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Time Period Unbundling and the Transformation of Auctions

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Abstract

Consider a firm that satisfies its demand for a specified time period by assigning it to a supplier via a procurement (reverse) auction; call this the standard auction. The firm is considering first breaking the demand down into smaller time periods and permitting bids for one or more of these sub-periods, either as independent bids or as package bids; call this the unbundled auction. Choosing the unbundled auction over the standard auction will tend to: (1) allow each supplier to choose a production plan in which it can satisfy buyer demand at lower cost, (2) increase competition among the suppliers, and (3) allow the buyer to combine bids from different suppliers in order to lower its purchase cost. All

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three effects might lead one to expect that the buyer's cost will always be lower in the unbundled auction than in the standard auction. We show that, to the contrary, there are cases in which the buyer will have a higher cost in the unbundled auction; further, we provide a bound on how much greater the buyer's cost can be. However, when the suppliers are not restricted by capacities, the buyer's cost in the unbundled auction will never exceed its cost in the standard auction.

Keywords: procurement, auction, unbundling, transformation, multi-period, supply chain, VCG mechanism, Vickrey auction

1 Introduction

Researchers and practitioners have been interested for some time in the idea of *bundling nonindentical items* for sale (see, e.g., Hanson and Martin (1990), Schoenherr and Mabert (2006).) In contrast, we are concerned here with *unbundling identical items by time period.* In this paper, we introduce the concept of auction *transformation*, which we define as the conversion of a standard procurement auction into a combinatorial auction (Cramton et al. (2006)) by the unbundling of buyer demand into component demands by time period.

Consider a firm called the *buyer* having demand for a specified time period for a product upstream in the supply chain, where the firm assigns orders to its suppliers via a second-price sealed-bid auction, i.e., the Vickrey auction. We will refer to as the *standard auction.* The firm is alternatively considering breaking down the period demand into smaller time periods and permitting bids for one or more of these sub-periods, either as independent bids or as package bids. In this alternative auction, which we call the *unbundled auction*, the buyer will assign orders to its suppliers via the generalized Vickrey auction, the *VCG mechanism*. Note that the unbundled auction is a combinatorial auction where otherwise–identical items being demanded in different periods need to be considered as distinct items.

Due to the multi-period setting, inventory holding costs play a role in both the standard auction and the unbundled auction. In the standard auction, the buyer requires delivery from the winning bidder at the beginning of the time period; here the buyer incurs the inventory costs. In the unbundled auction, the buyer requires delivery of the period demands from the winners at the beginning of each of the corresponding subperiods; here the winning suppliers incur the inventory costs. In both auctions, the term *supplier's total cost* will be used to refer to the sum of a specified supplier's production costs and its holding costs, while the term *buyer's total cost* will refer to the sum of the buyer's purchase costs and its holding costs.

Choosing the unbundled auction over the standard auction will have three effects. First, this will provide the opportunity in general for each supplier to choose a production plan in which it can satisfy buyer demand at lower cost, because the unbundled auction will allow each supplier to more fully incorporate its cost and capacity information when submitting its bids, a desirable property pointed out by Elmaghraby (2004). In particular, this includes the possibility of a supplier making use of its production capacity after the start of the time horizon—which is obviously of no value to it in the standard auction where all delivery is required at the start of the horizon. Second, suppliers can focus their bids on a specific period or periods within the horizon, and consequently will bid more competitively against each other. These first two effects will tend to result in lower bids from the suppliers, and consequently a lower purchase cost for the buyer. Third, the buyer might be able to combine bids from different suppliers to further lower its purchase cost. We call these three effects, respectively, the *supplier efficiency effect*, the *competition effect*, and the *buyer flexibility effect*. These effects might lead one to expect that the unbundled auction would always be preferred by the buyer, as it seems as though each one can only lower the buyer's cost.

In this paper we will show that, to the contrary, there are cases in which the buyer will be worse off with the unbundled auction; in these cases, the buyer needs to pay more to satisfy its demand. This situation arises from the fact that the competition effect has another side, viz., although the suppliers can bid more competitively against each other, they can also bid more competitively *against the buyer*. Thus, our result shows that the negative aspect (from the buyer's point of view) of the competition effect can dominate the positive aspect of this effect combined with the supplier efficiency effect and the buyer flexibility effect. (This somewhat counter-intuitive result is reminiscent of the well-known result of Hart (1975), who considers the consequences in a market structure of permitting trades that were previously prohibited, where "our intuition tell us that the introduction of additional markets ought to make people better off," but provides an example in which this is not the case.) However, we do more than this. We provide a bound on how much worse off the buyer can be with the unbundled auction.

A key factor in our scenario is supplier production capacity. We show that, when suppliers are not restricted by capacities, the buyer will always do at least as well in the unbundled auction as compared with the standard auction, i.e., will not increase its total cost and might lower it. (Again, there is an analogy with Hart (1975), where Hart points out that if enough new markets are opened to make the market structure complete, then his counter-intuitive result cannot occur.)

Our model is considerably general with respect to number of bidders, heterogeneity of bidder cost parameters, number of component periods resulting from the unbundling, as well as the component period demands. Specifically, in our model, we allow for an unlimited number of potential bidders (suppliers). The bidders all have fixed plus linear cost structures, but few assumptions are made on their cost parameters or the buyer's demands. Finally, the number of periods under consideration can be arbitrarily large.

The remainder of our paper is organized as follows. In the next section, we review the literature. In Section 3, we discuss the VCG mechanism. In Section 4 we introduce the two procurement auctions that we will be comparing in the paper. In Section 5 we compare the buyer's purchase cost resulting from running an unbundled multi-period auction versus running a standard auction. In Section 6 we discuss the special case where the capacities of the suppliers are not binding. Section 7 presents conclusions. Most of the proofs appear in the Appendices.

2 Literature review

In this section, we review the extant work on procurement auctions, especially those papers most relevant to the scenario considered in this paper. A recent exposition of procurement auctions is provided by Bichler et al. (2006), including a description of an industrial procurement auction conducted at Mars, Inc., which was presented by Hohner et al. (2003). Tunca and Wu (2006) provide a number of examples of companies and government organizations that make use of procurement auctions, including SUN Microsystems, Hewlett-Packard, IBM, Samsung, and Lucent. However, none of these papers focuses on the multi-period auction case. There are not many papers on this topic. In fact, in the procurement auction literature, the phrase "multi-period procurement" almost invariably refers to auctioning items *sequentially* via a series of single-period auctions, as in Elmaghraby (2005), rather than auctioning them *simultaneously* via a single auction for multiple periods, as we do here.

One exception is Kameshwaran et al. (2005), who touch on this idea in passing. They consider the procurement of heterogeneous items for a single period, where each supplier submits a single *discount bid* consisting of the cost for each item it offers to supply together with a discount based on the number of items actually supplied. The authors point out, however, that in a multi-period scenario these discount bids would not be appropriate, and that if period demand is considered to be an indivisible item, then the problem reduces to procurement of multiple items where a combinatorial bid can express a supplier's cost function more efficiently.

A line of research related to the scenario considered in this paper is presented in Elmaghraby (2005). In her model, a buyer seeks to purchase two units, and auctions off the second unit after the winner of the first auction has been announced. She is interested in how suppliers bid in the presence of competitors with asymmetric production capacity. She assumes that there are two types of suppliers: "global bidders," who have sufficient capacity to supply both units, and "small bidders," who can supply only one unit. Elmaghraby performs extensive numerical analysis and concludes that adding small bidders to the sequential auction with only global bidders may either increase or decrease the expected procurement costs. Her innovative works opens up an interesting line of enquiry regarding the relationship between the production capacities of suppliers and the procurement cost of the buyer. We find that there are two key factors that determine the buyer's best choice of auction scenario, viz., the supplier production capacities and setup costs. Elmaghraby's scenario is different than ours, since she considers a sequential auction rather than a multi-period auction.

3 The VCG mechanism

In the standard auction, the buyer assigns orders to suppliers via the Vickrey auction, see Vickrey (1961). In the unbundled auction, period demand is considered to be an indivisible item—in effect, an auction for heterogeneous goods—where the buyer assigns orders to suppliers via the generalization of the Vickrey auction to heterogeneous goods, the Vickrey-Clarke-Groves (VCG) mechanism. (See Ausubel and Milgrom 2006.) In the VCG mechanism, each supplier reports to the buyer his costs of supplying each possible subset of the buyer's desired collection of items. The buyer then combines all the bidder information to determine the cost-minimizing allocation, and then pays each winning bidder not his bid but the incremental surplus that he brings to the auction. (For several interesting variations on the VCG mechanism, see Bapna et al. (2005).)

As discussed by Ausubel and Milgrom (2006), the VCG mechanism has a number of virtues, foremost of which is that under VCG it is a dominant strategy for each bidder to bid its actual valuations, i.e., "truthful reporting." This is because, under the VCG mechanism, a supplier who bids higher than his true cost of supply will never end up with a higher payment and in fact might lose items that he otherwise would have won.

Another virtue of the VCG mechanism is that the outcome is efficient. Further, "the basic rules of the Vickrey auction can be further adapted if the auctioneer wishes to impose some extra constraints." Among the examples provided by Ausubel and Milgrom (2006) (p. 21): "[T]he buyer in a procurement auction might want to limit its total purchases from first-time bidders or might want to ensure security by requiring that the total relevant capacity of its suppliers is at least 200 percent of the amount ordered." As they explain, one can impose such constraints without affecting the theory in any essential way. Similarly, in the scenario of this paper, the additional costs that the buyer might need to incur from a bid from holding it in inventory will not affect the truthful reporting property of the VCG auction, since these costs are transparent to the bidders.

4 The two auctions

Let T denote the number of sub-periods into which the standard auction is unbundled in order to create the unbundled auction, where these periods are to be labeled $t = 1, 2, ..., T$. In both the standard auction and the unbundled auction, the buyer announces to the set of suppliers $\mathcal{S}$ its demand requirements prior to any point at which demand may be required. Period demand is considered to be an indivisible item. Supplier production in a period occurs at the beginning of that period and can be delivered in time to satisfy buyer demand in that period. Thus, in the standard auction, the single delivery occurs at the beginning of the horizon, which is also the beginning of what becomes period 1 in the unbundled auction. In the unbundled auction, delivery for each demand occurs by the beginning of the period in which it is required.

The suppliers are assumed to have private values, i.e., the supplier's payoff depends solely on its own estimate of value and not on the other suppliers estimates of value.

Supplier Cost Structure and Buyer Cost Structure. If supplier s produces in period t, then it faces a setup cost f_{st} and a unit production cost p_{st}, as well as a production capacity b_{st}. If supplier s carries inventory over from period t to period $t+1$, then it faces a unit inventory holding cost h_{st}. The buyer pays each supplier at the end of the auction, which occurs before the beginning of period 1. If the buyer carries inventory over from period t to period $t+1$, then it faces a unit inventory holding cost H_t. We make the standard supply chain assumption that $h_{st} \leq H_t$ for all $s \in \mathcal{S}$, $t \in \{1, 2, \ldots, T\}$.

Let $\mathbf{D} = (D_t) \in \mathbb{R}^T$ denote the vector of demands. The two auctions are defined as described below.

The Standard Auction. The buyer announces to the suppliers its demand D over the time period which is to be provided by a single supplier. After deciding to unbundle the demand in T time periods, consider this demand his *total T-period demand* $D = \sum_{t=1}^{T} D_t$. The suppliers individually submit bids in the form of a bid price representing an offer to supply the total T-period demand to be delivered at the beginning of period 1. Here, each supplier $s \in \mathcal{S}$ is restricted by its production capacity b_{s1} in period 1, the only period in which production is available to him. Let w denote the winner in the standard auction, and $C_{\mathcal{S}}$ denote the cost to w associated with delivering the total T-period demand.

The *buyer's total cost in the standard auction*, J, is given by

$$J \quad = \quad C_{\mathcal{S}\backslash\{w\}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t. \tag{1}$$

This expresses the lowest cost at which any one of the suppliers in $\mathcal{S}\backslash\{w\}$ can supply the total T-period demand $\sum_{t=1}^{T} D_t$, plus the sum of the buyer's inventory holding costs over the entire time horizon. (As already said, since all production occurs in period 1, inventory is only held at the buyer level.)

The Unbundled Auction. The buyer announces to the suppliers a demand D_t for each period t, where delivery is required at the beginning of the period. The suppliers individually submit bids in the form of a bid price together with a T-vector representing an offer to supply specific quantities of units in periods $1, \ldots, T$, where an offer in period t is not restricted to being zero or D_t, but can be the sum of any subset of the buyer's demands D_τ for $\tau \in \{t, t+1, \ldots, T\}$. We will assume that each bidder can submit an arbitrary number of bids on subsets, where he is willing to obtain *at most* one of these subsets. (For a discussion of this type of bid, called an *XOR bid*, see Nisan 2006). However, each supplier s is restricted by its production capacity b_{st} in period t. Let $\mathcal{W}$ be the set of winners in the unbundled auction, and let w_t denote the winner of demand D_t, for $t = 1, 2, \ldots, T$. Note that while the members of $\{s \mid s \in \mathcal{W}\}$ are distinct, the members of $\{w_t \mid t = 1, 2, \ldots, T\}$ might not be.

The *buyer's total cost in the unbundled auction*, J^{U}, is given by the buyer's sum total payment to the suppliers:

$$J^{\mathrm{U}} \;=\; C^{\mathrm{U}}_{\mathcal{S}} + \sum_{s \in \mathcal{W}} (C^{\mathrm{U}}_{\mathcal{S}\backslash\{s\}} - C^{\mathrm{U}}_{\mathcal{S}}), \tag{2}$$

where $C^{\mathrm{U}}_{\mathcal{S}}$ is the lowest cost at which the set of suppliers $\mathcal{S}$ can supply the vector of demands $\mathbf{D}$.

We will consider the difference in total cost in the standard and unbundled auctions,

$J - J^{\mathrm{U}}$, which we call the *savings*. However, the savings can be negative, as we will illustrate in Example 5.2.

5 Comparing the two auctions

To compare the buyer's cost in the two auctions, we first delimit a set of problem instances for which the unbundled auction is never more expensive than the standard auction. We then provide a lower bound on the savings. Specifically, we show that the buyer's total cost in the unbundled auction cannot exceed his cost in the standard auction by more than the maximum period 1 setup cost times the minimum of the number of suppliers and T, minus the minimum period 1 setup cost.

In the proposition below we derive, under mild assumptions, two conditions under which the unbundled auction is at least as good as the standard auction, depending on the winners in the auctions.

Proposition 5.1 *We have that $J^{\mathrm{U}} \leq J$ in the following cases:*

(i) the unbundled auction has one winner, or

(ii) the setup cost in period 1 is supplier-independent, i.e., $f_{s1} = f_{11}$ for all $s \in \mathcal{S}$, and the unbundled auction has at least two winners where $w_1 = w$.

The following example shows that the inequality of Proposition 5.1 will not in general hold when the setup cost in period 1 depends on the supplier.

Example 5.2 *Consider the scenario where the buyer requires an item over two time periods, with demand $D_t = 1$ in each period t, where the buyer faces a unit holding cost of*

Table 1: Proposition 5.1 requires identical period 1 setup costs

s	f_{s1}	p_{s1}	f_{s2}	p_{s2}
1	3.9	9	2	9
2	2	10	2	9
3	2	11	2	7

1.1 *in period 1. There are three suppliers, i.e.,* $\mathcal{S} = \{1, 2, 3\}$*, each of which faces a setup cost of 2 in each period, except for supplier 1 in period 1 who faces a setup cost there of* 3.9*. Each supplier faces a unit holding cost in period 1 of 1 and no capacity constraints. The other supplier data are shown in Table 1.*

First have a look at the standard auction. The winner of this auction is supplier 1 with cost 21.9:

$$C_{\mathcal{S}} = 3.9 + 2(9) = 21.9 \ \ [w = 1].$$

The unbundled auction allows three possibilities of supply: (i) D_1 *and* D_2 *are produced by a single supplier in period 1, as in the case of the standard auction, although now it is the supplier rather than the buyer who incurs the inventory cost; (ii)* D_1 *and* D_2 *are produced by two different suppliers in period 1, in which case the supplier for* D_2 *incurs an inventory cost; or (iii)* D_1 *is produced in period 1 and* D_2 *is produced in period 2, which involves one or two suppliers and no inventory costs.*

Since there are no period 1 capacity constraints, case (ii) is dominated by case (i), and only two possibilities of supply need to be considered in the unbundled auction: (i) D_1 *and* D_2 *are produced in period 1 by a single supplier; and (iii)* D_1 *is produced in period 1 and*

D_2 is produced in period 2. Therefore:

$$
\begin{aligned}
C_{\mathcal{S}}^{\mathrm{U}} &= \min\{\min_{s\in\mathcal{S}}\{(f_{s1}+p_{s1}(D_1+D_2))+(h_{s1}D_2)\}, \\
&\qquad \min_{s,s'}\{(f_{s1}+p_{s1}D_1)+(f_{s'2}+p_{s'2}D_2)\}\} \\
&= \min\{3.9+2(9)+1 \ \ [w_1\!=\!w_2\!=\!1], \\
&\qquad (2+10)+(2+7) \ \ [w_1\!=\!2,\ w_2\!=\!3]\} \\
&= \min\{22.9,\ 21\} = 21 \ \ [w_1\!=\!2,\ w_2\!=\!3].
\end{aligned}
$$

Thus, the lowest cost policy is policy (iii), with supplier 2 producing D_1 and supplier 3 producing D_2 in period 2.

The buyer's total cost in the standard auction is

$$J = C_{\mathcal{S}\setminus\{1\}} + H_1 D_2 = (2+2(10)) + 1.1(1) = 23.1.$$

The buyer's total cost in the unbundled auction is given by

$$J^{\mathrm{U}} = C^{\mathrm{U}}_{\mathcal{S}\setminus\{2\}} + C^{\mathrm{U}}_{\mathcal{S}\setminus\{3\}} - C^{\mathrm{U}}_{\mathcal{S}}.$$

After we eliminate supplier 2, the least-cost option is again to produce each unit in each corresponding period, but with $w_1 = 2$ at an increase of 0.9 *to* 21.9*. However, after we eliminate supplier* 3*, the best option is to produce both units in period 1 at a cost including inventory cost of* 22.9*. In summary, the buyer's total cost in the unbundled auction is equal to $J^{\mathrm{U}} = 21.9 + 22.9 - 21 = 23.8$, and the savings are negative: $23.1 - 23.8 = -0.7$.*

Example 5.2 illustrates that the standard auction can be better than the unbundled auction. In the following, we will show that the buyer's total cost in the unbundled auction cannot exceed the buyer's total cost in the standard auction by more than $(\max_{s\,\in\,\mathcal{S}} f_{s1})|\mathcal{W}| - \min_{s\,\in\,\mathcal{S}} f_{s1}$.

Proposition 5.3 *The savings is bounded by* $J - J^{\mathrm{U}} \geq -(\max_{s \in \mathcal{S}} f_{s1})|\mathcal{W}| + \min_{s \in \mathcal{S}} f_{s1}$.

The following theorem shows that the worst case bound found in Proposition 5.3 cannot be improved.

Theorem 5.4 *The bound given in Proposition 5.3 is tight.*

Proof: Consider the following scenario where the buyer requires demand vector $\mathbf{D} \in \mathbb{R}^T$, such that $D_1 > D_2 > \ldots > D_{T-1} > D_T > 1$, and the buyer faces a unit holding cost in each period t of $H_t = h$. The number of suppliers exceeds the number of periods by one, i.e., $\mathcal{S} = \{1, 2, \ldots, T+1\}$, where each supplier faces a setup cost f and a unit holding cost h in each period. The supplier unit production costs and capacities are given by:

$$
\begin{aligned}
p_{st} &= \begin{cases} p + Tf & \text{for} \quad t=1, \quad s \geq 2 \\ p + Tf + (t-1)h + \epsilon & \text{for} \quad t \geq 2, \quad s \neq t \\ p + (T-s)f & \text{for} \quad t = s \end{cases} \\
b_{st} &= \begin{cases} D_1 & \text{for} \quad t = 1, \quad s \leq T-1 \\ \sum_{\tau=1}^{T} D_\tau & \text{for} \quad t = 1, \quad s = T, T+1 \\ D_t & \text{for} \quad t \geq 2. \end{cases}
\end{aligned} \tag{3}
$$

All parameters are assumed to be positive.

Now we will analyze the optimal allocation of both auctions. First have a look at the case of the standard auction. Suppliers T and $T+1$ have identical cost and capacity structures, and are the only suppliers who have sufficient capacity to individually produce the total T-period demand in period 1. Therefore, in the standard auction, the winner is either supplier T or supplier $T+1$. Without loss of generality, we assume that it is supplier $T+1$.[1] It will face a cost of:

$$C_{\mathcal{S}} \;=\; f + (p + Tf)\sum_{t=1}^{T} D_t. \tag{4}$$

The unbundled auction allows D_t to be produced in any period $\tau \leq t$. It can be shown by contradiction that the lowest-cost unbundled allocation consists of assigning D_t to supplier t in period t, for each $t \in \{1, \ldots, T\}$ (see Appendix B). The cost of this allocation is:

$$\begin{aligned} C^{\mathrm{U}}_{\mathcal{S}} &= \sum_{t=1}^{T} (f + [p + (T-t)f]D_t), \\ &= Tf + \sum_{t=1}^{T} [p + (T-t)f]D_t, \end{aligned} \tag{5}$$

with supplier t producing period t demand in time, for each $t \in \{1, \ldots, T\}$, and $\mathcal{W} = \{1, \ldots, T\}$.

Now we will investigate $C^{\mathrm{U}}_{\mathcal{S}\setminus\{t\}}$, the cost of the optimal allocation in the unbundled auction after we eliminate supplier $t \in \mathcal{W}$. It can be shown by contradiction that demand D_τ, with $\tau \neq t$, will be assigned to supplier τ in period τ, while demand D_t will be allocated to supplier $T+1$ in period 1 (see Appendix B). This means that in determining $C^{\mathrm{U}}_{\mathcal{S}\setminus\{t\}}$ all demands, except for D_t, are allocated in the same manner as in determining $C^{\mathrm{U}}_{\mathcal{S}}$. Therefore,

$$\begin{aligned} C^{\mathrm{U}}_{\mathcal{S}\setminus\{t\}} - C^{\mathrm{U}}_{\mathcal{S}} &= (f + [p + Tf]D_t + h(t-1)D_t) - (f + [p + (T-t)f]D_t) \\ &= [h(t-1) + tf]D_t \qquad \text{for each } t \in \{1, \ldots, T\}. \end{aligned} \tag{6}$$

[1] In practice, ties are often settled with a random tie-breaking rule. This is the current practice at the Federal Communications Commission (Cramton 2006).

From (1), the buyer's total cost in the standard auction is:

$$\begin{aligned} J &= C_{\mathcal{S}\setminus\{T+1\}} + \sum_{t=2}^{T}(t-1)hD_t \\ &= f + (p+Tf)\sum_{t=1}^{T} D_t + h\sum_{t=2}^{T}(t-1)D_t \\ &= f + \sum_{t=1}^{T}[p+Tf+h(t-1)]D_t \end{aligned} \tag{7}$$

where we have used the fact that, in the absence of supplier $T+1$, supplier T delivers the total T-period demand in the standard setting. From (2), the buyer's total cost in the unbundled auction is:

$$\begin{aligned} J^{\mathrm{U}} &= C^{\mathrm{U}}_{\mathcal{S}} + \sum_{s\in\mathcal{W}}(C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} - C^{\mathrm{U}}_{\mathcal{S}}) \\ &= C^{\mathrm{U}}_{\mathcal{S}} + \sum_{t=1}^{T}(C^{\mathrm{U}}_{\mathcal{S}\setminus\{t\}} - C^{\mathrm{U}}_{\mathcal{S}}), \quad \text{and from (5) and (6) :} \\ &= Tf + \sum_{t=1}^{T}[p+(T-t)f]D_t + \sum_{t=1}^{T}[h(t-1)+tf]D_t \\ &= Tf + \sum_{t=1}^{T}[p+Tf+h(t-1)]D_t, \quad \text{and from (7) :} \\ &= (T-1)f + J, \end{aligned}$$

and the savings are equal to $-(T-1)f = -(|\mathcal{W}|-1)f = f - f|\mathcal{W}|$, since $|\mathcal{W}| = T$. The tightness of the bound follows. □

The following result is a corollary to Proposition 5.3. Note that the bound provided is independent of the unit product costs.

Corollary 5.5 *The savings is bounded by* $J - J^{\mathrm{U}} \geq -(\max_{s\in\mathcal{S}} f_{s1}) \cdot \min\{|\mathcal{S}|, T\} + \min_{s\in\mathcal{S}} f_{s1}$.

Table 2: Theorem 6.1 requires non-binding capacities

s	f_{s1}	p_{s1}	b_{s1}	f_{s2}	p_{s2}	b_{s2}
1	2	9	1	2	10.1	1
2	2	10	2	2	10.1	1
3	2	10.1	2	2	7	1

6 Non-binding capacities

In this section we assume that the capacities of the suppliers are not binding, i.e. $b_{st} \geq \sum_{\tau=t}^{T} D_\tau$, for all $s \in \mathcal{S}$ and $t \in \{1, \ldots, T\}$. We show under this condition that, if the setup costs in period 1 are supplier-independent, then the unbundled auction results in a cost to the buyer that is less than or equal to that which would result under the standard auction.

Theorem 6.1 *Suppose that the setup cost in period 1 is supplier-independent, i.e., $f_{s1} = f_{11}$ for all $s \in \mathcal{S}$. Then if the capacities are not binding, $J^{\mathrm{U}} \leq J$.*

The following example demonstrates that the specification of non-binding capacities in Theorem 6.1 cannot be relaxed.

Example 6.2 *Consider the scenario where the buyer requires an item over two time periods, with demand $D_t = 1$ in each period t, where the buyer faces a unit holding cost of 1.1 in period 1. There are three suppliers, i.e., $\mathcal{S} = \{1, 2, 3\}$, each of which faces a holding cost of 1 in period 1. The other supplier data are shown in Table 2.*

First have a look at the case of the standard auction, where demand for both periods must be produced in the first period. The winner of this auction is supplier 2, with cost

22:

$$C_{\mathcal{S}} = 2 + 2(10) = 22 \;\; [w = 2].$$

As before, the unbundled auction allows three possibilities of supply: (i) D_1 and D_2 are produced by a single supplier in period 1, in which case the supplier incurs the inventory cost; (ii) D_1 and D_2 are produced by two different suppliers in period 1, in which case the supplier for D_2 incurs an inventory cost; or (iii) D_1 is produced in period 1 and D_2 is produced in period 2, which involves one or two suppliers and no inventory costs. Therefore:

$$\begin{aligned}
C^{\mathrm{U}}_{\mathcal{S}} &= \min\{\min_{s\in\mathcal{S}}\{(f_{s1} + p_{s1}(D_1 + D_2)) + (h_{s1}D_2)\}, \\
&\qquad \min_{\substack{s,s'\\ s\neq s'}}\{(f_{s1} + p_{s1}D_1) + (f_{s'1} + p_{s'1}D_2 + h_{s'1}D_2)\}, \\
&\qquad \min_{s,s'}\{(f_{s1} + p_{s1}D_1) + (f_{s'2} + p_{s'2}D_2)\}\} \\
&= \min\{2 + 2(10) + 1 \;\; [w_1{=}w_2{=}2], \\
&\qquad (2 + 10) + (2 + 9 + 1) \;\; [w_1{=}2,\; w_2{=}1], \\
&\qquad (2 + 9) + (2 + 7) \;\; [w_1{=}1,\; w_2{=}3]\} \\
&= \min\{23,\; 24,\; 20\} = 20 \;\; [w_1{=}1,\; w_2{=}3].
\end{aligned}$$

Thus, the lowest cost policy is policy (iii) with supplier 1 producing period 1 demand and supplier 3 producing period 2 demand in period 2.

The buyer's total cost in the standard auction is

$$J = C_{\mathcal{S}\setminus\{2\}} + H_1 D_2 = 2 + 2(10.1) + 1.1(1) = 23.3.$$

The buyer's total cost in the unbundled auction is given by

$$J^{\mathrm{U}} = C^{\mathrm{U}}_{\mathcal{S}\setminus\{1\}} + C^{\mathrm{U}}_{\mathcal{S}\setminus\{3\}} - C^{\mathrm{U}}_{\mathcal{S}}.$$

After we eliminate supplier 1, we could purchase the period 1 unit from supplier 2 at an additional cost of 1 for a total cost for both periods of 21*, or could purchase both units in period 1 from supplier 2 at a total cost (including inventory) of* 23*, which is more expensive. However, after we eliminate supplier* 3*, the best option is to purchase both units in period 1 from supplier 2 at a total cost of* 23*. In summary, the buyer's total cost in the unbundled auction is equal to* $J^{\mathrm{U}} = 21 + 23 - 20 = 24$*, and the savings are negative:* $23.3 - 24 = -0.7$*.*

Example 5.2 from Section 5 demonstrates that Theorem 6.1 will not hold under supplier-dependent setup costs in period 1. The explanation is simple: Setup costs can act effectively like capacities to discriminate between lower and higher levels of production.

7 Conclusions

We have considered the case of a firm, called the buyer, who requires a product for a specified time period, where the firm will auction to his suppliers either his aggregate demand, i.e., a "standard auction," or will first break down his demand by time period and allow bids on one or more periods, including the possibility of package bids, i.e., an "unbundled auction." In the first case, he makes use of a (reverse) Vickrey auction; in the second case he employs the generalization, the VCG mechanism. We find that the unbundled auction can result in a higher cost to the buyer than the standard auction. Specifically, we find that the cost to the auctioneer of the unbundled auction can exceed the cost of the standard auction by as much as the maximum period 1 setup cost times the minimum of the number of suppliers and T, minus the minimum period 1 setup cost.

We also show via an example that this bound is sharp.

The explanation for the above result is that, in the unbundled auction, the suppliers will not only bid more competitively against each other, but will also bid more competitively against the buyer. This situation is only possible when there are capacity constraints on the suppliers. If they have no capacity constraints, the unbundled auction offers them no competitive advantage against the buyer as compared with the standard auction. We establish this by showing that without capacity constraints the unbundled auction will result in a lower or equal cost to the auctioneer than the standard auction. Our results depend on the relatively mild assumption that all the suppliers have the same setup cost in the first period. We show via an example that this assumption is required. Our results are quite general, with no restrictions on demand, and allowing for an arbitrary number of items, bidders, and periods.

The managerial implications are as follows. A firm that normally satisfies demand for a product upstream in a supply chain via a procurement auction should be judicious in it its choice of auction format. Specifically, if it knows that the quantities it requires from its suppliers would be comfortably under the production capacities of each of its suppliers, then it would generally be to its advantage to make use of an unbundled auction for procurement. However, where the production capacities of the suppliers may be binding, the standard auction would minimize the worst case supplier procurement cost. However, this worst case cost decreases in the number of bidders and the fixed cost of supplier production in the first period.

This paper introduced the concept of auction *transformation*, the conversion of a auction into a combinatorial auction by the unbundling of buyer demand into component time periods. We suspect that auction transformation can apply more generally, viz., the transformation of a standard auction to a combinatorial auction can occur in other contexts. We offer this up as an intriguing area for future research.

Finally, as pointed out by Ausubel and Milgrom (2006), since truthful reporting is a dominant strategy under the VCG mechanism, the suppliers have no incentive to spend resources learning about competitor values' or strategies'. Our results do not contradict this, or course. However, they bring in a new dimension from the point of view of the buyer. Specifically, in the choice of standard versus unbundled auction, the buyer would have a clear incentive to spend resources learning about the production capacities of its suppliers.

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Appendix

A Technical Results

In order to prove the main results of the paper, we first establish several technical results.

The following lemma relates the costs of the optimal allocation in both the standard and the unbundled auctions.

Lemma A.1 $C^{\mathrm{U}}_{\mathcal{S}} \leq C_{\mathcal{S}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t$.

Proof: Since the optimal allocation in the standard auction is always feasible in the unbundled auction, it follows that: *The lowest production and inventory holding cost at which the set of suppliers $\mathcal{S}$ can supply the vector of demands $\mathbf{D}$ in the unbundled auction will never exceed the lowest production cost at which any one supplier can supply the total T-period demand in the standard auction plus the inventory holding costs of the buyer.* The lemma is a concise re-statement of this. □

Let w_{-} be the supplier delivering the total T-period demand $\sum_{t=1}^{T} D_t$ in $C_{\mathcal{S}\setminus\{w\}}$. We may observe that when the setup costs in period 1 are supplier-independent then w is the supplier with the cheapest unit production costs in period 1 among all those suppliers who have sufficient capacity to produce the total T-period demand $\sum_{t=1}^{T} D_t$ in period 1, similarly w_{-} is the supplier with the second-cheapest unit production costs in period 1 among all those suppliers who have sufficient capacity to produce the total T-period demand $\sum_{t=1}^{T} D_t$ in period 1.

The following lemma proposes an upper bound on the total production costs incurred by w when producing a subset of demands.

Lemma A.2 *For any $T' \subseteq \{1, \ldots, T\}$, we have that:*

$$f_{w,1} + \sum_{t \in T'} (p_{w,1}) D_t \;\; \leq \;\; \max_{s \,\in\, \mathcal{S}} f_{s1} + \sum_{t \in T'} (p_{w\_,1}) D_t.$$

Proof: If $p_{w,1} \leq p_{w\_,1}$, the result holds trivially. Otherwise, if $p_{w,1} > p_{w\_,1}$, we have that

$$\begin{aligned}
C_{\mathcal{S}} &\leq C_{\mathcal{S} \setminus \{w\}} \\
f_{w,1} + \sum_{t=1}^{T} (p_{w,1}) D_t &\leq f_{w_,1} + \sum_{t=1}^{T} (p_{w_,1}) D_t \\
f_{w,1} + \sum_{t \in T'} (p_{w,1}) D_t + \sum_{t \notin T'} (p_{w,1}) D_t &\leq f_{w_,1} + \sum_{t \in T'} (p_{w_,1}) D_t + \sum_{t \notin T'} (p_{w_,1}) D_t \\
f_{w,1} + \sum_{t \in T'} (p_{w,1}) D_t + \sum_{t \notin T'} (p_{w,1} - p_{w_,1}) D_t &\leq f_{w_,1} + \sum_{t \in T'} (p_{w_,1}) D_t \\
f_{w,1} + \sum_{t \in T'} (p_{w,1}) D_t &\leq f_{w_,1} + \sum_{t \in T'} (p_{w_,1}) D_t \\
f_{w,1} + \sum_{t \in T'} (p_{w,1}) D_t &\leq \max_{s \,\in\, \mathcal{S}} f_{s1} + \sum_{t \in T'} (p_{w_,1}) D_t,
\end{aligned}$$

and the desired inequality follows. □

In the following, we obtain an upper bound on the costs $C^{\mathrm{U}}_{\mathcal{S} \setminus \{s\}}$. The following notation will be used throughout the rest of the appendix. Let T_s be the set of demands won by supplier s in the unbundled auction while $C^{\mathrm{U}}_{\mathcal{S},s}$ is the cost to s of supplying its allocation T_s.

Lemma A.3 *We have that:*

$$\max_{s \,\in\, \mathcal{S}} f_{s1} + \sum_{t \in T_s} (p_{w\_,1} + \sum_{\tau=1}^{t-1} H_\tau) D_t \; + \; (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) \;\; \geq \;\; C^{\mathrm{U}}_{\mathcal{S} \setminus \{s\}} \;\; s \in \mathcal{W}.$$

Proof: We will establish these inequalities by finding an allocation in the unbundled auction in the absence of supplier $s \in \mathcal{W}$, based on the optimal allocation of the unbundled auction.

Let us first assume that $s \neq w$. The right hand side of the inequality is by definition the lowest cost of supplying all the demands in the unbundled scenario when supplier s is not present. In the absence of s, all the demands allocated to him, T_s, can be assigned to w in period 1, while demand in the remaining periods is supplied under the optimal allocation $C^{\mathrm{U}}_{\mathcal{S}}$, at a cost of $C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}$. In this re-allocation of the demands T_s, a total production and inventory holding costs of at most $f_{w,1} + \sum_{t \in T_s}(p_{w,1} + \sum_{\tau=1}^{t-1} H_\tau)D_t$ is incurred. Note that, in this last expression, we have invoked the standard supply chain assumption presented in Section 4 that holding inventory at the buyer is always at least as expensive as holding it as the suppliers. Finally, using Lemma A.2, we have that this expression is bounded above by $\max_{s \in \mathcal{S}} f_{s1} + \sum_{t \in T_s}(p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau)D_t$.

The proof is similar for $s = w$. We just need to assign the demands in T_s to w_-. $\square$

We will denote by $p_t^{(n)}|_\alpha$ the n-th cheapest period-t unit production cost among all those suppliers who have sufficient capacity to produce α units in period t. When capacity is not binding, we will drop the subindex α. As pointed out above, if the setup cost in period 1 is supplier-independent, w_- is the supplier with the second-cheapest unit production costs in period 1 among all those suppliers who have sufficient capacity to produce the total T-period demand $\sum_{t=1}^{T} D_t$ in period 1, i.e., $p_{w_-,1} = p_1^{(2)}|_{\sum_{t=1}^{T} D_t}$, and we obtain the following corollary.

Corollary A.4 *Suppose that the setup cost in period 1 is supplier-independent, i.e., $f_{s1} = f_{11}$ for all $s \in \mathcal{S}$. Then:*

$$f_{11} + \sum_{t \in T_s} (p_1^{(2)}|_{\sum_{t=1}^{T} D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) \geq C_{\mathcal{S}\setminus\{s\}}^{\mathrm{U}} \quad s \in \mathcal{W}.$$

A similar result can be obtained when the capacity of supplier w_1 in period 1 is not binding.

Lemma A.5 *Suppose that the setup cost in period 1 is supplier-independent. If the capacity of supplier w_1 in period 1 is at least equal to $\sum_{t=1}^{T} D_t$, then*

$$\sum_{t \in T_s} (p_1^{(1)}|_{\sum_{t=1}^{T} D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) \geq C_{\mathcal{S}\setminus\{s\}}^{\mathrm{U}} \quad s \in \mathcal{W}\setminus\{w_1\}.$$

Proof: Similar to the proof of Lemma A.3, and using that the capacity of supplier w_1 period 1 is not binding, we know that in the absence of s (where $s \neq w_1$), all the demands allocated to s can be assigned to w_1 in period 1, the winner of D_1, and perhaps others, in the unbundled auction. Therefore, we have:

$$\sum_{t \in T_s} (p_{w_1,1} + \sum_{\tau=1}^{t-1} h_{w_1,\tau}) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) \geq C_{\mathcal{S}\setminus\{s\}}^{\mathrm{U}} \quad s \in \mathcal{W}\setminus\{w_1\},$$

where we observe that we did not need pay for the setup costs when reassigning demands T_s.

In the following we will show that

$$p_{w_1,1} + \sum_{\tau=1}^{t-1} h_{w_1,\tau} \leq p_1^{(1)}|_{\sum_{t=1}^{T} D_t} + \sum_{\tau=1}^{t-1} H_\tau \quad t \in T_s; s \in \mathcal{W}\setminus\{w_1\} \tag{8}$$

and the desired inequality will hold.

Denote by $T_{w_1}^1$ the set of consecutive demands in T_{w_1} containing D_1. The costs of assigning the demands in $T_{w_1}^1$ to w_1 are at least as cheap as the costs of assigning $T_{w_1}^1$ to

w and letting the buyer pay for the inventory holding costs, i.e.

$$\begin{aligned} f_{11} + \sum_{t \in T^1_{w_1}} (p_{w_1,1} + \sum_{\tau=1}^{t-1} h_{w_1,\tau}) D_t &\leq f_{11} + \sum_{t \in T^1_{w_1}} (p_1^{(1)}|_{\sum_{t=1}^T D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t \\ \sum_{t \in T^1_{w_1}} (p_{w_1,1} + \sum_{\tau=1}^{t-1} h_{w_1,\tau}) D_t &\leq \sum_{t \in T^1_{w_1}} (p_1^{(1)}|_{\sum_{t=1}^T D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t. \end{aligned}$$

This means that for at least one demand in $T^1_{w_1}$, say D_{t^*}, we have that

$$p_{w_1,1} + \sum_{\tau=1}^{t^*-1} h_{w_1,\tau} \leq p_1^{(1)}|_{\sum_{t=1}^T D_t} + \sum_{\tau=1}^{t^*-1} H_\tau,$$

and thus

$$p_{w_1,1} + \sum_{\tau=1}^{t-1} h_{w_1,\tau} \leq p_1^{(1)}|_{\sum_{t=1}^T D_t} + \sum_{\tau=1}^{t-1} H_\tau,$$

for all $t \geq t^*$.

Now inequality (8) follows by observing that any demand in T_s occurs after period t^*.

□

If $w_1 = w$, the capacity of supplier w_1 in period 1 is at least equal to $\sum_{t=1}^T D_t$. Therefore, we have that

Corollary A.6 *Suppose that the setup cost in period 1 is supplier-independent. If $w_1 = w$, then*

$$\sum_{t \in T_s} (p_1^{(1)}|_{\sum_{t=1}^T D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) \geq C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} \quad s \in \mathcal{W}\setminus\{w_1\}.$$

The following lemma gives a useful expression of the savings.

Lemma A.7 *The expression of the savings is equal to*

$$J - J^{\mathrm{U}} = f_{w_-,1} + \sum_{s \in \mathcal{W}} [\sum_{t \in T_s} (p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) - C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}}]. \tag{9}$$

Proof: From equations (1) and (2), we have that:

$$
\begin{aligned}
J - J^{\mathrm{U}} &= C_{\mathcal{S}\setminus\{w\}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t - [C^{\mathrm{U}}_{\mathcal{S}} + \sum_{s\in\mathcal{W}}(C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} - C^{\mathrm{U}}_{\mathcal{S}})] \\
&= f_{w_-,1} + p_{w_-,1}\sum_{t=1}^{T} D_t + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t - [C^{\mathrm{U}}_{\mathcal{S}} + \sum_{s\in\mathcal{W}}(C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} - C^{\mathrm{U}}_{\mathcal{S}})].
\end{aligned}
$$

We can rewrite this as a double summation over the winners in the unbundled auction $s \in \mathcal{W}$ and the index set of demands $t \in T_s$ won by each winner. Thus:

$$
\begin{aligned}
J - J^{\mathrm{U}} &= f_{w_-,1} + \sum_{s\in\mathcal{W}}\sum_{t\in T_s}(p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau)D_t - [\sum_{s\in\mathcal{W}} C^{\mathrm{U}}_{\mathcal{S},s} + \sum_{s\in\mathcal{W}}(C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} - C^{\mathrm{U}}_{\mathcal{S}})] \\
&= f_{w_-,1} + \sum_{s\in\mathcal{W}}[\sum_{t\in T_s}(p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau)D_t - C^{\mathrm{U}}_{\mathcal{S},s} - (C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} - C^{\mathrm{U}}_{\mathcal{S}})] \\
&= f_{w_-,1} + \sum_{s\in\mathcal{W}}[\sum_{t\in T_s}(p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau)D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) - C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}}].
\end{aligned}
$$

□

B Proofs of Main Results

Proof of Proposition 5.1

We will analyze the two cases separately.

(i) Since by assumption $\mathcal{W} = \{w_1\}$, the savings is given by:

$$
\begin{aligned}
J - J^{\mathrm{U}} &= (C_{\mathcal{S}\setminus\{w\}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t) - C^{\mathrm{U}}_{\mathcal{S}\setminus\{w_1\}} \\
&\geq (C_{\mathcal{S}\setminus\{w\}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t) - (C_{\mathcal{S}\setminus\{w_1\}} + \sum_{t=2}^{T}(\sum_{\tau=1}^{t-1} H_\tau)D_t)
\end{aligned}
$$

$$
\begin{aligned}
&= C_{\mathcal{S}\backslash\{w\}} - C_{\mathcal{S}\backslash\{w_1\}} \\
&\geq 0.
\end{aligned}
$$

The first inequality follows from Lemma A.1. The second inequality follows from the fact that the winner of the standard auction is the supplier who can deliver the total T-period demand at lowest cost.

(ii) From Corollary A.6:

$$\sum_{t \in T_s} (p_1^{(2)}|_{\sum_{t=1}^{T} D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) \geq C_{\mathcal{S}\backslash\{s\}}^{\mathrm{U}} \quad s \in \mathcal{W}\backslash\{w_1\}. \tag{10}$$

From Corollary A.4 for the case of $s = w_1$:

$$f_{11} + \sum_{t \in T_{w_1}} (p_1^{(2)}|_{\sum_{t=1}^{T} D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},w_1}^{\mathrm{U}}) \geq C_{\mathcal{S}\backslash\{w_1\}}^{\mathrm{U}}. \tag{11}$$

From (10), (11), and Lemma A.7 for the case of supplier-independent setup costs in period 1:

$$J - J^{\mathrm{U}} = f_{11} + \sum_{s \in \mathcal{W}} [\sum_{t \in T_s} (p_1^{(2)}|_{\sum_{t=1}^{T} D_t} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) - C_{\mathcal{S}\backslash\{s\}}^{\mathrm{U}}] \geq 0. \tag{12}$$

So, in both cases, $J^{\mathrm{U}} \leq J$. □

Proof of Proposition 5.3

Using Lemma A.7, the expression of the savings is equal to:

$$
\begin{aligned}
J - J^{\mathrm{U}} &= f_{w_{\_},1} + \sum_{s \in \mathcal{W}} [\sum_{t \in T_s} (p_{w_{\_},1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) - C_{\mathcal{S}\backslash\{s\}}^{\mathrm{U}}] \\
&= f_{w_{\_},1} + \sum_{s \in \mathcal{W}} [- \max_{s \in \mathcal{S}} f_{s1} + \max_{s \in \mathcal{S}} f_{s1} + \sum_{t \in T_s} (p_{w_{\_},1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) - C_{\mathcal{S}\backslash\{s\}}^{\mathrm{U}}]
\end{aligned}
$$

$$= \; f_{w_-,1} - (\max_{s\,\in\,\mathcal{S}} f_{s1})|\mathcal{W}| + \sum_{s\in\mathcal{W}} [\max_{s\,\in\,\mathcal{S}} f_{s1} + \sum_{t\in T_s} (p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) - C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}}]$$

$$= \; f_{w_-,1} - (\max_{s\,\in\,\mathcal{S}} f_{s1})|\mathcal{W}| + \sum_{s\in\mathcal{W}} [\max_{s\,\in\,\mathcal{S}} f_{s1} + \sum_{t\in T_s} (p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) - C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}}]$$

$$\geq \; \min_{s\,\in\,\mathcal{S}} f_{s1} - (\max_{s\,\in\,\mathcal{S}} f_{s1})|\mathcal{W}| + \sum_{s\in\mathcal{W}} [\max_{s\,\in\,\mathcal{S}} f_{s1} + \sum_{t\in T_s} (p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) - C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}}$$

Therefore, it is sufficient to prove that the following set of inequalities hold:

$$\max_{s\,\in\,\mathcal{S}} f_{s1} + \sum_{t\in T_s} (p_{w_-,1} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C^{\mathrm{U}}_{\mathcal{S}} - C^{\mathrm{U}}_{\mathcal{S},s}) \;\geq\; C^{\mathrm{U}}_{\mathcal{S}\setminus\{s\}} \quad s \in \mathcal{W}. \tag{13}$$

However, inequality (13) was proved in Lemma A.3. □

Proof of Theorem 5.4: The structure of $C^{\mathrm{U}}_{\mathcal{S}}$ and $C^{\mathrm{U}}_{\mathcal{S}\setminus\{t\}}$

Here we will discuss the structure of the optimal allocation of the unbundled auction for the class of problem instances introduced in Theorem 5.4, as well as the one of the optimal allocation in the unbundled auction in the absence of supplier t.

First, observe that for this class of problem instances the setup and the unit inventory holding costs are stationary and the same for all suppliers, therefore when comparing two (supplier,period) combinations we only need to discuss the unit costs. Now we will make some observations: (a) the most attractive supplier in period t is supplier t, since it has the lowest unit production costs; (b) suppliers T and $T+1$ will only produce in period 1 since they both have enough capacity to produce the total T-period demand while producing in period 1 and keeping in inventory until period t has lower unit costs than producing in period t; (c) supplier s will never produce in period t ($s \in \{1, \ldots, T-1\}$ and $s \neq t$) since the production can be done by supplier $T+1$ in period 1 and kept in inventory until the requested period at a savings of ε per unit.

We will start with $C^{\mathrm{U}}_{\mathcal{S}}$. In the following, we will show that in the lowest-cost unbundled allocation every demand will be produced in the same period it is demanded. Thus, by (a) we obtain the desired result, i.e., the lowest-cost unbundled allocation consists of assigning D_t to supplier t in period t, for each $t \in \{1, \ldots, T\}$. Now suppose that, in the optimal allocation of the unbundled auction, there exists at least one demand that is produced in advance. Let $\hat{t}$ be the largest element in $\{1, \ldots, T\}$ such that $D_{\hat{t}}$ is produced in advance. This means that any demand D_t, with $t > \hat{t}$, will be produced in the period in which it is demanded. Since $D_{\hat{t}}$ is produced in advance and the demands in the future are produced in the respective periods in which they are demanded, we know that none of the suppliers will produce during period $\hat{t}$. In the following, we will show that by allocating demand $D_{\hat{t}}$ to supplier $\hat{t}$ in period $\hat{t}$, we obtain a feasible allocation which is cheaper than the current one, and this will yield a contradiction. First, it is easy to see that this is a feasible allocation, since supplier $\hat{t}$ does not produce during period $\hat{t}$. Second, we will show that this yields a cheaper allocation. Because demand $D_{\hat{t}}$ is produced some time before period $\hat{t}$, the unit production cost paid for this demand will be at least $p+(T-\hat{t}+1)f$. Therefore the variable production cost incurred will be at least $[p + (T - \hat{t} + 1)f]D_{\hat{t}}$. Since $D_{\hat{t}} > 1$, we have that this cost is greater than $f + [p + (T - \hat{t})f]D_{\hat{t}}$, i.e., the cost of producing $D_{\hat{t}}$ by supplier $\hat{t}$ in period $\hat{t}$ (including setup costs), a contradiction.

We will now discuss $C^{\mathrm{U}}_{\mathcal{S}\setminus\{t\}}$. Using a similar argument as for $C^{\mathrm{U}}_{\mathcal{S}}$, we have that demand D_τ will be produced in time by supplier τ, for all $\tau > t$. In the following, we will show that demand D_t will be produced in period 1 by supplier $T+1$. Using a similar argument as for $\tau > t$, this will imply that, for all $\tau < t$, demand D_τ will also be produced in time by supplier τ, and the desired result will follow. Now consider the allocation of demand

D_t. Note that supplier $T+1$ belongs to $\mathcal{S} \setminus \{t\}$, thus using (b) and (c) we know that D_t will be produced in period 1 either by supplier 1 or supplier $T+1$. We will show that the first option is not possible. If D_t is produced by supplier 1 in period 1 then, because of the capacity of supplier 1 in period 1, D_1 will need to be produced by supplier $T+1$. Since $D_1 > D_t$, by exchanging the assignments of demands D_1 and D_t, we obtain an allocation that is cheaper than the current one, yielding a contradiction. $\square$

Proof of Theorem 6.1

The proof of this theorem resembles that of part (ii) of Proposition 5.1. Since by assumption the period 1 setup cost is the same for all suppliers, the winner of the standard auction will be the one with the cheapest unit production cost in period 1, and the buyer's total cost in the standard auction will be determined by the supplier with the second cheapest unit production cost in period 1.

Now, since the capacities are not binding, it follows from equation (9) in Lemma A.7:

$$J - J^{\mathrm{U}} = f_{11} + \sum_{s \in \mathcal{W}} [\sum_{t \in T_s} (p_1^{(2)} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) - C_{\mathcal{S}\setminus\{s\}}^{\mathrm{U}}].$$

Therefore, it is sufficient to prove that the following inequalities hold:

$$f_{11} + \sum_{t \in T_{w_1}} (p_1^{(2)} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},w_1}^{\mathrm{U}}) \geq C_{\mathcal{S}\setminus\{w_1\}}^{\mathrm{U}} \tag{14}$$

$$\sum_{t \in T_s} (p_1^{(2)} + \sum_{\tau=1}^{t-1} H_\tau) D_t + (C_{\mathcal{S}}^{\mathrm{U}} - C_{\mathcal{S},s}^{\mathrm{U}}) \geq C_{\mathcal{S}\setminus\{s\}}^{\mathrm{U}} \quad s \in \mathcal{W}\setminus\{w_1\}. \tag{15}$$

As in Proposition 5.1, inequality (14) follows immediately from Corollary A.4 for the case of $s = w_1$. Since the problem is uncapacitated, the capacity of supplier w_1 in period 1 is at least equal to $\sum_{t=1}^{T} D_t$. Thus, inequality (15) follows from Lemma A.5. $\square$