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Keywords Unit commitment, nonconvex pricing, mixed integer programming, market design

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Pricing in Day-Ahead Electricity Markets with Near-Optimal Unit Commitment

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Abstract

This paper revisits some peculiar pricing properties of near-optimal unit commitment solutions. Previous work has found that prices can behave erratically even as unit commitment solutions approach the optimal solution, resulting in potentially large wealth transfers due to suboptimality of the solution. Our analysis considers how recently proposed pricing models affect this behavior. Results demonstrate a previously unknown property of one of these pricing models, called approximate Convex Hull Pricing (aCHP), that eliminates erratic price behavior, and therefore limits wealth transfers with respect to the optimal unit commitment solution. The absence of wealth transfers may imply fewer strategic bidding incentives, which could enhance market efficiency.

1 Introduction

Changes to traditional pricing methodologies in electricity markets continue to stir controversy. Wholesale electricity markets are often conceptualized as a uniform price auction, such that setting the price equal to the marginal system cost provides the correct incentives for all participants to produce and consume electricity at their socially efficient levels. However, uniform prices are not guaranteed to clear the market because of important nonconvexities in the production capabilities of many generating facilities [1]. These nonconvexities create the need for side payments to ensure that generators do not suffer financial losses by following the socially efficient schedule [2] as well as rules to discourage production from generators who are not part of the least-cost schedule. Thus, the crux of the pricing controversy is whether to adhere to the usual marginal pricing policy or if an alternative pricing scheme with somehow lower side payments or better incentives can be formulated and adopted.

Price formation issues attracted interest from FERC following severe weather events in the winter of 2014-2015 that highlighted the role of prices in aligning dispatch incentives, maintaining reliability, signaling

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efficient investments, and maximizing the market surplus [3]. A subsequent Notice of Proposed Rulemaking (NOPR) highlighted the inclusion or exclusion of nonconvexities in pricing methodologies, i.e., start-up and no-load operating costs, minimum output levels, and minimum run times. This NOPR proposed to create uniform "fast-start" pricing rules for resources with quick response times [4]. Such resources are typically "block-loaded," operated at full capacity or not at all, and thus unable to set prices when the normal marginal cost criterion is used. All ISOs currently implement some form of fast-start pricing, but to varying degrees based on their resource mix and compatibility with existing ancillary service markets [4–10]. Rather than pursuing uniform rules in all six ISOs and RTOs under FERC's jurisdiction, FERC concluded the NOPR by opening new dockets to examine specific pricing rules for New York Independent System Operator (NYISO), PJM Interconnection (PJM), and Southwest Power Pool (SPP) [4]. While these issues are relevant in both the US and Europe, US markets solve nonconvexities in a centralized fashion whereas European markets require participants to internalize nonconvexities in their offer. This paper focuses on the treatment of nonconvexities as now undertaken in US markets.

The main contributions in this paper stem from Gribik, Hogan, and Pope's initial paper on convex hull pricing [11]. We address a previously unappreciated issue concerning how convex hull pricing affects a problem with centralized unit commitment, first discussed by Johnson *et al.* [12] and later by Sioshansi *et al.* [13]. These papers discuss how the selection of a near-optimal solution may entail significantly different financial outcomes for individual market participants despite little or no gains in productive efficiency compared to the actual optimal solution. Our paper relates these previous results to two computationally simplified versions of convex hull pricing. These two pricing models result in appreciably different levels of stability of financial outcomes relative to outcomes in the optimal unit commitment solution. The results have significant implications in the ongoing electricity pricing debate and, to our knowledge, have not been recognized previously in the convex hull pricing literature.

The remainder of Section 1 provides additional background about some of the economic and mathematical issues within the electricity market scheduling problem, called unit commitment. Section 2 formulates a standard unit commitment model and three pricing models. Section 3 summarizes market outcomes for each pricing model using a set of near-optimal unit commitment solutions to a test case based on PJM [14] and Section 4 concludes the paper.

1.1 Background

So-called "fast-start" pricing [4] is a specific instance of the nonconvex pricing problem, for which there is generally no completely accepted method for pricing and settlements. Difficulties in resolving nonconvex pricing issues stem from the presence of '*lumpiness*' or '*indivisibilities*' in the production sets of electric generators [1]. There are a few common instances of this problem, for example, a generator that:

- has a minimum output constraint such that it cannot feasibly produce power at a level less than some threshold value, unless it produces exactly zero,
- incurs fixed costs that are required to begin producing power but are otherwise independent of the amount of power produced, or
- must remain on-line or off-line for a specified amount of time before shutting off or coming back on-line.

Rather than being rare or pathological examples, the above features are common to most power generating units. These nonconvexities can cause the need for side payments, "make-whole" payments, that are paid to participants who are part of the optimal unit commitment schedule but do not recover their offered cost through the uniform energy price.

The settlement method of paying the locational marginal price (LMP) together with make-whole payments to suppliers who would otherwise not cover their variable costs, traditionally used in US markets, is formally presented in [2]. One of the objections to this approach is that too many side payments can cause poor incentives for participants to reveal truthful offers to the ISO, similar to a pay-as-bid auction. Reduction or elimination of these side payments (which may be make-whole payments or may fall into a broader category of "uplift" payments) has been proposed through various optimization models [15–19] and equilibrium models [20–22]. Some of these methods ensure that the optimal primal solution is supported, but others can result in a changed and possibly suboptimal schedule [22]. Because market efficiency is a primary goal of federal policy, US regulators are unlikely to approve schemes of the latter sort. Of the alternatives to LMP, convex hull pricing has attracted the most attention. This approach minimizes lost opportunity costs as defined by a computationally intensive Lagrangian dual problem that consists of separable profit maximization problems [11].

Schiro *et al.* describe hurdles to implementation of convex hull pricing [23]. For example, its properties cannot be guaranteed because the convex hull prices are difficult to calculate accurately in realistic market scheduling problems [23]. Instead of solving the Lagrangian dual directly, we use a computationally efficient primal approach by implementing tight UC constraints from [24–26]. Tightening the unit commitment problem to approach a primal convex hull formulation can become increasingly complex as additional time periods and resource details are considered. Examples of such details include combined cycle gas turbine (CCGT) transitions, time-dependent start up costs, hot- and warm-start-up types, and other operational details that won't be the focus of this paper. We refer to [28] for additional methods to tighten the unit commitment formulation.

Time constraints for publishing results of the day-ahead market often prevent modern MIP software from providing a provably optimal unit commitment solution in large electricity markets. Instead, ISO scheduling software runs until the solution meets a so-called "MIP gap" tolerance, an upper bound that defines acceptably near-optimal solutions [29]. Pricing issues stemming from such near-optimal solutions were first identified by Johnson *et al.* [12]. Their paper shows that each near-optimal commitment schedule that might be chosen by an ISO results in a different set of prices and therefore causes wealth transfers that depend on the ISO's discretion. Later, Sioshansi *et al.* showed that the same problem occurs in more efficient mixed integer programming software similar to what markets use today [13].

The price behavior shown by Johnson *et al.* and Sioshansi *et al.* in theory undermines incentives for participation in the ISO's centralized unit commitment [13]. Prices may have limited value for signaling efficient market behavior when there are large price differences between various near-optimal solutions. If market participants become aware of favorable financial outcomes in a lower cost solution, then they could argue that the ISO's actual schedule was arbitrary and capricious since it resulted in lower profits. This has not yet been argued in legal proceedings, but it is not clear if this is because such claims would be insubstantial or because market participants lack awareness of what their financial outcome would have been if the ISO had selected the true optimal solution. Interestingly, similar real-world disputes are described in [29], stemming from low-cost renewable resources that should have been "in-the-money" but were not selected by the ISO's scheduling software.

Whether the unit commitment solution is optimal or not, some unscheduled resources may appear profitable given the ISO's posted prices, and these resources may have a profit maximizing strategy to hedge against the opportunity cost of not being dispatched by "self-committing" or "self-scheduling." Self-commitments entail scheduling only the minimum output (i.e., submitting an offer with zero fixed operating costs), while self-scheduling typically schedules the full capacity of a resource (i.e. submitting an offer with zero total cost). Self-commitments and self-schedules account for about 41% of all offers in PJM's day-ahead market, including about 10% of combined cycle gas turbine generation and 40% of steam turbine generation [30]. Self-commitments and self-schedules account for 78% of the committed resources in Midcontinent ISO (MISO) [31]. Many self-commitments may be the result of insufficient modeling of resource configurations [32], but some resources may be exhibiting strategic behavior that undermines the efficiency of the ISO's centralized scheduling process. An example is provided in the Appendix.

While this paper focuses on the day-ahead market, recent work has made progress analyzing how pricing methodologies affect long-term investment decisions [33–35]. Herrero *et al.* shows that a "linear" pricing rule, based on convex hull pricing, results in an equilibrium investment plan that is closer to the socially optimal plan than when using a "nonlinear" pricing rule based on the LMP and that this gap widens as start-up costs become a higher portion of total costs [33]. Vazquez *et al.* proposes a pricing model with scalable generators, similar to the "dispatchable" model proposed in [11], and shows that it results in an efficient investment plan where the LMP does not [34]. Mays *et al.* calculates equilibrium investment plans in a realistically sized test system and shows that the convex hull price comes closest to supporting the efficient long term investment plan [35]. However, the results in each of these papers [33–35] may be case-dependent, and Mays *et al.* provides a simple example to show that the results are not generally applicable [35].

Good market design is multifaceted and requires careful analysis and balancing of a wider range of issues than are discussed here. The analysis in this paper compares the standard LMP methodology and two computationally simplified versions of convex hull pricing with respect to the problems described by Johnson *et al.* [12] and Sioshansi *et al.* [13]. In particular, both papers state that the problems in electricity pricing stem from the use of centralized unit commitment. Our results show that the magnitude of this problem depends crucially on the pricing methodology employed and is not an immutable property of centralized unit commitment itself.

2 Models

2.1 Unit Commitment

Scheduling software used by ISOs solves a mixed integer program (MIP) for the optimal unit commitment schedule. Each day, ISOs collect bids and offers that define consumer valuations and producer costs, respectively, and are used to calculate price and quantity schedules that will maximize the total market surplus. Bids are assumed to be equal to the value of a consumer's power demand, and offers equal to the cost of a producer's power generation. Demand will be assumed to be fixed, so market surplus is maximized by minimizing the as-bid cost of accepted offers.

The formulation below is broadly similar to models used by ISOs [14], except that transmission constraints, operating reserves, and price responsive demand are excluded for simplicity. Some constraints have been reformulated to give tighter lower bounds when the problem's binary constraints are relaxed [24–26]. The following variables describe the feasible production schedules for a generator $g \in \mathcal{G}$ in time period $t \in \mathcal{T}$:

p_{gt}: Quantity produced by g in period t

w_{gt}: 1 if g operates in period t, or 0 otherwise

x_{gt}: 1 if g starts up in period t, or 0 otherwise

y_{gt}: 1 if g shuts down in period t, or 0 otherwise

We now introduce constraints to define the feasible region of the UC problem. The first constraint balances the total production quantity with fixed demand D_t at time t.

$$\sum_{g \in \mathcal{G}} p_{gt} = D_t, \qquad \forall t \in \mathcal{T} \tag{1}$$

Additional constraints will satisfy generator minimum up and down times, ramp rates, and minimum and maximum output. They depend on the values of the operating status w_{gt}, start-up x_{gt}, and shutdown y_{gt}, which are related by the following constraints.

$$x_{gt} - y_{gt} = w_{gt} - w_{gt-1} \qquad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{2a}$$

$$x_{gt}, y_{gt} \geq 0 \qquad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{2b}$$

When w_{gt} and w_{gt-1} are constrained to be binary, the constraints in (2) will also force x_{gt} and y_{gt} to take on only binary values. These binary variables help define nonconvex constraints.

Minimum up-time and down-time constraints require generators to remain on-line for at least T_g^{up} periods after a start-up and offline for at least T_g^{dn} periods after a shutdown.

$$w_{gt} \geq \textstyle\sum_{\tau \in (t - T_g^{up} + 1, t)} x_{g\tau} \qquad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{3a}$$

$$w_{gt} \leq 1 - \textstyle\sum_{\tau \in (t - T_g^{dn} + 1, t)} y_{g\tau} \qquad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{3b}$$

Production in adjacent time periods is constrained by generator ramp rate limits up and down, R_g^{up} and R_g^{dn}, respectively. Generator minimum output is typically higher than R_g^{up} or R_g^{dn}, so to ensure feasibility, we assume the generator's ramping capability is equal to $P_g^{\min}$ during a start-up or shutdown sequence. We implement the two-period ramp inequalities proposed in [24] as follows.

$$p_{gt} \leq p_{gt-1} + (P_g^{\min} + R_g^{up}) w_{gt} - P_g^{\min} w_{gt-1} - R_g^{up} x_{gt}, \quad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{4a}$$

$$p_{gt-1} \leq p_{gt} + (P_g^{\min} + R_g^{dn}) w_{gt-1} - P_g^{\min} w_{gt} - R_g^{dn} y_{gt}, \quad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{4b}$$

Generator output is constrained to zero when $w_{gt} = 0$, or between $P_g^{\min}$ and $P_g^{\max}$ when $w_{gt} = 1$. Additionally, the formulation proposed in [25] lowers the upper bound to $P_g^{\min}$ during a start-up period or before a shutdown period. The constraint is implemented differently depending on minimum up-time, for two generator sets $g \in \mathcal{G}^+ = \{\mathcal{G} | T^{up} \geq 2\}$ and $g \in \mathcal{G}^- = \{\mathcal{G} | T^{up} < 2\}$.

$$p_{gt} \leq P_g^{\max} w_{gt} - \left(P_g^{\max} - P_g^{\min}\right) x_{gt} - \left(P_g^{\max} - P_g^{\min}\right) y_{gt+1}, \qquad \forall g \in \mathcal{G}^+, t \in \mathcal{T} \tag{5a}$$

$$p_{gt} \leq P_g^{\max} w_{gt} - \left(P_g^{\max} - P_g^{\min}\right) x_{gt}, \qquad \forall g \in \mathcal{G}^-, t \in \mathcal{T} \tag{5b}$$

$$p_{gt} \leq P_g^{\max} w_{gt} - \left(P_g^{\max} - P_g^{\min}\right) y_{gt+1}, \qquad \forall g \in \mathcal{G}^-, t \in \mathcal{T} \tag{5c}$$

Lastly, we introduce constraints to define piece-wise linear generator costs, c_{gt}, using the formulation proposed in [26].

$$c_{gt} - \hat{C}_{gk} w_{gt} \geq \hat{M}_{gk} \big(p_{gt} - \hat{P}_{gk} w_{gt} \big), \qquad \forall g \in \mathcal{G},\ t \in \mathcal{T}, k \in \mathcal{K} \tag{6}$$

The cost function constraints are formulated in point-slope form with sampled cost $\hat{C}_{gk}$ and quantity $\hat{P}_{gk}$ values at each step k. Sampled costs reflect no-load costs as well as variable or fuel costs. The generator's marginal cost at each step is $\hat{M}_{gk} = (\hat{C}_{gk+1} - \hat{C}_{gk})/(\hat{P}_{gk+1} - \hat{P}_{gk})$. So long as M_{gk} is monotonically increasing in each step k, minimization of c_{gt} in the objective will result in a convex cost function [36]. The terms $\hat{P}_{gk} w_{gt}$ and $\hat{C}_{gk} w_{gt}$ ensure that cost is equal to zero in time periods when the generator is not operating and also shrink the size of each step in the piecewise linear curve when w_{gt} is allowed to take values between zero and one.

The unit commitment model is formulated below for a 24-hour day-ahead market. Although not written explicitly, intertemporal constraints (2)-(5) have been implemented with circular referencing such that hour 1 is assumed to follow hour 24. For example, a generator with a minimum run time of four hours that starts up in hour 24 will also be required to remain on in hours 1, 2, and 3. The formulation highlights the binary constraint for w_{gt}, but constraining additional binary variables x_{gt} and y_{gt} resulted in substantially faster solution times.

$$\textbf{UC model:} \quad \min \quad z = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (c_{gt} + F_g x_{gt}) \tag{7a}$$

$$\text{s.t.} \quad \text{Constraints (1)-(6)} \tag{7b}$$

$$w_{gt} \in \{0, 1\}, \qquad \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{7c}$$

The objective value z includes piece-wise linear costs c_{gt} plus start-up costs $F_g x_{gt}$. Shutdown costs can be included in F_g if necessary. The optimal solution to (7) is denoted with an asterisk, e.g., w^*_{gt}.

2.2 Pricing Models

Since model (7) is a MIP, there is no standard method to calculate shadow prices from a dual problem [36]. In contrast, the following three linear programs (LPs) are based on (7) but have well-defined dual programs. Each pricing model is solved subsequent to the execution and solution of the unit commitment model (7), and the resulting prices are determined from the pricing model's dual variables.

First, the model proposed in [2] fixes integer values to their optimal value. Prices can only be set by the marginal cost of scheduled resources, as is the usual practice in ISO markets.

$$\textbf{LMP model:} \quad \min \quad z = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (c_{gt} + F_g x_{gt}) \tag{8a}$$

$$\text{s.t.} \quad \text{Constraints (1)-(6)} \tag{8b}$$

$$w_{gt} = w^*_{gt}, \ \forall g \in \mathcal{G},\ t \in \mathcal{T} \tag{8c}$$

The following two models relax binary constraints and are approximations of the approach proposed in [11]. This allows prices to reflect fixed operating costs or the cost of operating at minimum output for generators that are part of the ISO's schedule (in the "restricted" rCHP model) or for any generator that submits an offer to the ISO (in the "approximate" aCHP model).

$$\textbf{rCHP model:} \quad \min \quad z = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (c_{gt} + F_g x_{gt}) \tag{9a}$$

$$\text{s.t.} \quad \text{Constraints (1)-(6)} \tag{9b}$$

$$0 \leq w_{gt} \leq w^*_{gt}, \ \forall g \in \mathcal{G}, \ t \in \mathcal{T} \tag{9c}$$

$$\textbf{aCHP model:} \quad \min \quad z = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (c_{gt} + F_g x_{gt}) \tag{10a}$$

$$\text{s.t.} \quad \text{Constraints (1)-(6)} \tag{10b}$$

$$0 \leq w_{gt} \leq 1, \ \forall g \in \mathcal{G}, \ t \in \mathcal{T} \tag{10c}$$

The prices in each respective model are equal to the value of the dual variables of constraint (1) and are denoted λ^*_t.

2.3 Price Supports

Due to the non-convex nature of unit commitment, it often occurs that there is no set of uniform, nondiscriminatory prices such that all participants can maximize their profits by following the socially optimal schedule [1,2]. In the absence of side payments, generators receive *quasi*-linear[1] profits π^*_g, hereafter referred to simply as "linear" profits.

$$\pi^*_g = \sum_{t \in \mathcal{T}} \left(\lambda^*_t p^*_{gt} - c^*_g - F_g x^*_{gt} \right) \tag{11}$$

A generator's lost opportunity cost, LOC_g, is the increase in linear profits if it optimizes its schedule given prices λ^*_t.

$$LOC_g = \max \quad \sum_{t \in \mathcal{T}} (\lambda^*_t p_{gt} - c_g - F_g x_{gt}) - \pi^*_g \tag{12a}$$

$$\text{s.t.} \quad \text{Constraints (2)-(6) for } g \tag{12b}$$

$$w_{gt} \in \{0, 1\}, \quad \forall t \in \mathcal{T} \tag{12c}$$

Offering lost opportunity cost payments may lead to revenue inadequacy in the ISO. Simple examples show that lost opportunity costs may be higher than the market surplus; Mays *et al.* [35] gives an example to this effect using a zero-cost generator with minimum output greater than total demand in the market. Payment of lost opportunity costs also allows more opportunities to exercise market power because potentially many participants would be eligible to receive such payments. These participants can offer below true costs to inflate their lost opportunity cost payment.

While full lost opportunity cost payments may be problematic, higher lost opportunity costs may imply, in theory: (1) stronger incentives for strategic behavior that would result in market inefficiency, and (2) the need for additional market rules that contribute to increased overhead costs. This paper's results include

[1]Quasi-linearity denotes that revenues, $\lambda_t p_{gt}$, are linear and and costs, $c_g + F_g x_{gt}$, are nonlinear, each with respect to production level p_{gt}.

lost opportunity cost figures to measure how well each price model supports the commitment schedules. An example of strategic self-commitment behavior that decreases market efficiency is provided in the Appendix.

LMPs are equal to the strictly-defined marginal cost to supply an additional unit of demand, given the submitted bids and offers. Standard duality analysis of simple examples reveals that the LMP provides the right incentives for all dispatched generators in the form of supporting prices; that is, the LMP is lower than the marginal cost of all generators dispatched at their minimum output level, higher than the marginal cost of generators dispatched to their maximum output level, and equal to the marginal cost of generators dispatched somewhere in between their minimum and maximum. However, lost opportunity costs may still arise if the generator can increase its profits by changing to a different commitment period.

Convex hull pricing, proposed by Gribik *et al.* [11], has the property of minimizing lost opportunity costs. True convex hull is defined by minimizing the duality gap of the the Lagrangian relaxation of (7) [11], but it must be approximated for its practical application [26]. The rCHPs and aCHPs may differ from the system marginal cost as calculated by the LMP model (8). While the often-cited property of convex hull pricing is minimization of uplift payments defined by lost opportunity costs, minimization of lost opportunity costs itself may be an important aspect of auction design. Minimization of lost opportunity costs may encourage truthful cost revelation from market participants [16]. In contrast, restricting the relaxation to committed resources, as in the rCHP model, is almost entirely motivated by the reduction in uplift payments, and the relation to auction design principles is less clear.

Dispatch incentives can support the ISO's schedule either through lost opportunity cost payments or by assessing penalties, but a preferred approach is difficult to determine. Lost opportunity cost payments provide an additional mechanism for participants to influence the revenue they receive from the ISO and therefore additional gaming opportuities. A generator may be able to anticipate when it will be dispatched down to accommodate an inflexible fast-start that sets the price. The generator may then, with market power, lower its offer cost to inflate the amount of its lost-opportunity cost payment. To avoid this, an alternative approach is to assess penalties to generators that deviate from the ISO's dispatch instructions. For example, a generator will have incentive to increase its dispatch if it is dispatched below its maximum despite its marginal cost being less than the current price, but might be penalized for doing so. ISO-NE's Chief Economist points out that a penalty approach could result in participants offering less flexibility to the market by self-scheduling more frequently or submitting a higher minimum output constraint, thereby forcing the market to dispatch them at inefficient levels [38]. Participants may attempt to avoid lost opportunity costs by one means or another, so the potential for strategic or uneconomic behavior must be considered.

Standard practice in ISOs is not currently to pay a generator's full lost opportunity costs, but only to guarantee that each generator's revenues cover its as-bid costs. This "make-whole" payment, MWP_g, is defined as follows.

$$MWP_g^* = [-\pi_g^*]_+ \tag{13}$$

The make-whole payment is equal to the increase in linear profit if the generator is more profitable by producing nothing. Since producing nothing is a feasible solution to (12), $MWP_g \leq LOC_g$. All financial outcomes (profits) in this paper will include make-whole payments in addition to the linear profits defined by the LMP, rCHP, or aCHP models. Actual profits will be distinguished from linear profit with a tilde.

$$\tilde{\pi}_g^* = \pi_g^* + MWP_g^* \tag{14}$$

3 Results

The unit commitment and pricing models are applied to a test case based on an August 2009 snapshot of the PJM generation fleet with 1,011 generators and fixed demands for 24 time periods [14]. The formulations were modified from [39] and solved in GAMS with CPLEX 12.5 on a personal laptop with a 2.30 GHz Intel Core i5-6200U CPU with 8 GB RAM. Table 1 summarizes attributes of the unit commitment model and Figure 1 summarizes the resulting optimal solution.

Table 1: Test Case Attributes

Time Periods (hours)	24
Generators	1,011
Capacity (GW)	181.2
Peak Load (GW)	93.8
Energy Demand (GWh)	1,833
Variables	121,321
Binary Variables	72,792
Constraints	291,211
Non-Zeroes	1,115,738

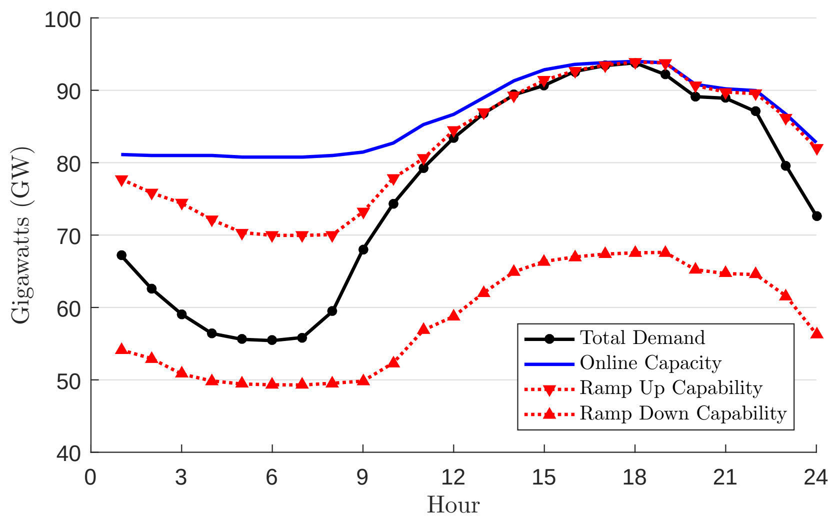


Figure 1: Solution Summary for 24-hour Unit Commitment Problem

The model was solved to complete optimality before filling a solution pool with 100 total integer solutions within a 0.1% optimality gap of the optimal solution, the same tolerance used by MISO [29], by specifying CPLEX options to emphasize and save integer solutions (`SolnPool, SolnPoolGap=0.001, SolnPoolPop 2, SolnPoolIntensity=4`).[2] The cost and optimality gap of each solution is provided in Table 2, ordered by descending optimality gap. Solutions 1-11 constitute the original branch-and-bound tree, denoted by an asterisk (*). Solutions 12-100 were found after the optimal solution. Solution 19 is an additional, alternative optimal solution. Although each solution is unique, many only contain a few minor differences because no diversity filter was used to determine which suboptimal solutions to include in the solution pool. That is,

[2]Default tolerances were also reduced due to the tight UC formulation (CPLEX options `epopt 1e-009, eprhs 1e-009`).

solutions 12-100 were merely found by continuing to explore the branch-and-bound tree and may not be representative of all solutions that are within 0.1% of the optimal solution.

In practice, ISOs solve their unit commitment problems within a predetermined MIP gap and not to full optimality. Thus, any solution in Table 2 would be a valid scheduling outcome if the ISO's software uses a 0.1% MIP gap stopping criteria. This paper differentiates the "optimality" gap that is calculated with respect to the true optimal solution (and shown in Table 2) with the "MIP" gap that is calculated during the branch-and-bound algorithm with respect to the best lower bound. The ISO's predetermined MIP gap tolerance is an upper bound to their actual optimality gap. The UC solution results show that the first integer solution is already close to optimal with a 0.023% MIP gap based on the lower bound found at the root node, and is within 0.018% of the true optimal solution.

3.1 Energy Payments and Side Payments

Summary statistics for total and individual make-whole payments and lost opportunity costs are shown in Table 3. Lost opportunity costs LOC_g are split into "on-line" if the generator is scheduled for at least one time period, or into "off-line" otherwise. Make-whole payments are only paid to scheduled units, so no distinction is made.

The aCHP model reduces total make-whole payments by about 50% compared to LMP. About the same number of participants receive make-whole payments given either pricing model, but the average individual make-whole payment is about half for aCHP compared to LMP. Despite this average behavior, LMP-based make-whole payments showed high variance and were actually lower than the aCHP make-whole payments in the majority of solutions. Decreasing the optimality gap generally corresponds to lowering the total LMP-based make-whole payments, albeit non-monotonically. The aCHP-based make-whole payments also behave non-monotonically but showed significantly less variance. Total lost opportunity cost for aCHP was the only price support to decrease monotonically with respect to the optimality gap.

The rCHP model raises prices enough to nearly eliminate the need for make-whole payments. However, higher prices lead to significantly higher lost opportunity costs (on-line and off-line) compared to the other two pricing models. A market design based on rCHP would therefore require additional mechanisms to address potential loss of market efficiency.

The aCHP model reduces total lost opportunity costs by about 25% compared to LMP. Although the aCHP model resulted in lower make-whole payments on average, it only resulted in lower make-whole payments in 19 of the 100 solutions. In other words, LMP often resulted in lower make-whole payments but also had a handful of solutions that required very high make-whole payments. Both models allocate about the same amount of lost opportunity costs to on-line units, but the difference is that the aCHP model allocates almost no lost opportunity costs to off-line units. That is, LMP and aCHP provide quantitatively similar incentives for scheduled resources while the aCHP model provides less incentive for unscheduled resources to self-commit. Lower offline lost opportunity costs may also help to avoid the type of unnecessary disputes described in [29], about whether a resource is "in-the-money".

Results also show that lost opportunity cost payments would vastly outstrip payments under the traditional make-whole payment scheme. This casts doubt into the practicality of adopting lost opportunity cost payments into a market design, or at least suggests that additional consideration be paid to the lost opportunity cost calculation. The calculation given by (12) would entitle over 40% of the market to some

Table 2: Unit Commitment Solutions

Soln.	Cost (k$)	Gap (k$ / %)		Soln.	Cost (k$)	Gap (k$ / %)	
27	29,517	28.324	0.0961%	87	29,489	0.575	0.0020%
26	29,517	28.315	0.0960%	100	29,489	0.556	0.0019%
90	29,512	23.078	0.0783%	76	29,489	0.544	0.0018%
79	29,504	15.671	0.0531%	82	29,489	0.528	0.0018%
85	29,504	15.626	0.0530%	37	29,489	0.487	0.0017%
78	29,503	14.432	0.0489%	75	29,489	0.455	0.0015%
84	29,503	14.387	0.0488%	61	29,489	0.449	0.0015%
47	29,502	13.599	0.0461%	81	29,489	0.438	0.0015%
53	29,502	13.092	0.0444%	28	29,489	0.413	0.0014%
88	29,501	12.668	0.0430%	74	29,489	0.401	0.0014%
29	29,500	11.431	0.0388%	57	29,489	0.385	0.0013%
99	29,498	9.426	0.0320%	80	29,489	0.385	0.0013%
46	29,494	5.952	0.0202%	66	29,489	0.371	0.0013%
45	29,494	5.604	0.0190%	93	29,489	0.371	0.0013%
*1	29,494	5.405	0.0183%	73	29,489	0.353	0.0012%
44	29,494	5.340	0.0181%	50	29,489	0.349	0.0012%
43	29,494	5.307	0.0180%	72	29,489	0.337	0.0011%
42	29,494	5.288	0.0179%	56	29,489	0.306	0.0010%
41	29,493	4.648	0.0158%	65	29,489	0.300	0.0010%
32	29,492	3.855	0.0131%	55	29,489	0.292	0.0010%
98	29,492	3.335	0.0113%	92	29,489	0.288	0.0010%
97	29,492	3.062	0.0104%	64	29,489	0.284	0.0010%
30	29,491	2.989	0.0101%	21	29,489	0.283	0.0010%
96	29,491	2.790	0.0095%	49	29,489	0.278	0.0009%
59	29,491	2.637	0.0089%	24	29,489	0.157	0.0005%
95	29,491	2.623	0.0089%	69	29,489	0.142	0.0005%
40	29,491	2.248	0.0076%	70	29,489	0.135	0.0005%
77	29,490	1.999	0.0068%	71	29,489	0.128	0.0004%
83	29,490	1.982	0.0067%	36	29,489	0.113	0.0004%
58	29,490	1.863	0.0063%	35	29,489	0.113	0.0004%
94	29,490	1.848	0.0063%	22	29,489	0.094	0.0003%
33	29,490	1.845	0.0063%	31	29,489	0.079	0.0003%
60	29,490	1.783	0.0060%	14	29,489	0.059	0.0002%
89	29,490	1.529	0.0052%	*6	29,489	0.059	0.0002%
23	29,490	1.469	0.0050%	34	29,488	0.041	0.0001%
68	29,490	1.404	0.0048%	15	29,488	0.036	0.0001%
52	29,490	1.382	0.0047%	*7	29,488	0.036	0.0001%
62	29,490	1.083	0.0037%	16	29,488	0.033	0.0001%
*2	29,489	0.940	0.0032%	*8	29,488	0.033	0.0001%
12	29,489	0.940	0.0032%	17	29,488	0.023	<0.0001%
*4	29,489	0.940	0.0032%	*9	29,488	0.023	<0.0001%
39	29,489	0.801	0.0027%	54	29,488	0.023	<0.0001%
25	29,489	0.786	0.0027%	91	29,488	0.019	<0.0001%
*3	29,489	0.762	0.0026%	63	29,488	0.015	<0.0001%
86	29,489	0.758	0.0026%	20	29,488	0.014	<0.0001%
13	29,489	0.752	0.0025%	*10	29,488	0.011	<0.0001%
*5	29,489	0.752	0.0025%	18	29,488	0.011	<0.0001%
38	29,489	0.635	0.0022%	48	29,488	0.009	<0.0001%
67	29,489	0.630	0.0021%	*11	29,488	-	-
51	29,489	0.608	0.0021%	19	29,488	-	-

Table 3: Price Support Statistics

	Payment	LMP	rCHP	aCHP
Market Total Mean (k$)	MWP	17.51	0.16	9.05
	LOC (total)	28.57	468.74	21.34
	LOC (on-line)	22.25	87.73	21.20
	LOC (off-line)	6.32	381.01	0.14
Market Total Std. Dev. (k$)	MWP	47.3	0.68	2.96
	LOC (total)	51.61	124.15	5.68
	LOC (on-line)	50.77	18.04	5.68
	LOC (off-line)	14.86	108.65	0.03
Market Total c.v.	MWP	2.70	4.25	0.33
	LOC (total)	1.81	0.26	0.27
	LOC (on-line)	2.28	0.21	0.27
	LOC (off-line)	2.35	0.29	0.21
Individual % Receiving	MWP	2.7%	0.2%	3.0%
	LOC (total)	12.9%	59.9%	43.4%
	LOC (on-line)	10.9%	47.7%	43.1%
	LOC (off-line)	2.0%	12.2%	0.3%
Individual Mean ($)	MWP	$2,010	$207	$956
	LOC (total)	$697	$2,460	$155
	LOC (on-line)	$644	$578	$155
	LOC (off-line)	$978	$9,805	$148

Table 4: Energy Payments (k$)

	Optimal	Mean	[max , min]	Range	c.v.
LMP	53,200	53,011	[55,869 , 48,171]	7,698	2.61%
rCHP	57,800	57,860	[60,726 , 55,545]	5,181	1.19%
aCHP	53,661	53,661	[53,661 , 53,661]	0	0.00%

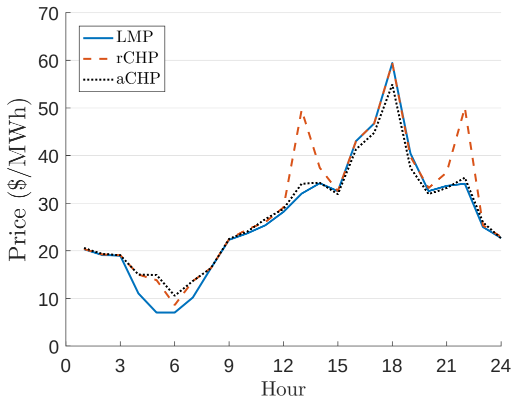
kind of side payment if the aCHP price is adopted, so a more restrictive calculation may be more reasonable.

3.2 Price Deviations

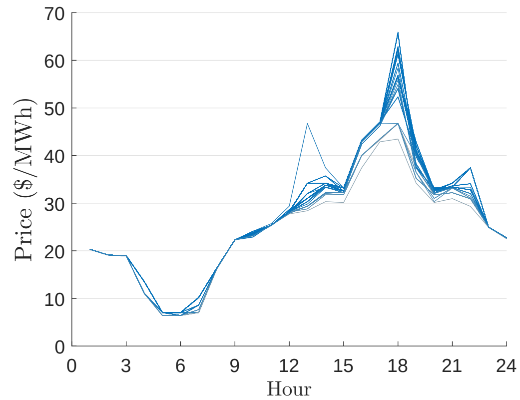
Total energy payments, exclusive of make-whole payments, are summarized in Table 4. These results show that the rCHP model led to a 9% increase in energy costs, or about $4.6 million. By contrast, the rCHP model's reduction in make-whole payments is no more than $300,000 in any LMP solution. The aCHP model resulted in similar total energy payments to the average LMP results. However, the LMP results showed high variance, replicating the results in [12] and [13]. Similar behavior is shown for the rCHP model, which has not been previously noted in the literature.

Prices for the optimal UC solution are shown in Fig. 2a, and for all 100 solutions in Figs. 2b-2d. Changes to the LMP price vector shown in Fig. 2b lead to a $7.7 million range of total energy payments, or about 25% of the system cost. For rCHP, the price vectors in Fig. 2c lead to a range of $5.2 million, or about 17% of the system cost. The aCHP model formulation (10) has no dependence on w^*_{gt} and therefore results in the same price for all solutions and no change in total energy payments in Table 4.

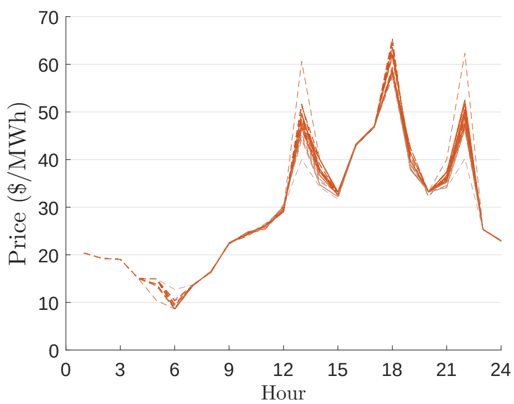
There were two main qualitative differences apparent when applying the three pricing models to the



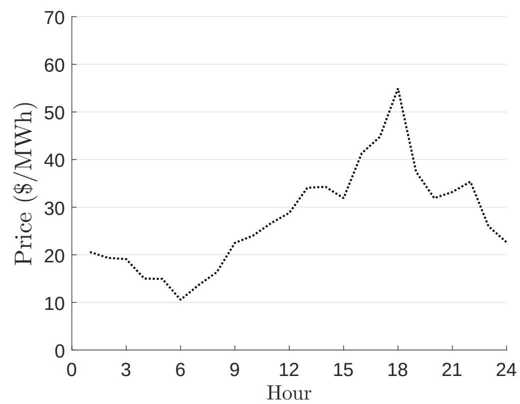
(a) Prices for the optimal solution



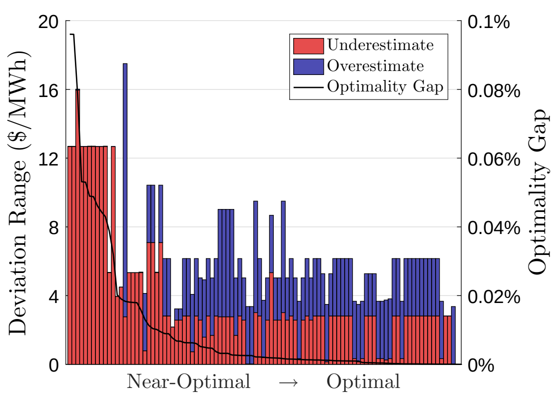
(b) LMP, all solutions



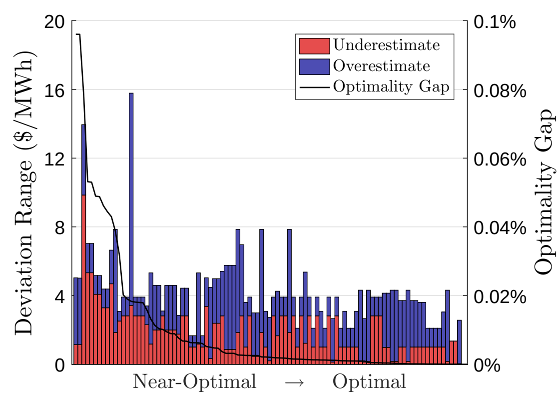
(c) rCHP, all solutions



(d) aCHP, all solutions



(e) LMP vs. Optimality Gap



(f) rCHP vs. Optimality Gap

Figure 2: Hourly prices (2a)-(2d) and deviation range per integer solution (2e),(2f)

optimal solution, shown in Fig. 2a. The LMP model results in lower prices during the early morning hours 4am-7am, and the rCHP model results in higher prices during ramp-up and ramp-down periods, roughly 1pm and 10pm. We note that some LMP solutions also display 1pm and 10pm price spikes, although this does not occur in the optimal solution. Although the aCHP model results in prices that are qualitatively more "moderate" than LMP or rCHP, the aCHP prices are not necessarily within the range of either LMP or rCHP prices.

Figs. 2e and 2f show the largest price deviations for LMPs and rCHPs compared the LMPs and rCHPs of the optimal solution, respectively. The price deviations are nonmontonic with the level of suboptimality, showing significant deviations even in solutions which have very small optimality gaps. In addition to the worst-case price deviations shown in Figs. 2e and 2f, the mean average percent deviations[3] (MAPE) for LMP, rCHP, and aCHP were 2.48%, 1.37% and 0%, respectively, when averaged across all 98 suboptimal solutions.[4] Like the deviation ranges, the MAPEs for each pricing model are nonmonotonic and not strongly related to the size of the optimality gap.

Lastly, note that price deviations are not obviously related to the narrowing optimality gap. LMPs show significant price deviations even in solutions which have very small optimality gaps (e.g., $< 0.005\%$). Like the make-whole payments before, this price deviation behavior is nonmonotonic and erratic.

3.3 Wealth Transfers Among Market Participants

Each near-optimal solution entails a series of wealth transfers between participants compared to financial outcomes in the optimal solution. The optimality gap represents a deadweight loss which removes market surplus without allocating it to another market participant. Changes to prices, commitment decisions, and make-whole payments result in additional wealth transfers. The transfers are related by the balance equation below.

$$z^{(s)} - z^* = \text{DW Loss}^{(s)} = \Delta\text{EnergyCost}^{(s)} + \Delta\text{UpliftCost}^{(s)} - \Delta\text{GenProfits}^{(s)} \tag{15}$$

These values are calculated for a specified price vector $\lambda_t^{(s)}$ (LMP, rCHP, or aCHP) in solution s, with associated make-whole payments $MWP_g^{(s)}$, linear profits $\pi_g^{(s)}$, and total cost $z^{(s)}$. Each component of (15) is defined and further subdivided below to accurately account for wealth transfers.

$$\Delta\text{EnergyCost}^{(s)} = \sum_t \left(\lambda_t^{(s)} - \lambda_t^*\right) D_t \qquad \text{where} \begin{cases} \text{Lower ECost}^{(s)} & = [\Delta\text{EnergyCost}^{(s)}]_- \\ \text{Higher ECost}^{(s)} & = [\Delta\text{EnergyCost}^{(s)}]_+ \end{cases} \tag{16}$$

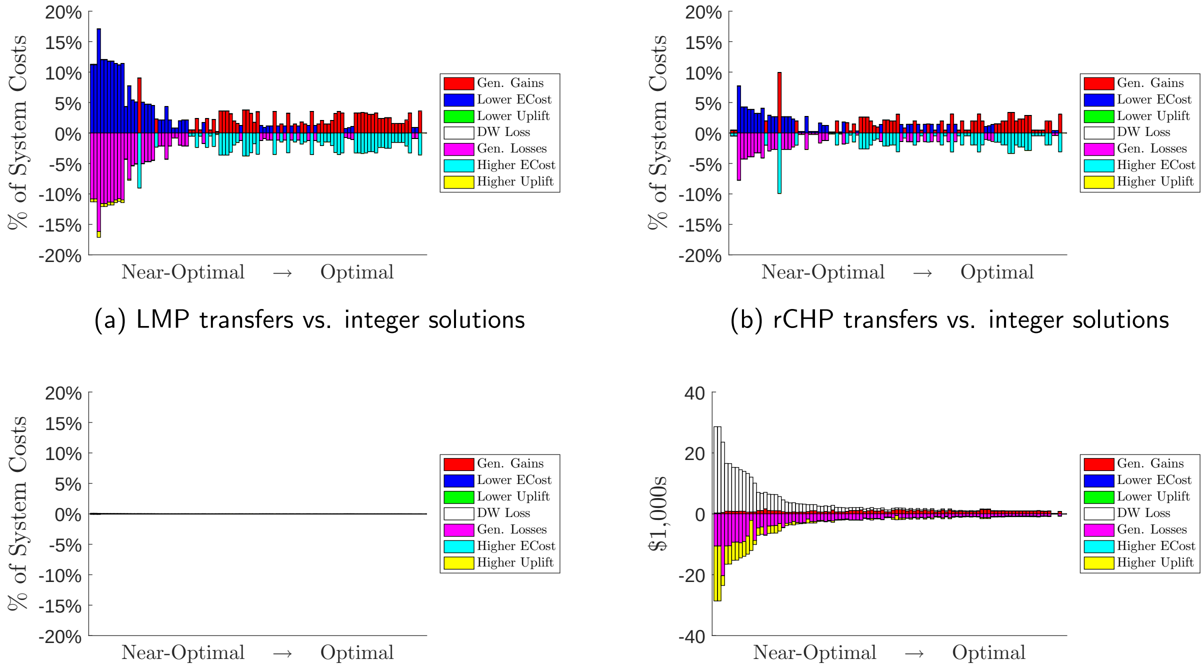
$$\Delta\text{UpliftCost}^{(s)} = \sum_g \left(MWP_g^{(s)} - MWP_g^*\right) \qquad \text{where} \begin{cases} \text{Lower Uplift}^{(s)} & = [\Delta\text{UpliftCost}^{(s)}]_- \\ \text{Higher Uplift}^{(s)} & = [\Delta\text{UpliftCost}^{(s)}]_+ \end{cases} \tag{17}$$

$$\Delta\text{GenProfits}^{(s)} = \sum_g \left(\tilde{\pi}_g^{(s)} - \tilde{\pi}_g^*\right) \qquad \text{where} \begin{cases} \text{Gen. Gains}^{(s)} & = \sum_g [\tilde{\pi}_g^{(s)} - \tilde{\pi}_g^*]_+ \\ \text{Gen. Losses}^{(s)} & = \sum_g [\tilde{\pi}_g^{(s)} - \tilde{\pi}_g^*]_- \end{cases} \tag{18}$$

The net change in consumer surplus is $\Delta\text{EnergyCost}^{(s)} + \Delta\text{UpliftCost}^{(s)}$, and the net change in producer surplus is $\Delta\text{GenProfits}^{(s)}$.

[3] Average percent deviation of all 24 hourly prices (LMP or rCHP) compared to the respective prices in the optimal UC solution.

[4] Solutions 11 and 19 were both optimal and resulted in the same LMPs and rCHPs.



(a) LMP transfers vs. integer solutions

(b) rCHP transfers vs. integer solutions

(c) aCHP transfers vs. integer solutions

(d) aCHP transfers vs. integer solutions, detail

Figure 3: Inter-solution wealth transfers

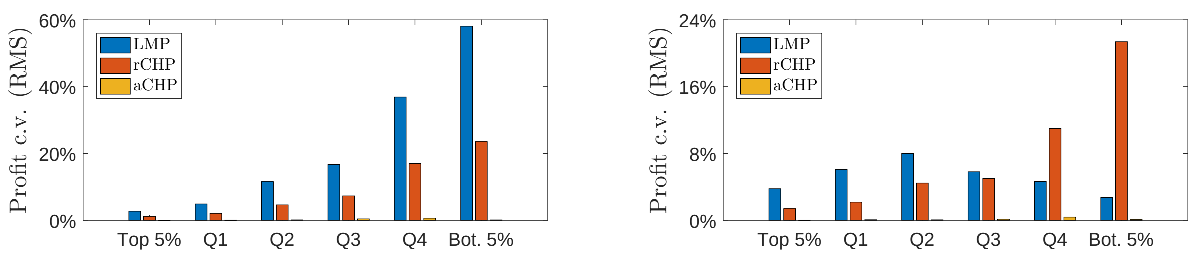
Fig. 3 shows each of these inter-solution wealth transfers relative to settlements in the optimal solution for each pricing model. From Figs. 3a-3c, it can be seen that the LMP and rCHP pricing models create very large wealth transfers, often 3-4% of the system cost, while aCHP has comparatively much smaller wealth transfers. The majority of transfers for LMP and rCHP are simply between consumers and generators as a result of price deviations.

The majority of transfers for LMP and rCHP are simply between consumers and generators as a result of price deviations. Table 5 compares solution 23 with solution 11 (i.e., the optimal solution) to exhibit typical transfers between generators with the LMP pricing model. Generators 102, 154, 816, 902 and 912 are a subset of the nine total generators whose commitment status has changed in any period between the two solutions. This is not shown due to space limitations, but generator 120 is only dispatched during hours 11-23 in the solution 11 and all 24 hours in solution 23. Low prices in the early morning hours do not cover it's additional operating costs in solution 24, so the sub-optimal solution reduces generator 102's profits by \$496. Generator 154, which is not part of the optimal solution, is dispatched during hour 18 in solution 23 and earns a profit of \$187. The remaining generators 816, 902, and 912 are each committed for an additional hour in solution 23 and earn additional profits of \$138, \$786, and \$148, respectively. These individual changes in profit are less than the optimality gap of \$1469, and are overshadowed by much larger wealth transfers caused by the change in prices.

In contrast, Fig. 3d shows that the aCHP wealth transfers occur primarily among generators. Unlike results for LMP and rCHP, these transfers for aCHP pricing are typically around the same order of magnitude as the optimality gap, and in fact, the optimality gap is the dominant source of wealth transfers in the most costly solutions.

Table 5: Generator Commitment and LMP Settlement Changes in Solutions 11 and 23

	Gen	Hour (Dispatch in MWh, LMP in $/MWh) 13	14	15	16	17	18	19	20	21	Profit
Soln. 11	102	120	120	120	127	127	127	127	120	120	$10,497
	154										$0
	816			30	44	44	44	30			$2,035
	902		94	154	223	223	223	172	94		$9,822
	912					30	44	30			$633
	LMP	$32.01	$34.19	$32.53	$43.04	$46.74	$59.42	$40.44	$32.53	$33.65	
Soln. 23	102	126	126	120	127	127	127	127	120	120	$10,001
	154						45				$187
	816		30	30	44	44	44	44	30		$2,173
	902		94	154	223	223	223	223	145	94	$10,608
	912				30	44	44	30			$782
	LMP	$34.20	$34.20	$33.19	$43.04	$46.74	$62.58	$40.77	$32.20	$33.39	



(a) Profitability Quartiles

(b) Capacity Quartiles

Figure 4: Aggregate c.v. by Generator Characteristics

Wealth transfers are not distributed randomly among market participants. Fig. 4 shows aggregate coefficient of variance (c.v.) of individual generator profits. Fig. 4a groups generators by profitability quartiles and Fig. 4b by capacity quartiles. Individual generator c.v. statistics are aggregated for each quartile using the root-mean square (RMS).

$$\text{c.v.}(\mathcal{Q}, \star) = \frac{\sqrt{\sum_{g \in \mathcal{Q}} (\sigma_g^\star)^2}}{\sum_{g \in \mathcal{Q}} \mu_g^\star} \ , \quad \mathcal{Q} \subset \mathcal{G} \ , \ \star \in \{\text{LMP, rCHP, aCHP}\} \tag{19}$$

High c.v. values imply that generator profits are highly sensitive to the particular unit commitment solution and pricing model. Generators with lower profits could be expected to have a high c.v. simply due to lowering the c.v.'s denominator, and this is exactly what is shown for LMP and rCHP in Figure 4a. In comparison, aCHP's c.v. remained low even among the least profitable generators.

Near-optimal unit commitment solutions pose some risk for small resources that may remain unscheduled by the ISO merely because their benefits to the system are smaller than the size of the optimality gap. Results in Figure 4b show that this problem is exacerbated when using rCHP. Profit variances for LMP are actually largest for resources in the second capacity quartile. The variance in profits for aCHP remained low regardless of generator capacity.

4 Conclusion

It has long been recognized that sub-optimal solutions can have significant distributional implications in markets with nonconvexities [12, 13]. What hasn't been explored is whether those implications are very different under the various pricing systems that have been proposed as alternatives to the traditional LMP. This paper is the first to compare pricing and income distribution outcomes of near-optimal unit commitment solutions for two alternative practical approximations of convex hull pricing.

One model, which we call aCHP, is a convex primal formulation that approximates convex hull pricing by relaxing all binary constraints regardless of the UC solution. The main result in this paper is that the aCHP model results in *de minimus* wealth transfers among near-optimal UC solutions. Traditional LMP pricing models do not possess this property. Small wealth transfers and the lower lost opportunity costs among participants may help reduce strategic opportunities for generators to self-schedule or self-commit and avoid unnecessary disputes about the ISO's schedule.

The relative stability of aCHP model's financial outcomes therefore appear to diminish incentives for gaming, which may yield some modest efficiency gains in the day-ahead electricity market. Due to the size of wholesale electricity markets, even efficiency gains that are less than 1% of total production costs could be significant in the absolute amount of savings. The small relative savings and the computational complexity involved in analyzing bidder behavior make it difficult to determine the magnitude of the possible efficiency gains in a realistic test case. Thus, whether newly proposed electricity pricing methodologies should be adopted remains an open question. Improved computational or empirical[5] approaches to perform such analysis may be a valuable area for future work.

Appendix

A Examples of Self-Commitment Equilibrium

Optimal self-commitment decisions for nonconvex generators are analyzed in the following two examples. The following will be assumed: (1) the nonconvex generators are block loaded, so that, if committed, then its minimum operating level is equal to its maximum operating level, (2) further, there is no distinction between self-scheduling and self-committing, (3) generators are only able to behave strategically only with respect to their decision to self-commit, in which case they offer to the market at zero cost and forego the possibility of receiving any make-whole payment, and (4) all generators are owned separately. If a generator does not self-commit, then it is assumed that it will offer its true costs.

The first example looks at a market with fixed demand and three generators with distinct costs and production capabilities. Price dynamics of LMP ensure that self-scheduling is not part of any generator's optimal strategy. The second example creates a replicated economy by including five of each generator type and multiplying demand by five. Growing the market in this way creates opportunity for generators to strategically self-commit. The behavior can be characterized in a mixed-strategy Bayesian Nash equilibrium.

[5] An anonymous referee pointed out that the implementation of the new Integrated Single Electricity Market (I-SEM) in Ireland may provide a natural experiment concerning the effect of alternative market structures on the bidding of commitment and other costs.

A.1 Simple Example

Demand and generator characteristics for the first example are provided in Table 6, and its only two feasible integer solutions are provided in Table 7. The three generator types are: Gen1, Gen2, and Gen3. Gen1 is nonconvex due to being block loaded, while Gen2 and Gen3 are convex generators with simple linear cost curves. A small increment ϵ is added to demand to avoid issues with degeneracy as well as to make easier identification of the price-setting generator. However, this ϵ will be ignored when calculating costs.

Table 6: Simple Market Characteristics, Single Period

Resource Parameters	Min. Qty.	Max. Qty.	Cost
Gen1 Start-up, x_1	0	1	\$0
Gen1 Output, p_1	$25x_1$	$25x_1$	$\$15p_1$
Gen2 Output, p_2	0	25	$\$10p_2$
Gen3 Output, p_3	0	25	$\$25p_3$
Demand, D	$45+\epsilon$	$45+\epsilon$	N/A

Table 7: Simple Market Optimal Schedules

Solution:	Gen1 Startup, $x_1 = 1$	Gen1 Startup, $_x1 = 0$
Gen1 Output, p_1	$20+\epsilon$ MWh	0 MWh
Gen2 Output, p_2	25 MWh	25 MWh
Gen3 Output, p_3	0 MWh	$20+\epsilon$ MWh
Gen1 Cost	\$320	\$0
Gen2 Cost	\$250	\$250
Gen3 Cost	\$0	\$500
Total Cost, z	\$570	\$750
LMP, λ	\$10/MWh	\$25/MWh

Table 7 shows that this market has two integer solutions. In the first solution, Gen1 is started, sets the LMP and requires a \$100 make-whole payment to cover its start-up costs. In the second solution, Gen1 is not started but is replaced by Gen3, which sets the LMP at \$25/MWh and does not require a make-whole payment. The first solution is optimal. The second solution is suboptimal and Gen1 sees that it would have been profitable at a price of \$25/MWh, resulting in a lost opportunity cost of (25-11)25-100 = \$250.

Because of the focus on self-commitment, it will be assumed that the possibility for strategic behavior only exists for Gen1. Gen2 and Gen3 are convex and are therefore less likely to self-commit or self-schedule.

If the first solution is selected, no generator is able to improve its profits at the given price, so there is no incentive to self-schedule. If the second solution is selected (erroneously) by the ISO, Gen1 could conceivably self-schedule to 20 MW. This would keep the price at \$25/MWh but would be physically infeasible for the block-loaded generator. If Gen1 self-commits its entire 25 MW, then the price would fall to \$10/MWh, the marginal cost of Gen2, and Gen1 would not be eligible to receive a make-whole payment to cover its losses. Thus, there is no optimal decision to self-schedule in the small market given by Table 6.

A.2 Replicated Example

Demand and generator characteristics for the replicated economy are provided in Table 8. Replicated generators are denoted by $n \in \{1, \ldots, 5\}$. Assuming truthful marginal cost offers, optimal schedules are provided in Table 9 with explicit reference to the number of Gen1s that self-commit, given by N.

Table 8: Replicated Market Characteristics, Single Period

Resource Parameters $n \in \{1, \ldots, 5\}$	Min. Qty.	Max. Qty.	Cost
Gen1 Start-up, $x_{1(n)}$	0	1	\$0
Gen1 Output, $p_{1(n)}$	$25x_{1(n)}$	$25x_{1(n)}$	$\$15p_{1(n)}$
Gen2 Output, $p_{2(n)}$	0	25	$\$10p_{2(n)}$
Gen3 Output, $p_{3(n)}$	0	25	$\$25p_{3(n)}$
Demand D	$225 + \epsilon$	$225 + \epsilon$	N/A

Table 9: Replicated Market Optimal Schedules

Gen1 Self-Commits	$N < 5$	$N = 5$
Gen1 Start-ups, $\sum_n x_{1(n)}$	4	5
Gen1 Output, $\sum_n p_{1(n)}$	100 MW	125 MW
Gen2 Output, $\sum_n p_{2(n)}$	125 MW	$100 + \epsilon$ MW
Gen3 Output, $\sum_n p_{3(n)}$	ϵ MW	0 MW
Gen1 Cost as Offered	\$375(4-N)	\$0
Gen2 Cost as Offered	\$1250	\$1000
Gen3 Cost as Offered	\$0	\$0
Obj. Cost, z	$\$(2750 - 375N)$	\$1000
Actual Cost, $\hat{z}$	\$2750	\$2875
LMP, λ	\$25/MWh	\$10/MWh

The optimal unit commitment is simple enough to solve by hand. Self-committed units are considered "free" to the ISO's scheduling software and are each scheduled to their maximum output. When $N < 5$, Gen2s are dispatched to full capacity and four Gen1s are committed. The last ϵ demand is cheaper to serve with Gen3 than committing the last Gen1 and dispatching down a Gen2 by $25 - \epsilon$. When $N = 5$, all Gen1s are committed and the Gen2s are collectively dispatched to $100 + \epsilon$, less than their aggregate capacity.

The market described in Table 8 has five socially optimal solutions: each which entails not scheduling one Gen1 but scheduling the other four Gen1s to their maximum output. Of the five Gen1s in the market, only four can be part of the socially optimal solution. Therefore, we will assume that there is a probability of $(4 - N)/(5 - N)$ that a Gen1 is not selected, given $N \in \{0, \ldots, 4\}$. The market has an LMP of \$25/MWh when $N < 5$ or \$10/MWh if $N = 5$. Moving from one optimal solution to the other creates a \$250 wealth transfer from the decommitted Gen1 to the newly committed Gen1 and results in a \$250 lost opportunity cost for the now decommitted Gen1.

It is profitable for Gen1 to self-commit in the replicated market, and this behavior can be described in a mixed strategy Nash equilibrium. Each Gen1's expected profit depends on two things: (1) its own decision to self-commit (no-SC or SC), and (2) the total number of Gen1s that are self-committed ($N = \{0, 1, 2, 3, 4, 5\}$). A Gen1 strategy is defined as the probability that the generator decides to self-commit and is denoted α_1.

Table 10: Expected Profits Given Self-Scheduling Decisions

Event, $\omega \in \Omega$	$Pr(\omega)$	$E[\pi_{1(n)}\|\omega]$
no-SC, $N = 0$	$(1-\alpha_1) \times (1-\alpha)^4$	\$200.00
no-SC, $N = 1$	$(1-\alpha_1) \times 4(1-\alpha)^3\alpha$	\$187.50
no-SC, $N = 2$	$(1-\alpha_1) \times 6(1-\alpha)^2\alpha^2$	\$166.67
no-SC, $N = 3$	$(1-\alpha_1) \times 4(1-\alpha)\alpha^3$	\$125.00
no-SC, $N = 4$	$(1-\alpha_1) \times \alpha^4$	\$0.00
SC, $N < 5$	$\alpha_1 \times (1-\alpha^4)$	\$250.00
SC, $N = 5$	$\alpha_1 \times \alpha^4$	-\$125.00

Each Gen1 has identical characteristics to the other Gen1s, so it can be assumed that the other Gen1s all choose the same mixed strategy α. We maintain a distinction between α_1 and α to emphasize that the Gen1s are competing and thus do not coordinate their strategies (i.e. collude). The expected profits for each combination of events is given in Table 10. Note that "no-SC" and "$N = 5$" are mutually exclusive because $N = 5$ entails self-scheduling each Gen1. The probabilities in Table 10 are given by the binomial distribution and result in the following expected profit function.

$$\begin{aligned}E[\pi_1|\Omega] = (1-\alpha_1)&\left[200(1-\alpha)^4 + 750(1-\alpha)^3\alpha + 1000(1-\alpha)^2\alpha^2 + 500(1-\alpha)\alpha^3\right] \\ &+ \alpha_1\left[250(1-\alpha^4) - 125\alpha^4\right]\end{aligned}$$

The first order condition for $E[\pi_1|\omega]$ with respect to α_1 is,

$$\frac{\partial E[\pi_1]}{\partial \alpha_1} = -200(1-\alpha)^4 - 750(1-\alpha)^3\alpha - 1000(1-\alpha)^2\alpha^2 - 500(1-\alpha)\alpha^3 + 250(1-\alpha^4) - 125\alpha^4 = 0$$

Solving the fourth-order polynomial results in two real solutions and two imaginary solutions. The only valid solution to the first order condition is $\alpha = 0.831$. The other real solution is $\alpha = -0.548$ and can be disregarded along with the imaginary solutions since they are outside the possible range of α, from 0 to 1. The Gen1 strategies are assumed symmetrical, so we also have $\alpha_1 = 0.831$. The expected profit at equilibrium is $E[\pi_1|\alpha = 0.831] = \$71.53$, less than half of the optimal coordinated (collusive) strategy $E[\pi_1|\alpha = 0] = \$200$. There is no incentive to deviate when each Gen1 chooses this strategy since $E[\pi_1|\text{no-SS}, \alpha = 0.831] = E[\pi_1|\text{SS}, \alpha = 0.831] = \$71.53$.

If we suppose all other Gen1s choose the strategy $\alpha = 0$, then a strategic Gen1's will choose $\alpha_1 = 1$ and will increase expected profits from \$200 to \$250. Thus, this strategy is not a Nash equilibrium. On the other hand, considering the strategies $\alpha_1 = 0$ and $\alpha = 1$ reveals an asymmetric Nash equilibrium. Since the generator with strategy $\alpha_1 = 0$ is arbitrary, this represents five asymmetric equilibria in addition to the mixed strategy equilibrium. While these alternative equilibria satisfy the Nash criteria, they may be unsustainable if the Gen1 with $\alpha_1 = 0$ can anticipate a change in its competitor's strategies after it begins a new strategy $\alpha_1 > 0$. Additional asymmetric equilibria are also possible but will be ignored in this example.

Finally, consider the market outcome if the ISO calculates the convex hull price (CHP) instead of the LMP. Instead of \$25/MWh, the CHP is set by a partial commitment of an uncommitted Gen1. Relaxing x_1 sets the price at \$15/MWh. The Gen1s break even at this price and therefore have no self-scheduling incentives, so we will assume $N = 0$ and the ISO selects the socially efficient schedule. The price is higher than the marginal cost of Gen2, which is dispatched to its maximum, and lower than the marginal cost of Gen3, which is only dispatched to ϵ. The make-whole payment required by Gen3 is negligible.

Market efficiency implications are summarized in Table 11. The only event that causes a suboptimal schedule is if all 5 Gen1s self-commit, given by the probability $Pr(N = 5)$. System costs are taken from Table 9: \$2750 if the schedule is efficient ($N < 5$) and \$2789.53 if all Gen1s self-commit ($N = 5$). Similarly, prices are \$25/MWh (if $N < 5$) and \$10/MWh (if $N = 5$). In the replicated market example there is a 1.44% increase in costs and 27.14% increase in price, both in expectation, due to the optimal Gen1 self-commitment mixed strategy. Since make-whole payments are negligible, the expected cost to consumers also increases by 27.14%.

The self-commitment incentives of non-convex generators reveals the following non-intuitive results:

- Market power can be exercised by becoming a price taker.
- Participants can collude by offering truthfully.
- Marginal cost pricing can result in economic inefficiency.

While these results do not consider strategic marginal cost offers, it is reasonable to assume that doing so only would make the auction results appear less efficient. Growing the market further, letting $n \to \infty$, would serve to increase competitive behavior with respect to marginal cost offers. However, strategic self-commitment behavior would remain due to the presence of uncommitted Gen1s when a Gen3 sets the LMP. Lost opportunity costs represent the incentive of these uncommitted generators to self-commit, and such incentives can be lessened by the adoption of convex hull pricing.

Table 11: Efficiency in the Replicated Market

Price model	$Pr(N < 5)$	$Pr(N = 5)$	Cost, $E[\hat{z}]$	Price, $E[\lambda]$
LMP	0.395	0.605	\$2789.53	\$19.07/MWh
CHP	1.000	0.000	\$2750.00	\$15.00/MWh
Diff. (%)			1.44%	27.14%

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