FISCAL POLICY IN A DEPRESSED ECONOMY: A COMMENT

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Abstract

In an influential article, Delong and Summers (2012) consider the implications of hysteresis for government debt. They derive an upper limit for the after-tax real interest rate. If the interest rate is below this limit, the debt incurred during a one-off fiscal stimulus will be automatically repaid without the need for higher taxes. Their analysis assumes that a one-off stimulus leaves an infinite legacy of future benefits (hysteresis effects) that increase through time. This note extends their analysis to situations where hysteresis effects remain constant or decay in the course of time. By highlighting the hysteresis time profile, it provides a more transparent treatment of debt dynamics.

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1. Introduction

Economists have been aware of the potential importance of hysteresis since Blanchard and Summers (1984) wrote on the subject. A recession may leave a harmful legacy lasting for many years. Conversely, a temporary fiscal stimulus may leave a beneficial legacy in the form of higher output or employment that continues long after the stimulus is removed. Such a legacy has fiscal implications since it will generate a future stream of extra tax revenue and thereby influence the evolution of government debt. This raises an important question. Under what conditions will the debt resulting from the stimulus be sustainable, assuming no change in the tax rate? Sustainability in this context has two meanings. It may refer to a situation in which the ratio of stimulus-related debt to GDP remains positive but becomes vanishingly small in the course of time. Alternatively, the term may refer to a situation in which this debt is fully repaid within a finite period.

In an influential article, Delong and Summers (2012) explored the implications of hysteresis for sustainability in the latter sense. Their aim was to find the conditions under which a temporary fiscal stimulus is self-financing. To this end, they derived an upper limit for the after-tax real interest rate. If the interest rate is below this limit, the debt incurred during the stimulus period will spontaneously disappear within a finite length of time. Their formula for the upper limit depends on the tax rate, the multiplier, a coefficient specifying the initial scale of hysteresis and the trend growth rate of the economy. To derive this formula they assume that a one-off fiscal stimulus generates a stream of future benefits (hysteresis effects) that increase in line with trend growth in economy.[1] If this assumption is dropped, the trend growth rate becomes irrelevant. What matters is the time profile of hysteresis effects. DeLong and Summers refer to this issue, but only as a qualification to their main results. This note highlights the role of the hysteresis time profile and thereby provides a more transparent treatment of debt dynamics.

Throughout the paper I assume that the hysteresis effects following a stimulus are positive. However, these effects could in principle be negative. Government expenditure might be used to delay desirable structural change or to subsidise ailing companies, thereby damaging medium-term economic growth and tax revenue. This possibility should be borne in mind.

2. The reference path

Consider the path which the economy would have followed if there had been no stimulus. Government expenditure is E_t in year t and tax revenue is T_t. Both items exclude government interest payments and taxes thereon. Government debt at the end of the year satisfies the following difference equation:

$$D_t = E_t - T_t + (1 + r_t) D_{t-1} \qquad (1)$$

where r_t is the real after-tax real interest rate.

Suppose that expenditure and tax revenue are perturbed by ΔE_t and ΔT_t respectively. Assuming no change in the interest rate, perturbed government debt satisfies the equation

$$D_t + \Delta D_t = \left(E_t + \Delta E_t\right) - \left(T_t + \Delta T_t\right) + (1 + r_t)\left(D_{t-1} + \Delta D_{t-1}\right) \qquad (2)$$

Subtracting equation (1) from equation (2) yields

$$\Delta D_t = \Delta E_t - \Delta T_t + (1 + r_t)\Delta D_{t-1} \qquad (3)$$

3. Fiscal stimulus

In year 0 government expenditure is perturbed by an amount $x > 0$ and then at the end of the year returns to its normal path. Thus,

$$\begin{aligned} \Delta E_0 &= x \\ \Delta E_t &= 0 \;\; \text{for } t > 0 \end{aligned} \qquad (4)$$

Through the multiplier effect, the additional expenditure leads to additional output $\Delta Y_0 = \mu x$ in year 0. Hysteresis implies that some of this additional output carries over to the following year. We assume that $\Delta Y_1 = \eta \Delta Y_0$ for some constant $\eta > 0$. In future years the hysteresis effect changes at an annual rate *h*. Thus, $\Delta Y_t = (1+h)\Delta Y_{t-1}$. DeLong and Summers assume that *h* is equal to *g*, the trend rate of growth of the economy, but this is not necessary. Indeed, *h* could be negative, indicating that the hysteresis effect decays in the course of time. Working backwards, it follows that $\Delta Y_t = (1+h)^{t-1}\Delta Y_1 = (1+h)^{t-1}\eta\mu x$. Thus,

$$\begin{aligned} \Delta Y_0 &= \mu x \\ \Delta Y_t &= (1+h)^{t-1}\eta\mu x \;\; \text{for } t > 0 \end{aligned} \qquad (5)$$

The effects on tax revenue are as follows:

$$\begin{aligned} \Delta T_0 &= \tau \Delta Y_0 = \tau\eta\mu x \\ \Delta T_t &= \tau \Delta Y_t = (1+h)^{t-1}\tau\eta\mu x \;\; \text{for } t > 0 \end{aligned} \qquad (6)$$

where $\tau > 0$ is the tax rate on income excluding government interest payments. There is no need to specify the tax rate on the latter.

To close the system note that

$$\Delta D_{-1} = 0 \qquad (7)$$

Substitution in equation (3) yields

$$\Delta D_0 = (1-\tau\mu)x \qquad (8)$$

Assume that $1-\tau\mu > 0$ and also that $r_t = r$ for $t > 0$. Equations (3) and (8) imply that for $t > 0$

$$\Delta D_t = -(1+h)^{t-1}\tau\eta\mu x + (1+r)\Delta D_{t-1} \qquad (9)$$

Consider the solution to the above difference equation. If $r \le h$, it is easily shown that $\Delta D_t < 0$ for large *t*. If $r > h$ the solution is

$$\Delta D_t = A(1+h)^t + B(1+r)^t \qquad (10)$$

Where

$$A = \left(\frac{\tau\eta\mu}{r-h}\right)x \text{ and } B = \frac{1-\tau\mu}{r-h}\left(r-h-\frac{\tau\eta\mu}{1-\tau\mu}\right)x$$

Since $r > h$ it follows that $\Delta D_t < 0$ for large *t* provided $B < 0$. This will be the case if:

$$r < r^* = h + \frac{\tau\eta\mu}{1-\tau\mu} \qquad (11)$$

If the above condition is satisfied, the debt ΔD_t will become negative in the long-run. Thus, the fiscal stimulus will eventually pay for itself without the need for a higher tax rate. The above condition is identical to the formula given by DeLong and Summers if $h = g$, as they assume,

4. Debt to GDP ratio

If $r^* \le r < g$ the debt ΔD_t will remain positive and its long-run growth rate will be equal to *r*. Moreover, the long-run growth rates of $Y_t + \Delta Y_t$ and ΔY_t will both be equal to *g*. Hence, assuming the limit on the right-hand side of the following equation exists and is finite,

$$\lim_{t\to\infty}\left(\frac{D_t+\Delta D_t}{Y_t+\Delta Y_t}\right)=\lim_{t\to\infty}\left(\frac{D_t}{Y_t}\right) \quad (12)$$

In this case, the stimulus-related debt will have no long-run effect on the overall debt to GDP ratio.

5. Simulations

Table 1 shows values of the upper limit r^* for various parameter values. It is assumed that the tax rate = 0.333. The entries in the top panel, where h = 2.5 percent, are almost identical to the simulation results reported by Delong and Summers[2]. If r is less than the value of r^* shown in Table 1, the stimulus-related debt will eventually become negative. If $r^* \leq r < g$ the ratio of stimulus-related debt to GDP will remain positive but converge to zero.

For example, suppose that the multiplier $\mu = 1.5$, the hysteresis coefficient $\eta = .025$ and the trend growth rate is g = 2.5 percent. With h = +2.5, 0,-2.5 or -10 percent the corresponding values of r^* are +5.00, +2.50, 0, -7.50 percent respectively. The contrast between these cases illustrates the importance of persistence or growth in hysteresis effects. In the penultimate case if $r < 0$ the stimulus-related debt will eventually become negative. If $0 \leq r < 2.5$ percent this debt will remain positive but become vanishingly small relative to GDP.

Figure 1 illustrates the dynamics of stimulus-related debt following a one-off government expenditure boost equal to 1 unit. It is assumed that $h = -2.5$ percent $\mu = 1.5$ and $\eta = 0.025$. The above formula implies that $r^* = 0$. Thus, if $r < 0$, the debt will be repaid within a finite length of time. Given the interest rates over the past seventy years, this is a stringent condition, although it is not out of line with recent experience. If $r = -1$ percent or -2 percent, the debt is repaid completely within 61 years and 44 years respectively. If $r > 0$, the debt will eventually explode, but in some cases it may take many decades for this to happen. In the meantime the stimulus-related debt may actually fall.

It is instructive to compare the hysteresis time profiles identified in Table 1. These are illustrated in Figure 2. In this diagram the initial hysteresis effect is normalised to 1. When $h = +2.5$ percent the hysteresis effect after 20 years is 64 percent larger than it was initially. This is the case considered by DeLong and Summers. When $h = -10$ percent the hysteresis effect after 20 years is only 11 percent of its initial value.

Table 1: Values of r* (percent)

	μ = 0.5	μ = 1	μ = 1.5
h = 2.5%			
η = 0.025	3.00	3.75	5.00
η = 0.05	3.50	5.00	7.49
η = 0.1	4.50	7.49	12.48
η = 0.2	6.50	12.49	22.46
h = 0			
η = 0.025	0.50	1.25	2.50
η = 0.05	1.00	2.50	4.99
η = 0.1	2.00	4.99	9.98
η = 0 .2	4.00	9.99	19.96
h = -2.5%			
η = 0.025	-2.00	-1.25	0.00
η = 0.05	-1.50	0.00	2.49
η = 0.1	-0.50	2.49	7.48
η = 0 .2	1.50	7.49	17.46
h = -10.0%			
η = 0.025	-9.50	-9.00	-7.50
η = 0.05	-9.00	-7.50	-5.01
η = 0.1	-8.00	-5.01	-0.02
η = 0 .2	-6.00	-0.01	9.96

Note: This table indicates the conditions under which the incremental debt due to a fiscal stimulus eventually becomes negative in the course of time. This occurs when $r < r^*$.

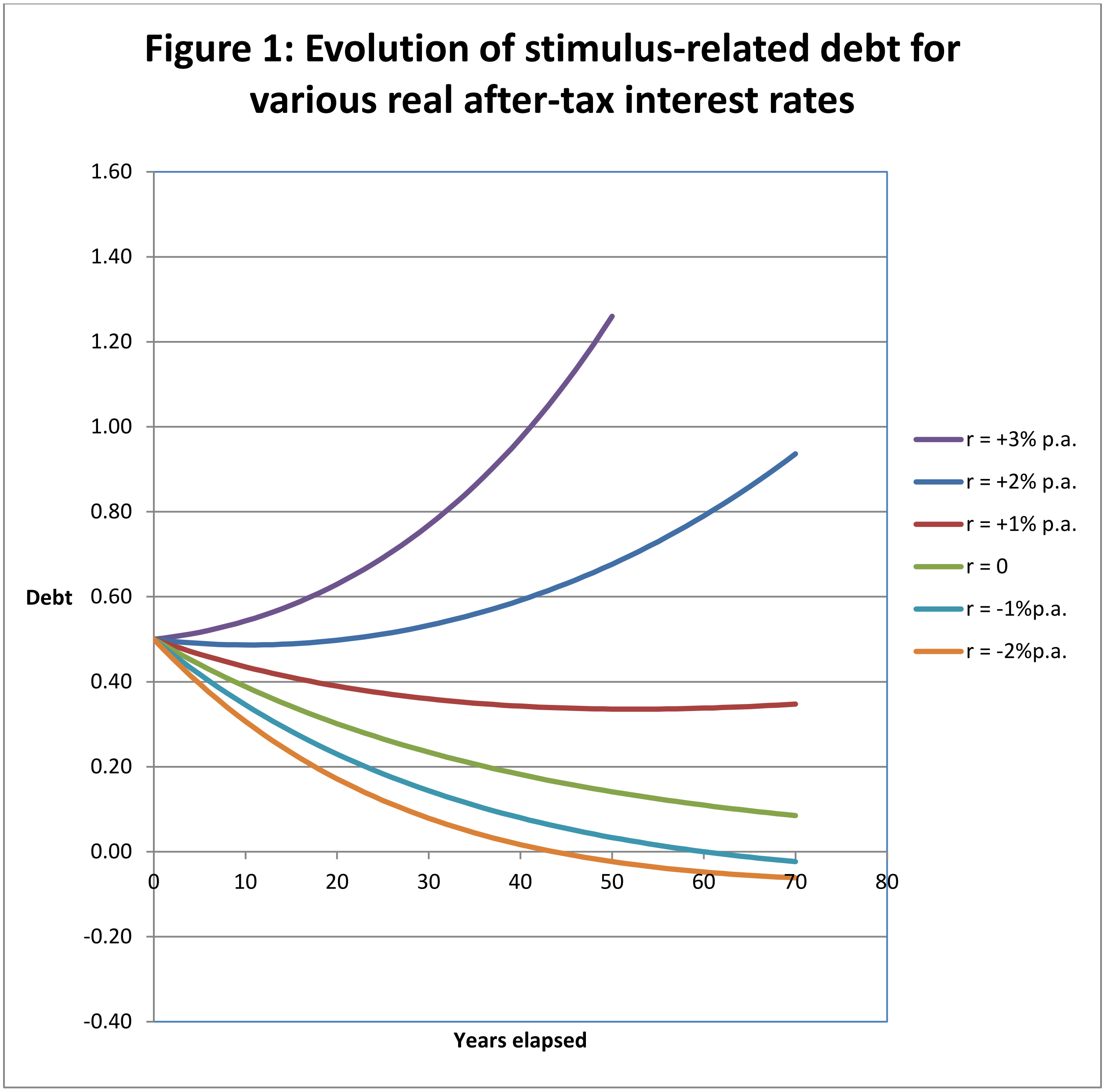
Figure 1: Evolution of stimulus-related debt for various real after-tax interest rates
Debt
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
0
10
20
30
40
50
60
70
80
Years elapsed
r = +3% p.a.
r = +2% p.a.
r = +1% p.a.
r = 0
r = -1%p.a.
r = -2%p.a.

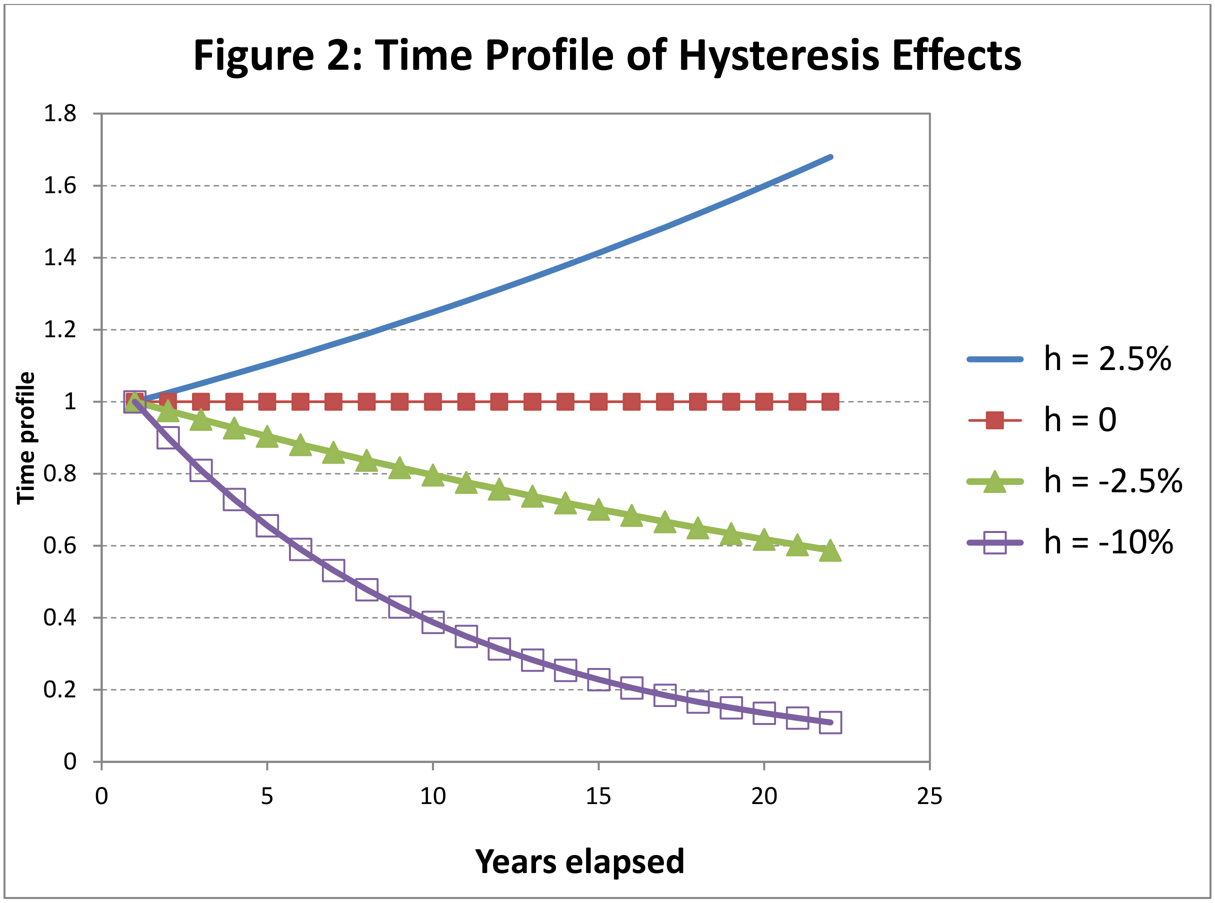

Figure 2: Time Profile of Hysteresis Effects
Time profile
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Years elapsed
h = 2.5%
h = 0
h = -2.5%
h = -10%

6. Conclusion

The time profile of hysteresis effects plays a central role in debt dynamics following a temporary fiscal stimulus. The strong results obtained by DeLong and Summers depend on their assumption that the hysteresis effects increase geometrically in line with the growth of the economy. However, even with some decay in hysteresis effects, their general point regarding the importance of hysteresis for debt dynamics still holds. The after-tax real interest rate also plays an important role. The relevant interest rate in this context is the rate after the fiscal stimulus has ended. If the interest remains low, as in recent years, hysteresis may ensure that the stimulus-related debt is paid off completely within a few decades without the need to raise taxes. Even if the debt is not paid off completely, hysteresis may considerably retard its growth.

If hysteresis effects are sufficiently large, a stimulus package will be self-financing and the associated debt will be automatically repaid within a few decades. If this is widely accepted by market opinion then, as DeLong and Summers point out, there should be no need to reassure markets by announcing a future deficit reduction programme. Hysteresis will spontaneously generate the necessary tax revenue without the need for austerity. In practice, of course, hysteresis effects may not be large, and even if they are, market opinion may be sceptical of their importance.

Notes

1 Delong and Summers do not explicitly assume that the hysteresis benefit grows at the same rate as GDP However, the simulation results reported in their Table 2 assume that the hysteresis benefit is a constant fraction of trend GDP, which amounts to the same thing.

2 There are some minor discrepancies between the entries in our Table 1 (for $h = +0.25$) and those reported in Table 2 of DeLong and Summers. This is hard to understand since they are supposedly based on the same formula and the same parameter values.

References

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