How does renewables competition affect forward contracting in electricity markets?

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How does renewables competition affect forward contracting in electricity markets?

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Abstract

Higher renewables penetration reduces the incentive of conventional electricity generators to make forward commitments via forward- or retail-market contracts. This can undermine the role of forward contracting in mitigating market power. More renewables raise wholesale electricity prices in states of the world where their capacity utilization is low due to intermittency.

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1 Introduction

Renewables such as solar and wind already account for up to 30% of power generation in the UK, Germany and parts of the US (Pollitt & Anaya 2015), and global decarbonization objectives will require further large-scale investment. Due to their near-zero marginal costs, renewables come with a well-known “merit-order effect” by which they displace conventional electricity generators (e.g., Green & Léautier 2015; Liski & Vehvilainen 2015).

The literature on wholesale electricity markets places significant emphasis on how forward contracting can mitigate market power (e.g., Wolak 2000; Ausubel & Cramton 2010). Such forward commitments can take the form of forward contracting (Allaz & Vila 1993) or retail market sales (Bushnell, Mansur & Saravia 2008).¹ In practice, power generators indeed sell forward a significant fraction of production (Anderson, Hua & Winchester 2007).

This paper examines the equilibrium interaction between renewables competition and forward contracting. The model generalizes Allaz & Vila (1993) to (i) incorporate the intermittent

¹My thanks are due to Anette Boom, David Newbery and EPRG colleagues for their comments. All views and any errors are mine.

¹This paper takes the same approach as this literature in that it examines the strategic incentive for forward contracting rather than the hedging motive driven by risk aversion.
nature of renewables production, and (ii) allow for \( n > 2 \) strategic players, with cost heterogeneity to represent different generation technologies (such as coal- or gas-fired plant).

2 Model

Consider a wholesale electricity market with a set \( N = \{1, 2, ..., n\} \) of \( n \geq 2 \) active “incumbent” electricity generators. Renewables are installed with capacity \( R \), with zero marginal operating costs and zero carbon emissions.

There are \( M \geq 2 \) states of the world, reflecting the intermittency of renewables production. State \( m \) occurs with probability \( \delta_m \in (0, 1) \) where \( \sum_{k=1}^{M} \delta_k = 1 \). In state \( m \), the rate of capacity utilization of renewables is \( \gamma_m \in (0, 1] \), ordered such that \( \gamma_1 > \gamma_2 > ... > \gamma_M \). Firm \( i \in N \) sells \( x_i^m \) units with a marginal cost \( c_i \), so total conventional output \( X_m = \sum_{i \in N} x_i^m \).

Electricity buyers form a linear demand curve \( p(Q) = \alpha - \beta Q \), where \( Q \) is consumption and \( (\alpha, \beta) > 0 \). There is market clearing in each state of the world, so prices are state-contingent: in state \( m \), total output satisfies \( Q_m = X_m + \gamma_m R \), and electricity trades at a price \( p_m \).

The timing of the game is as follows. In Stage 1, each incumbent chooses its forward commitment \( y_i \), where the forward market is competitive with rational expectations. Then the state of the world \( \gamma_m \) is revealed. In Stage 2, each incumbent chooses its output \( x_i^m \). Incumbents each maximize their profits, while interacting strategically; renewables production is non-strategic. The game is solved backwards for the subgame-perfect Nash equilibrium.

3 Results

The main question is, what is the equilibrium impact of more renewables capacity \( R \)? This could arise because of an increase in renewables subsidies or due to technological progress which reduces their investment costs.

First-order conditions

In Stage 2, the state of the world \( m \) is known. Firm \( i \)’s problem is to:

\[
\max_{x_i^m} \{ (x_i^m - y_i)p_m - c_i x_i^m \}
\]

where \( y_i \) is its forward commitment made in Stage 1, and demand \( p_m = \alpha - \beta(X_m + \gamma_m) \). The firm here only makes revenues on its uncommitted units \( (x_i^m - y_i) \). The first-order condition is:

\[
0 = (p_m - c_i) - \beta(x_i^m - y_i) = [\alpha - \beta(X_m + \gamma_m R) - c_i] - \beta(x_i^m - y_i).
\]

These \( n \) first-order conditions define incumbents’ optimal output choices as a function of contracts. Let \( Y = (y_1, y_2, ..., y_n) \) denote forward positions, leading to outputs \( x_i^m = x_i(Y; \gamma_m) \) for

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2 For simplicity, renewables are grouped into a single capacity figure.

3 Firms’ choices are assumed to be observable and there is no discounting.
each \( i \in N \), and thus \( X_m = X(Y; \gamma_m) \) and \( p_m = p(Y; \gamma_m) \) at the market-level for each state \( m = 1, 2, \ldots, M \).

In Stage 1, the state of the world is not yet known, so firm \( i \) maximizes its expected profits:

\[
\max_{y_i} E \pi_i = \sum_{k=1}^{M} \delta_k \left\{ (p_k - c_i)x_i^k + (p^f - p_k)y_i \right\}.
\]

The first term reflects its spot market profits and the second term represents its forward-market profits at price \( p^f \). With a competitive forward market, the latter term is zero since the forward price \( p^f = \sum_{k=1}^{M} \delta_k p_k \) equals the expected spot price by the no-arbitrage condition.

Thus firm \( i \)'s problem boils down to:

\[
\max_{y_i} E \pi_i = \sum_{k=1}^{M} \delta_k \left[ p(Y; \gamma_k) - c_i \right] x_i(Y),
\]

which makes explicit the dependencies on forward contract position arising in Stage 2. The first-order condition is:

\[
0 = \sum_{k=1}^{M} \delta_k \left\{ \left[ p(Y; \gamma_k) - c_i \right] - \beta x_i(Y; \gamma_k) \left( 1 + \frac{dX_{-i}(Y; \gamma_k)}{dx_i} \right) \right\} \frac{dx_i(Y; \gamma_k)}{dy_i}.
\]

This reflects how firm \( i \)'s forward commitment \( y_i \) affects its subsequent production \( x_i^m \), in each of the \( M \) states of the world. It also incorporates the strategic effect that changes in its own production \( dx_i \) have on the best-response outputs of its rivals \( dX_{-i}(Y; \gamma_m) = \sum_{j \in N \setminus i} dx_j(Y; \gamma_m) \).

The following result is useful in simplifying this condition:

**Lemma 1.** In state \( m \), the incumbent firms’ output responses in Stage 2 satisfy:

\[
\frac{dX_{-i}(Y; \gamma_m)}{dx_i} = -\frac{(n-1)}{n} < 0 \quad \text{and} \quad \frac{dx_i(Y; \gamma_m)}{dy_i} = \frac{n}{(n+1)} > 0.
\]

**Proof.** Summing the Stage 2 first-order conditions from (1) for all firms but firm \( i \) gives:

\[
0 = (n-1) \left[ \alpha - \beta (X(Y; \gamma_m) + \gamma_m R) \right] - \sum_{j \in N \setminus i} c_j - \beta \left[ X_{-i}(Y; \gamma_m) - Y_{-i} \right].
\]

Differentiating this expression shows that \( dX_{-i}(Y; \gamma_m)/dx_i = -(n-1)/n < 0 \), as claimed, since \( n \geq 2 \). Rearranging the first-order condition for firm \( i \) from (1) shows that

\[
x_i(Y; \gamma_m) = y_i + \left( \frac{\alpha - c_i}{\beta} - \left[ X(Y; \gamma_m) + \gamma_m R \right] \right) \Rightarrow \frac{dx_i(Y; \gamma_m)}{dy_i} = 1 - \frac{dX(Y; \gamma_m)}{dy_i}.
\]

Summing (1) over all \( n \) firms gives:

\[
0 = n \left[ \alpha - \beta (X(Y; \gamma_m) + \gamma_m R) \right] - \sum_{i \in N} c_i - \beta \left[ X(Y; \gamma_m) - Y \right].
\]
Solving this for aggregate output gives:

\[ X(Y; \gamma_m) = \frac{n(\alpha - \beta \gamma_m R) - \sum_{i \in N} c_i + \beta Y}{\beta (n + 1)} \leftrightarrow \frac{dX(Y; \gamma_m)}{dy_i} = \frac{dX(Y; \gamma_m)}{dY} = \frac{1}{n + 1} \]  

(3)

since \( Y \equiv \sum_{i \in N} y_i \), and so \( dY/dy_i = 1 \). Using this expression in \( dx_i(Y; \gamma_m)/dy_i = [1 - dX(Y; \gamma_m)/dy_i] \) confirms that \( dx_i(Y; \gamma_m)/dy_i = n/(n + 1) > 0 \), as claimed.

The first part shows that competition in Stage 2 is in **strategic substitutes**: if firm \( i \) raises its output, then it is optimal for its rivals to cut back. The second part is the pro-competitive effect of forward contracting identified by Allaz & Vila (1993); Lemma 1 shows that it survives under the presence of renewables.

The key observation is that these responses are **state-independent**: they do not vary with the capacity utilization of renewables generation \( \gamma_m \), which has an impact on the levels of prices and quantities, but not on the strength of firms’ strategic responses at the margin.\(^4\)

### Equilibrium

The equilibrium is defined implicitly by the \( n \times (M + 1) \) first-order conditions for \( \{x^m_i\}^{n}_{i=1} \) in each of \( M \) states plus \( \{y_i\}^{n}_{i=1} \). Label this equilibrium \( \hat{x}^m_i = x_i(\hat{Y}; \gamma_m) \) and \( \hat{y}_i \) for each \( i \in N \), and thus \( \hat{X}_m = X(\hat{Y}; \gamma_m) \), and \( \hat{P}_m = p(\hat{Y}; \gamma_m) \).

**Lemma 2.** In equilibrium, firm \( i \) engages in forward contracting according to:

\[ \hat{y}_i = \frac{(n - 1)}{n} \sum_{k=1}^{M} \delta_k \hat{x}^k_i. \]

**Proof.** By (1), optimality in Stage 2 implies \((\hat{p}_m - c_i) = \beta(\hat{x}^m_i - \hat{y}_i)\), in equilibrium, for firm \( i \), and using this in the first-order condition for Stage 1 from (2) gives:

\[
0 = \beta \sum_{k=1}^{M} \delta_k \left\{ \hat{x}^k_i - \hat{y}_i \right\} - \hat{x}^k_i \left( 1 + \frac{dX_{-i}}{dx_i} \right) \left|_{\{x^k_i\}^{n}_{i=1}} \right| \frac{dx_i}{dy_i} \left|_{\{x^k_i\}^{n}_{i=1}} \right|
\]

\[ = \beta \sum_{k=1}^{M} \delta_k \left\{ \hat{x}^k_i - \hat{y}_i \right\} - \hat{x}^k_i \frac{n}{n + 1}, \]

where the second line uses Lemma 1. Further rearranging gives:

\[ 0 = \sum_{k=1}^{M} \delta_k \left\{ \frac{(n - 1)}{n} \hat{x}^k_i - \hat{y}_i \right\} \implies \hat{y}_i = \frac{(n - 1)}{n} \sum_{k=1}^{M} \delta_k \hat{x}^k_i, \]

as claimed, since \( \sum_{k=1}^{M} \delta_k \equiv 1 \). \( \blacksquare \)

Each firm would like to sell forward a fraction \((n - 1)/n\) of its subsequent output in each state, which exceeds 50% but falls short of complete contracting (Allaz & Vila, 1993). However, because of uncertainty about renewables intermittency during forward contracting, its optimal strategy here is to sell forward this fraction of its **expected** output.

\(^4\)This is a feature of the linear-quadratic setup of the model.
This degree of forward contracting is broadly in line with real-world practice: contract cover has ranged from 73 to 95% across the UK, New Zealand, and various US electricity markets (Anderson, Hua & Winchester 2007).

**Lemma 3.** The equilibrium output choices for each state $m$ and the equilibrium forward contracting position of firm $i$ are given by:

$$
\hat{y}_i = \frac{(n-1)}{\beta} \left[ (\alpha - c_i) - \frac{n^2}{(n^2+1)}(\alpha - \bar{\tau}) \right] - \frac{(n-1)}{(n^2+1)} R \sum_{k=1}^{M} \delta_k \gamma_k
$$

$$
\hat{x}_i^m = \frac{n}{\beta} \left[ (\alpha - c_i) - \frac{n^2}{(n^2+1)}(\alpha - \bar{\tau}) \right] - \frac{R}{(n+1)} \left[ \gamma_m + \frac{(n-1)}{(n^2+1)} \sum_{k=1}^{M} \delta_k \gamma_k \right],
$$

where $\bar{\tau} = \frac{1}{n} \sum_{i \in N} c_i$ is the (unweighted) average unit cost of firms.

**Proof.** See the Appendix. 

Note that firm $i$’s output $\hat{x}_i^m$ in state $m$ depends individually on the renewable load factor $\gamma_m$, while its forward position $\hat{y}_i$ can depend only on the average $\sum_{k=1}^{M} \delta_k \gamma_k$. Firm $i$ is indeed “active”, as assumed, in state $m$ as long as $\hat{p}_m > c_i \iff \hat{x}_i^m > \hat{y}_i$; its cost disadvantage cannot be too pronounced, $(\alpha - c_i) > \left[ \frac{n^2}{(n^2+1)} \right] (\alpha - \bar{\tau})$. A sufficient condition for all $n$ firms to be active in all $M$ states is that:

$$
R < \frac{(n+1)}{\beta} \left[ (\alpha - \max_i \{c_i\})^n - \frac{n^2}{(n^2+1)}(\alpha - \bar{\tau}) \right] \equiv \overline{R}.
$$

Lemma 3 leads to the following main results:

**Proposition 1.** More renewables competition:

(i) reduces the equilibrium volume of forward contracting by firm $i$, $d\hat{y}_i/dR < 0$;

(ii) leads to the equilibrium displacement of firm $i$’s production in each state $m$, $d\hat{x}_i^m/dR < 0$.

**Proof.** Follows by inspection of Lemma 3. 

Proposition 1 identifies the *forward-contracting effect* of renewables competition. More renewables displace incumbent producers according to the well-known merit-order effect. However, this makes the market less attractive to incumbent producers, which reduces their incentive to make forward commitments.

Renewables thus *directly* raise the intensity of competition in the wholesale market but they *indirectly* reduce the intensity of rivalry amongst incumbents.

**Proposition 2.** (i) More renewables competition raises the equilibrium price in state $m$ if and only if the forward-contracting effect outweighs the merit-order effect; this holds in all states of
the world for which renewables’ capacity utilization is sufficiently low:

\[
\frac{d\hat{p}_m}{dR} = -\frac{\beta}{(n+1)} \left( \gamma_m \frac{d\hat{Y}}{dR} \right) > 0 \iff \gamma_m < \left( -\frac{d\hat{Y}}{dR} \right) \iff \gamma_m < \frac{n(n-1)}{(n^2+1)} \sum_{k=1}^{M} \delta_k \gamma_k \equiv \gamma,
\]

while the equilibrium price falls in all other states, with \( \gamma_m \geq \gamma \).

(ii) More renewables competition decreases the average equilibrium price as measured by the forward price:

\[
\frac{d\hat{p}_f}{dR} = -\frac{\beta}{(n+1)} \left( 1 - \frac{n(n-1)}{(n^2+1)} \right) \sum_{k=1}^{M} \delta_k \gamma_k < 0.
\]

**Proof.** For part (i), the price impact of more renewables, in general, is given by:

\[
\frac{d\hat{p}_m}{dR} = \frac{\partial \hat{p}_m}{\partial R}_{\hat{Y} \text{ fixed}} + \frac{\partial \hat{p}_m}{\partial \hat{Y}}_{\hat{Y} = \hat{\gamma}} \frac{d\hat{Y}}{dR}.
\]

Since demand curve in state \( m \), at equilibrium, is \( \hat{p}_m = \alpha - \beta [\hat{X}_m + \gamma_m R] \), it follows that:

\[
\frac{d\hat{p}_m}{dR} \Big|_{\hat{Y} \text{ fixed}} = -\beta \left( \frac{d\hat{X}_m}{dR} \Big|_{\hat{Y} \text{ fixed}} + \gamma_m \right) = -\frac{\beta \gamma_m}{(n+1)} < 0,
\]

and

\[
\frac{\partial \hat{p}_m}{\partial \hat{Y}} \Big|_{\hat{Y} = \hat{\gamma}} = -\beta \frac{d\hat{X}_m}{d\hat{Y}} \Big|_{\hat{Y} = \hat{\gamma}} = -\frac{\beta}{(n+1)} < 0
\]

which both use (3), at equilibrium. Putting these results together in (4) yields:

\[
\frac{d\hat{p}_m}{dR} = -\frac{\beta}{(n+1)} \left( \gamma_m + \frac{d\hat{Y}}{dR} \right).
\]

Using the result for \( \frac{d\hat{Y}_i}{dR} < 0 \) from Proposition 1 confirms that:

\[
\frac{d\hat{Y}}{dR} = \sum_{i \in N} \frac{d\hat{Y}_i}{dR} = -\frac{n(n-1)}{(n^2+1)} \sum_{k=1}^{M} \delta_k \gamma_k < 0,
\]

and the claims follow. For part (ii), the equilibrium forward price equals the expected market price, and so:

\[
\hat{p}_f = \sum_{k=1}^{M} \delta_k \hat{p}_k \implies \frac{d\hat{p}_f}{dR} = \sum_{k=1}^{M} \delta_k \frac{d\hat{p}_k}{dR}
\]

Using (5) gives:

\[
\frac{d\hat{p}_f}{dR} = -\frac{\beta}{(n+1)} \left( 1 - \frac{n(n-1)}{(n^2+1)} \right) \sum_{k=1}^{M} \delta_k \gamma_k < 0,
\]

which proves the result since the term in brackets is positive for all \( n \geq 2 \).

Renewables can raise the electricity price. The merit-order effect is always present but weaker for states with lower \( \gamma_m \). The forward-contracting effect is equally strong because commitments are not state-contingent. So prices rise for “low” \( \gamma_m \), and fall for “high” values of \( \gamma_m \).
Specifically, price rises if \( \gamma_m < \varphi \sum_{k=1}^M \delta_k \gamma_k \) by the fraction \( \varphi \equiv n(n - 1)/(n^2 + 1) \in [\frac{2}{5}, 1) \).

With six incumbents, states with utilization below \( \varphi \approx 80\% \) of the average experience higher prices. In the binary case where renewables are either at capacity or inactive, the condition is always met in the inactive state (for any \( n \geq 2 \)).

Large spreads in renewables’ capacity factors are borne out in practice (Borenstein 2012; Pollitt & Anaya 2015). Averages for wind are typically \( \approx 30–40\% \) while they are as low as \( 10\% \) for solar. Peak capacity factors for wind can be above \( 80\% \) while utilization in Germany has been as low as \( 5\% \) on some days, with a zero contribution by solar.

4 Conclusion

Renewables competition can weaken the role of forward contracting in mitigating market power in wholesale electricity markets—and lead to higher prices in states of the world with strong intermittency.

These results would be robust to demand uncertainty in form of state-contingent \( \{\alpha_k\}_{k=1}^M \).
Similar to renewables output in the above, this would not affect strategic responses at the margin, so the comparative statics still hold. Renewables competition \( R > \overline{R} \) could induce exit of higher-cost incumbents, altering the set of firms \( N \). Exit raises prices across all states and reduces the degree of forward contracting—which would exacerbate the price-increasing effect.

Increasing marginal costs would reduce the degree of forward contracting, relative to the standard Allaz-Vila model with constant unit costs. This would likely dampen the above comparative statics but not overturn them. Risk-averse conventional generators also have a hedging motive for forward contracting. In general, more renewables could increase or reduce their risk exposure depending, e.g., on the correlation between renewables output \( \{\gamma_k\}_{k=1}^M \) and demand \( \{\alpha_k\}_{k=1}^M \) across states. In any case, the above results apply as long as risk aversion is not too pronounced.

The knock-on effects of renewables penetration on competition via forward commitments may deserve more attention from policymakers and analysts. These results should lend themselves naturally to empirical and experimental testing. Future research could also pursue a welfare analysis that incorporates the cost side of renewables investment as well as the social value of the carbon emissions reductions achieved.
Appendix

Proof of Lemma 3. The proof begins by determining the market-level equilibrium quantities for \( \widehat{X}_m \) and \( \widehat{Y} \), and then turns to deriving to the firm-level analogs. From (3), in equilibrium:

\[
\widehat{X}_m = \frac{n[(\alpha - \overline{c}) - \beta \gamma_m R] + \beta \widehat{Y}}{\beta(n + 1)},
\]

(7)

where \( \overline{c} \equiv \frac{1}{n} \sum_{i \in N_i} c_i \) is the (unweighted) average unit cost of firms. Lemma 2 implies \( \widehat{Y} = \frac{(n-1)}{n} \sum_{k=1}^{M} \delta_k \hat{X}_k \) at the market-level; using (10) repeatedly in it gives:

\[
\widehat{Y} = \frac{(n-1)}{n} \sum_{k=1}^{M} \delta_k \frac{n[(\alpha - \overline{c}) - \beta \gamma_k R] + \beta \widehat{Y}}{\beta(n + 1)} = \frac{(n-1)}{\beta n(n + 1)} [n \left( (\alpha - \overline{c}) - \beta R \sum_{k=1}^{M} \delta_k \gamma_k \right) + \beta \widehat{Y}],
\]

(8)

which uses \( \sum_{k=1}^{M} \delta_k \equiv 1 \). Solving (8) for \( \widehat{Y} \) yields:

\[
\widehat{Y} = \frac{n(n-1)}{\beta(n^2 + 1)} \left( (\alpha - \overline{c}) - \beta R \sum_{k=1}^{M} \delta_k \gamma_k \right).
\]

(9)

Finally, using (9) in (7) and solving out gives:

\[
\widehat{X}_m = \frac{n(\alpha - \overline{c}) \left[ 1 + \frac{(n-1)}{(n^2 + 1)} \right] - \beta n R \left[ \gamma_m + \frac{(n-1)}{(n^2 + 1)} \sum_{k=1}^{M} \delta_k \gamma_k \right]}{\beta(n + 1)} = \frac{\frac{n}{\beta(n+1)} \left( \frac{n(n+1)}{(n^2 + 1)} (\alpha - \overline{c}) - \beta R \left[ \gamma_m + \frac{(n-1)}{(n^2 + 1)} \sum_{k=1}^{M} \delta_k \gamma_k \right] \right)}{\beta(n + 1)}.
\]

(10)

Now turning to the firm-level results, the Stage 2 first-order condition (1) for firm \( i \) in state \( m \) implies that, in equilibrium, \( \widehat{x}_i^m = \hat{y}_i + (\alpha - c_i) / \beta - (\widehat{X}_m + \gamma_m R) \). Inserting (10) and rearranging gives:

\[
\widehat{x}_i^m = \hat{y}_i + \frac{(\alpha - c_i)}{\beta} - \gamma_m R - \frac{n}{\beta(n + 1)} \left[ \frac{n(n+1)}{(n^2 + 1)} (\alpha - \overline{c}) - \beta R \left[ \gamma_m + \frac{(n-1)}{(n^2 + 1)} \sum_{k=1}^{M} \delta_k \gamma_k \right] \right] = \hat{y}_i + \frac{1}{\beta} \left[ (\alpha - c_i) - \frac{n^2}{(n^2 + 1)} (\alpha - \overline{c}) \right] - \frac{R}{(n + 1)} \left[ \gamma_m - \frac{n(n-1)}{(n^2 + 1)} \sum_{k=1}^{M} \delta_k \gamma_k \right].
\]

(11)

Recalling from Lemma 2 that \( \hat{y}_i = \frac{(n-1)}{n} \sum_{k=1}^{M} \delta_k \hat{x}_i^k \), and using (11) in it repeatedly gives:

\[
\hat{y}_i = \frac{(n-1)}{n} \left\{ \hat{y}_i + \frac{1}{\beta} \left[ (\alpha - c_i) - \frac{n^2}{(n^2 + 1)} (\alpha - \overline{c}) \right] - \frac{R}{(n^2 + 1)} \sum_{k=1}^{M} \delta_k \gamma_k \right\} \Rightarrow \hat{y}_i = (n-1) \left\{ \frac{1}{\beta} \left[ (\alpha - c_i) - \frac{n^2}{(n^2 + 1)} (\alpha - \overline{c}) \right] - \frac{R}{(n^2 + 1)} \sum_{k=1}^{M} \delta_k \gamma_k \right\}.
\]

(12)

Finally, using (12) in (11) and solving yields the formula for \( \widehat{x}_i^m \), as claimed.
References


