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Analytic Solutions for Supply Function Equilibria: Uniqueness and Stability

EPRG Working Paper 0824

Cambridge Working Paper in Economics 0848

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Electricity liberalisation breaks up previously vertically integrated franchise monopoly electricity industries by unbundling the potentially competitive activities of generation and supply (retailing) from the natural monopoly activities of transmission and distribution. This requires a wholesale market into which generators can offer power and from which consumers or their agents, the supply companies, can buy power. (In a fully liberalised market other services that deliver reliability, stability and security services may also be traded on markets, otherwise they are likely contracted by the system operator.) The move from franchise monopoly to competitive generation industry is, however, often slow and may even be reversed where previously independent electricity companies are encouraged to merge to create 'national champions'. The early experience of electricity liberalisation has therefore been one of oligopoly or more extreme forms of market dominance, rather than workably competitive wholesale markets.

Imperfectly competitive electricity markets raise important issues for regulators and competition authorities wishing to monitor performance, improve market design, adjudicate on mergers and propose remedies. Electricity is homogenous, non-storable, inelastically demanded, produced by similar firms with similar and known cost functions, all properties that amplify market power and create opportunities for collusion. Understanding how firms may be tempted to behave in such markets is therefore an essential pre-requisite for intelligent market monitoring and the design of market remedies.

The standard workhorse of Industrial Organization is the Nash Cournot oligopoly model, which can be justified in a price-setting world provided capacity constrains output and that output is fairly stable from period to period (or can be stored). Electricity markets

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experience widely varying levels of demand over periods of 24 hours, and from season to season, so that the winter peak demand may be a multiple of the summer off-peak demand (at least in higher latitudes with little air conditioning demand). Capacity clearly constrains demand in peak periods, but unless markets are both isolated and highly concentrated, generating companies are unlikely to be pivotal (that is, essential to meet demand) in off-peak periods, where competition may be expected to be more intense.

Klemperer and Meyer (1989) proposed a new equilibrium concept to deal with uncertain demand that might, in some states of the world, lead to tight capacity-constrained market situations for which the Cournot equilibrium was appropriate, but might in other states of the world lead to very competitive or Bertrand-like equilibria. They envisaged firms offering continuous and differentiable supply functions that would be profit maximising given all possible states of demand and the supply functions offered by rivals. A Supply Function Equilibrium (SFE) would be a Nash Equilibrium in these supply functions, and instead of simple first order conditions to find the profit maximising level of output, the equilibrium would typically be described by a linked set of differential equations.

Green and Newbery (1992) adapted the Klemperer and Meyer model for wholesale electricity markets, noting that in the English electricity pool (one of the first examples of a liberalised electricity market) generators had to offer supply functions that would be binding for the 48 half-hour periods of the following day, over which the possible range of residual demands facing any one company could be very wide. This (largely predictable) time variation in demand replaced the variation in uncertain demands of Klemperer and Meyer's model, and allowed one to use essentially the same differential equations to describe optimal bidding behaviour in an electricity pool. Subsequent work by Holmberg, Newbery and Ralph (2008) showed that even if firms were required to offer stepped bids (with successive amounts of capacity offered at successively increasing but step-wise constant prices), rather than smooth differentiable supply functions, the resulting equilibria would converge on the SFE for the continuous representation as the size of the priuce increments became smaller and the number of steps increased.

If SFE are the appropriate equilibrium concept for wholesale electricity markets, then it would be helpful to have a set of analytical solutions for reasonable simplifications to the cost functions, so that one can investigate the properties of various possible industry structures (in terms of concentration, contract cover, and capacity adequacy, for example), just as the constant marginal cost Cournot oligopoly solution has been a useful test-bed for standard Industrial Organization analyses.

This admittedly rather mathematically dense paper derives analytic solutions for the case of linear and quadratic costs, and discusses the question of their uniqueness and stability. It collects together and extends results scattered in earlier working papers to make them more accessible. It shows that the standard model can be readily adapted to handle contracts, and demonstrates the potential problem that there may be a whole continuum of possible SFE, leaving open the question of how firms might select from this set. That raises the question of what might happen if a firm selected a supply function that would only be optimal if other firms chose consistent supply functions, when in fact they were choosing other potentially valid supply inconsistent with the firm's choice.

Baldick and Hogan (2006) have argued that such deviations severely limit the set of stable and hence acceptable SFE to a unique linear solution. The paper argues instead that capacity constraints and entry conditions combined with contracting are a more fruitful source of uniqueness, and that out-of-equilibrium behaviour raises difficult issues that do not necessarily argue for instabilities leading to uniqueness.

Analytic solutions for symmetric oligopolies with the same linear cost functions (constant marginal costs) can be obtained in a simple form with supply offered as a function of the price that the firm is willing to accept. With quadratic costs (affine marginal costs) the resulting solutions are implicit functions than can be graphed but are no longer so easy to manipulate. Nevertheless, they are closely related to the constant marginal cost case, and to the readily solved linear solution, providing some reassurance about the robustness of these two simpler examples. The more general case in which firms differ in their cost functions (which is analytically soluble for the simple Cournot equilibrium) can normally only is solved by numerical methods.

Contact	dmgn@econ.cam.ac.uk
Publication	August 2008
Financial Support	ESRC, EPRG, Swedish Energy Agency, Jan Wallander's foundation

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