higher expected returns is provided by Roll (1981). Roll argues that the risk measures in Banz (1981) are biased downward due to autocorrelation in the returns of small firms which are infrequently traded.[8] Barry and Brown (1984), on the other hand,

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4 See e.g. E. Fama and K. French, "Value versus Growth," *JF* 53 (1998): 1975-1999; S. Heston et al., "The Role of Beta and Size in the Cross-Section of European Stock Returns," *EFM* 5 (1999): 9-27.

5 While testing the explanatory power of the price earnings ratio, Reinganum (1981) also confirms that after controlling for market value the P/E-ratio loses its significance. M. Reinganum, "Misspecification of capital asset pricing," *JFE* 9 (1981): 20.

6 See R. Banz, "The Relationship Between Return and Market Value of Common Stock," *JFE* 9 (1981): 9.

7 He follows a procedure first introduced by Black et al. (1972) and Fama and MacBeth (1973). See F. Black et al., "The Capital Asset Pricing Model," in *Studies in the Theory of Capital Markets*, ed. M. Jensen (New York: Praeger, 1972): 84-86; E. Fama, and J. MacBeth, "Risk, return and equilibrium," *JPE* 71 (1973): 614-618. This procedure will be discussed in more detail in the methodology section.

8 See R. Roll, "A Possible Explanation of the Small Firm Effect," *JF* 36 (1981): 884. However, this bias is more severe when daily returns are used instead of monthly returns as in Banz's

provide evidence that the size effect is at least partly associated with differential information about small and large firms and thus related to the perceived riskiness of small firm stocks.[9] This hypothesis was also tested by Amihud and Mendelson (1989), who proxy the information factor of an asset by its bid-ask spread. Their results suggest that only systematic risk and a stock's illiquidity, measured by its bid-ask spread, affect returns. In fact, the size factor in their model appears to be insignificant and changing in sign over different sample periods.[10]

Further evidence on the predictability of stock returns based on firm size is provided by Chan and Chen (1988 and 1991). Chan and Chen (1988) show that the size effect is merely an outcome of measurement errors in the estimation of betas and thus just a proxy for underlying risk factors captured by the true beta.[11] They elaborate this assertion in their follow-up paper, in which they find that small firms often suffered from inefficient production and cash flow problems in preceding years causing their share price to fall, which in turn causes higher financial leverage.[12] Hence they conclude that small firms do indeed represent riskier investments justifying higher expected returns. Fama and French (1992, 1995 and 1996) complete this view of size and book-to-market factors as being part of rational asset pricing within the efficient markets framework in a series of papers in which they finally reject the validity of the CAPM in favour of a three factor model.[13] However, they admit that it is difficult to find an economic reason for size and book-to-market as risk factors in asset pricing.[14]

Berk (1995 and 1997) provides a theoretical explanation for the relation of expected return and size. He argues that size is always inversely related to

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(1981) study. For a detailed discussion refer to M. Scholes and J. Williams, "Estimating betas from nonsynchronous data," *JFE* 5 (1977): 309-328; E. Dimson, "Risk measurement," *JFE* 7 (1979): 197-226.

9 See Ch. Barry and S. Brown, "Differential Information and the Small Firm Effect," *JFE* 13 (1984): 288-289.

10 See Y. Amihud and H. Mendelson, "The Effect of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns," *JF* 44 (1989): 483.

11 See K. Chan and N. Chen, "An Unconditional Asset-Pricing Test," *JF* 43 (1988): 309-325.

12 See K. Chan and N. Chen, "Structural and Return Characteristics of Small and Large Firms," *JF* 46 (1991): 1480 f.

13 See E. Fama and K. French,"The Cross-Section of Expected Stock Returns," *JF* 47 (1992): 427-465; E. Fama and K. French,"Size and Book-to-Market Factors in Earnings and Returns," *JF* 50 (1995): 131-155; Fama, E., and K. French. "Multifactor explanations of asset-pricing anomalies." *JF* 51 (1996): 55-84.

14 See E. Fama and K. French, "Size and Book-to-Market Factors in Earnings and Returns," *JF* 50 (1995): 131.

expected returns, since stocks with high expected returns also have high discount rates which in turn automatically cause lower market values.[15] The size factor therefore does not proxy any specific other risk variable, but captures any residual risk not explained by the factor model under scrutiny.[16] Berk's hypothesis it tested by Fan and Liu (2005), who estimate a simultaneous equation model in order to find the characteristic components of size and book-to-market.[17] Their findings suggest that size and book-to-market are driven by financial distress, the firm's growth options and momentum and contrarian effects in the market.[18] Similarly, Dissanaike (2002) argues that the small-firm effect is merely an indication of investor overreaction and provides evidence for the UK that small size firms are also those with relatively negative stock price performance over the past.[19]

III. Methodology

Fama-MacBeth Procedure

I use a modification of the time series cross-section regression method of Fama and MacBeth (1973).[20] The entire sample period between 1996 and 2006 is divided into approximately two five year periods, the pre- and the post-ranking period. In June 2001 all stocks are ranked based on market value and beta estimates. The pre-ranking period is used to estimate asset betas on the basis of at least 24 months of security returns as described in detail below. In each of the 57 months in the post-ranking period a cross-sectional asset pricing test is conducted in which stock returns are regressed on different variables that are assumed to

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[15] See J. Berk, "A Critique of Size-Related Anomalies," *RFS* 8 (1995): 277.
[16] Ibid. 281.
[17] See X. Fan and M. Liu, "Understanding Size and Book-to-Market Ratio," *JFR* 28 (2005): 503-518.
[18] For a empirical findings of momentum and contrarian effects in stock markets refer to N. Jegadeesh and S. Titman, "Returns to Buying Winners and Selling Losers," *JF* 48 (1993): 65-91; G. Dissanaike, "Do Stock Market Investors Overreact?" *JBFA* 24 (1997): 32 ff; W. De Bondt and R. Thaler, "Does the Stock Market Overreact?" *JF* 40 (1985): 793-805. For evidence on the German market refer to D. Schiereck et al., "Contrarian and Momentum Strategies in Germany," *FAJ* Nov/Dec (1999): 104-116.
[19] See G. Dissanaike, "Does the Size Effect Explain the UK Winner-Loser Effect?" *JBFA* 29 (2002): 147-150.
[20] See E. Fama, and J. MacBeth, "Risk, return and equilibrium," *JPE* 71 (1973): 607-636. For a formal discussion of the methodology see also E. Fama, *Foundations of Finance* (New York: Basic Books, 1976): 326 ff.

explain the variation in expected returns using a two-stage least squares method, in which

$$R_{it} = \sum_j \beta^j x_{it}^j + \varepsilon_{it} \quad i = 1,.., N; t = 1,..., T$$

is the second-step multivariate regression equation of the two-stage least squares with j explanatory variables and the projection of the explanatory variable (such as size) from the first-step regressions. These equations have a cross-sectional element for N firms and a time-series element for T observations for each firm. According to Fama and MacBeth the regression estimates and their standard errors can be obtained by taking the time-series means of the cross-sectional estimates and their standard deviations, respectively:

$$\hat{\beta}^j = \frac{1}{T} \sum_{t=1}^{T} \hat{\beta}_t^j$$
$$\sigma^2\left(\hat{\beta}^j\right) = \frac{1}{T(T-1)} \sum_{t=1}^{T} \left(\hat{\beta}_t^j - \hat{\beta}^j\right)^2$$

These time-series means of the monthly regression estimates are then used in standard tests for the explanatory power of the independent variables, i.e. to test whether the different variables are on average priced in asset returns.[21] Since all explanatory variables like size, book-to-market, leverage, etc. except asset beta are directly observable, there is no need to estimate the Fama-MacBeth regressions for portfolios if more precise estimates for individual asset betas can be obtained. This is done following a procedure suggested by Fama and French (1992) described in the subsequent section.[22]

Beta Estimation

In order to run cross-sectional regressions to explain security returns on basis of the CAPM, estimations of asset betas are necessary since the security's systematic risk is not directly measurable. However, these estimates are affected by a

[21] Assuming normally distributed returns that are IID over time, the regression factors will also be IID, so that the test statistic is distributed student-t and asymptotically normal. For a detailed discussion of the Fama-MacBeth procedure and a comparison with other methods refer to J. Cochrane, *Asset Pricing* (Princeton: University Press, 2001): 244-251 or J. Campbell et al., *The Econometrics of Financial Markets* (Princeton: University Press, 1997): 215-217.

[22] See E. Fama, and K. French, "The Cross-Section of Expected Stock Returns," *JF* 47 (1992): 430-432.

sampling error for individual assets causing the so called error-in-variables-problem.[23] To mitigate this problem I follow the common approach introduced by Friend and Blume (1970) and Black et al. (1972) and estimate betas for portfolios of individual assets.[24] Although this procedure improves the precision of the estimations, it is also inefficient, since grouping reduces the range of betas. I alleviate this problem by sorting the securities into five equally weighted portfolios based on their estimated individual betas after having sorted them into five equally weighted portfolios based on their market values.[25] Hence, twenty-five portfolios containing a similar number of securities are formed.

While the first five years of return data was used to estimate the pre-ranking betas of individual securities, the next five years of equally-weighted monthly portfolio returns is calculated to re-estimate portfolio betas. These portfolios are rebalanced monthly to allow securities to be assigned to different portfolios based on their change in market value. After re-estimating portfolio betas for each of the twenty-five size-beta portfolios, these post-ranking betas are allocated to each stock in that portfolio. This procedure allows the Fama-MacBeth regressions to be used for individual securities rather than portfolios in order to retain the security specific information.[26]

Two-Stage Least Squares

Fama and French (1995 and 1996) as well as Chan and Chen (1991) argue that the explanatory power of size for stock returns in earlier studies captures the compensation for an underlying unknown risk factor, for which size serves as a

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[23] For a detailed analysis of the resulting attentuation bias refer to M. Merton, and M. Scholes, “Rates of Return in Relation to Risk,” in *Studies in the Theory of Capital Markets*, ed. M. Jensen (New York: Praeger, 1972): 60-63; J. Johnston and J. DiNardo, *Econometric Methods*, 4th ed. (New York: McGraw-Hill, 1997): 153-155.

[24] See I. Friend and M. Blume, “Measurement of Portfolio Performance under Uncertainty,” *AER* 60 (1970): 564-568; F. Black et al., “The Capital Asset Pricing Model,” in *Studies in the Theory of Capital Markets*, ed. M. Jensen (New York: Praeger, 1972): 84-87.

[25] Forming portfolios based on size produces a wide spread of returns and betas, but does not take into account the possible high correlation between size and beta. Therefore a second division of the size portfolios based on betas is useful to allow for variations in beta that are unrelated to size. See R. Banz, “The Relationship between Return and Market Value of Common Stock,” *JFE* 9 (1981): 7-8; E. Fama, and K. French, ”The Cross-Section of Expected Stock Returns,” *JF* 47 (1992): 430.

[26] For theoretical justification of this approach refer to E. Fama, and K. French, ”The Cross-Section of Expected Stock Returns,” *JF* 47 (1992): 432.

proxy.[27] They particularly mention financial distress as one possible risk factor that is not captured by the market factor in the CAPM. However, the positive risk premium on size could equally be due to several other factors omitted in the original CAPM. This unobserved heterogeneity leads to biased and inconsistent estimators when an ordinary least square regression estimation is used to find the linear relationship between risk and return, since the error terms in the regression will be correlated with one or more explanatory variables. Moreover, Berk (1995) shows that since size is usually measured by the market value of equity it is not only endogenous, but simultaneously related to expected returns, thus inducing the so called simultaneity bias.[28]

In the light of these findings I use a set of instrumental variables in a two stage least squares regression to control for the endogenous variable size. All accounting variables used in the regressions are matched with returns with a six month lag in order to ensure that these variables are publicly available and incorporated in stock prices.[29] All ratios like book to market, price earnings ratio, etc. are calculated on a monthly basis using monthly market values and quarterly accounting data. Table 1 summarizes the variables that are included in the modified Fama-MacBeth regressions. The basic asset pricing model used here to explain stock returns follows the findings of Fama and French (1995, 1996) and Jegadeesh (1992), that book-to-market and size play a significant role as factors in addition to the stock's beta.[30] Its empirical expression is given by

$$R_{it} = \gamma_0 + \gamma_1 BETA_{it} + \gamma_2 BM_{it} + \gamma_3 SIZE_{it} + \varepsilon_{it} \, , \qquad (1)$$

where BETA is the portfolio beta assigned to the individual stock in the way explained above, BM is the ratio of book value of equity to its market value and SIZE is the market value of equity. Since earlier studies also report a relation

[27] See e.g. E. Fama and K. French, "Size and Book-to-Market Factors in Earnings and Returns," JF 50 (1995): 131; K. Chan and N. Chen, "Structural and Return Characteristics of Small and Large Firms," *JF* 46 (1991): 1468.

[28] See J. Berk, "A Critique of Size-Related Anomalies," *RFS* 8 (1995): 280.

[29] Banz (1983) only assumes a three month lag until accounting data is known in the market, but Fama and French (1992) give a rationale for the conservative six month lag. See E. Fama, and K. French, "The Cross-Section of Expected Stock Returns," *JF* 47 (1992): 429.

[30] See E. Fama and K. French, "Size and Book-to-Market Factors in Earnings and Returns," JF 50 (1995): 143-144; E. Fama and K. French, "Multifactor explanations of asset-pricing anomalies," *JF* 51 (1996): 55-84; N. Jedadeesh, "Does Market Risk Really Explain the Size Effect?" *JFQA* 27 (1992): 348.

Table 1: List of variables of the OLS and two stage least square regressions

return	arithmetic monthly return including dividends
beta	monthly estimated portfolio beta assigned to individual asset
size	natural logarithm of monthly market value of equity
bm	natural logarithm of ratio of book value of equity to market value of equity
ba	natural logarithm of book value of total assets
sal	natural logarithm of total sales
lev	financial leverage calculated as the ratio of long-term debt to market value of equity
div	dividend yield calculated as the ratio of dividends per share to price per share on a monthly basis
prft	profitability ratio calculated as the ratio of earnings before interest and taxes to sales
pe	price to earnings ratio calculated as price market value divided by earnings before interest and taxes
bas	bid ask spread calculated as the difference in the average monthly bid and ask price
vol	monthly trading volume

between risk-adjusted returns and dividend yield and price-to-earnings ratios the model in (1) is extended to

$$R_{it} = \gamma_0 + \gamma_1 BETA_{it} + \gamma_2 BM_{it} + \gamma_3 SIZE_{it} + \gamma_4 DIV_{it} + \gamma_5 PE_{it} + \varepsilon_{it} , \qquad (2)$$

where DIV is the dividend yield of the stock and PE its price-earnings ratio.[31]

[31] See S. Basu, "Investment Performance of Common Stock in Relation to Their Price-Earnings Ratios," *JF* 12 (1977): 129-156. R. Litzenberger and K. Ramaswamy, "The Effect of Personal Taxes and Dividends on Capital Asset Prices," *JFE* 7 (1979): 184; refer also to F. Black and M. Scholes (1974) for contrary results on the effect of dividends on stock prices. See F. Black and

Choice of Instrumental Variables

In the first step of the two-stage least square regression SIZE is regressed on its set of instrumental variables **Z**. It is crucial for the choice of instruments that they are exogenous in the structural equation (2) and correlated to the endogenous explanatory variable SIZE, i.e. $E[\mathbf{z_{it}}, \varepsilon_{it}] = 0$ and $E[\mathbf{z_{it}}, SIZE_{it}] = \delta$ for some nonzero δ, respectively. Furthermore, the reduced form regression must have at least as many instruments as there are explanatory variables in the structural equation (2).[32]

Berk (2000) emphasizes the misspecification of size measured by the market value of equity in explaining stock returns due to the simultaneous relation with expected returns. He thus suggests non-market measures of size in asset pricing tests.[33] I therefore use the book value of total assets (BA) and total sales (SAL) as instrumental variables. Moreover, Fama and French (1996), Chan and Chen (1991) and more recently Vassalou and Xing (2004) suggest that smaller firms suffer from financial distress and thus are expected to experience higher stock returns.[34] To model for default risk, leverage (LEV), measured by the ratio of long-term debt to market equity and the profitability ratio (PRFT), measured by the ratio of earnings before interest and taxes to total sales, are also included in the reduced form equation. Finally, bid-ask-spread (BAS) and trading volume (VOL) are used as IVs for size as well, since smaller stocks are usually also illiquid stocks and thus traded less frequently giving rise to higher holding period risk.[35] Hence, the reduced form regression is obtained as

$$SIZE_{it} = \lambda_0 + \lambda_1 BA_{it} + \lambda_2 SAL_{it} + \lambda_3 LEV_{it} + \lambda_4 PRFT_{it} + \lambda_5 BAS_{it} + \lambda_6 VOL_{it} + \eta_{it}, \qquad (3)$$

containing the variables explained above.

M. Scholes, "The Effects of Dividend Yield and Dividend Policy on Common Stock Prices and Returns," *JFE* 1 (1974): 18.

32 Although the asymptotic efficiency increases with the number of instruments, the finite sample bias also increases. See P. Phillips, "The Exact Distribution of Instrumental Variable Estimators in an Equation Containing n+1 Endogenous Variables, " *Econometrica* 48 (1980): 875.

33 See J. Berk, "A View on the Current Status of the Size Anomaly," in *Security Market Imperfections*, ed. D. Keim and W. Ziemba (Cambridge: University Press, 2000): 95-97.

34 See e.g. M. Vassalou, Y. Xing, "Default Risk in Equity Returns," *JF* 59 (2004): 847.

35 See Y. Amihud and H. Mendelson, "The Effect of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns," *JF* 44 (1989): 483.

The orthogonal projection of SIZE onto the matrix of instrumental variables **Z** gives the fitted values for SIZE. Replacing SIZE in equation (2) with its fitted values of the first-step regression in (3) results in the second-step regression

$$R_{it} = \gamma_0 + \gamma_1 BETA_{it} + \gamma_2 BM_{it} + \gamma_3 P * SIZE_{it} + \gamma_4 DIV_{it} + \gamma_5 PE_{it} + e_{it}\,, \qquad (4)$$

where **P** is the projection matrix of the reduced form regression defined by

$$\mathbf{P} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\,.$$

Although the IV estimation gives consistent estimators when the covariance conditions stated above hold, the 2SLS estimators will be asymptotically biased towards OLS if the endogenous explanatory variable and the instruments are only weakly correlated. Nelson and Startz (1990) derive that

$$\frac{OLS\,bias}{2SLS\,bias} \leq 1$$

whenever the inequality

$$\frac{1}{\hat{\rho}_{xz}^2} >> n \qquad (5)$$

holds, where $\hat{\rho}_{xz}^2$ is the bias-corrected R^2 statistic of the reduced form regression and n is the number of observations.[36] Moreover, low estimators in the first-step regression induce an upward bias in the t-statistic, rejecting the null in regression (3), $H_0 : \lambda_j = 0 \quad j = 1,...,6,$ where in fact it is true. Hence, a choice of poor IVs might lead to erroneous inference.

Additionally, Hahn and Hausman (2003) state that even with a high R^2 in the first-step regression the 2SLS-estimators will be biased if the disturbances of the structural form and the reduced form equations are correlated.[37] A high correlation might particularly occur when many instruments are used. This study therefore

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[36] See C. Nelson and R. Startz, “The Distribution of the Instrumental Variable Estimator,” *JB* 63 (1990): 127-131. The authors also show that the R^2 of the first-step regression is upward biased by approximately 1/n, hence suggesting a downward correction. See also C. Nelson and R. Startz, “Some Further Results on the Exact Small Sample Properties of the Instrumental Variable Estimator,” *Econometrica* 58 (1990): 971-972.

[37] See J. Hahn and J. Hausman, “Weak Instruments,” *AER* 93 (2003): 119.

mainly differs from Fan and Liu (2005) with respect to the number of instrumental variables. While Fan and Liu use up to 152 IVs in their simultaneous equation model in order to explain the cross-section of expected returns, not only including idiosyncratic components but also macroeconomic variables, I only implement the six most promising IVs hypothesized to be highly related to firm size. Adding less important variables like macroeconomics variables will only add little explanatory power, but increase the bias dramatically.[38]

IV. Sample Selection and Descriptive Data

I obtained monthly stock return and accounting data for all companies listed in the German composite index CDAX through Thomson Financials Datastream database for a 10 year period between March 1996 and December 2005.[39] The initial sample includes 674 companies. Those with less than 24 months of return data for beta estimation purposes before July 2001 or missing accounting data are excluded from the final sample resulting in 447 German firms with complete return and accounting data. The summary statistics for the sample over the entire period is given in table 2.

[38] See A. Buse, "The Bias of Instrumental Variable Estimators," Econometrica 60 (1990): 178.

[39] The CDAX index includes all domestic companies of the Prime and General Standard of the Frankfurt Stock Exchange comprised of DAX, MDAX, SDAX, TecDAX and other stocks of the General Standard segment and thus represents the whole breadth of the German stock market.

Table 2: Summary Statistics (all values in million euros except ratios)

Companies	447				
Months	57				
Variables	**Obs.**	**Mean**	**Std. Dev.**	**Min**	**Max**
return	25468	0.0082	0.2470	-0.9355	19.1429
beta	25468	0.1477	0.1417	-0.2531	0.6210
market value	25467	1592.57	6488.90	0.06	110148.90
book-to-market	23982	4.66	15.73	0.00	100.00
book assets	21421	1042.72	4138.92	0.00	96500.00
sales	21421	4417.15	14400.00	0.06	162000.00
leverage	17178	3.23	21.20	0.00	617.86
dividend yield	25468	2.26	7.53	0.00	294.12
EBIT	25468	199.09	1247.01	-21100.00	15300.00
P/E-ratio	12884	25.73	135.08	0.00	3887.06
profitability ratio	21421	0.02	1.42	-69.45	29.75
bid-ask-spread	23763	0.26	0.42	-2.05	2.99
trading volume	25468	336.06	1283.82	0.00	84982.10

Table 3 shows the annualized buy-and-hold returns of the 25 size-beta portfolios during the post-ranking period. Buy-and-hold returns are calculated following Roll's (1983) 'rebalanced returns' according to

$$BHR_p = \left[\prod_t (1 + \frac{1}{N} \sum_{i=1}^{N} R_i) \right] - 1 ,$$

where R is the monthly stock return of each individual firm i.[40] Intriguingly, the results show that the larger firm portfolios outperformed the smaller firm portfolios on a raw return basis. Similar results are obtained comparing risk-adjusted portfolio returns (not reported here).[41] However, the small firm portfolios show higher negative abnormal returns than portfolios of larger firms giving rise to the assumption that the CAPM does not appropriately capture the risk inherent in small firms. These informal results for portfolios highlight the importance of an investigation based on the cross-section of individual securities.

40 See R. Roll, "On Computing Mean Returns and the Small Firm Premium," *JFE* 12 (1983): 373. Roll finds out that using buy-and-hold returns mitigates the upward bias introduced by arithmetic returns when calculating the size premium for mean portfolio returns.

41 The results for risk-adjusted buy-and-hold returns are available on request. A possible explanation for this 'reverse' size effect can be the exuberant losses of smaller high tech firms, which are more vulnerable to business cycles than large firms, after the burst of the stock market bubble. See e.g. M. R. Reinganum, "A Revival of the Small-Firm Effect," *JPM* 18 (1992): 55-62. For similar results after sorting according to size and book-to-market see K. Daniel and S. Titman, "Evidence on the characteristics of cross sectional variation in stock returns," *JF* 52 (1997): 6.

Table 3: Annualized Buy-and-Hold Returns of the 25 Size-Beta Portfolios During the Post-Ranking Period

		Beta Quintile Portfolios				
		1	2	3	4	5
Size Quintile Portfolios	1	-0.0382	-0.1284	0.0104	-0.1424	-0.0858
	2	0.0579	0.0625	0.1241	0.1733	-0.2003
	3	0.1132	0.1145	0.0603	0.0474	0.0680
	4	0.1527	0.1955	0.1610	0.1327	0.0069
	5	0.1457	0.1195	0.0880	0.0884	-0.0057

1 = small size / low beta 5 = large size / high beta

The size-beta portfolios are constructed by first sorting the entire sample into five quintiles based on their monthly market values beginning with the smallest followed by a second sort into five quintiles based on their pre-ranking beta beginning with the lowest. Portfolios are rebalanced each month and buy-and-hold returns are calculated on equally weighting the stocks in each portfolio. The returns represent the actual investment return of each portfolio after maintaining equal weighting assuming an investment horizon of one year.

V. Empirical Results

First-Step Regressions

Given the rationale for a two-stage least squares estimation discussed above, I estimate the regression in (3), in order to investigate whether a linear relation between size and the hypothesized instruments exists. The results of the reduced form regression are presented in table 4, panel A. All explanatory variables are highly statistically significant on the 0.01-level (two-tailed test) in explaining the variation in firm size with an overall goodness of fit of 88%. Such a high correlation between firm size and the IVs is a reasonable indicator for an adequate choice of instruments. Consistent to Fan and Liu (2005), particularly book value of assets and sales as physical size measures capture the variation in size based on market value.[42] However, the parameter for trading volume is extremely low indicating a biased t-statistic mentioned above.[43]

Moreover, since a high R-squared is not a guarantor for non-spurious IV estimators, the downward adjustment and simple test given in inequality (5) suggested by Nelson and Startz (1990) is conducted.[44] The results confirm the justification of an IV approach. With multiple explanatory variables in the first-step regression spurious estimators might also result from high correlation among the variables. The correlation coefficients of the instrumental variables are given in table 4, panel B. Only the correlations between SAL and BA could give rise to concern. With the number of instruments large enough that the first and second moments estimators exist,[45] but still moderate enough to avoid overidentifying problems and correlation among the disturbances between the structural and reduced form equations, the two-stage least squares estimation appears to be appropriate to produce unbiased and consistent estimators.

Second-Step Regressions

Having confirmed the endogeneity of the size factor, the second step of the two-stage least squares regression is estimated containing the predictions of size as regressors as in equation (4). The mean slopes of the Fama-MacBeth regressions are presented in table 5. Panel A shows the results of the ordinary single step OLS regressions. In the extended form (2) none of the parameters is statistically significantly different from zero, indicating that the model does not capture the variation in the cross section of stock returns. However, consistent with the findings of Fama and French (1992, 1996) and Jegadeesh (1992) the basic three-factor model reveals significant explanatory power of book-to-market on the 0.05-level whereas CAPM beta shows none, yet the sign of book-to-market is negative.

The second stage results of the 2SLS regression are given in panel B of table 5. In both models, the extended and the basic equation, the parameters on book-to-

[42] See X. Fan and M. Liu, "Understanding Size and Book-to-Market Ratio," *JFR* 28 (2005): 513.
[43] See C. Nelson and R. Startz, "The Distribution of the Instrumental Variable Estimator," *JB* 63 (1990): 127-131.
[44] See C. Nelson and R. Startz, "The Distribution of the Instrumental Variable Estimator," *JB* 63 (1990): 138.
[45] For a derivation of the conditions for the existence of moments in simultaneous equation models refer to T. Kinal, "The Existence of Moments of k-class Estimators," *Econometrica* 48 (1980): 245-249.

market and size are significantly negative. Their t-statistic is -4.27 and -2.42 in the basic equation, respectively. Moreover, the constant is also statistically significant in the basic equation with a t-statistic of 2.11.

The results confirm prior empirical findings that controlling for book-to-market and size, other factors like beta, dividend yield and P/E-ratio do not have any explanatory power with respect to average stock returns. Consistent with prior findings on the size effect, stock returns seem to be inversely related to firm size, but contrary to prior findings, also inversely related to book-to-market. The negative slope on book-to-market is somewhat intriguing, but not fully surprising. In the aftermath of the burst of the stock market bubble and its following recession many so called “new economy” firms lost tremendously in market value. Since these firms often consisted of small family-owned mostly non-profitable companies, which had issued relatively little equity, their book-to-market ratio probably did not increase as much relative to large industrial firms, which also lost in market value, but had higher book values of equity. Hence, the smaller firms in this sample are most likely also those firms with relatively smaller book-to-market ratios.

The findings presented here highlight the fact that size and book-to-market cannot be risk factors on their own, but rather represent economic mechanisms not properly specified in current asset pricing models. Moreover, the results of this study corroborate the theoretical and empirical findings of Berk (1995, 2000), Fama and French (1992, 1996), Vassalou and Xing (2004) and Fan and Liu (2005) of an economic rationale behind size. As shown above, the size factor does indeed capture the higher default risk of smaller firms and the illiquidity risk of trading their stocks as well as a physical size measure based on the book value of assets or sales value.[46]

[46] Asset value and sales are related to the growth component in returns hypothesized by Berk et al. (1999) and therefore assumed to serve as proxy for expected cash flows. See J. Berk et al., “Optimal Investment,” *JF* 54 (1999): 1561-1662.

VI. Interpretation of Results and Concluding Remarks

The size effect is still hotly debated after twenty years of its first detection. It is often mentioned together with the value premium, the positive risk premium on the book-to-market ratio. Although size and book-to-market are widely accepted as components in asset pricing, there is still disagreement on the role they play in explaining expected stock returns.

While Fama and French (1993, 1996) and Davis et al. (2000) suggest that these factors display systematic risk not captured by the CAPM beta,[47] Daniel and Titman (1997) and Fan and Liu (2005) propose a characteristics-based explanation for size and book-to-market, stating that these factors are not part of the covariance structure of returns, but rather represent idiosyncratic components.[48] Furthermore, Lakonishok et al. (1994) suggest a behavioral explanation for the small firm and book-to-market effect based on investor's irrational extrapolation of past earnings growth.[49] Likewise Dissanaike observes that the higher returns on small capitalization stocks might be driven by the mean reverting behavior of stock prices.[50]

The findings in this paper on size related anomalies in the German stock market indicate idiosyncratic risk priced and captured in size, leaving room for two interpretations. Either the results confirm Berk's (1995) assertion that size and book-to-market variables capture all misspecification errors left in asset pricing models, as such for instance the use of a market portfolio that is not mean-variance efficient.[51] Or the results corroborate market irrationality, suggesting that investors fail to diversify away idiosyncratic risk, as Fan and Liu and Daniel and Titman propose. A promising avenue for future research is a deeper analysis of the economic rationales behind size and book-to-market factors, in order to finally

[47] See E. Fama and K. French, "Common Risk Factors on the Returns on Stocks and Bonds, " *JFE* 33 (1993): 51-53; J. Davis et al., "Characteristics, Covariances, and Average Returns," *JF* 55 (2000): 403.
[48] See e.g. K. Daniel and S. Titman, "Evidence on the characteristics of cross sectional variation in stock returns," *JF* 52 (1997): 3.
[49] See J. Lakonishok et al., "Contrarian Investment, Extrapolation, and Risk." *JF* 49 (1994): 1541-1578.
[50] See G. Dissanaike, "Does the Size Effect Explain the UK Winner-Loser Effect?" *JBFA* 29 (2002): 152.
[51] See Roll's general critique of tests of asset pricing models, R. Roll, "A Critique of the Asset Pricing Theory's Test," *JFE* 4 (1977): 129-176.

be able to relate the behaviour of stock returns to real economic factors. Furthermore, the interaction between different anomalies has to be further investigated to find out common influential factors that might explain some of these anomalies. As such for example it might be fruitful to examine whether the size effect might be explained by investor overreaction and thus serve as a proxy for past stock returns as proposed by Dissanaike (2002). This study made a preliminary step towards a possible explanation for size as an explanatory factor for stock returns.

Table 4: Average Slopes from Month-by-Month First-Step Regressions of Equation (3)

Panel A: First-Step Regression Results

	CONS	BA	SAL	LEV	VOL	BAS	PRFT
Mean Coefficient	-6.4280	0.8377	0.1286	-0.0225	0.0001	0.2192	0.0634
Standard Error	0.0734	0.0087	0.0053	0.0015	0.0000	0.0207	0.0163
t-Statistic	-87.6233**	96.7886**	24.1053**	-14.8088**	10.6550**	10.5852**	3.8920**
Mean R-squared	0.88						
Mean R-squared (adjusted)	0.87						

Panel B: Correlation Coefficients

	BA	SAL	LEV	VOL	BAS	PRFT
BA	1					
SAL	0.87	1				
LEV	0.23	0.01	1			
VOL	0.59	0.41	0.24	1		
BAS	0.46	-0.47	-0.07	-0.25	1	
PRFT	0.42	0.43	-0.01	0.56	-0.23	1

* statistically significant on the 0.05-level (two-tailed test) ** statistically significant on the 0.01-level (two-tailed test)

Panel A shows the regression results of the first-step regression of SIZE on the set of instrumental variables following the Fama-MacBeth methodology. SIZE is regressed on the IVs in each of the post-ranking months. The mean coefficients are the time series means of the month-by-month OLS regressions. Standard errors are calculated as described in section III assuming homoscedasticity of the regression residuals.

Panel B shows the time series mean correlation coefficients between each independent variable of the OLS regression.

Table 5: OLS and Two-Stage Least Squares Regression Results of Fama-MacBeth Regressions

Panel A: Average Slopes from Month-by-Month OLS Regressions According to Fama-MacBeth Method								
	CONS	BETA	BM	SIZE	DIV	PE	R-Squared	R-Squared (adj.)
Mean Coefficient	0.0113	0.0380	-0.0052	-0.0016	0.0000	0.0000	0.05	0.03
Standard Error	0.0084	0.0345	0.0036	0.0011	0.0004	0.0000		
t-Statistic	1.3440	1.0995	-1.4498	-1.4231	-0.1374	-0.7022		
Mean Coefficient	0.0050	0.0212	-0.0072	-0.0005			0.05	0.04
Standard Error	0.0079	0.0272	0.0032	0.0009				
t-Statistic	0.6361	0.7803	-2.2863*	-0.5957				
Panel B: Average Slopes from Month-by-Month Two-Stage Least Squares Regressions According to Fama-MacBeth Method								
	CONS	BETA	BM	P*SIZE	DIV	PE		
Mean Coefficient	0.0160	0.0330	-0.0106	-0.0026	0.0001	0.0000	0.06	0.03
Standard Error	0.0088	0.0290	0.0024	0.0011	0.0007	0.0000		
t-Statistic	1.8294	1.1387	-4.3402**	-2.3759*	0.1673	-0.3981		
Mean Coefficient	0.0170	0.0329	-0.0102	-0.0026			0.04	0.03
Standard Error	0.0081	0.0294	0.0024	0.0011				
t-Statistic	2.1088*	1.1195	-4.2688**	-2.4200*				

* statistically significant on the 0.05-level (two-tailed test) ** statistically significant on the 0.01-level (two-tailed test)

Panel A shows the OLS regression results of monthly returns on beta, book-to-market, size, dividends and price-earnings ratio following the Fama-MacBeth methodology. Beta is first estimated for portfolios as described in section III and then assigned to each individual stock in that portfolio in each of the post-ranking months. The mean coefficients are the time series means of the month-by-month OLS regressions. Standard errors are calculated as described in section III assuming homoscedasticity of the regression residuals.

Panel B shows the regression results of the second-step regression of monthly returns on beta, book-to-market, the projection of size, dividends and price-earnings ratio following the Fama-MacBeth methodology. The mean coefficients are the time series means of the month-by-month 2SLS regressions. Standard errors are calculated as described in section III assuming homoscedasticity of the regression residuals.

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