How to Escape the Statistical Purgatory of Stress Testing

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Berkowitz' statistical purgatory

- Stress testing is about imperfect predictability. When it comes to imperfect predictability, dealing with the 'known unknowns' is relatively easy. What is far from easy is dealing with the rare, atypical, unprecedented events that do not lend themselves to traditional statistical analysis, yet can create havoc to portfolios, businesses, economies (and lives): the break-up of the Euro, a geopolitical event, a global pandemic.
- These are the dismal shores of scenario analysis and stress testing. Why dismal? Because traditional stress testing has inhabited the awkward land of the unquantifiably possible, and this lack of quantifiability has greatly hampered its acceptance – and, indeed, its usefulness. Switching to Berkowitz's (1999) metaphor, '*stress testing is in a statistical purgatory. We have some loss numbers, but who is to say whether we should be concerned about them?*'

The changing landscape

- Things are beginning to change, and new ways are being proposed to escape this statistical purgatory. One of the most attractive possibilities is to harness the intuitional appeal of modern graphical methods, such as Bayesian nets, with the power of causal connection afforded by structural models – and, indeed, by the way the human mind naturally works.
- The children of this marriage are
  - (literally) a 'picture' of the scenario at hand,
  - a logically coherent assessment of the probabilities attaching to the various outcomes, and
  - the ability to assess how dependent our results are on the assumptions we have to make about how the world works.

What is the proposed approach?

- First, we put **causation** squarely at the heart of our analysis. This causal information comes from our understanding, imperfect as it may be, of how the world works.
- When we adopt a purely correlation-based description we relinquish this precious information. What is worse, we force ourselves to work against our cognitive grain, which is deeply rooted in causality.
- This cannot be good. Stress testing is difficult enough as it is, and throwing away not just useful information, but our most powerful mode of thinking, is a luxury that we just cannot afford.

The proposed approach - ctd

- Second, we believe that, at least ‘locally', co-dependencies are relatively stable during normal market conditions.
- These local and relatively co-dependencies can therefore be profitably extracted using traditional statistical (frequentist) techniques.

The proposed approach - ctd

- Third, we believe that in conditions of market turmoil the codependence among changes in asset prices is radically altered. With many econophysicists, we believe that extreme events are ‘in a (qualitative) class of their own'.

The proposed approach - ctd

- Fourth, we believe that **each crisis unfolds according to its own, idiosyncratic dynamics**. We do not assume that what happened during any one particular crisis carries a lot of information about how prices will move together in the next.
- It is not clear to us, for instance, where to find useful *statistical* (frequentist) information to guide us in modelling what would happen if the Euro were to break up.

The proposed approach - ctd

- Fifth, we combine the statistical information that pertains to the normal market conditions with the information that we have extracted from our understanding of how the world may work today.
- Once this combined joint distribution has been obtained, we finally know whether we should ‘worry’ or not. We are beginning to escape Berkowitz’s purgatory.
- *The tools we use to ‘splice’ the scenario-specific tails onto the business-as-usual body of the distribution are provided by the Bayesian net technology.*

Constraints that inform the approach

- The first constraint is that we want our results to be intuitively understandable and challengeable by the intelligent-but-not-mathematically-versed senior professional (where ‘senior professional’ is a blanket term to encompass the CRO, the regulator, the Chief Investment Officer, the central banker, the CEO, etc).
- We believe that the time of black-boxes is rapidly running out both for asset allocation models and for risk management in general.

- The second self-imposed constraint is the robustness of the output. We are ambitious but modest. We know that we are trying something very difficult, and we therefore want our final results to be forgiving, in the sense of not displaying a high sensitivity to small changes in the necessarily approximate inputs.
- To use a hackneyed expression, ***we want to be approximately right, not precisely wrong***.

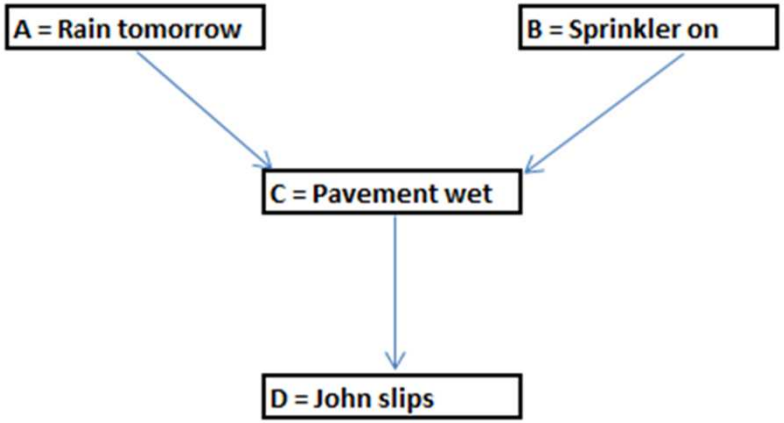
What Bayesian Nets can offer

- Bayesian nets are unsurpassed when it comes to intuitional appeal. They have a rigorous logical and mathematical underpinning.
- They allow the parsimonious factorization of a complex joint probability distribution in a handful of important conditional probabilities (ie, probabilities of the type 'If A has happened, what is the probability of B happening?').
- They speak the cognitive language of the human mind – ie the language of causation.
- They lend themselves to ready sensitivity analysis – which means that the intelligent-but-not-quantitative senior professional mentioned above will be able to get a feel for how much she can trust the results.
- ***They give us an idea of whether we 'should worry'.***

The intuition behind Bayesian nets

- Understanding the intuition behind their power is disarmingly : suppose that you are interested in the probability of John slipping tomorrow on the pavement outside his front door – this is our 'stress event'.
- The probability of John slipping will depend (*inter alia*) on whether the pavement is wet or dry. Whether the pavement is wet depends, in turn, on the probability of rain tomorrow, and on the probability of the garden sprinkler being active.
- This is shown in the Bayesian net in Fig 1, in which A represents the probability of rain tomorrow; B represents the probability of the of the sprinkler being on; C denotes the probability of the pavement being wet; and D is the probability of John slipping.

Our stress event: John slipping



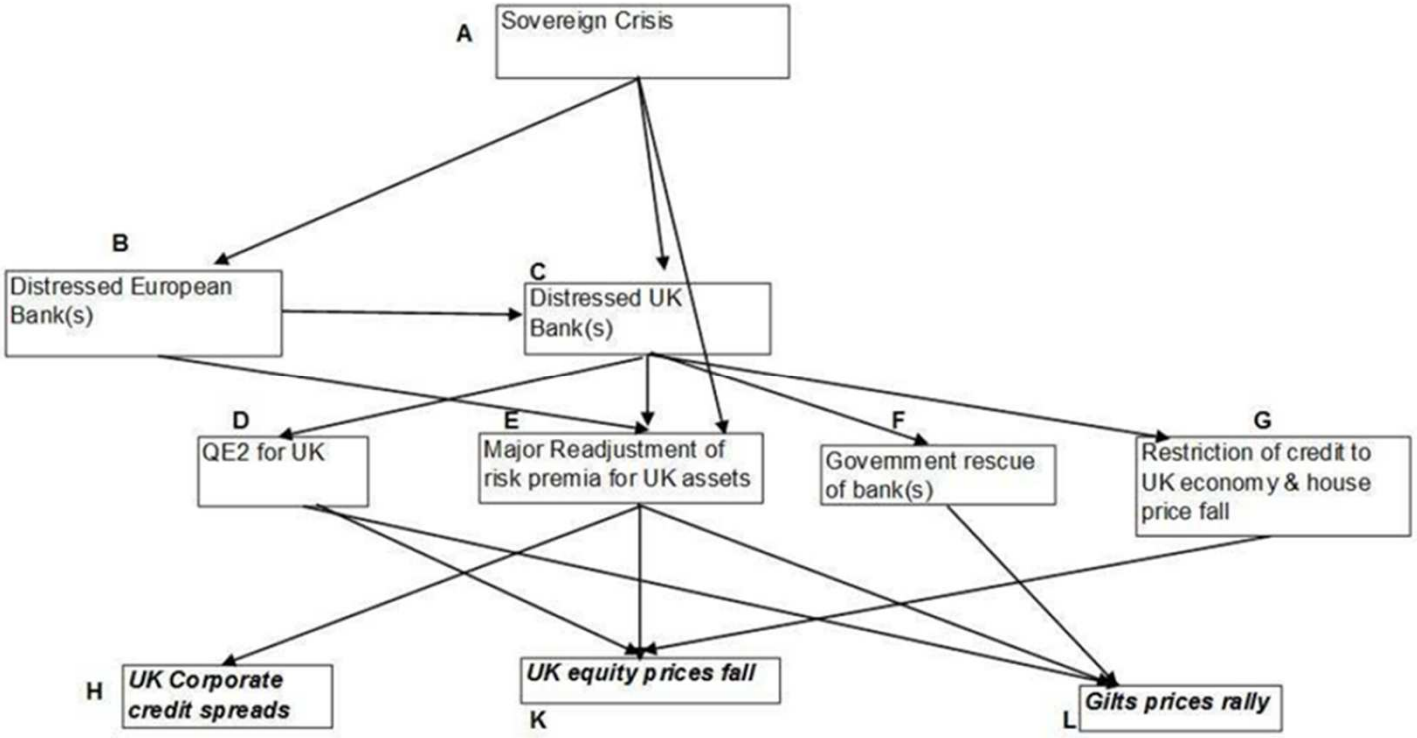
The intuition - ctd

- The central insight behind the probability factorization produced by Bayesian nets (an insight that goes under the name of *conditional independence*) is that, *once we know whether the path is wet or dry*, it does not matter if it is wet because of the rain or because of the sprinkler.
- The wet/dry status of the path contains all the necessary information needed to determine the probability we are interested in – the variable "John slips" is 'screened' from the variables rain and sprinkler by the variable "Pavement wet".

A more interesting example

- If the simplification in the cognitive and computational burden required to arrive at the probability we are interested in seems in this case rather underwhelming, one may care to ponder on the advantages associated with a more complex scenario, such as the one depicted in Fig 2.

A more interesting example



Rebutting the objections

- One objections often raised is that Bayesian nets require the assignment of (supposedly difficult to specify) conditional probabilities. But consider the following.
- You would be probably hard pressed if asked to provide the probability of you being hit by a car tomorrow.
- You would find it no easier to assign the probability of ending up in hospital tomorrow.
- However I think you can safely venture a guess of 40-60 per cent for the probability of you ending up in hospital, *given that you have been hit by a car*.
- Once again, when used in the causal direction, **conditional probabilities are natural and cognitively 'easy' to assign**.

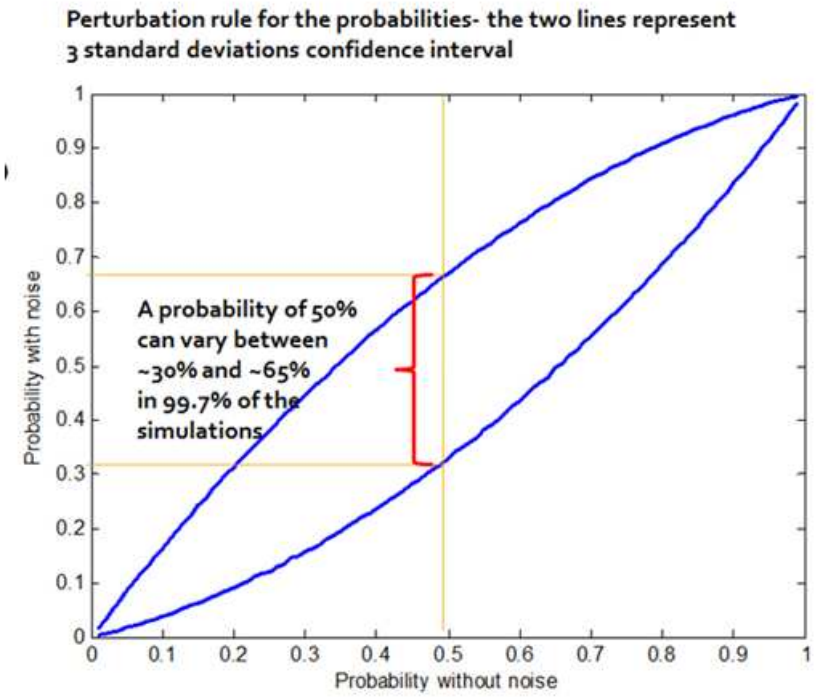
The subjective nature of the inputs

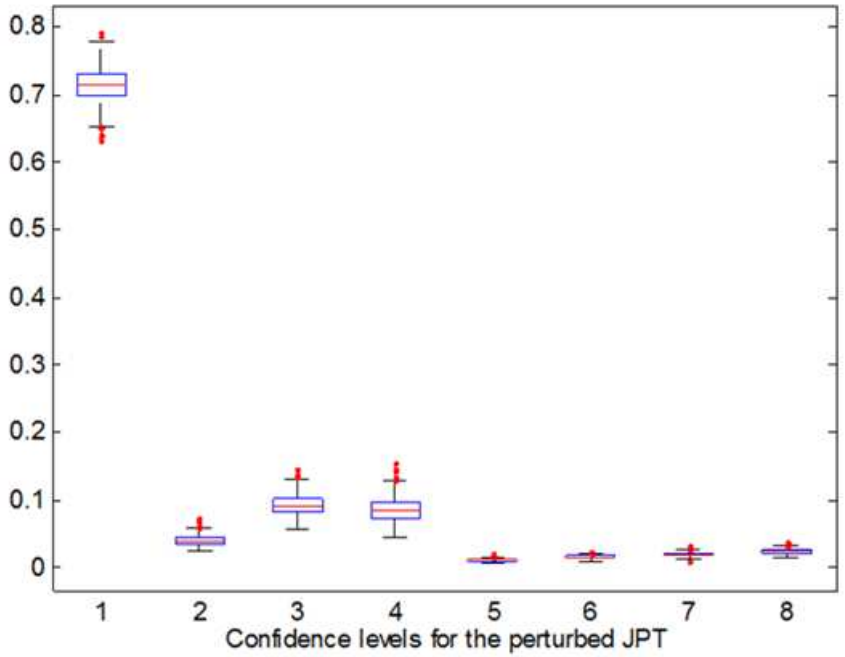
- Of course, many (or most) of the conditional probabilities a Bayesian net requires are subjective in nature.
- **The agnosticism of the approach as to where the probabilities come from, should, however, be seen as an advantage, not a drawback.**
- This is because the user of the net is unshackled by the constraints of a statistical (frequentist, past-data-based) approach that is of little or no use to analyze novel situations: where would we look up in our data series, for instance, if we wanted to glean information about what would happen if Greece left the Euro next Monday?

Sensitivity to subjective probabilities

- Subjective probabilities, by their very nature, never attain the same precision frequentist probabilities can. Fortunately, **it is very easy to asses the sensitivity of the output from a Bayesian net to its uncertain input probabilities**.
- In the left panel of Fig 3 we show the perturbation of an input conditional probabilities for a 3-standard deviation shock: so, for instance, a 50 per cent input conditional probability will lie between approximately 30% and 65% in 99.7% of the simulations.
- The panel on the right then shows the uncertainty in the outputs joint probabilities generated by the uncertainty in the inputs conditional probabilities.

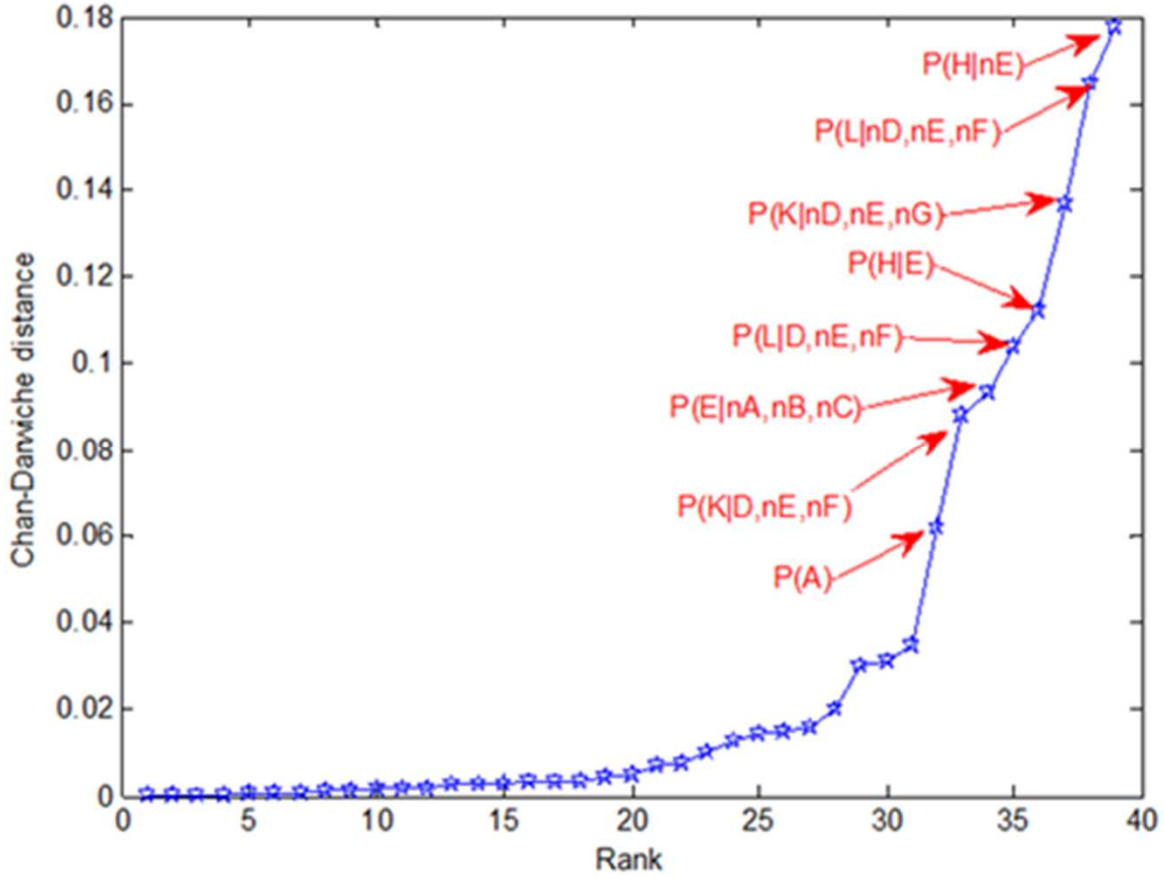
Sensitivity analysis





Focussed dependence on the inputs

- But there is more: when one perturbs the uncertain input conditional probabilities one, of course, obtains different output probabilities for the scenario of interest. In principle, changing every input conditional probability changes the answer we are interested in.
- However, as Fig 4 shows, for a reasonable and realistic net, *only a handful* if input probabilities (those labelled in red) significantly affect the answer.
- The Bayesian net tool shows with great clarity the input conditional probabilities that must be carefully assessed when building the net.


Chan-Darwiche distance
Rank
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0
5
10
15
20
25
30
35
40
P(H|nE)
P(L|nD,nE,nF)
P(K|nD,nE,nG)
P(H|E)
P(L|D,nE,nF)
P(E|nA,nB,nC)
P(K|D,nE,nF)
P(A)

Focussed dependence - ctd

- *These* are the essential quantities the answer depends on, and *these* are therefore the few but important quantities the senior decision-maker will do well to focus on in analyzing the output of the stress test.
- *Bayesian nets lay bare with unparalleled clarity what the ultimate drivers of a scenario really are.*
- Most importantly, by doing so, they point to the 'neuralgic' pressure points of a portfolio, thereby enabling **focussed** and **targeted** remedial action.

Conclusions - 1

- Both the industry and the regulators are paying more and more attention to stress testing. To mention one important document, in May 2009 the BIS issued the *Principles for sound stress testing practices and supervision*, which deals with the use of stress testing and its integration in risk governance.
- In particular, under Pillar 2 (the supervisory review process) it says: “supervisors should examine a bank’s stress testing results as part of a supervisory review *of both their internal capital assessment and its liquidity risk management*”.

Conclusions - 2

- A stress-test-based review of capital adequacy and liquidity resilience can only be achieved if stress testing exits Berkowitz's purgatorial state – if, that is, an approximate, but order-of-magnitude-meaningful, assessment of the probability and severity of a stress is possible.
- A stress test must pass the "*Should we worry?*" test to be of any use.

Conclusions - 3

- The Bayesian net technology (which, by the way, has already found applications in risk control environments such as the nuclear industry) has to date been woefully underutilized in the financial and macroprudential context.
- If this state of affairs were to change, we believe that significant benefits could be reaped.