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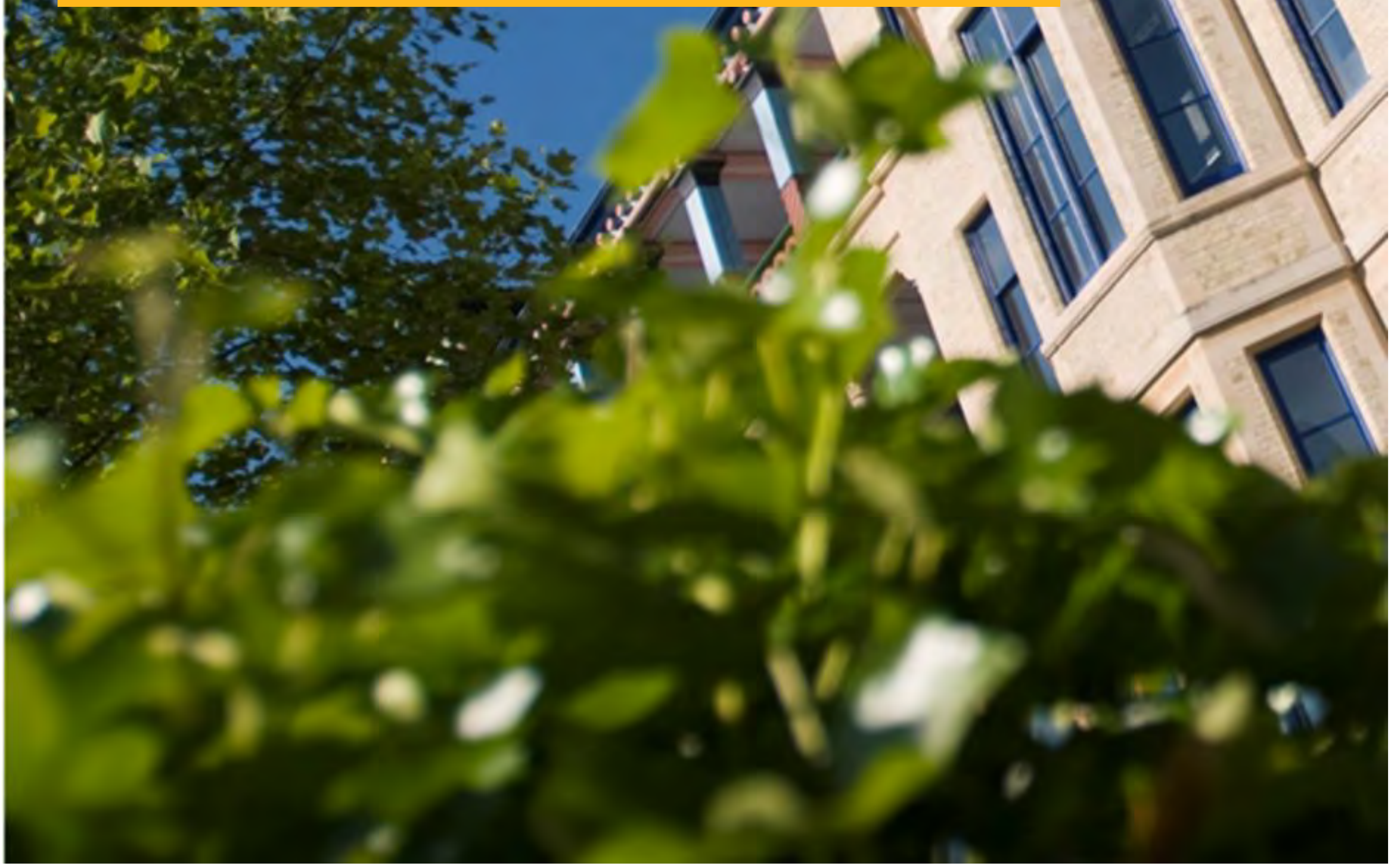
OIL, GAS, PANDEMICS, AND WAR: THE DRIVERS OF INFLATION

Luisa Corrado , Stefano Grassi , Aldo Paolillo and Francesco Ravazzolo



Energy Policy Research Group

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Abstract : We study how the COVID-19 pandemic and Russia's invasion of Ukraine reshaped energy prices and macroeconomic conditions in the Euro area. We develop and estimate a two-sector model in which oil, coal, and gas are combined to produce refined energy used by households and firms. The model allows for complementarities between energy and non-energy inputs, so shocks to individual energy markets propagate broadly through produc tion, consumption, and inflation. Focusing on shocks specific to oil, coal, and gas from the onset of the pandemic to 2022:Q3, we find that they raised energy inflation by about 36 percentage points and headline inflation by 1.8 percentage points. Complementarities, wage indexation, and monetary policy amplify these effects, while subsidies offset them only partially.

Keywords : Fossil energy; supply shocks; inflation; complementarities; monetary policy; fiscal policy.

JEL Classification : E31, E32, E52, E62, Q43

Oil, Gas, Pandemics, and War: The Drivers of Inflation*

By Luisa Corrado†, Stefano Grassi‡, Aldo Paolillo§ and Francesco Ravazzolo¶

We study how the COVID-19 pandemic and Russia's invasion of Ukraine reshaped energy prices and macroeconomic conditions in the Euro area. We develop and estimate a two-sector model in which oil, coal, and gas are combined to produce refined energy used by households and firms. The model allows for complementarities between energy and non-energy inputs, so shocks to individual energy markets propagate broadly through production, consumption, and inflation. Focusing on shocks specific to oil, coal, and gas from the onset of the pandemic to 2022:Q3, we find that they raised energy inflation by about 36 percentage points and headline inflation by 1.8 percentage points. Complementarities, wage indexation, and monetary policy amplify these effects, while subsidies offset them only partially.

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I. Introduction

On 26 August 2022, natural gas in Europe reached a record price of 342 euros (EUR) per megawatt-hour (MWh).[1] Less than a year earlier, the price was only 50 EUR per MWh (EUR/MWh), and from 2010 to 2020, it was between 8 and 32 EUR/MWh. This unprecedented price increase followed Russia's invasion of Ukraine on February 24, 2022.[2] Similar price movements affected other energy commodities as well, see Figure 1. Two

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[1] As measured by the Dutch TTF Natural Gas Futures, which is Europe's benchmark for natural gas prices. The series is available at `https://tradingeconomics.com/commodity/eu-natural-gas`.

[2] For an assessment of the energy market in time of war, see Pollitt (2022).

weeks after the beginning of hostilities, oil, coal, and gas prices had risen by approximately 40%, 130%, and 180% respectively, see Adolfsen et al. (2022). In the Euro area (EA), these increases in crude energy prices were rapidly transmitted to retail energy prices. The Harmonized Index of Consumer Prices (HICP) for energy products increased by approximately 57% between 2020:Q1 (normalized to one) and its peak in 2022:Q4, as shown in Figure 2.

These developments occurred while the EA economy was recovering from the COVID-19 pandemic, one of the most severe and rapid recessions in its history. Real GDP declined by about 11% in 2020:Q2 relative to 2020:Q1 and then rebounded sharply in 2020:Q3, almost returning to its pre-pandemic level, as shown in Figure 2. As emphasized by Cardani et al. (2022) and Ferroni et al. (2024), this episode differed from standard recessions because it reflected the closure and subsequent reopening of the economy.

The recovery was also unusual. By 2022:Q3, real GDP had increased by approximately 7% relative to 2020:Q1, while the HICP had risen by about 9% over the same period, marking the most pronounced inflationary episode since the introduction of the euro.

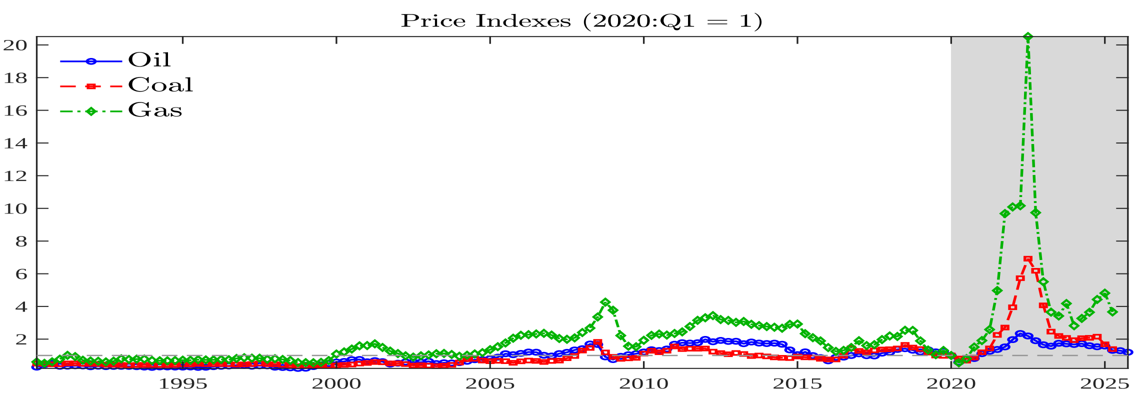

*Figure 1. : **Price Indexes of the Three Main Fossil Energy Sources**. The figure reports oil (solid blue line with circles), coal (dashed red line with squares), and natural gas (dash-dotted green line with diamonds). All series are normalized to one in 2020:Q1. The oil price is measured by the Brent crude oil price (DCOILBRENTEU) from the Federal Reserve Bank of St. Louis; the coal price is the global coal price (Australia) from the International Monetary Fund; and the gas price is the European Union natural gas price from the International Monetary Fund. See Section III for further details on data sources.*

This paper studies the combined effects of the pandemic and energy shocks and the channels through which they propagated to prices and economic activity. We propose a closed-economy two-sector Dynamic Stochastic General Equilibrium (DSGE) model with a core sector and an energy sector. In addition to much of the earlier literature, which typically focuses on oil alone (Kim and Loungani, 1992), our model includes oil, coal, and gas. This distinction matters for economies such as the EA, where refined energy is produced from an energy mix rather than a single primary source. On the supply side, the energy sector combines crude energy inputs (oil, coal, and gas) to produce refined

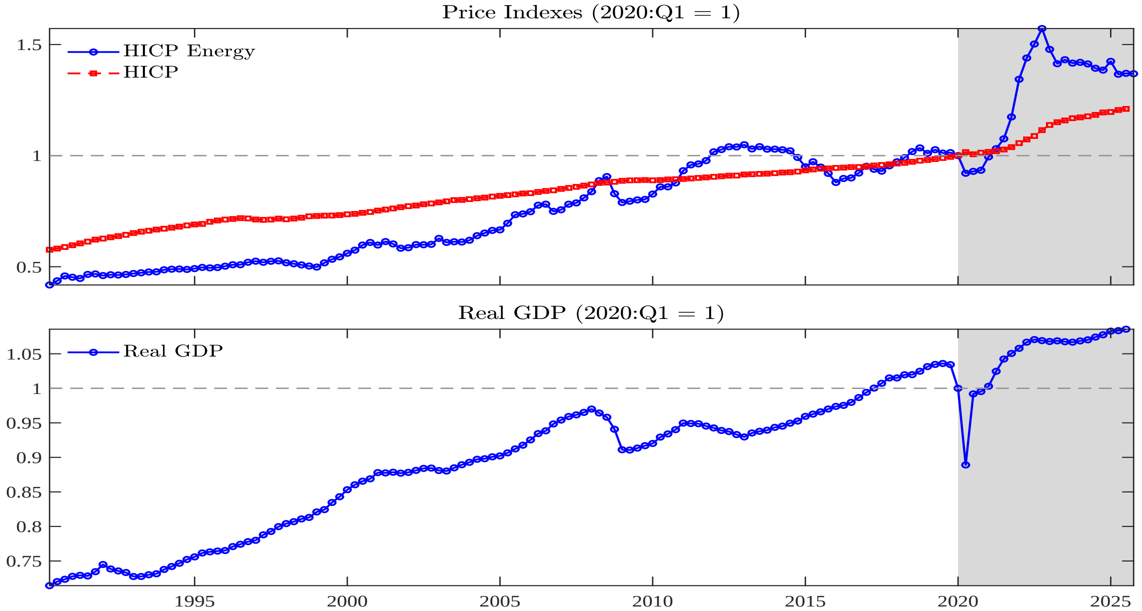

Figure 2. : ***HICP Price Indexes and Real GDP Growth in the EA****. The top panel reports the HICP (solid blue line with circles) and the HICP for energy (dashed red line with squares). The bottom panel reports real GDP (solid blue line with circles). All series are normalized to one in 2020:Q1. The HICP series are obtained from the Area-Wide Model (AWM) database and the OECD. Real GDP data are obtained from the AWM database and Eurostat. See Section III for details.*

energy.[3] On the demand side, refined energy is used both by households for consumption and by firms in the core sector for production.

A central feature of the model is that energy is allowed to be imperfectly substitutable with other inputs in both production and consumption. Because energy is a peculiar component of expenditure and production, its substitutability with non-energy inputs is ultimately an empirical question. The literature has considered cases ranging from no substitutability (Finn, 2000), to unit elasticity (Bodenstein et al., 2007), to unrestricted elasticities (Kim and Loungani, 1992; Labandeira et al., 2017). We adopt the unrestricted specification and estimate the relevant substitution parameters for households, core-sector firms, and energy firms.

We estimate the model on EA real and nominal data up to 2025:Q4. The framework identifies the contribution of two sets of disturbances: the recession and rebound shocks in 2020 associated with the pandemic, and the subsequent shocks to the prices of oil, coal, and gas in 2020–2022.[4] Because these energy markets are highly interconnected and linked to global economic conditions, we model their joint behavior explicitly, see Baumeister

[3]In the EA, refined energy includes electricity, gas, liquid fuels, solid fuels, heat energy, and fuels and lubricants for personal transport equipment, and electricity alone accounted for 25% of final energy consumption in the residential sector and 33% in the industrial sector in 2021 see Jacquinot et al. (2009) and Donoval et al. (2010) and Eurostat's Energy Balances, see Eurostat (2026a,c)

[4]We consider only shocks to crude energy, to isolate the effects of commodity price increase in 2020-2022. A broader definition of these shocks could also include shocks to demand and supply of refined energy.

et al. (2022) and Bjørnland et al. (2018). Specifically, we first extract a latent factor from the prices of oil, coal, and gas and then combine this factor with global GDP in a Structural Vector Autoregression (SVAR) embedded in the DSGE model. This approach allows us to capture the endogeneity of crude energy prices with respect to global activity, as in Bjørnland et al. (2018). At the same time, we introduce wedges between the global prices implied by the SVAR and the corresponding prices in the DSGE model so that energy prices in the EA can also respond to domestic demand conditions and shocks.

Our first result is that shocks specific to oil, coal, and gas played a central role in the surge in refined energy prices. Over the period 2020:Q2–2022:Q3, refined energy prices (HICP Energy) would have increased by about 16% in the absence of these shocks, compared with an observed increase of 57%. Thus, approximately 71% of the rise in refined energy prices can be attributed to energy shocks. The effect on headline inflation is smaller, but still economically meaningful. Over the same period, the total HICP would have increased by about 9% in the absence of these shocks, compared with an observed increase of 12%, implying a contribution of approximately 25%. These findings are consistent with a strong direct effect of oil, coal, and gas price disturbances on retail energy prices, together with a smaller but non-negligible pass-through to headline inflation. Oil and gas account for the largest shares of these effects, reflecting the larger role of oil in energy use and the greater volatility of gas prices, while coal contributes comparatively less.

Our second result is that these energy shocks have a negative effect on economic activity. By 2022:Q3, real GDP is about 0.75% lower than in the counterfactual without oil, coal, and gas shocks. In this sense, recent energy disturbances act as negative supply shocks for the EA economy. Our analysis of the interaction between post-pandemic recovery forces and energy shocks complements Amiti et al. (2024), who emphasize the interaction between domestic pressures, especially labor market conditions, and external disruptions through supply chains.

Our third result is that complementarities between energy and non-energy inputs are central to the transmission mechanism. The estimated elasticities of substitution in both production and consumption are below one, implying that energy and non-energy inputs are complements rather than substitutes. As a consequence, firms and households are limited in their ability to reoptimize their input mix when energy prices rise. This amplifies the effects of energy shocks on output and demand, in line with the mechanisms emphasized by Ramey and Vine (2011), Bachmann et al. (2022), and Moll et al. (2023). Counterfactual exercises confirm that lower substitutability generates larger and more persistent declines in economic activity.

Other mechanisms further shape the transmission of energy shocks. Lower substitutability in production increases the sensitivity of output to energy price changes, while lower substitutability in consumption reduces the ability of households to reallocate expenditure between energy and non-energy goods. In addition, a high degree of wage indexation amplifies the effects of energy shocks by transmitting price increases into labor costs, reinforcing inflationary pressures and generating more persistent output losses.

We also discuss whether energy shocks behave as Keynesian supply shocks, namely shocks that originate on the supply side but are transmitted with demand-like features, see Guerrieri et al. (2022) and Kharroubi and Smets (2024).[5] Our estimates imply that the relevant rigidities are not consistent with such a transmission mechanism.

A further result concerns monetary policy. The model includes a central bank interest-rate rule that responds to inflation, which allows us to evaluate how policy affects the transmission of energy shocks to activity and prices, see Bernanke et al. (1997), Leduc and Sill (2004), Blanchard and Gali (2007), Kormilitsina (2011) and Ramey and Vine (2011). Although the energy sector is modest in size relative to total output, recent energy shocks generate substantial inflationary pressure. Motivated by this fact, we simulate tighter and looser monetary policy responses by varying the inflation coefficient in the policy rule. When a gas price shock is accompanied by a more aggressive monetary response, the decline in GDP is about 2.5 times as large as under a looser response. When the central bank does not raise the interest rate in response to energy shocks until 2022:Q3, GDP is about 1.5% higher.

Our final set of results concerns fiscal policy. We analyze an intervention that effectively fixes the price of refined energy faced by households and core-sector firms at its 2020:Q2 level over the period 2020:Q2–2022:Q3. This policy is implemented through fiscal shocks that offset fluctuations in energy prices and therefore shield consumption and production decisions from the increase in energy costs. We find that such interventions mitigate the adverse effects of energy shocks on GDP by sustaining energy demand and reducing marginal costs. However, the associated fiscal multipliers are below one for household-targeted interventions. Distinguishing between policy instruments, subsidies directed to firms generate larger multipliers than subsidies directed to households because they operate more directly through production costs and supply conditions. At the same time, the effectiveness of these interventions is weakened by equilibrium feedbacks. Since EA demand affects the prices of oil, coal, and gas in our framework, the increase in energy demand induced by subsidies pushes up crude energy prices and partially offsets the policy. Accounting for this channel is therefore important when evaluating the effectiveness of energy-relief measures.

[5] Kharroubi and Smets (2024) analyze the Keynesian feature of energy shocks in a theoretical model with flexible prices and heterogeneous households. They find that energy shocks can have Keynesian effects when income heterogeneity is intermediate and the fraction of credit-constrained households is high.

Related literature

This paper contributes to several strands of literature. It contributes to the macroeconomic literature on the transmission of large supply shocks and to the literature on complementarities and their implications for fiscal and monetary policy, Guerrieri et al. (2022), Kharroubi and Smets (2024), Dew-Becker (2023) and Baqaee and Farhi (2022). Our framework combines multiple sectors, complementarities in production and consumption, and both real and nominal rigidities, allowing us to study the dynamic propagation of large energy shocks and the effects of policy interventions in a unified setting. In the case of the large energy shocks observed after the pandemic, complementarities amplify the transmission of these shocks and shape the effectiveness of fiscal and monetary policy responses.

Our paper is also related to the literature on the role of energy in macroeconomic models. Kim and Loungani (1992) study energy in a single-sector Real Business Cycle (RBC) model in which energy enters production and its price follows an exogenous process.[6] Rotemberg and Woodford (1996) examine energy in a model with imperfect competition, while Finn (2000) analyze energy price shocks in a perfectly competitive model with complementarities in production. By contrast, we consider a two-sector model with multiple energy sources, real and nominal rigidities, and complementarities in both production and consumption. Jacquinot et al. (2009) refine this framework by distinguishing between crude and refined energy. Relative to their setup, we also include coal and gas, in addition to oil, as crude inputs in the energy sector, reflecting the major developments in these markets over our sample period.

Blanchard and Gali (2007) analyze oil price shocks in a model where oil is used both in production and consumption, highlighting the roles of wage rigidity, monetary policy, and oil shares. Golosov et al. (2014) develop a DSGE model with fossil energy sources to study the optimality of carbon taxes in the presence of environmental externalities. Like them, we consider multiple fossil energy sources, but we focus on historical shock transmission rather than environmental damage. Dissou and Karnizova (2016) use a multi-sector model with coal, oil, and electricity to evaluate emission caps and emission taxes. Unlike their calibrated framework, we abstract from carbon emissions and instead study energy subsidies as a policy response to realized shocks.

Recent research has analyzed the large energy shocks associated with the Russo-Ukrainian War using multi-sector models. Bachmann et al. (2022) study these shocks in a multi-sector setting. Albrizio et al. (2022) and Di Bella et al. (2024) examine how integrating the gas market and access to global liquefied gas markets help buffer gas supply shocks

[6] Using a similar single-sector model, Dhawan et al. (2010) study the link between energy prices and the volatility of macroeconomic variables during the Great Moderation. Bjørnland et al. (2018) analyze the effects of oil price fluctuations on macroeconomic volatility in a Markov-switching New Keynesian model.

in European countries. As in our paper, they show that complementarities amplify the effects of negative energy supply shocks. Unlike our setup, these papers focus on supply-side effects through input-output models that abstract from business cycle amplification mechanisms. Di Giovanni et al. (2022) analyze the impact of the COVID-19 pandemic on EA inflation using an input-output model with international trade linkages and Keynesian effects as in Baqaee and Farhi (2022). In contrast, our approach uses a more aggregated model that captures both dynamics and policy feedback.

In this paper, we build and estimate a structural model to study the propagation of energy shocks, whereas much of the recent literature relies on VAR methods to assess their macroeconomic effects. Casoli et al. (2024) analyze the effects of energy shocks on EA inflation using a Bayesian SVAR. Adolfsen et al. (2024) study shocks in the European gas market and find that gas supply shocks and changes in world economic activity affect both energy and core-sector prices. Boeck and Zörner (2025) analyze gas price shocks in the EA and document positive pass-through to inflation and inflation expectations. Alessandri and Gazzani (2025) identify gas supply shocks using external instruments and find stagflationary effects. Finally, Caldara et al. (2022) and Bruhin et al. (2023) show that the increase in geopolitical risk associated with the Russo-Ukrainian War generated stagflationary effects at the global and European levels. Related to our approach, Pataracchia et al. (2025) also use a structural DSGE model to analyze the effects of energy commodity price shocks, complementing the VAR-based evidence in the literature.

The remainder of the paper is organized as follows. Section II describes the model. Section III discusses the empirical analysis. Section IV presents the economic results. Finally, Section V concludes. Derivations, data description, and further empirical results are reported in the Appendix.

II. Model Description

The economy consists of households, a core sector, an energy sector, unions, the government, and the central bank, see Figure 3. In the energy sector, firms combine crude energy sources to produce refined energy. Following Bernanke et al. (1999), and for tractability, firms in both sectors are divided into wholesalers (responsible for production processes) and retailers (responsible for selling goods and setting prices). Wholesale firms produce goods using sector-specific production technologies and transfer output to retail firms, which sell goods and set prices in monopolistically competitive final-goods markets. In the core sector, wholesale production uses refined energy (m_e), labor (n_c), and capital (k_c). In the energy sector, wholesale production uses a composite basket of crude energy sources (V_e), labor (n_e), and capital (k_e).

The composite basket of crude energy sources (V_e) consists of oil (V_o), coal (V_c) and gas (V_g), and the price of these sources also depends on economic conditions in the rest of the world (RoW). Households consume the core sector good (c_c) and the refined energy

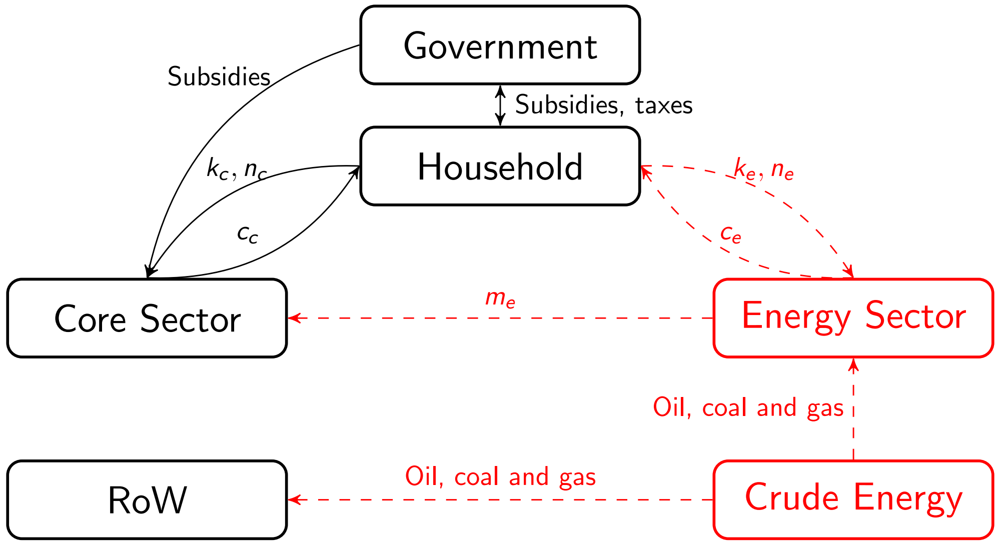

Figure 3. : ***Flowchart of the Economy****. The Core Sector produces general goods; Crude Energy represents the sources of crude energy (oil, coal, and gas) that are refined by the Energy Sector to produce refined energy; the prices of crude energy sources also depend on the Rest of the World (RoW). Arrows represent the flow of the indicated variables. k_c and k_e represent the capital stocks that are lent by the households to the Core Sector and Energy Sector firms; n_c and n_e are the hours of work that are supplied by the households and employed by the Core Sector and Energy Sector firms; c_c and c_e represent the consumption variables demanded by the households to the Core Sector and Energy Sector firms; m_e represents the intermediate energy used for production purposes by the Core Sector firms. The arrows connected to the government block represent the subsidies to the price of energy provided to households and firms, and the taxes that the households must pay to finance these subsidies. The Core Sector and the Energy Sector include unions and retail/wholesale branches, which are omitted from the flowchart for brevity. Finally, the central bank is also omitted for convenience.*

good (c_e), and supply labor (n_c and n_e) and capital (k_c and k_e) to wholesale firms in the two sectors. Labor markets are subject to frictions, and unions (not shown in the flow chart for convenience) intermediate between households and wholesale firms, introducing contractual wage stickiness in both sectors, see Smets and Wouters (2007) and Iacoviello and Neri (2010). The government sets taxes and subsidies for households and firms, while the central bank conducts monetary policy. In what follows, we refer to the core sector as the 'c' Sector ($\mathcal{S}_c$), and to the energy sector as the 'e' Sector ($\mathcal{S}_e$).

Energy Sector

Energy firms supply refined energy ($Y_{e,t}$) and are divided into wholesale and retail branches. Wholesale energy firms rent capital ($k_{e,t}$) from households, hire labor ($n_{e,t}$) from unions, and employ crude energy sources (oil, $V_{o,t}$, gas, $V_{g,t}$, and coal, $V_{c,t}$) taking as given the input prices (the real wage rate $w_{c,t}$, the real capital rental rate $r_{k_c,t}$), the utilization rate of capital $u_{k_e,t}$,[7] and the prices of crude energy sources ($p_{o,t}$, $p_{g,t}$ and $p_{c,t}$). These prices are expressed relative to the price of refined energy, $P_{e,t}$, so that $p_{o,t} = P_{o,t}/P_{e,t}$,

[7]As in Christiano et al. (2005), firms take the utilization rate of capital as given because this choice is made by households, which own the capital stock.

$p_{c,t} = P_{c,t}/P_{e,t}$, and $p_{g,t} = P_{g,t}/P_{e,t}$ for oil, coal, and gas, respectively. Capital ($k_{e,t}$) is also used to refine crude energy sources, and can be interpreted more broadly as including other forms of capital employed in energy production, such as renewable energy plants. Wholesale energy firms sell refined energy at a wholesale price ($P^w_{e,t}$), and we denote the markup between the final-good price and the wholesale price as $X_{e,t} = \frac{P_{e,t}}{P^w_{e,t}}$.

The profit maximization problem, expressed in real terms by dividing nominal profits by $P_{e,t}$, is:

$$\max \frac{Y_{e,t}}{X_{e,t}} - w_{e,t}n_{e,t} - r_{k_e,t}u_{k_e,t}k_{e,t-1} - p_{o,t}V_{o,t} - p_{g,t}V_{g,t} - p_{c,t}V_{c,t}, \tag{1}$$

subject to the production function:

$$Y_{e,t} = a_{z_e,t}a^{ss}_{z_e}\left(n_{e,t}\right)^{1-\alpha_e}\left[\left(u_{k_e,t}k_{e,t-1}\right)^{\omega_{k_e}} V_{e,t}^{1-\omega_{k_e}}\right]^{\alpha_e}. \tag{2}$$

In equation (2), $a_{z_e,t}$ denotes energy-sector productivity, while $a^{ss}_{z_e}$ scales the steady-state size of the sector (Y_e), see Appendix B. The parameter α_e determines the cost share of capital and crude energy relative to labor in the production of refined energy, whereas ω_{k_e} measures the cost share of capital relative to crude energy within the capital–energy composite. As in Golosov et al. (2014), energy firms use multiple fossil fuels as intermediate inputs to produce refined energy. They optimally choose the composition of this energy mix, effectively acting on behalf of households and firms that demand refined energy. To allow for flexible substitution patterns across these inputs, the composite crude energy basket ($V_{e,t}$) is specified as a nested Constant Elasticity of Substitution (CES) aggregator of oil, gas, and coal. Because gas and coal play similar roles in electricity generation and industrial applications, they are first combined in an inner CES nest, $V_{gc,t}$, which is then aggregated with oil in the outer nest $V_{e,t}$:

$$V_{e,t} = \left[a_{V_o,t}\,\omega_o^{\frac{1}{\sigma_{o,gc}}}\,V_{o,t}^{\frac{\sigma_{o,gc}-1}{\sigma_{o,gc}}} + \left(1-\omega_o\right)^{\frac{1}{\sigma_{o,gc}}}\,V_{gc,t}^{\frac{\sigma_{o,gc}-1}{\sigma_{o,gc}}}\right]^{\frac{\sigma_{o,gc}}{\sigma_{o,gc}-1}}. \tag{3}$$

$$V_{gc,t} = \left[a_{V_g,t}\,\omega_g^{\frac{1}{\sigma_{gc}}}\,V_{g,t}^{\frac{\sigma_{gc}-1}{\sigma_{gc}}} + a_{V_c,t}\left(1-\omega_g\right)^{\frac{1}{\sigma_{gc}}}\,V_{c,t}^{\frac{\sigma_{gc}-1}{\sigma_{gc}}}\right]^{\frac{\sigma_{gc}}{\sigma_{gc}-1}}. \tag{4}$$

This structure allows gas and coal to be more substitutable with each other than oil is with the gas–coal bundle, consistent with the distinct role of oil in transportation and refining and with evidence on interfuel substitution, see Stern (2012). In equations (3)–(4), σ_{gc} and $\sigma_{o,gc}$ denote the elasticities of substitution between gas and coal, and between oil and the gas–coal composite, respectively. The parameters ω_o and ω_g determine the steady-state cost shares of oil and gas in the energy mix, while $a_{V_o,t}$, $a_{V_g,t}$, and $a_{V_c,t}$ are exogenous productivity shifters for the three fossil fuels. The corresponding price indices for the inner and outer nests are

$$p_{gc,t} = \left[\omega_g \, p_{g,t}^{1-\sigma_{gc}} + (1-\omega_g)\, p_{c,t}^{1-\sigma_{gc}}\right]^{\frac{1}{1-\sigma_{gc}}},$$

$$p_{v,t} = \left[\omega_o \, p_{o,t}^{1-\sigma_{o,gc}} + (1-\omega_o)\, p_{gc,t}^{1-\sigma_{o,gc}}\right]^{\frac{1}{1-\sigma_{o,gc}}}, \tag{5}$$

where $p_{v,t}$ denotes the aggregate price index of crude energy. Solving the optimization problem in equation (1), subject to equations (2)–(5), yields the optimal demand schedules for labor, capital, and each crude energy source. These conditions are reported in Appendix A.

Retailers in the energy sector buy wholesale output ($Y_{e,t}$) at a price $P^w_{e,t}$ and differentiate it costlessly into a continuum of varieties with constant elasticity of substitution (ϵ_{π_e}. Demand for each variety j is $Y_{e,t}(j) = \left(\frac{P_{e,t}(j)}{P_{e,t}}\right)^{-\epsilon_{\pi_e}} Y_{e,t}$. Retailers face quadratic adjustment costs *à la* Rotemberg in retail price $P_{e,t}(j)$, that depend on the previous quarter's inflation rate, with relative weight governed by the indexation parameter ι_{π_e}. Adjustment costs ($\Xi_{\pi_e,t}$) generate price stickiness, and are given by:

$$\Xi_{\pi_e,t} = \frac{\eta_{\pi_e}}{2}\left(\frac{P_{e,t}(j)}{P_{e,t-1}(j)} - \pi_{e,t-1}^{\iota_{\pi_e}}\right)^2 Y_{e,t},$$

so that deviations of individual varieties inflation rates, $\frac{P_{e,t}(j)}{P_{e,t-1}(j)}$, from lagged inflation, $\pi_{e,t-1}^{\iota_{\pi_e}}$, are penalized according to the rigidity parameters η_{π_e}. This pricing problem is standard, see Appendix A.

Concerning the supply of crude energy sources, we distinguish between international prices and EA prices. We denote international prices by an asterisk, $p^*_{x,t}$ for $x \in \{o, g, c\}$, while EA prices without an asterisk, $p_{x,t}$. To capture comovement in international energy markets, we assume that international prices depend on a common unobserved factor ($p_{f,t}$) plus idiosyncratic components ($a_{p_o,t}$, $a_{p_g,t}$, and $a_{p_c,t}$):

$$\begin{bmatrix} p^*_{o,t} \\ p^*_{g,t} \\ p^*_{c,t} \end{bmatrix} = \begin{bmatrix} \lambda_o \\ \lambda_g \\ \lambda_c \end{bmatrix} p_{f,t} + \begin{bmatrix} a_{p_o,t} \\ a_{p_g,t} \\ a_{p_c,t} \end{bmatrix},$$

where λ_o, λ_g and λ_c are factor loadings.[8]

[8]These loadings scale the steady state of the common factor (p_f^{ss}) so that the steady states of the international prices match the calibration targets, namely $\lambda_o = p_o^{*,ss}/p_f^{ss}$, $\lambda_g = p_g^{*,ss}/p_f^{ss}$ and $\lambda_c = p_c^{*,ss}/p_f^{ss}$, see Appendix B.

We assume that the idiosyncratic components follow autoregressive (AR) processes:

$$
\begin{aligned}
a_{p_o,t} &= \rho_{p_o} a_{p_o,t-1} + \varepsilon_{p_o,t}, \\
a_{p_g,t} &= \rho_{p_g} a_{p_g,t-1} + \varepsilon_{p_g,t}, \\
a_{p_c,t} &= \rho_{p_c} a_{p_c,t-1} + \varepsilon_{p_c,t},
\end{aligned} \tag{6}
$$

where, ρ_{p_o}, ρ_{p_g} and ρ_{p_c} are persistence parameters, and $\varepsilon_{p_o,t}$, $\varepsilon_{p_g,t}$ and $\varepsilon_{p_c,t}$ are Gaussian white-noise shocks, with standard deviations σ_{p_o}, σ_{p_g} and σ_{p_c}, respectively.
To allow crude energy prices to respond endogenously to global economic conditions, we model the common factor ($p_{f,t}$) jointly with global GDP, GDP_t^{W}, in an SVAR:[9]

$$
\boldsymbol{A}_0 \begin{bmatrix} \Delta \log \left(GDP_t^{\mathrm{W}} \right) \\ \log \left(p_{f,t} \right) \end{bmatrix} = \boldsymbol{c} + \sum_{q=1}^{2} \boldsymbol{A}_q \begin{bmatrix} \Delta \log \left(GDP_{t-q}^{\mathrm{W}} \right) \\ \log \left(p_{f,t-q} \right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{W,t} \\ \varepsilon_{p_f,t} \end{bmatrix}, \tag{7}
$$

where $\boldsymbol{A}_0$, $\boldsymbol{A}_1$, $\boldsymbol{A}_2$ are coefficient matrices, $\boldsymbol{c}$ is the vector of constants and $\varepsilon_{W,t}$ and $\varepsilon_{p_f,t}$ the Gaussian white-noise shocks to global GDP and crude energy price factor, with standard deviations σ_{p_f} and σ_W. As in Bjørnland et al. (2018) and similarly to Kilian (2009) we assume two lags in the SVAR and a lower triangular matrix $\boldsymbol{A}_0$, implying that global GDP responds to crude energy price shocks only with a lag, while crude energy prices can respond contemporaneously to global GDP shocks.[10] The SVAR block is estimated jointly with the rest of the DSGE model and directly identifies the latent common factor driving crude energy prices.[11]

A bivariate SVAR with only global activity and the common factor for crude energy prices rules out endogenous effects from EA activity. This assumption is reasonable because the EA represents only a modest share of the global economy.[12] The SVAR block delivers an international component for crude energy prices, $\{p^*_{o,t}, p^*_{g,t}, p^*_{c,t}\}$, that captures global comovement. We allow the prices relevant for the EA model, $\{p_{o,t}, p_{g,t}, p_{c,t}\}$ (see equation 1), to also reflect local market conditions. In particular, even if the EA is small relative to the world economy, its demand for specific energy inputs may affect the price it effectively faces when markets are imperfectly integrated. To capture this reduced-form

Consistent with the model, relative to the price of refined energy, i.e. $p_{x,t} = P_{x,t}/P_{e,t}$ for $x \in \{o, g, c\}$, ensuring stationarity and consistency with the factor model structure. All processes are estimated jointly with the rest of the model rather than in a two-step procedure.

[9]Exogenous oil supply has been considered by Leduc and Sill (2004), Blanchard and Gali (2007), and Bodenstein et al. (2011), among others. The gains from endogenizing the price of oil have been stressed by Kilian (2009) and Nakov and Pescatori (2010), among others.

[10]Unlike Bjørnland et al. (2018) and Kilian (2009), we use a common factor for crude energy prices rather than the price of oil alone. Also, as in those papers, we include global GDP in log differences and the energy price factor in logs.

[11]Technically, the parameters of the VAR in equation (7) are estimated jointly with the remaining model parameters using the Metropolis–Hastings algorithm, and the common factor $p_{f,t}$ is extracted as a latent variable with the Kalman smoother.

[12]Precisely, 12% of world GDP in PPP terms in 2022, according to the ECB. Source: `https://www.ecb.europa.eu/mopo/eaec/html/index.en.html`

local price endogeneity, we assume that EA prices are elastic with respect to EA quantities relative to their steady states:

$$(8) \qquad p_{o,t} = p^*_{o,t}\left(\frac{V_{o,t}}{V_o^{ss}}\right)^{\eta_o}, \qquad p_{g,t} = p^*_{g,t}\left(\frac{V_{g,t}}{V_g^{ss}}\right)^{\eta_g}, \qquad p_{c,t} = p^*_{c,t}\left(\frac{V_{c,t}}{V_c^{ss}}\right)^{\eta_c}.$$

Here V_o^{ss}, V_g^{ss}, and V_c^{ss} denote the steady-state quantities, and η_o, η_g, and η_c are the corresponding price elasticities with respect to EA demand. In log terms, $\eta_j = \partial \log p_{j,t}/\partial \log V_{j,t}$ for $j \in \{o, g, c\}$, so higher values of η_j imply a stronger response of prices to local demand conditions. This mechanism improves robustness by allowing for imperfect market integration and fuel-specific segmentation. We estimate η_o, η_g, and η_c jointly with the remaining model parameters in Section III.

Core Sector

The core sector is divided into wholesale and retail branches. Wholesale firms produce goods and sell them to retail firms at flexible wholesale prices. They rent raw capital ($k_{c,t}$) from households, hire labor ($n_{c,t}$) from unions, and employ intermediate energy goods ($m_{e,t}$), taking as given the input prices (the real wage rate $w_{c,t}$, the real rental rate of raw capital $r_{k_c,t}$, and the relative price of energy goods in terms of core sector goods, $p_{e,t} = P_{e,t}/P_{c,t}$) to maximize their profits. Wholesale core firms sell goods at the wholesale price $P^w_{c,t}$, and we define the markup between the final-good price and the wholesale price as $X_{c,t} = \frac{P_{c,t}}{P^w_{c,t}}$. The resulting profit-maximization problem expressed in real terms by dividing by $P_{c,t}$, is:

$$(9) \qquad \max \frac{Y_{c,t}}{X_{c,t}} - w_{c,t}n_{c,t} - r_{k_c,t}u_{k_c,t}k_{c,t-1} - p_{e,t}m_{e,t},$$

subject to the production function:

$$(10) \qquad Y_{c,t} = a_{z_c,t}\left(n_{c,t}\right)^{1-\alpha_c}\left(\bar{k}_{c,t}\right)^{\alpha_c},$$

where $\bar{k}_{c,t}$ denotes the capital used in the core sector, and is defined as a CES function[13] of raw capital and energy, with elasticity of substitution σ_{k_c}. The expression for $\bar{k}_{c,t}$ is:

$$(11) \qquad \bar{k}_{c,t} = \left[\omega_{k_c}^{\frac{1}{\sigma_{k_c}}}\left(u_{k_c,t}k_{c,t-1}\right)^{\frac{\sigma_{k_c}-1}{\sigma_{k_c}}} + \left(1-\omega_{k_c}\right)^{\frac{1}{\sigma_{k_c}}} m_{e,t}^{\frac{\sigma_{k_c}-1}{\sigma_{k_c}}}\right]^{\frac{\sigma_{k_c}}{\sigma_{k_c}-1}},$$

[13] Cobb-Douglas functions (implying an elasticity of substitution equal to one) have been proposed for instance by Bodenstein et al. (2007), Blanchard and Gali (2007), Bjørnland et al. (2018), Argentiero et al. (2018); CES functions (implying an unrestricted elasticity of substitution) have been used by Kim and Loungani (1992), Jacquinot et al. (2009), Dhawan et al. (2010), Bodenstein et al. (2011), Bodenstein and Guerrieri (2011), Natal (2012), Balke and Brown (2018), Di Bella et al. (2024) among others; functions with a fixed energy requirement tied to the level of production (analog to Leontief function, implying an elasticity of substitution equal to zero) have been proposed by Finn (2000), Leduc and Sill (2004), Kormilitsina (2011), Dissou and Karnizova (2016).

where ω_{k_c} pins down the steady-state expenditure share of raw capital relative to total capital-related costs in production. The CES specification, which we also use for the household consumption bundle in the next subsection, nests the limiting cases of perfect substitutability ($\sigma_{k_c} \to +\infty$), Cobb-Douglas function ($\sigma_{k_c} \to 1$), and perfect complementarity ($\sigma_{k_c} \to 0$). We estimate σ_{k_c} so that the degree of substitutability is determined by the data; see Section III. Solving the profit maximization problem faced by the core firms in equation (9) subject to (10) and (11) yields the optimal factor demand schedules in the core sector, see Appendix A.

As in the energy sector, retailers in the core sector purchase wholesale goods $Y_{c,t}$ at a price $P^w_{c,t}$ and differentiate it costlessly into a continuum of varieties with constant elasticity of substitution ϵ_{π_c}. The demand for variety j is $Y_{c,t}(j) = \left(\frac{P_{c,t}(j)}{P_{c,t}}\right)^{-\epsilon_{\pi_c}} Y_{c,t}$. Retailers face quadratic adjustment costs *à la* Rotemberg in retail price, $P_{c,t}(j)$, that depend on the previous quarter's inflation rate, with relative weight governed by the indexation parameter ι_{π_c}. The adjustment costs ($\Xi_{\pi_c,t}$) are given by:

$$\Xi_{\pi_c,t} = \frac{\eta_{\pi_c}}{2}\left(\frac{P_{c,t}(j)}{P_{c,t-1}(j)} - \pi_{c,t-1}^{\iota_{\pi_c}}\right)^2 Y_{c,t},$$

so that deviations of individual variety inflation rates, $\frac{P_{c,t}(j)}{P_{c,t-1}(j)}$, from lagged inflation, $\pi_{c,t-1}^{\iota_{\pi_c}}$, are penalized according to η_{π_c}. The maximization problem and the resulting Phillips curve are reported in Appendix A.

Households

At each time t, households choose the basket of core and energy consumption ($\bar{c}_t$), hours worked in the two sectors ($n_{c,t}$ and $n_{e,t}$), investment in sectoral capital stocks ($i_{c,t}$ and $i_{e,t}$), capital utilization rates ($u_{k_c,t}$ and $u_{k_e,t}$), and bond holdings (b_t) to maximize lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta)^t a_{\zeta,t}\left[\frac{1-h}{1-\beta h}\log(\bar{c}_t - h\bar{c}_{t-1}) - a_{\varphi,t}\varphi^c \frac{n_{c,t}^{1+\nu_c}}{1+\nu_c} - a_{\varphi,t}\varphi^e \frac{n_{e,t}^{1+\nu_e}}{1+\nu_e}\right]. \tag{12}$$

Equation (12) describes the discounted utility flow from consumption net of the disutility from supplying labor in the two sectors. The parameter β is the intertemporal discount rate, while $a_{\zeta,t}$ is a discount-factor shock that captures changes in household patience. The parameters ν_c and ν_e govern the curvature of labor disutility and therefore measure the elasticity of labor supply with respect to wages. The term $a_{\varphi,t}$ is a labor-supply shock (e.g., new social insurance programs) that shifts hours worked in both sectors. The coefficients φ^c and φ^e scale steady-state hours to match their historical averages. Finally, h denotes the external habit parameter, which increases the persistence of consumption over time.

The consumption basket $\bar{c}_t$ is a CES aggregate of core ($c_{c,t}$) and energy ($c_{e,t}$) goods:

$$\bar{c}_t = \left[\omega_{c_c}^{\frac{1}{\sigma_c}} c_{c,t}^{\frac{\sigma_c-1}{\sigma_c}} + a_{j,t}^{\frac{1}{\sigma_c}} (1-\omega_{c_c})^{\frac{1}{\sigma_c}} c_{e,t}^{\frac{\sigma_c-1}{\sigma_c}}\right]^{\frac{\sigma_c}{\sigma_c-1}}. \tag{13}$$

In equation (13), σ_c is the elasticity of substitution between core and energy goods, and ω_{c_c} controls the steady-state expenditure share of non-energy relative to energy consumption. The variable $a_{j,t}$ is a taste shifter reflecting changes in the relative preference of energy and non-energy goods in consumption, and therefore shifts exogenously household demand for energy. We refer to this shock as the *'energy demand'* shock.

In the utility maximization problem households face the following budget constraint:

$$\begin{aligned} c_{c,t} + p_{e,t}c_{e,t} + \frac{i_{c,t}}{a_{k,t}} + p_{e,t}\frac{i_{e,t}}{a_{k,t}} + b_t =& \frac{R_{t-1}b_{t-1}}{\pi_{c,t}} + \frac{w_{c,t}n_{c,t}}{X_{w_c,t}} + p_{e,t}\frac{w_{e,t}n_{e,t}}{X_{w_e,t}} \\ &+ r_{k_c,t}u_{k_c,t}k_{c,t-1} + p_{e,t}r_{k_e,t}u_{k_e,t}k_{e,t-1} \\ &+ \Pi_t - \frac{\Psi_t}{a_{k,t}} \\ &+ p_{e,t}(1-s_o^*)p_oV_o + p_{e,t}(1-s_g^*)p_gV_g + p_{e,t}(1-s_c^*)p_cV_c, \end{aligned} \tag{14}$$

expressed in real terms using the relative price of energy goods, $p_{e,t} = P_{e,t}/P_{c,t}$. In this constraint, $r_{k_c,t}$ and $r_{k_e,t}$ denote the real rental rates of capital in the two sectors, while $w_{c,t}$ and $w_{e,t}$ denote real wages.

The right-hand side (r.h.s.) of equation (14) collects the sources of funds: wage income in the core sector ($\frac{w_{c,t}n_{c,t}}{X_{w_c,t}}$), wage income in the energy sector ($p_{e,t}\frac{w_{e,t}n_{e,t}}{X_{w_e,t}}$), returns on capital rented to core and energy firms ($r_{k_c,t}u_{k_c,t}k_{c,t-1}$ and $p_{e,t}r_{k_e,t}u_{k_e,t}k_{e,t-1}$), income from the domestic component of crude energy endowments ($p_{e,t}(1-s_o^*)p_oV_o + p_{e,t}(1-s_g^*)p_gV_g + p_{e,t}(1-s_c^*)p_cV_c$), and liquidity from maturing bonds (b_{t-1}). The parameters s_o^*, s_g^*, and s_c^* denote the import shares of oil, gas, and coal, so that only the domestically available share of crude energy generates income for households. Bonds are denominated in nominal units of the numeraire good and pay a gross risk-free return R_t, implying real gross returns equal to $R_{t-1}b_{t-1}/\pi_{c,t}$, where $\pi_{c,t} = P_{c,t}/P_{c,t-1}$ is core inflation. The terms $X_{w_c,t}$ and $X_{w_e,t}$ are wage markups, that is, wedges between the wages paid by wholesale firms and those received by households, with the difference accruing to unions that enforce monopolistic competition in labor markets. The term Π_t collects profits from retailers and labor unions, which are rebated lump-sum to households; its expression is reported in Appendix A. The term Ψ_t collects investment and capacity adjustment costs; see Appendix A. These costs imply that reallocating capital across sectors is not frictionless, making adjustments between the energy and core sector costly.

The left-hand side of equation (14) shows the uses of funds: core and energy consumption ($c_{c,t}$ and $c_{e,t}$), investment in core and energy capital ($i_{c,t}$ and $i_{e,t}$), and new bond purchases (b_t). Net investment in the two sectors is defined by:

$$i_{c,t} = k_{c,t} - (1-\delta_{k_c})\, k_{c,t-1}, \qquad \text{and} \qquad i_{e,t} = k_{e,t} - (1-\delta_{k_e})\, k_{e,t-1}, \tag{15}$$

where δ_{k_c} and δ_{k_e} are the depreciation rates of capital in the core and energy sectors. Finally, $a_{k,t}$ is an investment-specific technology shock that changes the relative cost of investment goods; see Justiniano et al. (2010). Solving the household problem in equation (12), subject to equations (13), (14), and (15), yields optimal demands for core and energy goods, optimal bond holdings, and optimal supplies of labor and capital to both sectors; see Appendix A.

Unions

Unions buy homogeneous labor services from households and costlessly differentiate them into varieties. Differentiated labor inputs, $n_{i,t}(j)$, where j indexes the variety and i the sector, are aggregated into CES composites and sold to wholesale firms. In each sector, unions face labor demand schedules of the form $n_{i,t}(j) = \left(\frac{W_{i,t}(j)}{W_{i,t}}\right)^{-\epsilon_w} n_{i,t}$, $i \in \{c, e\}$, and incur quadratic Rotemberg adjustment costs when changing wages. These costs depend on lagged inflation, with the degree of indexation governed by ι_w. The adjustment-cost terms are

$$\Xi_{w_{i,t}} = \frac{\eta_w}{2}\left(\frac{W_{i,t}(j)}{W_{i,t-1}(h)} - \pi_{i,t-1}^{\iota_w}\right)^2 w_{i,t} n_{i,t}, \qquad i \in \{c, e\},$$

where η_w determines the degree of wage rigidity. Solving this problem yields the wage Phillips curves for the two sectors; see Appendix A.

Monetary Policy

The central bank sets the nominal interest rate according to the Taylor rule:

$$R_t = R_{t-1}^{r_R} R_{ss}^{1-r_R} \pi_t^{(1-r_R)r_\pi} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{(1-r_R)r_Y} (a_{r,t}) \exp(\varepsilon_{e,t}). \tag{16}$$

In equation (16), R_{ss} denotes the steady-state gross interest rate, $\varepsilon_{e,t}$ is a Gaussian white-noise shock capturing transitory deviations from the monetary policy target, and $a_{r,t}$ is an autocorrelated process that captures more persistent shifts in monetary policy:

$$\log(a_{r,t}) = \rho_r \log(a_{r,t-1}) + \varepsilon_{r,t},$$

where ρ_r is the persistence parameter and $\varepsilon_{r,t}$ is a Gaussian white-noise innovation with standard deviation σ_r. Equation (16) implies that the central bank responds to inflation and GDP growth with coefficients r_π and r_Y, respectively, and smooths interest rates with inertia r_R.

We define gross domestic product, GDP_t, in equation (16) as total production in the two sectors, $Y_{c,t}$ and $Y_{e,t}$, net of intermediate inputs used in production, namely refined energy $m_{e,t}$ in the core sector and crude energy $V_{e,t}$ in the energy sector:

$$GDP_t = Y_{c,t} - p_{e,t} m_{e,t} + p_{e,t} \left(Y_{e,t} - p_{v,t} V_{e,t} \right).$$

Headline inflation, π_t, is defined as a weighted average of core inflation, $\pi_{c,t} = P_{c,t}/P_{c,t-1}$, and energy inflation, $\pi_{e,t} = P_{e,t}/P_{e,t-1}$, where the weights are the value-added shares of the two sectors:

$$\pi_t = \pi_{c,t}^{s_{c,t}} \pi_{e,t}^{1-s_{c,t}}, \tag{17}$$

where $s_{c,t}$ is the share of core-sector value added in GDP at time t, given by $s_{c,t} = \left(Y_{c,t} - p_{e,t} m_{e,t} \right) / GDP_t$.

Aggregation and Equilibrium

For convenience, aggregate consumption and aggregate investment are defined as

$$c_t = c_{c,t} + p_{e,t} c_{e,t}, \qquad i_t = i_{c,t} + p_{e,t} i_{e,t}.$$

The evolution of the relative price of $\mathcal{S}_e$ is related to inflation in the two sectors:

$$\frac{p_{e,t}}{p_{e,t-1}} = \frac{P_{e,t}/P_{c,t}}{P_{e,t-1}/P_{c,t-1}} = \frac{\pi_{e,t}}{\pi_{c,t}}.$$

Similarly, the evolution of the relative prices of oil, coal, and gas, $p_{o,t}$, $p_{c,t}$, and $p_{g,t}$, is linked to the inflation rates of their nominal prices, π_t^o, π_t^c, and π_t^g:

$$\frac{p_{o,t}}{p_{o,t-1}} = \frac{P_{o,t}/P_{e,t}}{P_{o,t-1}/P_{e,t-1}} = \frac{\pi_t^o}{\pi_{e,t}}, \qquad \frac{p_{c,t}}{p_{c,t-1}} = \frac{\pi_t^c}{\pi_{e,t}}, \qquad \frac{p_{g,t}}{p_{g,t-1}} = \frac{\pi_t^g}{\pi_{e,t}}.$$

The resource constraint for $\mathcal{S}_c$ equates consumption and investment to production net of adjustment costs:[14]

$$c_{c,t} + i_{c,t} + p_{e,t} i_{e,t} = Y_{c,t} - \Psi_t - \Xi_{c,t}. \tag{18}$$

[14]Investment in both the core and energy sectors ($i_{c,t}$ and $i_{e,t}$) appears in the resource constraint of the core sector because investment goods are produced using core-sector output ($Y_{c,t}$) rather than refined energy ($Y_{e,t}$). See Iacoviello and Neri (2010) for a similar assumption in a model with a core sector and a housing sector. Consistently, the real adjustment costs associated with capital accumulation and capacity utilization (Ψ_t) also enter the core-sector resource constraint.

Here, Ψ_t collects real adjustment costs related to capital and capacity utilization, while $\Xi_{c,t}$ denotes nominal adjustment costs in the core sector; their expressions are reported in Appendix A.

The resource constraint for $\mathcal{S}_e$ requires that energy used by households, $c_{e,t}$, and by core firms, $m_{e,t}$, equals energy production net of nominal adjustment costs:

$$c_{e,t} + m_{e,t} = Y_{e,t} - \Xi_{e,t} - s_o^* p_o V_o - s_g^* p_g V_g - s_c^* p_c V_c. \tag{19}$$

Following Smets and Wouters (2003), we define employment as an auxiliary variable, l_t, that adjusts sluggishly to changes in hours worked, with rigidity governed by θ_l:

$$l_t - l_{t-1} = \mathbb{E}_t l_{t+1} - l_t + \left[\frac{(1-\theta_l)(1-\beta\theta_l)}{\theta_l}\right](n_{c,t} + n_{e,t} - l_t).$$

This variable is needed for estimation because, for the EA, aggregate hours worked are not available, whereas employment is observed; see Section III.

Finally, aggregate wage inflation, ω_t, is defined as the weighted average of wage inflation in the two sectors, with weights given by steady-state hours worked, n_c^{ss} and n_e^{ss}:

$$\omega_t = \frac{n_c^{ss}}{n_c^{ss} + n_e^{ss}}\omega_{c,t} + \frac{n_e^{ss}}{n_c^{ss} + n_e^{ss}}\omega_{e,t}.$$

Exogenous Processes

The remaining exogenous variables are assumed to follow AR processes. These include the discount-factor shock ($a_{\zeta,t}$), the energy-demand shock ($a_{j,t}$), the labor-disutility shock ($a_{\varphi,t}$), the productivity of crude oil, coal, and natural gas ($a_{V_o,t}$, $a_{V_c,t}$, $a_{V_g,t}$), the marginal productivity of investment ($a_{k,t}$), the core sector productivity ($a_{z_c,t}$), and the energy sector productivity ($a_{z_e,t}$). Specifically,

$$\log(a_{\zeta,t}) = \rho_\zeta \log(a_{\zeta,t-1}) + \varepsilon_{\zeta,t}, \qquad \log(a_{j,t}) = \rho_j \log(a_{j,t-1}) + \varepsilon_{j,t},$$
$$\log(a_{\varphi,t}) = \rho_\varphi \log(a_{\varphi,t-1}) + \varepsilon_{\varphi,t}, \qquad \log(a_{V_o,t}) = \rho_{V_o} \log(a_{V_o,t-1}) + \varepsilon_{V_o,t},$$
$$\log(a_{V_c,t}) = \rho_{V_c} \log(a_{V_c,t-1}) + \varepsilon_{V_c,t}, \qquad \log\left(a_{V_g,t}\right) = \rho_{V_g} \log\left(a_{V_g,t-1}\right) + \varepsilon_{V_g,t},$$
$$\log(a_{k,t}) = \rho_k \log(a_{k,t-1}) + \varepsilon_{k,t}, \qquad \log(a_{z_c,t}) = \rho_{z_c} \log(a_{z_c,t-1}) + \varepsilon_{z_c,t},$$
$$\log(a_{z_e,t}) = \rho_{z_e} \log(a_{z_e,t-1}) + \varepsilon_{z_e,t}.$$

The innovations $\varepsilon_{\zeta,t}$, $\varepsilon_{j,t}$, $\varepsilon_{\varphi,t}$, $\varepsilon_{V_o,t}$, $\varepsilon_{V_c,t}$, $\varepsilon_{V_g,t}$, $\varepsilon_{k,t}$, $\varepsilon_{z_c,t}$ and $\varepsilon_{z_e,t}$ are Gaussian white-noise shocks with standard deviations σ_ζ, σ_j, σ_φ, σ_{V_o}, σ_{V_g}, σ_{V_c}, σ_k, σ_{z_c}, and σ_{z_e}, respectively.

III. Empirical Analysis

Data

We solve the model by a first-order approximation around the steady state in Dynare, see Adjemian et al. (2022). The steady-state derivation is reported in Appendix B. We estimate the model with Bayesian methods using EA data on real GDP, real consumption, real investment, employment, GDP deflator growth, HICP of energy products, oil price inflation, coal price inflation, natural gas price inflation, wage inflation, gross available energy of oil, coal, and natural gas,[15] the nominal short-term interest rate, and global GDP growth. Including both energy prices and energy quantities among the observables helps discipline demand- and supply-side responses in the energy block and improves identification of the substitution parameters. Appendix C provides a detailed description of the data sources.

The measurement equations are:

$$
\begin{aligned}
&\Delta GDP_t^{data} = \log(GDP_t) - \log(GDP_{t-1}), && \Delta c_t^{data} = \log(c_t) - \log(c_{t-1}),\\
&\Delta i_t^{data} = \log(i_t) - \log(i_{t-1}), && \Delta l_t^{data} = \log(l_t) - \log(l_{t-1}),\\
&\pi_t^{data} = \gamma_\pi + \log(\pi_t), && \pi_{e,t}^{data} = \gamma_{\pi_e} + \log(\pi_{e,t}),\\
&\pi_t^{o,data} = \gamma_{\pi_o} + \log(\pi_t^o), && \pi_t^{c,data} = \gamma_{\pi_c} + \log(\pi_t^c),\\
&\pi_t^{g,data} = \gamma_{\pi_g} + \log(\pi_t^g), && \omega_t^{data} = \gamma_\omega + \omega_t,\\
&V_{o,t}^{data} = \gamma_{V_o} \times V_{o,t}, && V_{c,t}^{data} = \gamma_{V_c} \times V_{c,t},\\
&V_{g,t}^{data} = \gamma_{V_g} \times V_{g,t}, && R_t^{data} = 400 \times (R_t - 1),\\
&\Delta GDP_t^{W,data} = \log(GDP_t^W) - \log(GDP_{t-1}^W).
\end{aligned}
\tag{20}
$$

In equation (20), the constants γ_π, γ_{π_e}, γ_{π_o}, γ_{π_c}, γ_{π_g}, γ_ω, γ_{V_o}, γ_{V_c} and γ_{V_g} align the means of model variables with those of the corresponding data series. In particular, the parameters γ_{V_o}, γ_{V_c} and γ_{V_g} rescale the model-implied quantities of crude energy to match their empirical counterparts.[16]

Calibrated Parameters

Following standard practices in the DSGE literature (see, among others, Smets and Wouters, 2007, Ireland, 2004, Christiano et al., 2005, and Herbst and Schorfheide, 2015),

[15]For each energy item, we use Eurostat's *gross available energy* from the Simplified Energy Balances which measures the quantity of each fuel available in the EA for domestic energy use at the frequency of the data, expressed in tonnes of oil equivalent (Eurostat, 2026a,c).

[16]Note that the data variables ΔGDP_t^{data}, Δc_t^{data} and Δi_t^{data} have been demeaned (see Appendix C). As a result, they are zero-mean variables, like their model counterparts, and therefore do not require constants in equation (20). Moreover, energy quantities ($V_{o,t}$, $V_{c,t}$ and $V_{g,t}$) are included in levels, as they do not exhibit a pronounced deterministic trend over the sample.

we calibrate a subset of parameters on the basis of economic theory. We also target the parameters governing the steady state to match key features of the EA energy market.[17] Table 1 reports the calibrated parameters and their values.

We begin by matching labor input across sectors. After normalizing steady-state hours in the core sector to one, $n_c = n_c^{ss} = 1$, we set steady-state hours in the energy sector to $n_e = n_e^{ss} = 0.0116$, corresponding to 1.1% of total hours worked in the energy sector; see Donoval et al., 2010 and the EU KLEMS database.[18]). Following Christiano et al. (2005), we set the inverse Frisch elasticities of labor supply, ν_c and ν_e, equal to 1. The discount factor is set to $\beta = 0.991$, implying an annual steady-state interest rate of 3.60%. Quarterly depreciation rates are fixed at $\delta_{k_c} = \delta_{k_e} = 0.025$, which corresponds to annual depreciation rates of 10%. The elasticities of substitution across core goods, energy goods, and labor varieties, ϵ_{π_c}, ϵ_{π_e}, and ϵ_w, are set equal to 7.68, implying steady-state markups of 15%, see Iacoviello and Neri (2010). For the production side, we set the capital share in the core sector, α_c, to the standard value of 0.30 (Smets and Wouters, 2003), and the capital share in the energy sector, α_e, to 0.55, which implies a labor share $\left(\frac{w_e n_e}{Y_e} X_e = 1 - \alpha_e\right)$ of 0.45, see Jacquinot et al. (2009).

We next calibrate the energy-related parameters. We normalize the steady-state relative price of oil to one, $p_o = p_o^{ss} = 1$. The steady-state gas price (p_g) is set equal to the average ratio of natural gas to oil prices (both per Ton of Oil Equivalent, TOE) computed over the estimation sample[19], yielding $p_g = p_g^{ss} = 0.66$. Analogously, the steady-state relative price of coal is calibrated as the average ratio of coal to oil prices (per TOE), implying $p_c = p_c^{ss} = 0.30$ (see Appendix C).

Following Jacquinot et al. (2009), we target a steady-state household energy expenditure share of 6%, which implies $\omega_{c_c} = 0.94$. Since average intermediate energy inputs account for 3% of production in the EA (see Donoval et al., 2010 and the EU KLEMS database), and the ratio of production to capital is 17% (see EU KLEMS database), we set the steady-state share of intermediate energy in capital-related production costs to $s_{m_e,k_c} \equiv \frac{m_e}{k_c} = 0.03 \times 0.17 = 0.0052$. Again following Jacquinot et al. (2009), we target a steady-state cost share of crude energy in energy-sector production equal to 33.5%, $s_{p_v V_e, Y_e} \equiv \frac{p_v V_e}{Y_e} = 0.335$. Using the estimation sample and expressing all quantities in TOE, we calibrate the ratio of gas to oil to $s_{V_g,V_o} \equiv \frac{V_g}{V_o} = 0.51$, and the ratio of coal to oil to $s_{V_c,V_o} \equiv \frac{V_c}{V_o} = 0.33$. We calibrate the steady-state import shares of oil, natural gas, and coal ($s_o^\star, s_g^\star, s_c^\star$) to match the EA import dependency rates by fuel type, obtaining $s_o^\star = 0.95$, $s_g^\star = 0.87$, and $s_c^\star = 0.60$ (Eurostat, 2026b). Appendix B describes in detail how these calibration targets map into the steady state of the model.

[17]EA countries are heterogeneous in both economic structure and energy mix. Because our focus is on aggregate variables, such as prices, that reflect developments in the energy sector and are affected by ECB monetary policy, we model the aggregate EA economy.

[18]*Source:* EU KLEMS data: `www.euklems.net`.

[19]Data from 2022 are excluded to avoid distortions from large shocks.

*Table 1—: **Calibrated Parameters**. The table reports the parameter's name (Full Name), the associate symbol (Symbol), and the calibrated value (Value).*

Economic parameters		
Full Name	Symbol	Value
Hours worked in $\mathcal{S}_c$	n_c^{ss}	1.000
Hours worked in $\mathcal{S}_e$	$100 \times n_e^{ss}$	1.160
Depreciation $\mathcal{S}_c$	δ_{k_c}	0.025
Inverse Frisch elast. $\mathcal{S}_c$	ν_c	1.000
Inverse Frisch elast. $\mathcal{S}_e$	ν_e	1.000
Capital share $\mathcal{S}_c$	α_c	0.300
Depreciation $\mathcal{S}_e$	δ_{k_e}	0.025
Capital share $\mathcal{S}_e$	α_e	0.550
Discount factor	β	0.991
Elast. substitution of goods $\mathcal{S}_c$	ϵ_{π_c}	7.677
Elast. substitution of goods $\mathcal{S}_e$	ϵ_{π_e}	7.677
Elast. substitution of labor varieties	ϵ_w	7.677
Energy parameters		
Full Name	Symbol	Value
Relative price of oil	p_o^{ss}	1.000
Relative price of natural gas	p_g^{ss}	0.660
Relative price of coal	p_c^{ss}	0.300
Share of imported oil	$s_o^\star$	0.950
Share of imported natural gas	$s_g^\star$	0.870
Share of imported coal	$s_c^\star$	0.600
Share of non-energy expenditure in consumption	ω_{c_c}	0.940
Share of refined energy in $\mathcal{S}_c$ production	s_{m_e,k_c}	0.005
Share of raw energy in $\mathcal{S}_e$ production	$s_{p_v V_e, Y_e}$	0.335
Ratio of gas to oil	s_{V_g,V_o}	0.510
Ratio of coal to oil	s_{V_c,V_o}	0.330
Steady-state common factor	p_f^{ss}	0.826

Estimation

We estimate the model on quarterly EA data from 1990:Q1 to 2025:Q4. For observations in 2020, we treat some entries as missing to limit the effect of pandemic outliers on the likelihood. We then use the estimated parameters to recover the shocks realized between 2020:Q2 and 2025:Q4, similarly to Boragan Aruoba et al. (2018), Brinca et al. (2021) and Faria-e Castro (2021).

We use a standard Random Walk Metropolis-Hastings (RWMH) algorithm to target the posterior distribution of the model parameters. Prior choices follow the literature on medium-scale DSGE models (Smets and Wouters, 2003) and are reported in Tables 2 and 3. We map the Rotemberg adjustment-cost parameters for prices and wages, η_{π_e}, η_{π_c}, and η_w, into the corresponding probabilities of non-adjustment in an equivalent Calvo setting; see Richter and Throckmorton (2016). These probabilities are denoted by θ_{π_e}, θ_{π_c}, and θ_w, respectively, and satisfy

$$\eta_i = \frac{\theta_i(\epsilon_i - 1)}{(\beta\theta_i - 1)(\theta_i - 1)}, \qquad i = \pi_e, \pi_c, w.$$

For θ_{π_e} and θ_{π_c}, we use Beta ($\mathcal{B}$) priors with mean 0.75 and standard deviation 0.05, as in Smets and Wouters (2003), implying an average price duration of one year. For the wage rigidity (θ_w) we use a $\mathcal{B}$ prior centered at 0.50, consistent with a moderate degree of wage rigidity. For the habit parameter (h), we set a $\mathcal{B}$ prior with a mean 0.80 and standard deviation 0.05. The priors for θ_l, r_R, r_Y, r_π, ι_{π_c}, andι_{π_e} also follow Smets and Wouters (2003). For the investment adjustment-cost parameter, η_k, we adopt the prior in Iacoviello and Neri (2010). For the capacity-utilization rigidity parameter, η_u, we use a $\mathcal{B}$ prior centered at 0.50, as in Smets and Wouters (2007) and Iacoviello and Neri (2010). The prior for wage indexation, ι_w, is set equal to that for price indexation; see Smets and Wouters (2003). For the substitution elasticities in the nested fossil-energy aggregator in (3) and (4), we use informative but relatively diffuse Gamma ($\mathcal{G}$) priors based on the macro-level elasticities reported by Stern (2012), which reviews 47 studies of interfuel substitution. For the inner coal-gas elasticity σ_{gc} in (4), we center the prior at Stern's macro predicted coal-gas value, $\sigma_{CG} = 1.114$ (s.e. 0.904). For the outer elasticity $\sigma_{o,gc}$ in (3), Stern reports similar macro predicted elasticities for oil–gas and coal–oil, $\sigma_{og} = 1.783$ (s.e. 0.319) and $\sigma_{CO} = 1.744$ (s.e. 0.384), which supports interpreting the outer nest as oil versus a fossil composite. We therefore center $\sigma_{o,gc}$ at their average, $\mu_{o,gc} = (1.783 + 1.744)/2 = 1.764$, and set the prior dispersion equal to the corresponding average uncertainty, $s_{o,gc} = (0.319 + 0.384)/2 = 0.352$.

For η_o, η_g, and η_c, we use $\mathcal{G}$ priors. Motivated by the view that gas markets in Europe are more segmented than coal markets, and coal markets are more segmented than oil markets, we assign prior means that are highest for η_g, followed by η_c, and then η_o. This reflects the idea that more segmented markets display a stronger response of local prices to European demand. At the same time, we remain agnostic by keeping these priors sufficiently diffuse.

For the elasticity of substitution between energy and core consumption goods, σ_c, and the elasticity of substitution between intermediate energy and raw capital in production, σ_{k_c}, we use $\mathcal{G}$ priors centered at 0.54, as in Acurio Vásconez (2015). The second part of Table 2 reports the priors for the constants in the measurement equations (20). For these parameters, we use Normal ($\mathcal{N}$) priors with means equal to the sample averages of the corresponding observables and standard deviations equal to 10 percent of those averages. Table 3 reports the priors for the parameters governing the exogenous shock processes: $\mathcal{B}$ for the persistence parameters and Inverse Gamma ($\mathcal{IG}$) for the standard deviations of the shocks. The parameters of the SVAR in equation (7) are estimated jointly with the remaining model parameters. Their priors are $\mathcal{N}$ and are based on a preliminary estimation over a presample. The corresponding prior and posterior distributions are reported in Appendix D.

Tables 2 and 3 report posterior means and standard deviations for the estimated parameters. The posterior estimates point to a high degree of consumption habits (h).

*Table 2—: **Estimation Results**. The table shows the endogenous propagation parameters and the measurement-equation parameters. The table reports each parameter name (Full Name), the associate symbol (Symbol), the prior distribution (Prior), prior mean and standard deviation (Mean, St. Dev), and the posterior mean (Post. Mean) and standard deviation (Post. St. Dev). $\mathcal{B}$ denotes the Beta distribution, $\mathcal{N}$ the Normal distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.*

Endogenous propagation parameters					
Full Name	Symbol	Prior	Mean, St. Dev.	Post. Mean	Post. St. Dev
Habits	h	$\mathcal{B}$	(0.80, 0.05)	0.78	0.04
Price Rigidity $\mathcal{S}_c$	θ_{π_c}	$\mathcal{B}$	(0.75, 0.05)	0.45	0.03
Price Rigidity $\mathcal{S}_e$	θ_{π_e}	$\mathcal{B}$	(0.75, 0.05)	0.72	0.01
Employment Rigidity	θ_l	$\mathcal{B}$	(0.50, 0.15)	0.80	0.02
Wage Rigidity	θ_w	$\mathcal{B}$	(0.50, 0.05)	0.78	0.02
Price Indexation $\mathcal{S}_c$	ι_{π_c}	$\mathcal{B}$	(0.75, 0.15)	0.65	0.12
Price Indexation $\mathcal{S}_e$	ι_{π_e}	$\mathcal{B}$	(0.75, 0.15)	0.03	0.02
Wage Indexation	ι_w	$\mathcal{B}$	(0.75, 0.15)	0.50	0.13
Cap. Adj. Cost	η_k	$\mathcal{G}$	(10.00, 2.50)	27.32	2.61
Utiliz. Adj. Cost	η_u	$\mathcal{B}$	(0.50, 0.05)	0.55	0.05
Taylor Rule Inertia	r_R	$\mathcal{B}$	(0.80, 0.10)	0.89	0.01
Taylor Rule Output	r_Y	$\mathcal{N}$	(0.13, 0.05)	0.21	0.05
Taylor Rule Inflation	r_π	$\mathcal{N}$	(1.70, 0.10)	1.74	0.09
El. Subst. Energy Consumption	σ_c	$\mathcal{G}$	(0.54, 0.10)	0.25	0.04
El. Subst. Energy Production	σ_{k_c}	$\mathcal{G}$	(0.54, 0.10)	0.30	0.05
El. Subst. Gas vs Coal	σ_{gc}	$\mathcal{G}$	(1.11, 0.90)	0.77	0.03
El. Subst. Oil vs (Gas,Coal)	$\sigma_{o,gc}$	$\mathcal{G}$	(1.76, 0.35)	0.52	0.04
Feedback (oil price)	η_o	$\mathcal{G}$	(0.05, 0.05)	0.46	0.12
Feedback (gas price)	η_g	$\mathcal{G}$	(0.40, 0.20)	3.88	1.16
Feedback (coal price)	η_c	$\mathcal{G}$	(0.08, 0.08)	1.11	0.23
Measurement equations parameters					
Full Name	Symbol	Prior	Mean, St. Dev.	Post. Mean	Post. St. Dev
Meas. Const. Inflation	$100 \times \gamma_\pi$	$\mathcal{N}$	(0.54, 0.05)	0.51	0.04
Meas. Const. Inflation $\mathcal{S}_e$	$100 \times \gamma_{\pi_e}$	$\mathcal{N}$	(0.91, 0.09)	0.87	0.06
Meas. Const. Oil Price	$100 \times \gamma_{\pi_o}$	$\mathcal{N}$	(1.14, 0.11)	1.27	0.08
Meas. Const. Gas Price	$100 \times \gamma_{\pi_g}$	$\mathcal{N}$	(1.34, 0.13)	1.48	0.10
Meas. const. Coal Price	$100 \times \gamma_{\pi_c}$	$\mathcal{N}$	(0.85, 0.09)	0.87	0.08
Meas. Const. Wage Inflation	$100 \times \gamma_\omega$	$\mathcal{N}$	(0.73, 0.07)	0.91	0.04

Price-setting frictions are sizable in both sectors, with θ_{π_c} and θ_{π_e} implying non-negligible nominal rigidity. Price indexation is moderate in the core sector (ι_{π_c}) and close to zero in the energy sector (ι_{π_e}). The estimate of θ_l implies employment rigidity, suggesting that employment adjusts only gradually to changes in hours worked. The Taylor-rule estimates imply strong interest-rate smoothing, through (r_R), and moderate response of the nominal interest rate to GDP (r_Y) and inflation (r_π). Wage-setting estimates indicate a high degree of wage rigidity, θ_w, together with a moderate degree of wage indexation, ι_w. We also estimate sizable investment adjustment costs, η_k, and a moderate degree of rigidity in capital utilization, η_u. Finally, the estimated substitution elasticities across fossil fuels, σ_{gc} and $\sigma_{o,gc}$, and the substitution elasticities of energy in consumption and production, σ_c and σ_{k_c}, are all below one, implying limited substitutability of energy inputs. We discuss the role of these estimates for model dynamics in the next section.

*Table 3—: **Estimation Results**. The table reports the parameters of the exogenous processes. The table reports each parameter name (Full Name), the associate symbol (Symbol), the prior distribution (Prior), prior mean and standard deviation (Mean, St. Dev), and the posterior mean (Post. Mean) and standard deviation (Post. St. Dev). $\mathcal{B}$ denotes the Beta distribution, $\mathcal{N}$ the Normal distribution, $\mathcal{G}$ the Gamma distribution, and $\mathcal{IG}$ the Inverse-Gamma distribution.*

Exogenous processes parameters					
Full Name	Symbol	Prior	Mean, St. Dev.	Post. Mean	Post. St. Dev
Persistence parameters					
Persistence Prod. $\mathcal{S}_c$	ρ_{z_c}	$\mathcal{B}$	(0.50, 0.10)	0.98	0.00
Persistence Prod. $\mathcal{S}_e$	ρ_{z_e}	$\mathcal{B}$	(0.50, 0.10)	0.62	0.03
Preference for $\mathcal{S}_e$	ρ_j	$\mathcal{B}$	(0.50, 0.10)	0.98	0.01
Persistence Lab. Supply	ρ_φ	$\mathcal{B}$	(0.50, 0.10)	0.96	0.01
Persistence Intertemp.	ρ_ζ	$\mathcal{B}$	(0.50, 0.10)	0.83	0.04
Persistence Prod. Inv.	ρ_k	$\mathcal{B}$	(0.50, 0.10)	0.92	0.02
Persistence Oil Price	ρ_{p_o}	$\mathcal{B}$	(0.50, 0.10)	0.96	0.01
Persistence Gas Price	ρ_{p_g}	$\mathcal{B}$	(0.50, 0.10)	0.86	0.05
Persistence Coal Price	ρ_{p_c}	$\mathcal{B}$	(0.50, 0.10)	0.92	0.02
Persistence Oil Prod.	ρ_{V_o}	$\mathcal{B}$	(0.50, 0.10)	0.75	0.03
Persistence Gas Prod.	ρ_{V_g}	$\mathcal{B}$	(0.50, 0.10)	0.39	0.05
Persistence Coal Prod.	ρ_{V_c}	$\mathcal{B}$	(0.50, 0.10)	0.90	0.02
Standard deviations of shocks					
St. Dev. Prod. $\mathcal{S}_c$	$100 \times \sigma_{z_c}$	$\mathcal{IG}$	(0.001, 0.01)	0.87	0.08
St. Dev. Temp. Mon. Policy	$100 \times \sigma_e$	$\mathcal{IG}$	(0.001, 0.01)	11.47	0.82
St. Dev. Prod. $\mathcal{S}_e$	$100 \times \sigma_{z_e}$	$\mathcal{IG}$	(0.001, 0.01)	10.37	0.87
St. Dev. Intratemp.	$100 \times \sigma_j$	$\mathcal{IG}$	(0.001, 0.01)	9.38	0.74
St. Dev. Pers. Mon. Policy	$100 \times \sigma_r$	$\mathcal{IG}$	(0.001, 0.01)	0.22	0.17
St. Dev. Lab. Supply	$100 \times \sigma_\varphi$	$\mathcal{IG}$	(0.001, 0.01)	2.77	0.34
St. Dev. Pref.	$100 \times \sigma_\zeta$	$\mathcal{IG}$	(0.001, 0.01)	3.40	0.53
St. Dev. Oil Price	$100 \times \sigma_{p_o}$	$\mathcal{IG}$	(0.001, 0.01)	10.04	1.09
St. Dev. Gas Price	$100 \times \sigma_{p_g}$	$\mathcal{IG}$	(0.001, 0.01)	17.56	3.03
St. Dev. Coal Price	$100 \times \sigma_{p_c}$	$\mathcal{IG}$	(0.001, 0.01)	5.60	0.59
St. Dev. Oil Prod.	$100 \times \sigma_{V_o}$	$\mathcal{IG}$	(0.001, 0.01)	15.48	2.04
St. Dev. Gas Prod.	$100 \times \sigma_{V_g}$	$\mathcal{IG}$	(0.001, 0.01)	10.25	1.19
St. Dev. Coal Prod.	$100 \times \sigma_{V_c}$	$\mathcal{IG}$	(0.001, 0.01)	13.78	1.36
St. Dev. Prod. Inv.	$100 \times \sigma_k$	$\mathcal{IG}$	(0.001, 0.01)	1.62	0.15
St. Dev. Common Factor	$100 \times \sigma_{p_f}$	$\mathcal{IG}$	(0.001, 0.01)	18.62	1.60
St. Dev. Global GDP	$100 \times \sigma_W$	$\mathcal{IG}$	(0.001, 0.01)	0.36	0.02

IV. Economic Results

IRFs

Figure 4 reports the Impulse Response Functions (IRFs) to an unexpected increase in the price of natural gas, (p_g), generated by a shock (ε_{p_g}) to the idiosyncratic gas-price component a_{p_g}, see equation (6). The shock reduces the quantity of natural gas, V_g, used by firms in the energy sector to produce refined energy. This raises both the crude-energy price index, p_v, and the refined-energy price, p_e. Higher energy prices reduce firms' demand for energy, m_e, and households' energy consumption, c_e. As a result, GDP declines because lower energy use depresses production in the core sector and household demand. At the same time, headline inflation, π, increases, implying that the shock operates as a negative supply shock. At the same time, headline inflation, π, increases, implying that the shock operates as a negative supply shock. Appendix E reports similar IRFs for idiosyncratic shocks to the prices of oil and coal, as well as for shocks to the common factor in crude-energy prices.[20]

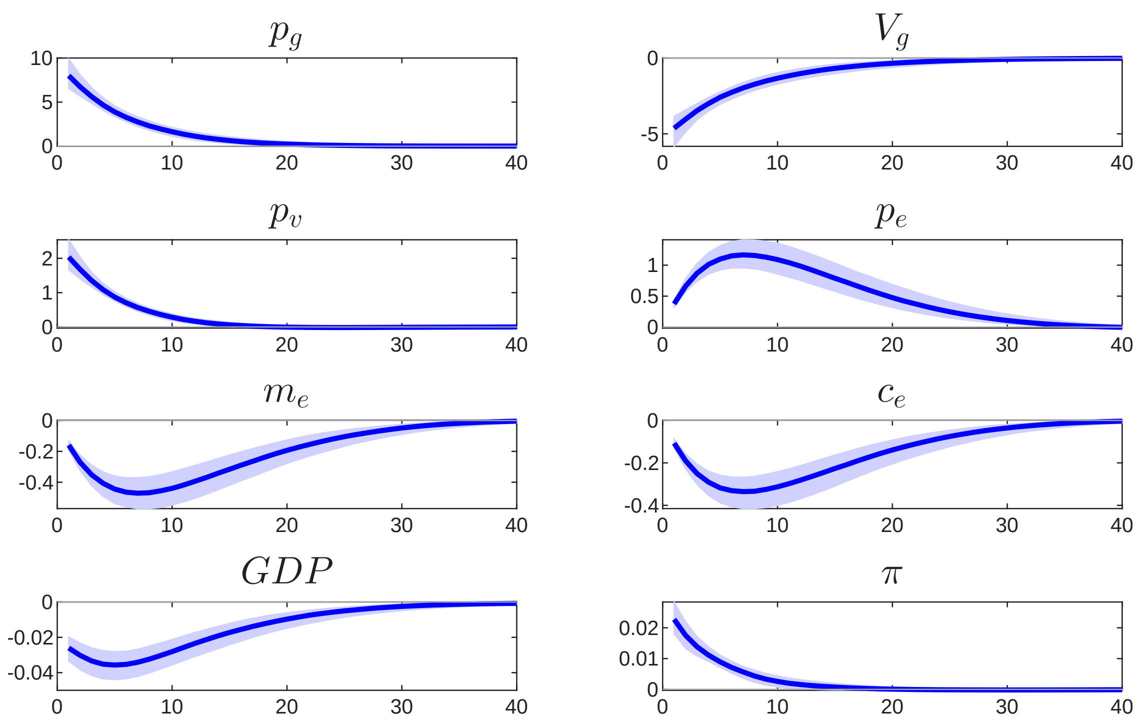

Figure 4. : ***Impulse Responses Functions (IRFs) to a Natural Gas Price Shock.*** *The figure reports the responses of the natural gas price (p_g), natural gas use in energy production (V_g), the crude-energy price index (p_v), the refined-energy price (p_e), intermediate energy demand by core-sector firms (m_e), household energy consumption (c_e), real GDP (GDP), and headline inflation (π) to a positive one-standard-deviation shock to the idiosyncratic component of the gas price. Shaded areas denote the 20th and 80th percentile credible bands associated with parameter uncertainty. The IRFs represent percentage deviations from the steady state.*

Smoothed shocks

The estimated model provides a historical decomposition of the shocks associated with the pandemic in 2020 and the subsequent energy-price shocks. Figure 5 reports the smoothed shocks over the sample, with a focus on the period from 2015:Q1 to 2025:Q4.[21] The main shocks driving the COVID-19 recession and the subsequent recovery are concentrated in the final part of the sample. The pandemic is associated with a large increase in labor disutility (ε_φ) in 2020:Q2, followed by a sharp reversal in 2020:Q3, consistent with the collapse and subsequent rebound in hours worked. The discount-factor shock (ε_ζ)

[20]We focus on gas price shocks in the main text because gas prices display the largest increases in our sample. The corresponding simulations for oil, coal, and the common crude-energy factor are reported in Appendix E.

[21]The smoothed shocks are obtained by running the Kalman smoother on the full sample, from 1990:Q1 to 2025:Q4, using the posterior mean of the parameters reported in Tables 2 and 3.

also fluctuates markedly during this period, turning negative in 2020:Q2 and positive in 2020:Q3, in line with a temporary rise in savings followed by a recovery in consumption.

The pandemic period is also characterized by a large negative global shock (ε_W) in 2020:Q2, capturing the contraction in world economic activity, followed by a sharp rebound in 2020:Q3. Energy-price shocks display a more heterogeneous pattern across commodities. In particular, gas and coal price shocks (ε_{p_g} and ε_{p_c}) become strongly positive and remain persistently elevated in the post-pandemic period, especially around 2022, consistent with the sustained rise in EA energy prices (see Figure 1 in the introduction). By contrast, oil price shocks (ε_{p_o}) are more volatile and less persistent over the same period.

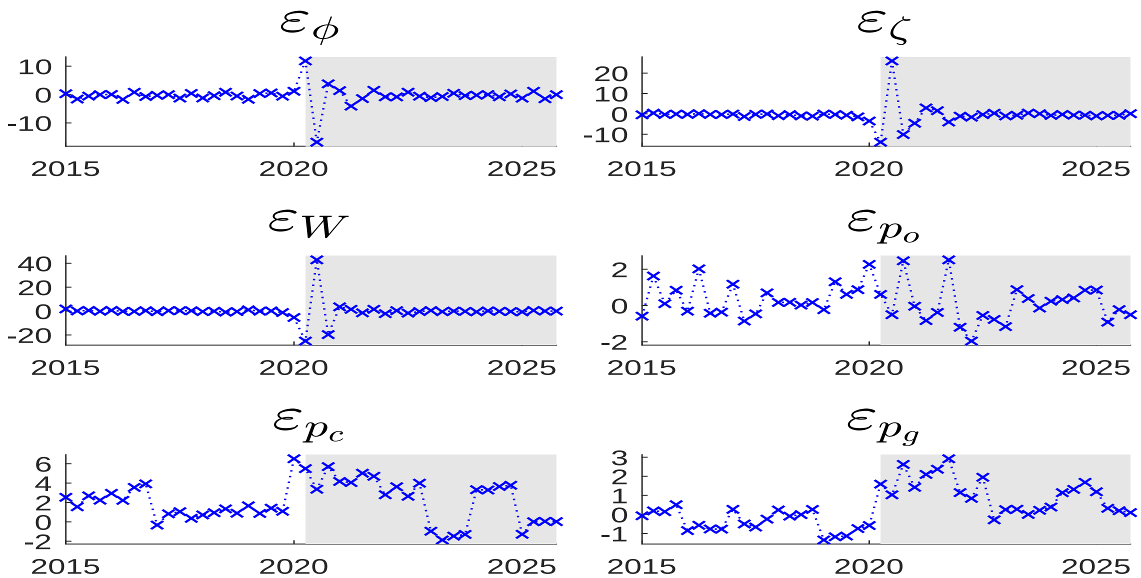

Figure 5. : ***Smoothed Shocks Over the Sample****. The shocks are standardized by their respective standard deviations and are computed using the posterior mean of the model parameters reported in Tables 2 and 3. The gray-shaded area denotes the period after the outbreak of the pandemic, starting in 2020:Q2. ε_φ denotes the labor-supply shock, ε_ζ the discount-factor shock, ε_W the global-GDP shock, ε_{p_o} the oil-price shock, ε_{p_c} the coal-price shock, and ε_{p_g} the gas-price shock.*

Energy shocks

To isolate the contribution of energy-price shocks, we perform a counterfactual exercise in which we shut down shocks to oil, coal, and gas prices (ε_{p_o}, ε_{p_c}, ε_{p_g}) over the period 2020:Q2–2022:Q3; see Figure 6. As shown in Figure 5, these shocks are predominantly positive over this interval and capture the unusually large increase in energy prices observed in the EA. We focus on these shocks as they directly drive energy price fluctuations in the model, allowing us to isolate their contribution to the dynamics of prices and economic activity during this period.

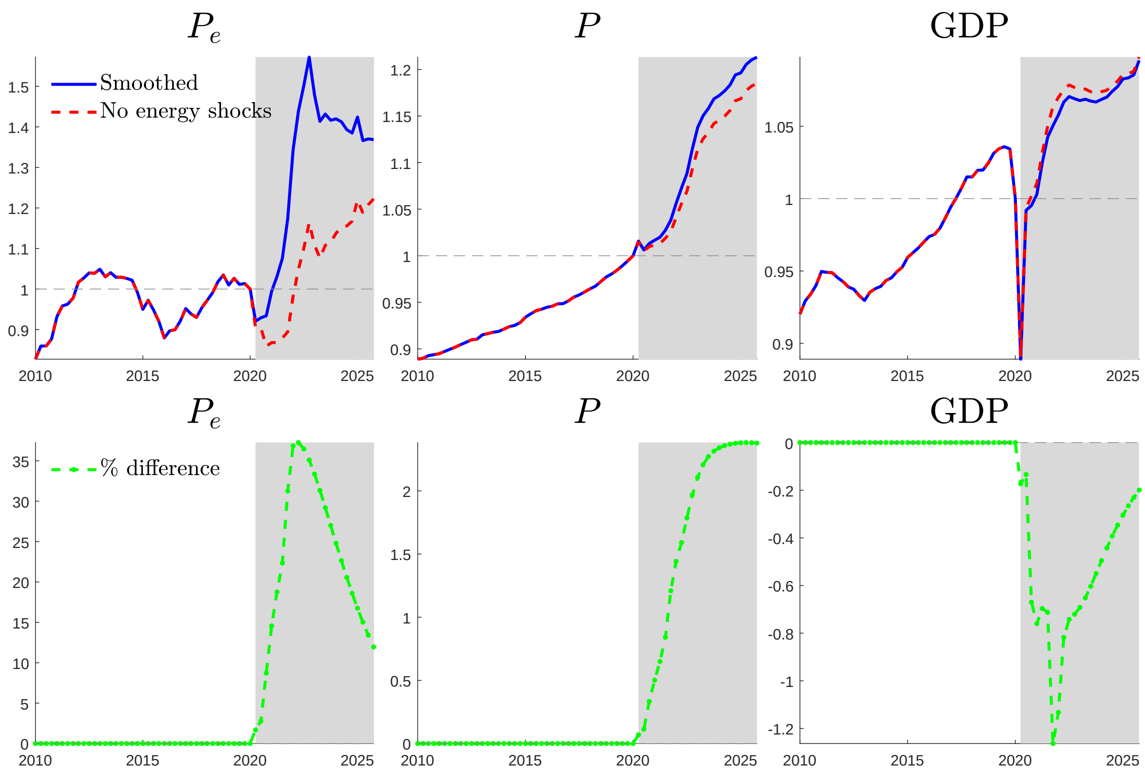

Figure 6. : ***Energy Counterfactual****. The top row reports the smoothed (realized) series (solid blue lines) together with their counterfactual counterparts (dashed red lines), obtained by shutting down shocks to oil, coal, and gas prices, ε_{p_o}, ε_{p_c}, and ε_{p_g}, over the period 2020:Q2–2022:Q3. The three columns report, respectively, the HICP of energy products (P_e), the aggregate HICP (P), and real gross domestic product (GDP). The bottom row reports the percentage difference between the smoothed and counterfactual series (green dash-dotted lines), which measures the contribution of energy-specific shocks over time. All level variables are normalized to one in 2020:Q1, the quarter immediately preceding the start of the counterfactual, so that deviations can be interpreted as percentage changes relative to the pre-counterfactual level. The shaded area denotes the period following the onset of the COVID-19 pandemic.*

Figure 6 reports the results of the energy counterfactual. The first row shows the smoothed paths of the main variables together with their counterfactual counterparts obtained by shutting down oil, coal, and gas shocks from 2020:Q2 to 2022:Q3. The second row reports the corresponding percentage differences, which make the contribution of these shocks over time easier to visualize. For comparability with the figures in the introduction, all level variables are normalized to one in 2020:Q1, the quarter immediately preceding the start of the counterfactual.

The first column reports the effect on the HICP for energy products (P_e). Consistent with the evidence discussed in the introduction, energy prices increase by about 57% in the data, while in the absence of energy shocks they would have risen by only about 16%. Thus, a large share of the increase in refined energy prices is directly attributable to shocks to oil, coal, and gas. The second column reports the effect on the total HICP (P). Over the period 2020:Q2 to 2022:Q3, headline inflation rises by about 12% in the data, while in the absence of energy shocks it would have increased by approximately 9%.

Energy shocks therefore account for a substantial share of the overall rise in consumer prices. First, energy prices enter headline inflation directly through the energy component of the consumption basket, see equation (17). Second, higher energy prices raise firms' marginal costs because energy is used as an intermediate input (m_e), thereby increasing core prices (*production channel*). Third, they affect households' consumption choices, as higher energy prices induce substitution effects that alter the demand for non-energy goods and, in turn, their prices (P) (*consumption channel*). We examine the latter two mechanisms in more detail below. The third column reports the response of real GDP (GDP). Energy shocks depress economic activity over most of the period, so that by 2022:Q3 output is about 0.75% lower than in the counterfactual without these shocks.This evidence is consistent with interpreting recent energy disturbances as adverse supply shocks for the EA economy, in line with Liadze et al. (2022), who estimate that the Russo-Ukrainian war reduced European GDP by between 1% and 2%.

Figure 7 reports the contribution shares of oil, gas, and coal to the movements in headline inflation (P) and in the energy price level (P_e). These shares are obtained by comparing the baseline economy with counterfactual scenarios in which shocks to one energy source at a time are shut down over the period 2020:Q2 to 2022:Q3. More specifically, for each energy source we construct a counterfactual path by shutting down the corresponding idiosyncratic shock while leaving all other shocks unchanged. The contribution of each source is then measured as the difference between the baseline and the corresponding counterfactual path, expressed as a share of the total contribution of all energy shocks. At the end of the sample, oil and gas account for the largest shares in the increase in headline inflation, while coal plays a smaller role. Oil contributes substantially due to its larger weight in energy use and higher relative price, whereas gas, makes a comparable contribution because of the magnitude of its price increase A similar pattern emerges for the energy price level (P_e), where oil and gas account for most of the variation and coal contributes relatively less.

The role of complementarities

The estimated elasticities of substitution between energy and non-energy goods in production (σ_{k_c}) and consumption (σ_c) are both below one, implying complementarities. To assess their quantitative importance, we compute IRFs to a gas price shock under alternative values of σ_{k_c}. Figure 8 shows that lower substitutability amplifies the decline in GDP. Quantitatively, the impact response of GDP is about 60% larger at the estimated value of σ_{k_c} than under unit elasticity. The reason is that, when relative energy prices rise, firms have limited scope to substitute away from energy inputs. As a result, the share of energy costs in production, $s_{m_e} \equiv \frac{p_e m_e}{r_{k_e} u_{k_e} k_e + p_e m_e}$, increases after the shock when $\sigma_{k_c} < 1$, further amplifying the contraction in output.

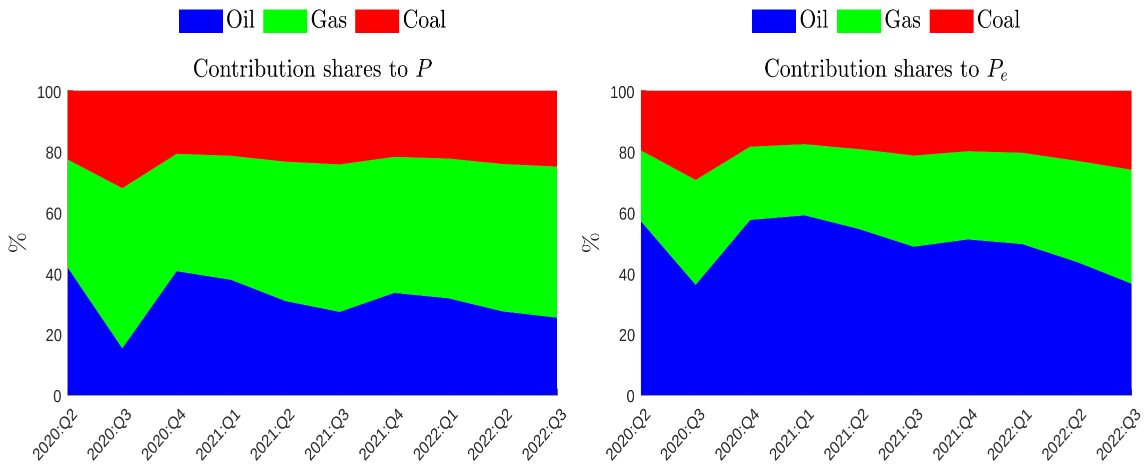

Figure 7. : ***Contribution Shares of Oil, Gas, and Coal****. The figure reports the percentage contributions of oil (blue), gas (green), and coal (red) to movements in headline inflation (P, left panel) and in the energy price level (P_e, right panel) over the period 2020:Q2-2022:Q3. Contributions are computed by comparing the baseline economy with counterfactual scenarios in which shocks to each energy source are shut down one at a time. The vertical axis is normalized so that 100% corresponds to the total contribution of energy shocks.*

These findings indicate that complementarities are quantitatively important for the transmission of large energy shocks. Additional results, including alternative shocks and the role of σ_c, are reported in Appendix E, where we also discuss how wage indexation amplifies energy shocks.

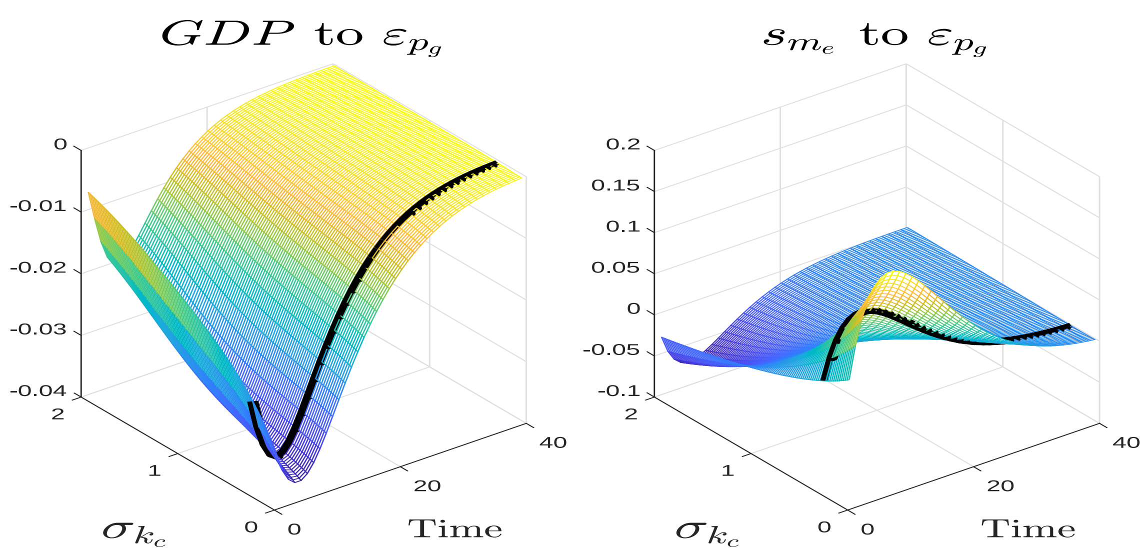

Figure 8. : ***The Role of the Elasticity of Substitution in the Transmission of a Natural Gas Price Shock.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_g}) to the price of natural gas (p_g), computed for alternative values of the elasticity of substitution parameter σ_{k_c}. The black line corresponds to the response at the estimated value of σ_{k_c}. IRFs represent percentage deviations from the steady state.*

Keynesian supply shocks

Keynesian supply shocks (KSS) arise when a negative supply disturbance, such as a supply-chain disruption, triggers a decline in aggregate demand that is larger than the initial supply contraction; see Guerrieri et al. (2022). In that case, the shock is effectively transmitted as a demand shock, leading to a fall in prices. Guerrieri et al. (2022) emphasize several mechanisms that can generate KSS, including complementarities in consumption, incomplete markets, and firm exit. Because negative energy supply shocks could, in principle, have this characterization in our model, we apply the framework of Guerrieri et al. (2022), originally developed for pandemic shocks, to our setting.

As shown in Section IV, the overall effect of energy shocks on core-sector prices (P_c) is positive, which rules out a Keynesian supply-shock interpretation in our baseline model. To highlight the mechanisms that distinguish our setup from that of Guerrieri et al. (2022), Figure 9 reports the IRFs of core-sector inflation (π_c) to a generic supply shock to the common factor in crude energy prices. At the estimated parameter values (Baseline model, blue solid line), π_c increases after the energy shock, implying positive pass-through from energy prices to core-sector prices and therefore excluding KSSs.

To connect our framework to the key features of the stylized model in Guerrieri et al. (2022), we next consider a parameterization with no consumption habits ($h \approx 0$), flexible wages ($\theta_w \approx 0$), and no use of energy in production ($s_{m_e,k_c} \approx 0$). We also set the elasticity of substitution between energy and core goods in consumption to a very low value ($\sigma_c = 0.01$), so that the two goods are strong complements. Under this configuration, shown by the dashed red line, core-sector inflation falls after the energy shock, mimicking the effect of a negative demand shock. When consumption goods are strong complements and habits are absent, a decline in energy consumption generates a sufficiently large decline in demand for core goods (*consumption channel*) to reduce their prices. At the same time, because intermediate energy in production is shut down ($s_{m_e,k_c} \approx 0$), higher energy prices do not raise marginal costs in the core sector through the production function in equation (10), so the *production channel* implying higher final good prices is absent.

To further study the interaction between the consumption and production channels, we perform a second exercise in which we vary the elasticity of substitution in consumption (σ_c) and the share of energy in production (s_{m_e,k_c}). Figure 10 reports the impact response of core-sector inflation (π_c) to a positive shock (ε_{p_f}) to the common crude-energy factor (p_f). We focus on this shock because it generates a broad increase in energy prices and avoids substitution effects across individual fuels. The responses are computed under the KSS parameterization described above, with σ_c ranging from 0.01 (strong complementarity) to 1.50 (strong substitutability), and s_{m_e,k_c} ranging from 0.0001 (almost no energy in production) to 0.0156 (three times the baseline value). When substitutability is low and the energy share in production is small, the response of π_c is negative, generating KSS. In

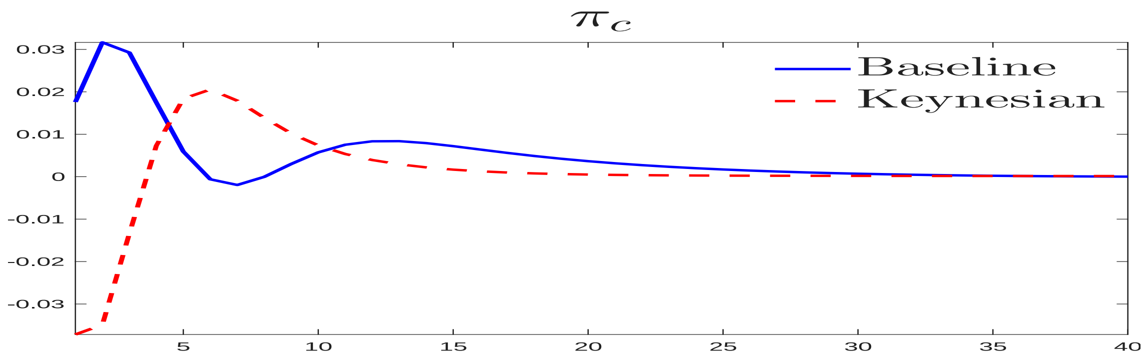

Figure 9. : ***Energy Shocks as Keynesian Supply Shocks.*** *The figure reports the impulse response function (IRFs) of core-sector inflation (π_c) to a positive one-standard-deviation shock (ε_{p_f}) to the common crude-energy component (p_f), computed at the posterior mean of the estimated parameters (Baseline) and under the parameter configuration that generates Keynesian supply shocks (Keynesian). The IRFs represent percentage deviations from the steady state.*

this case, the negative demand-complementarity effect dominates the positive supply-side effect of higher energy prices on core prices. By contrast, when demand complementarities are weak and the energy share in production is large, core-sector prices rise after the shock.

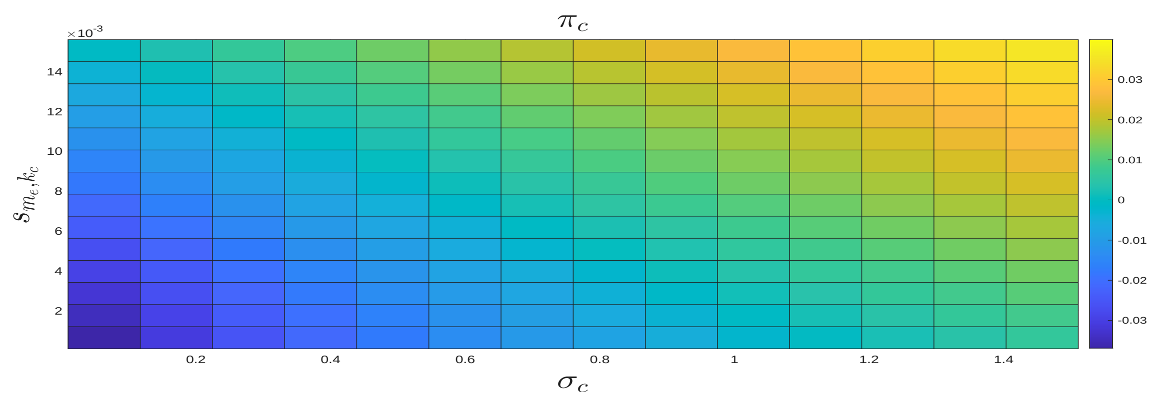

Figure 10. : ***Energy shocks as Keynesian Supply Shocks Heatmap****. The figure shows the impact response of core-sector inflation (π_c) to a positive one-standard-deviation shock (ε_{p_f}) to the common crude-energy component (p_f) for alternative values of the elasticity of substitution in consumption, σ_c, and the energy share in production, s_{m_e,k_c}. Darker colors correspond to more negative responses of π_c, consistent with a Keynesian supply-shock transmission, while lighter colors correspond to positive responses. Responses are expressed as percentage deviations from steady state.*

To isolate the role of complementarities in a simplified setting, we consider a stripped-down version of the model that removes capital, intermediate energy in production, habits, and nominal rigidities in both goods and labor markets, and excludes energy as an input in production. This setup closely resembles the framework in Guerrieri et al. (2022). Figure 11 reports the response of core-sector inflation for different values of σ_c, shown in the color bar: values of σ_c below one generate KSS effects, whereas values above one do not. By contrast, we do not find KSS effects in the estimated full model, where real and nominal rigidities, together with pass-through from energy used in production, dampen the Keynesian transmission mechanism.

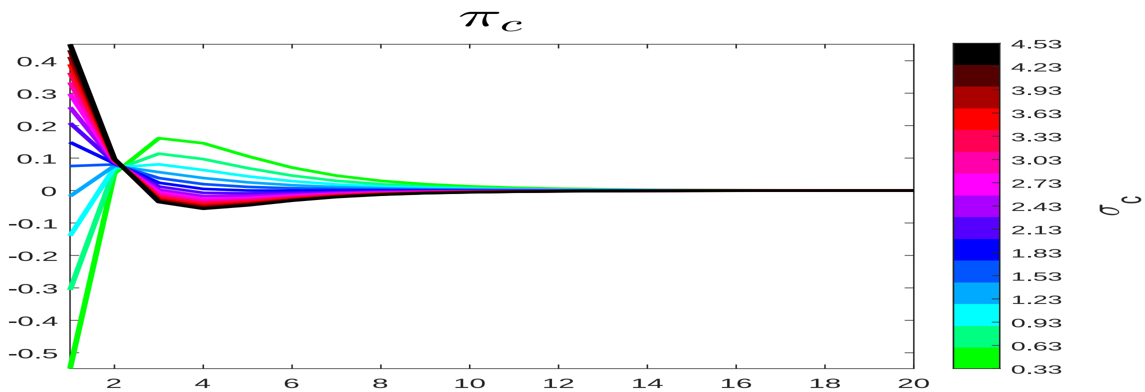

Figure 11. : ***Impulse Responses of Core Inflation to an Energy Price Shock****. The figure shows the impulse responses of core-sector inflation (π_c) to a one-standard-deviation shock to the common crude-energy component (p_f), computed under alternative values of the complementarity parameter σ_c (see color bar). The model abstracts from capital and nominal rigidities, allowing for a characterization of energy disturbances as Keynesian supply shocks. All responses are expressed as percentage deviations from steady state.*

Monetary policy

The literature has long emphasized the role of monetary policy in amplifying the effects of energy shocks, see Bernanke et al. (1997), Leduc and Sill (2004), Blanchard and Gali (2007), Kormilitsina (2011) and Ramey and Vine (2011). As shown in Section III, the energy sector, despite its modest share in total production, plays an important role in inflation dynamics. In our framework, energy shocks therefore affect GDP not only directly, but also indirectly through the monetary policy response to higher inflation. By contrast, Bodenstein and Guerrieri (2011) find that monetary policy plays only a limited role in the transmission of oil price shocks, because in their estimated model most oil price fluctuations originate abroad.

The top panel of Figure 12 reports the IRF of headline inflation (π) to an exogenous increase in the price of natural gas under alternative values of the inflation coefficient in the Taylor rule, r_π, ranging from 1.25 (a loose response) to 2.50 (an aggressive response). The figure also reports the IRF at the estimated value, $r_\pi = 1.74$ (solid black line). A stronger monetary response reduces the inflationary effect of the gas price shock. On impact, a loose response with $r_\pi = 1.25$ implies an increase in inflation that is 62% larger than under a more aggressive response with $r_\pi = 2.50$. This tighter inflation stabilization comes at the cost of a larger decline in output, as shown in the bottom panel of Figure 12. The impact decline in GDP is about three times larger when $r_\pi = 2.50$ than when $r_\pi = 1.25$. Appendix E reports analogous results for shocks to individual crude energy prices and to the common factor in crude energy prices under alternative monetary policy responses.[22]

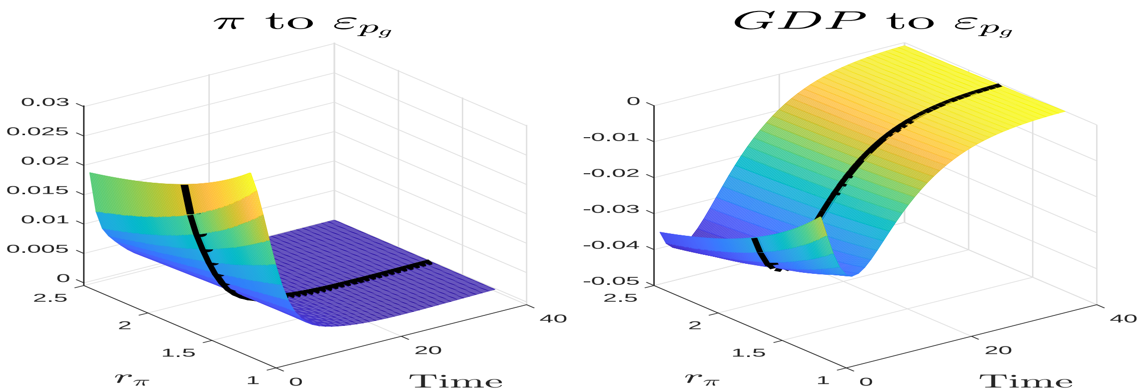

Figure 12. : ***Monetary Policy Transmission and the Response to a Natural Gas Price Shock****. The figure displays the impulse responses of inflation and output to a one-standard-deviation shock to the natural gas price (ε_{p_g}), computed under alternative values of the central bank's responsiveness to inflation (r_π). All responses are expressed as percentage deviations from steady state.*

Figure 13 reports a counterfactual scenario in which the policy rate (upper panel) is held constant at its 2021:Q4 level, thereby shutting down the subsequent monetary tightening. To implement this experiment, we simulate a sequence of monetary policy shocks, $\varepsilon_{e,t}$, that offsets the observed increase in the interest rate between 2022:Q1 and 2022:Q4.

By the end of the sample, 2022:Q4, headline inflation (π) and core inflation (π_c) are about 80 basis points higher than in the baseline with tightening. Energy inflation (π_e), by contrast, is about 20 basis points lower. Real GDP is approximately 1.60% higher under the counterfactual. These results highlight the trade-off faced by the central bank between containing inflation and supporting economic activity in the presence of large energy shocks.

Fiscal policy

We next consider a fiscal intervention aimed at shielding users of refined energy from the increase in energy prices. The fiscal authority introduces a subsidy, $\tau_{e,t}$, that lowers the price of refined energy paid by households and firms in the core sector.[23] The policy is financed through a balanced-budget lump-sum tax, T_t, levied on households. Under this

[22] Analyzing how beliefs about the central bank's future stance on inflation affect the transmission of energy shocks, in the spirit of Bianchi (2013), is left for future research.

[23] For instance, several Euro area countries implemented energy price caps and subsidies during 2021–2022. In Germany, the *Energiepreisbremsen* introduced in 2022 capped retail electricity and gas prices for households and firms, while Italy and France adopted similar measures to limit the pass-through of wholesale energy prices to final consumers. These policy responses are discussed in a broader European perspective by Pollitt (2024) and Pollitt et al. (2022).

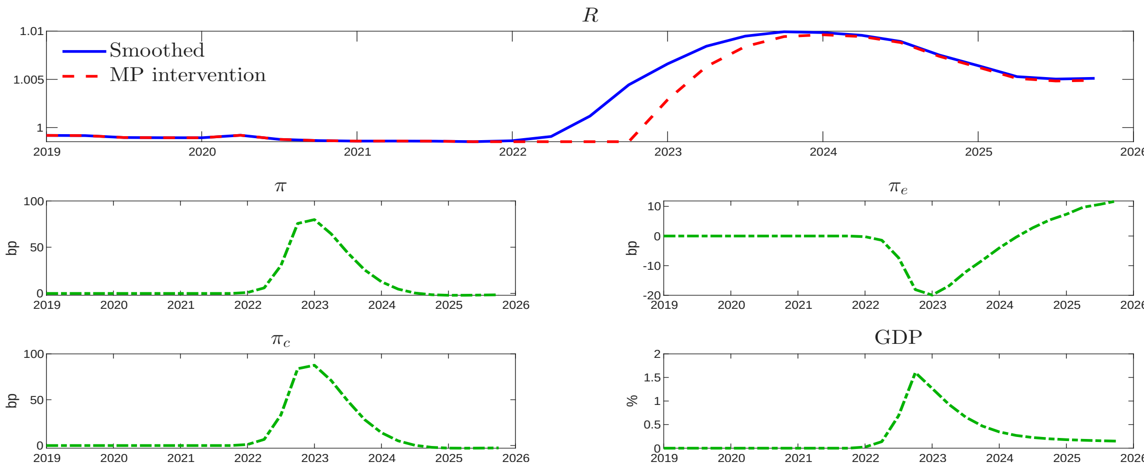

Figure 13. : ***Effects of Accommodative Monetary Policy on Energy Price Shocks.*** *The top panel reports the nominal interest rate (R), with the solid blue line denoting the smoothed series and the dashed red line the counterfactual path under monetary policy intervention. The remaining panels report the differences in headline inflation (π), energy inflation (π_e), core inflation (π_c), and real gross domestic product (GDP) between the counterfactual and the baseline scenario. The green dash-dotted lines denote these differences, measured in basis points (bp) for inflation rates and in percentage terms (%) for GDP.*

policy, the household budget constraint in equation (14) becomes

$$\begin{aligned} c_{c,t} + (1-\tau_{e,t})\, p_{e,t} c_{e,t} + \frac{i_{c,t}}{a_{k,t}} + p_{e,t}\frac{i_{e,t}}{a_{k,t}} + b_t = & \frac{R_{t-1} b_{t-1}}{\pi_{c,t}} + \frac{w_{c,t} n_{c,t}}{X_{w_c,t}} + p_{e,t}\frac{w_{e,t} n_{e,t}}{X_{w_e,t}} \\ & + r_{k_c,t} u_{k_c,t} k_{c,t-1} + p_{e,t} r_{k_e,t} u_{k_e,t} k_{e,t-1} \\ & + \Pi_t - \frac{\Psi_t}{a_{k,t}} + p_{e,t}(1-s_o^*) p_o V_o \\ & + p_{e,t}(1-s_g^*) p_g V_g + p_{e,t}(1-s_c^*) p_c V_c - T_t, \end{aligned}$$

and the profit function of core sector firms (see equation 9) becomes:

$$\max \frac{Y_{c,t}}{X_{c,t}} - w_{c,t} n_{c,t} - r_{k_c,t} u_{k_c,t} k_{c,t-1} - (1-\tau_{e,t})\, p_{e,t} m_{e,t}.$$

The subsidy ($\tau_{e,t}$) follows an exogenous AR(1) process,

$$\tau_{e,t} = \rho_{\tau_e} \tau_{e,t-1} + \varepsilon_{\tau_e,t},$$

where ρ_{τ_e} is the persistence parameter and the innovation $\varepsilon_{\tau_e,t}$ captures new policy interventions. The balanced-budget lump-sum tax (T_t) satisfies:

$$T_t = \tau_{e,t} p_{e,t} m_{e,t} + \tau_{e,t} p_{e,t} c_{e,t}. \tag{21}$$

In the quantitative exercise, we assume that the fiscal authority chooses interventions so as to keep the effective price of energy at its 2020:Q1 level, denoted t^*. Operationally, this requires increasing the subsidy rate to offset subsequent price increases. We set the persistence parameter to $\rho_{\tau_e} = 0.99$, so that the policy is perceived as long-lasting, although agents do not expect the fiscal authority to offset future energy price increases beyond the realized intervention path.

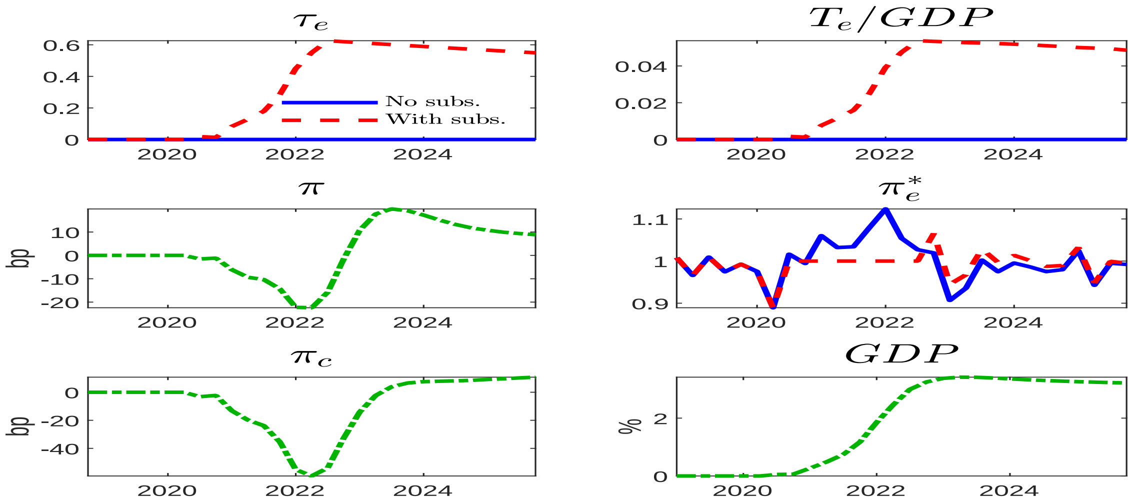

Figure 14. : ***Effects of Fiscal Policy in Response to Energy Price Shocks.*** *The figure reports the subsidy rate ($\tau_{e,t}$), the ratio of lump-sum taxes to gross domestic product (T_e/GDP), headline inflation (π), effective energy inflation ($\pi_e^* \equiv [p_{e,t}(1-\tau_{e,t})]/[p_{e,t-1}(1-\tau_{e,t-1})]$), core inflation ($\pi_c$), and real gross domestic product (GDP). In the panels for $\tau_{e,t}$, T_e/GDP, and π_e^*, the solid blue lines denote the smoothed series under no subsidy shock, while the dashed red lines denote the counterfactual with the subsidy policy in place. In the panels for π, π_c, and GDP, the dashed red lines denote the difference between the subsidy counterfactual and the no-subsidy baseline.*

Figure 14 reports the evolution of selected variables under the subsidy program and in the no-intervention baseline. For comparability with the monetary policy exercise, we plot the same variables as in Figure 13, except that the first row reports the fiscal instruments, namely the subsidy rate and the lump-sum tax. The subsidy rate, $\tau_{e,t}$, must rise by 62% to offset the cumulative increase in energy prices by 2022:Q4 (first panel). Financing this intervention requires a lump-sum tax equal to 5.3% of GDP in 2022:Q4 (second panel). The fourth panel shows that the policy is designed to stabilize effective energy inflation, $\pi_{e,t}^* \equiv \frac{p_{e,t}(1-\tau_{e,t})}{p_{e,t-1}(1-\tau_{e,t-1})}$, (red dashed line) rather than allowing the large positive energy inflation observed in the data (blue solid line). By lowering the effective price of energy, the policy mitigates the reduction in household energy consumption, $c_{e,t}$, and firms' demand for intermediate energy, $m_{e,t}$, thereby raising GDP through both the consumption and production channels. By 2022:Q4, GDP is about 3.25% higher than in the no-subsidy scenario. Figure 14 also shows that the subsidy reduces both headline inflation, π, and core inflation, π_c, relative to the no-intervention benchmark.

To understand these effects more clearly, we examine the role of feedback from EA

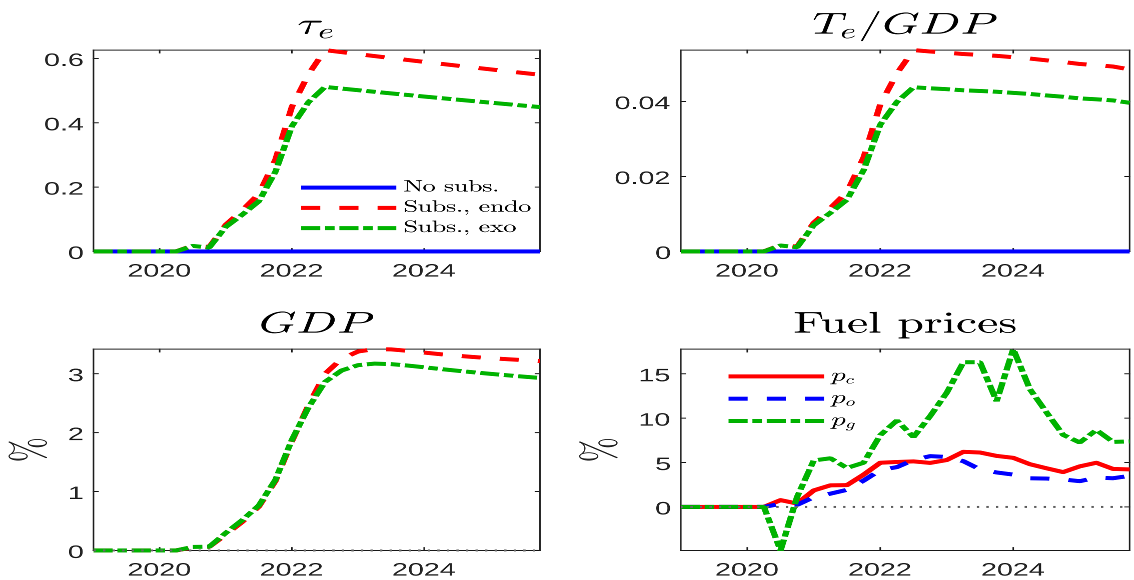

Figure 15. : ***The Role of Endogenous Energy Prices in Fiscal Policy****. The figure compares the effects of an energy subsidy under two alternative assumptions: fuel prices (oil, gas, and coal) are either endogenous and respond to EA demand (baseline model), or exogenous. The top panels report the subsidy rate (τ_e) and the ratio of lump-sum taxes to GDP (T_e/GDP), while the bottom-left panel reports the percentage deviation of real GDP relative to the no-subsidy baseline within each specification. Solid blue lines denote the smoothed series without subsidies, dashed red lines the subsidy counterfactual with endogenous prices, and dash-dotted green lines the counterfactual with exogenous prices. The bottom-right panel reports the difference in fuel-price responses between the two specifications for coal (p_c), oil (p_o), and gas (p_g). Positive values indicate that the subsidy raises fuel prices more when prices are endogenous, reflecting feedback from EA demand.*

demand to energy prices. Figure 15 compares the subsidy under two alternative specifications. The first is the baseline estimated model, in which oil, gas, and coal prices are endogenous and respond to EA demand according to equation (8), through the feedback elasticities η_o, η_c, and η_g. The second shuts down this channel by treating energy prices as exogenous.

When energy prices are exogenous, the subsidy lowers the effective price of refined energy without generating any feedback on crude fuel prices. In that case, the policy is more effective because it does not induce further increases in input costs. By contrast, when energy prices are endogenous, the subsidy raises the demand for refined energy by households and firms in the core sector. Energy producers respond by increasing the use of oil, gas, and coal, which pushes up the prices of these inputs. As shown in the bottom-right panel of Figure 15, this feedback is particularly strong for gas, whose price rises by as much as 18%, while oil and coal prices also increase, although by less. Energy firms pass these higher costs through to refined energy prices, which in turn requires a larger subsidy to stabilize the effective price of energy.

This creates a feedback loop: higher subsidies stimulate energy demand, stronger demand raises crude fuel prices, and higher input costs require additional fiscal support. Although this mechanism increases the subsidy rate and the associated tax burden, it also supports aggregate demand and output. At the same time, it makes the policy less efficient, because part of the intended relief is offset by the induced increase in energy prices. We quantify this inefficiency using fiscal multipliers.

We compare the economic gains resulting from this policy (increases in GDP) with its costs (increases in taxes), quantifying the fiscal multipliers (Mountford and Uhlig, 2009 and Zubairy, 2014):

$$M_{\tau_e,h} = \frac{\mathbb{E}_t \sum_{i=0}^{h} \Delta \left(\frac{1}{R_{t|t+i}}\right) GDP_{t+i}}{\mathbb{E}_t \sum_{i=0}^{h} \Delta \left(\frac{1}{R_{t|t+i}}\right) T_{t+i}}, \tag{22}$$

where the discount factor is the product of gross risk-free policy rates from t to $t+i$:

$$R_{t|t+i} = \prod_{j=0}^{i} R_{t+j}.$$

In equation (22), at each horizon i after the subsidy shock ($\varepsilon_{\tau_e,t}$), we compute (i) the gain in GDP relative to the no-subsidy case, ΔGDP_{t+i}, and (ii) the corresponding increase in taxes, ΔT_{t+i}, which equals the increase in subsidies under the balanced-budget rule in equation (21).

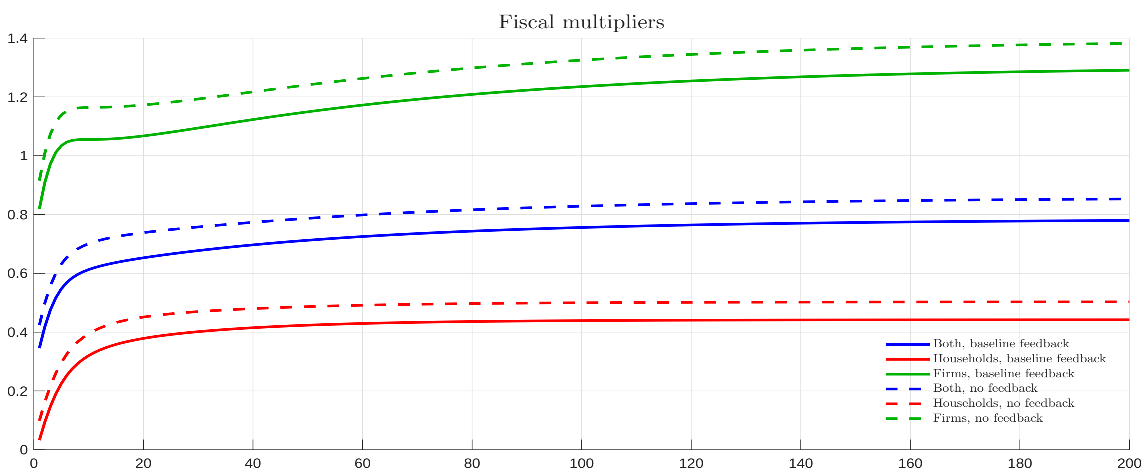

Figure 16. : ***Fiscal Multipliers.*** *The figure reports the cumulative fiscal multipliers associated with the energy-subsidy policy at different horizons after a shock to the subsidy rate, ε_{τ_e}. The horizontal axis measures the number of quarters after the shock. The two upper green lines correspond to subsidies targeted to firms, with the solid line referring to the baseline model with endogenous energy prices and the dashed line to the specification with exogenous energy prices. The two middle blue lines correspond to joint subsidies to households and firms, again with the solid line denoting the baseline model and the dashed line the exogenous-price case. The two lower red lines correspond to subsidies targeted to households, with the same distinction between solid and dashed lines.*

Figure 16 reports the cumulative multipliers $M_{\tau_e,h}$ at different horizons. Multipliers are positive on impact and increase over time, with substantial heterogeneity across policy designs. We distinguish between interventions targeted to households, firms, and both jointly, and between the baseline model with endogenous energy prices and the case with exogenous prices. In the baseline specification, the multipliers converge to 0.40 for household-targeted interventions, 1.30 for firm-targeted interventions, and 0.80 when both policies are implemented jointly. The larger multipliers for firms reflect stronger supply-side effects, since lower energy costs directly reduce production costs.

Multipliers are systematically larger when energy prices are exogenous. In that case, they converge to 0.50 for household-targeted interventions, 1.40 for firm-targeted interventions, and 0.85 when both policies are active. Without feedback from EA demand to energy prices, the subsidy does not raise crude input costs and therefore becomes more effective. By contrast, when energy prices are endogenous, the subsidy increases demand for fossil fuels, raises their prices, and partially offsets the reduction in effective energy costs. This feedback weakens transmission and lowers the associated multipliers. These findings suggest that endogenous energy-price feedback is a key component in the evaluation of energy-relief policies. At the same time, our framework abstracts from mechanisms that could amplify the effectiveness of subsidies, such as imperfect consumption smoothing by debt-constrained households that benefit most from the policy (Bhattarai and Trzeciakiewicz, 2017).[24]

V. Conclusion

This paper develops and estimates a two-sector macroeconomic model in which an energy sector combines crude energy sources—coal, oil, and gas—to produce refined energy used by households and a core sector. The model is estimated on EA data covering the surge in energy prices following the COVID-19 pandemic, which allows us to distinguish between the effects of pandemic-related shocks and those of energy shocks over 2020–2022.

We find that shocks to oil, coal, and gas are the main drivers of refined energy prices, while their contribution to headline inflation is smaller but still economically meaningful. Oil and gas account for most of the variation in energy prices, whereas coal plays a more limited role. The model also highlights several amplification mechanisms. Complementarities in energy use, labor market inertia, and monetary policy all strengthen the effects of energy shocks. At the same time, the use of energy in production, together with the estimated nominal and real rigidities, implies positive pass-through to core-sector prices, ruling out a standard Keynesian transmission of these shocks.

We also study the role of fiscal policy in mitigating the effects of energy shocks. The effectiveness of energy subsidies depends on the response of energy prices to domestic demand. When this feedback is present, stronger demand raises fuel prices and partially offsets the decline in effective energy costs, weakening the transmission of the policy.

[24] At the same time, introducing constraints on public finance could reduce the effectiveness of this policy.

When energy prices are exogenous, this channel is absent and fiscal interventions are more effective. Future work could extend the model to allow for limited consumption smoothing by households, which may increase the effectiveness of fiscal policy. Another extension would be to incorporate green energy sources and study how energy shocks interact with the transition toward cleaner technologies.

REFERENCES

Acurio Vásconez, V. (2015). What If Oil Is Less Substitutable? A New-Keynesian Model with Oil, Price and Wage Stickiness Including Capital Accumulation. Technical report, Université Panthéon-Sorbonne (Paris 1), Centre d'Economie de la Sorbonne.

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Mihoubi, F., Mutschler, W., Pfeifer, J., Ratto, M., Rion, N., and Villemot, S. (2022). Dynare: Reference Manual Version 5. Dynare Working Papers 72, CEPREMAP.

Adolfsen, J. F., Kuik, F., Schuler, T., and Lis, E. (2022). The Impact of the War in Ukraine on Euro Area Energy Markets. *Economic Bulletin Boxes*, 4.

Adolfsen, J. F., Minesso, M. F., Mork, J. E., and Van Robays, I. (2024). Gas Price Shocks and Euro Area Inflation. *Journal of International Money and Finance*, 149:103183.

Albrizio, S., Bluedorn, J. C., Koch, C., Pescatori, A., and Stuermer, M. (2022). Market Size and Supply Disruptions: Sharing the Pain From a Potential Russian Gas Shut-off to the EU. *IMF Working Papers*, 143:2022.

Alessandri, P. and Gazzani, A. (2025). Natural Gas and the Macroeconomy: Not All Energy Shocks are Alike. *Journal of Monetary Economics*, 151:103749.

Amiti, M., Heise, S., Karahan, F., and Şahin, A. (2024). Inflation Strikes Back: The Role of Import Competition and the Labor Market. *NBER Macroeconomics Annual*, 38(1):71–131.

Argentiero, A., Bollino, C. A., Micheli, S., and Zopounidis, C. (2018). Renewable Energy Sources Policies in a Bayesian DSGE Model. *Renewable Energy*, 120:60–68.

Bachmann, R., Baqaee, D., Bayer, C., Kuhn, M., Löschel, A., Moll, B., Peichl, A., Pittel, K., Schularick, M., et al. (2022). What If? The Economic Effects for Germany of a Stop of Energy Imports from Russia. *ECONtribute Policy Brief*, 28:2022.

Balke, N. S. and Brown, S. P. (2018). Oil Supply Shocks and the US Economy: An Estimated DSGE Model. *Energy Policy*, 116:357–372.

Baqaee, D. and Farhi, E. (2022). Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis. *American Economic Review*, 112(5):1397–1436.

Baumeister, C., Korobilis, D., and Lee, T. K. (2022). Energy Markets and Global Economic Conditions. *Review of Economics and Statistics*, 104(4):828–844.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The Financial Accelerator in a Quantitative Business Cycle Framework. *Handbook of Macroeconomics*, 1:1341–1393.

Bernanke, B. S., Gertler, M., Watson, M., Sims, C. A., and Friedman, B. M. (1997). Systematic Monetary Policy and the Effects of Oil Price Shocks. *Brookings Papers on Economic Activity*, 1997(1):91–157.

Bhattarai, K. and Trzeciakiewicz, D. (2017). Macroeconomic Impacts of Fiscal Policy Shocks in the UK: A DSGE Analysis. *Economic Modelling*, 61:321–338.

Bianchi, F. (2013). Regime Switches, Agents' Beliefs, and Post-World War II US Macroeconomic Dynamics. *Review of Economic Studies*, 80(2):463–490.

Bjørnland, H. C., Larsen, V. H., and Maih, J. (2018). Oil and Macroeconomic (in) Stability. *American Economic Journal: Macroeconomics*, 10(4):128–51.

Blanchard, O. J. and Gali, J. (2007). The Macroeconomic Effects of Oil Shocks: Why are the 2000s so different from the 1970s?

Bodenstein, M., Erceg, C., and Guerrieri, L. (2007). Optimal Monetary Policy in a Model with Distinct Core and Headline Inflation Rates. *Journal of Monetary Economics, forthcoming*.

Bodenstein, M., Erceg, C. J., and Guerrieri, L. (2011). Oil Shocks and External Adjustment. *Journal of International Economics*, 83(2):168–184.

Bodenstein, M. and Guerrieri, L. (2011). Oil Efficiency, Demand, and Prices: A Tale of Ups and Downs. Technical report, Board of Governors of the Federal Reserve System (US).

Boeck, M. and Zörner, T. O. (2025). Natural Gas Prices, Inflation Expectations, and the Pass-through to Euro Area Inflation. *Energy Economics*, 141:108061.

Borağan Aruoba, S., Cuba-Borda, P., and Schorfheide, F. (2018). Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries. *The Review of Economic Studies*, 85(1):87–118.

Brinca, P., Duarte, J. B., and Faria-e Castro, M. (2021). Measuring Labor Supply and Demand Shocks During COVID-19. *European Economic Review*, 139:103901.

Bruhin, J., Scheufele, R., and Stucki, Y. (2023). The Economic Impact of Russia's Invasion of Ukraine on European Countries-a SVAR Approach. Technical report, Swiss National Bank.

Caldara, D., Conlisk, S., Iacoviello, M., and Penn, M. (2022). The Effect of the War in Ukraine on Global Activity and Inflation. Technical report, Board of Governors of the Federal Reserve System (US).

Cardani, R., Croitorov, O., Giovannini, M., Pfeiffer, P., Ratto, M., and Vogel, L. (2022). The Euro Area's Pandemic Recession: A DSGE-based Interpretation. *Journal of Economic Dynamics and Control*, 143:104512.

Casoli, C., Manera, M., and Valenti, D. (2024). Energy shocks in the Euro Area: Disentangling the Pass-through From Oil and Gas Prices to Inflation. *Journal of International Money and Finance*, 147:103154.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113:1–45.

Dew-Becker, I. (2023). Tail Risk in Production Networks. *Econometrica*, 91(6):2089–2123.

Dhawan, R., Jeske, K., and Silos, P. (2010). Productivity, Energy Prices and the Great Moderation: A New Link. *Review of Economic Dynamics*, 13(3):715–724.

Di Bella, G., Flanagan, M., Foda, K., Maslova, S., Pienkowski, A., Stuermer, M., and Toscani, F. (2024). Natural Gas in Europe: the Potential Impact of Disruptions to Supply. *Energy Economics*, 138:107777.

Di Giovanni, J., Kalemli-Özcan, Silva, A., and Yildirim, M. A. (2022). Global Supply Chain Pressures, International Trade, and Inflation. Technical report, National Bureau of Economic Research.

Dissou, Y. and Karnizova, L. (2016). Emissions Cap or Emissions Tax? A Multi-Sector Business Cycle Analysis. *Journal of Environmental Economics and Management*, 79:169–188.

Donoval, M., Gautier, E., Nuño, G., Nakov, A., Jiménez, N., de los Llanos Matea, M., Estrada, Á., Zioutou, P., Bragoudakis, Z., Weymes, L., et al. (2010). Energy Markets and the Euro Area Macroeconomy. Technical report, European Central Bank.

Eurostat (2026a). Glossary: Gross Available Energy. Accessed: 2026-02-12.

Eurostat (2026b). Import Dependency on Third Countries by Fuel Type. Online data code: nrg_ind_id3cf. Accessed: 2026-02-12.

Eurostat (2026c). Simplified Energy Balances. Online data code: nrg_bal_s. Accessed: 2026-02-12.

Fagan, G., Henry, J., and Mestre, R. (2005). An Area-Wide Model for the Euro Area. *Economic Modelling*, 22(1):39–59.

Faria-e Castro, M. (2021). Fiscal Policy During a Pandemic. *Journal of Economic Dynamics & Control*, 125:104–135.

Ferroni, F., Fisher, J. D., and Melosi, L. (2024). Unusual Shocks in Our Usual Models. *Journal of Monetary Economics*, 147:103598.

Finn, M. G. (2000). Perfect Competition and the Effects of Energy Price Increases on Economic Activity. *Journal of Money, Credit and Banking*, pages 400–416.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal Taxes on Fossil Fuel in General Equilibrium. *Econometrica*, 82(1):41–88.

Guerrieri, V., Lorenzoni, G., Straub, L., and Werning, I. (2022). Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages? *American Economic Review*, 112(5):1437–1474.

Herbst, E. P. and Schorfheide, F. (2015). *Bayesian Estimation of DSGE Models*. Princeton University Press.

Iacoviello, M. and Neri, S. (2010). Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics*, 2:125–64.

Ireland, P. N. (2004). A Method for Taking Models to the Data. *Journal of Economic Dynamics and Control*, 28(6):1205–1226.

Jacquinot, P., Kuismanen, M., Mestre, R., and Spitzer, M. (2009). An Assessment of the Inflationary Impact of Oil Shocks in the Euro Area. *The Energy Journal*, 30(1).

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2010). Investment Shocks and Business Cycles. *Journal of Monetary Economics*, 57(2):132–145.

Kharroubi, E. and Smets, F. (2024). Energy Shocks as Keynesian Supply Shocks: Implications for Fiscal Policy. *European Economic Review*, 170:104875.

Kilian, L. (2009). Not All Oil Price Shocks are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market. *American Economic Review*, 99(3):1053–69.

Kim, I.-M. and Loungani, P. (1992). The Role of Energy in Real Business Cycle Models. *Journal of Monetary Economics*, 29(2):173–189.

Kormilitsina, A. (2011). Oil Price Shocks and the Optimality of Monetary Policy. *Review of Economic Dynamics*, 14(1):199–223.

Labandeira, X., Labeaga, J. M., and López-Otero, X. (2017). A Meta-Analysis on the Price Elasticity of Energy Demand. *Energy Policy*, 102:549–568.

Leduc, S. and Sill, K. (2004). A Quantitative Analysis of Oil-Price Shocks, Systematic Monetary Policy, and Economic Downturns. *Journal of Monetary Economics*, 51(4):781–808.

Liadze, I., Macchiarelli, C., Mortimer-Lee, P., and Sanchez Juanino, P. (2022). Economic Costs of the Russia-Ukraine War. *The World Economy*.

Moll, B., Schularick, M., and Zachmann, G. (2023). The Power of Substitution: The Great German Gas Debate in Retrospect. *Brookings Papers on Economic Activity*, 2023(2):395–455.

Mountford, A. and Uhlig, H. (2009). What are the Effects of Fiscal Policy Shocks? *Journal of Applied Econometrics*, 24(6):960–992.

Nakov, A. and Pescatori, A. (2010). Oil and the Great Moderation. *The Economic Journal*, 120(543):131–156.

Natal, J.-M. (2012). Monetary Policy Response to Oil Price Shocks. *Journal of Money, Credit and Banking*, 44(1):53–101.

Pataracchia, B., Pfeiffer, P., Ratto, M., and Teresinski, J. (2025). Energy Commodity Price Shocks in the Euro Area: Evidence from a Large-Scale Structural Model. *Available at SSRN 5910584*.

Pollitt, M. (2022). The Energy Market in Time of War. Technical report, Brussels: Centre on Regulation in Europe.

Pollitt, M. G. (2024). Energy Markets Under Stress: Some Reflections on Lessons From the 2021–2023 Energy Price Crisis in Europe. *Economics of Energy & Environmental Policy*, 13(2).

Pollitt, M. G., Reiner, D. M., and Newbery, D. M. G. (2022). The Energy Price Guarantee: What Principles Should the UK Government Apply in Thinking About How to Implement This? Cambridge Working Paper, Energy Policy Research Group (EPRG), University of Cambridge. Energy Policy Research Group Working Paper.

Ramey, V. A. and Vine, D. J. (2011). Oil, Automobiles, and the US Economy: How Much have Things Really Changed? *NBER Macroeconomics Annual*, 25(1):333–368.

Richter, A. W. and Throckmorton, N. A. (2016). Is Rotemberg Pricing Justified by Macro Data? *Economics Letters*, 149:44–48.

Rotemberg, J. J. and Woodford, M. (1996). Imperfect Competition and the Effects of Energy Price Increases on Economic Activity. *Journal of Money, Credit, and Banking*, 28(4):549.

Smets, F. and Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1:1123–1175.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97:586–606.

Stern, D. I. (2012). Interfuel Substitution: a Meta-analysis. *Journal of Economic Surveys*, 26(2):307–331.

Zubairy, S. (2014). On Fiscal Multipliers: Estimates from a Medium Scale DSGE Model. *International Economic Review*, 55:169–195.

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Supplementary Material for "Energy Shocks, Pandemics and the Macroeconomy"

By Luisa Corrado, Stefano Grassi, Aldo Paolillo and Francesco Ravazzolo.

A. The Model Equations

Wholesale Energy Sector Firms

The first-order conditions of the wholesale firms in the energy sector are:

- *Labor demand*:

(A1)
$$\frac{1}{X_{e,t}}\frac{(1-\alpha_e)Y_{e,t}}{n_{e,t}} = w_{e,t}.$$

- *Raw capital demand*:

(A2)
$$\frac{1}{X_{e,t}}\frac{\alpha_e\omega_{k_e}Y_{e,t}}{k_{e,t-1}} = r_{k_e,t}u_{k_e,t}.$$

- *Crude oil demand*:

  Maximizing profits in equation (1) with respect to $V_{o,t}$ yields:

(A3)
$$\frac{1}{X_{e,t}}\frac{\alpha_e(1-\omega_{k_e})Y_{e,t}}{V_{e,t}}\, a_{V_o,t}\,\omega_o^{\frac{1}{\sigma_{o,gc}}}\left(\frac{V_{e,t}}{V_{o,t}}\right)^{\frac{1}{\sigma_{o,gc}}} = p_{o,t}.$$

  Using the definition of the composite-energy price $p_{v,t} = \frac{1}{X_{e,t}}\alpha_e(1-\omega_{k_e})\frac{Y_{e,t}}{V_{e,t}}$, equation (A3) can be written more compactly as:

$$V_{o,t} = \omega_o V_{e,t}\left(\frac{p_{v,t}\, a_{V_o,t}}{p_{o,t}}\right)^{\sigma_{o,gc}}.$$

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- *Natural gas demand:*
  Maximizing profits in equation (1) with respect to $V_{g,t}$ requires differentiating $Y_{e,t}$ with respect to $V_{g,t}$, taking into account the dependence of $Y_{e,t}$ on the inner and outer CES aggregators in equations (4)–(3). By the chain rule,

$$\frac{\partial Y_{e,t}}{\partial V_{g,t}} = \frac{\partial Y_{e,t}}{\partial V_{e,t}} \cdot \frac{\partial V_{e,t}}{\partial V_{gc,t}} \cdot \frac{\partial V_{gc,t}}{\partial V_{g,t}}.$$

These terms are:

$$\frac{\partial Y_{e,t}}{\partial V_{e,t}} = \alpha_e(1-\omega_{k_e})\,\frac{Y_{e,t}}{V_{e,t}} \qquad \frac{\partial V_{e,t}}{\partial V_{gc,t}} = (1-\omega_o)^{\frac{1}{\sigma_{o,gc}}} \left(\frac{V_{e,t}}{V_{gc,t}}\right)^{\frac{1}{\sigma_{o,gc}}},$$

$$\frac{\partial V_{gc,t}}{\partial V_{g,t}} = a_{V_g,t}\,\omega_g^{\frac{1}{\sigma_{gc}}} \left(\frac{V_{gc,t}}{V_{g,t}}\right)^{\frac{1}{\sigma_{gc}}}.$$

Combining these three components gives:

$$\frac{\partial Y_{e,t}}{\partial V_{g,t}} = \alpha_e(1-\omega_{k_e})\,\frac{Y_{e,t}}{V_{e,t}}\,(1-\omega_o)^{\frac{1}{\sigma_{o,gc}}} \left(\frac{V_{e,t}}{V_{gc,t}}\right)^{\frac{1}{\sigma_{o,gc}}} a_{V_g,t}\,\omega_g^{\frac{1}{\sigma_{gc}}} \left(\frac{V_{gc,t}}{V_{g,t}}\right)^{\frac{1}{\sigma_{gc}}}.$$

Equating this marginal product to the marginal cost $p_{g,t}$ yields the natural gas demand:

$$\text{(A4)}\quad \frac{1}{X_{e,t}}\,\alpha_e(1-\omega_{k_e})\,\frac{Y_{e,t}}{V_{e,t}}\,(1-\omega_o)^{\frac{1}{\sigma_{o,gc}}} \left(\frac{V_{e,t}}{V_{gc,t}}\right)^{\frac{1}{\sigma_{o,gc}}} a_{V_g,t}\,\omega_g^{\frac{1}{\sigma_{gc}}} \left(\frac{V_{gc,t}}{V_{g,t}}\right)^{\frac{1}{\sigma_{gc}}} = p_{g,t}.$$

Using

$$p_{v,t} = \frac{1}{X_{e,t}}\alpha_e(1-\omega_{k_e})\frac{Y_{e,t}}{V_{e,t}}$$

and the outer-nest relation

$$p_{gc,t} = p_{v,t}(1-\omega_o)^{1/\sigma_{o,gc}} \left(\frac{V_{e,t}}{V_{gc,t}}\right)^{1/\sigma_{o,gc}},$$

equation (A4) becomes

$$p_{g,t} = p_{gc,t}\,a_{V_g,t}\,\omega_g^{\frac{1}{\sigma_{gc}}} \left(\frac{V_{gc,t}}{V_{g,t}}\right)^{\frac{1}{\sigma_{gc}}}.$$

Solving for $V_{g,t}$ gives the explicit demand for natural gas:

$$V_{g,t} = \omega_g \, V_{gc,t} \, \left(\frac{p_{gc,t} \, a_{V_g,t}}{p_{g,t}} \right)^{\sigma_{gc}} .$$

- *Coal demand:*
  The demand for coal is derived analogously. Maximizing profits in equation (1) with respect to $V_{c,t}$ yields:
  (A5)
  $$\frac{1}{X_{e,t}} \, \alpha_e (1-\omega_{k_e}) \, \frac{Y_{e,t}}{V_{e,t}} \, (1-\omega_o)^{\frac{1}{\sigma_{o,gc}}} \left(\frac{V_{e,t}}{V_{gc,t}} \right)^{\frac{1}{\sigma_{o,gc}}} a_{V_c,t} \, (1-\omega_g)^{\frac{1}{\sigma_{gc}}} \left(\frac{V_{gc,t}}{V_{c,t}} \right)^{\frac{1}{\sigma_{gc}}} = p_{c,t}.$$
  Using again
  $$p_{v,t} = \frac{1}{X_{e,t}} \alpha_e (1-\omega_{k_e}) \frac{Y_{e,t}}{V_{e,t}}$$
  and
  $$p_{gc,t} = p_{v,t} (1-\omega_o)^{1/\sigma_{o,gc}} \left(\frac{V_{e,t}}{V_{gc,t}} \right)^{1/\sigma_{o,gc}} ,$$
  equation (A5) can be rewritten as
  $$p_{c,t} = p_{gc,t} \, a_{V_c,t} \, (1-\omega_g)^{\frac{1}{\sigma_{gc}}} \left(\frac{V_{gc,t}}{V_{c,t}} \right)^{\frac{1}{\sigma_{gc}}} . \tag{A6}$$
  Solving (A6) for $V_{c,t}$ gives:
  $$\left(\frac{V_{gc,t}}{V_{c,t}} \right)^{\frac{1}{\sigma_{gc}}} = \frac{p_{c,t}}{p_{gc,t} \, a_{V_c,t} \, (1-\omega_g)^{\frac{1}{\sigma_{gc}}}},$$
  hence the explicit coal demand:
  $$V_{c,t} = (1-\omega_g) \, V_{gc,t} \, \left(\frac{p_{gc,t} \, a_{V_c,t}}{p_{c,t}} \right)^{\sigma_{gc}} .$$

Retail Energy Sector Firms

Retail firms in the energy sector choose the price of variety j, $P_{e,t}(j)$, to maximize the expected discounted stream of profits net of quadratic Rotemberg adjustment costs. These costs depend on the price-adjustment parameter η_{π_e} and on lagged inflation, with indexation governed by ι_{π_e}:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{u_{c_c,t}}{u_{c_c,0}} \left[\frac{P_{e,t}(j)}{P_{e,t}} Y_{e,t}(j) - \frac{1}{X_{e,t}} Y_{e,t}(j) - \frac{\eta_{\pi_e}}{2} \left(\frac{P_{e,t}(j)}{P_{e,t-1}(j)} - \pi_{e,t-1}^{\iota_{\pi_e}} \right)^2 Y_{e,t} \right] \right\}, \tag{A7}$$

subject to:

$$Y_{e,t}(j) = \left(\frac{P_{e,t}(j)}{P_{e,t}}\right)^{-\epsilon_{\pi_e}} Y_{e,t}.$$

The adjustment-cost term penalizes deviations of the price change of an individual variety, $\frac{P_{e,t}(j)}{P_{e,t-1}(j)}$, from lagged indexed inflation, $\pi_{e,t-1}^{\iota_{\pi_e}}$. Equation (A7) also shows that retailer profits are discounted by the stochastic discount factor, $\beta^t \frac{u_{c_c,t}}{u_{c_c,0}}$, which depends on the marginal utility of consumption. When prices are fully flexible, that is, when $\eta_{\pi_e} = 0$, the markup is constant at its steady-state value, $X_e = \frac{\epsilon_{\pi_e}}{\epsilon_{\pi_e}-1}$. Profits of retail firms in the energy sector are given by

$$\Pi_{r_e,t} = \left(1 - \frac{1}{X_{e,t}}\right) Y_{e,t} - \frac{\eta_{\pi_e}}{2}\left(\pi_{e,t} - \pi_{e,t-1}^{\iota_{\pi_e}}\right)^2 Y_{e,t}.$$

The first-order condition of the retailer's problem yields the following Phillips curve for energy prices:

$$\begin{aligned} &1 - \pi_{e,t}\eta_{\pi_e}\left(\pi_{e,t} - \pi_{e,t-1}^{\iota_{\pi_e}}\right) + \beta\eta_{\pi_e}\mathbb{E}_t\left[\pi_{e,t+1}\frac{u_{c_c,t+1}}{u_{c_c,t}}\left(\pi_{e,t+1} - \pi_{e,t}^{\iota_{\pi_e}}\right)\frac{Y_{e,t+1}}{Y_{e,t}}\right] \\ &= \left(1 - \frac{1}{X_{e,t}}\right)\epsilon_{\pi_e}. \end{aligned} \tag{A8}$$

Wholesale Core Sector Firms

Profit maximization by wholesale firms in the core sector yields the following input-demand schedules:

- *Labor demand:*

$$\frac{1}{X_{c,t}}\frac{(1-\alpha_c)Y_{c,t}}{n_{c,t}} = w_{c,t}. \tag{A9}$$

- *Raw capital demand:*

$$\frac{1}{X_{c,t}}\alpha_c\,\omega_{k_c}^{\frac{1}{\sigma_{k_c}}}\frac{Y_{c,t}}{\bar{k}_{c,t}}\left(\frac{\bar{k}_{c,t}}{u_{k_c,t}k_{c,t-1}}\right)^{\frac{1}{\sigma_{k_c}}} u_{k_c,t} = r_{k_c,t}u_{k_c,t}. \tag{A10}$$

- *Energy demand:*

$$\frac{1}{X_{c,t}}\frac{\alpha_c Y_{c,t}}{\bar{k}_{c,t}}(1-\omega_{k_c})^{\frac{1}{\sigma_{k_c}}}\left(\frac{\bar{k}_{c,t}}{m_{e,t}}\right)^{\frac{1}{\sigma_{k_c}}} = p_{e,t}. \tag{A11}$$

Retail Core Sector Firms

Retail firms in the core sector face quadratic Rotemberg price-adjustment costs, governed by the rigidity parameter η_{π_c} and the indexation parameter ι_{π_c}, analogously to retailers in the energy sector. Their problem is to choose $P_{c,t}(j)$ to maximize the expected discounted stream of profits net of adjustment costs:

(A12)

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta)^t \left\{ \frac{u_{c_c,t}}{u_{c_c,0}} \left[\frac{P_{c,t}(j)}{P_{c,t}} Y_{c,t}(j) - \frac{1}{X_{c,t}} Y_{c,t}(j) - \frac{\eta_{\pi_c}}{2} \left(\frac{P_{c,t}(j)}{P_{c,t-1}(j)} - \pi_{c,t-1}^{\iota_{\pi_c}} \right)^2 Y_{c,t} \right] \right\},$$

subject to

$$Y_{c,t}(j) = \left(\frac{P_{c,t}(j)}{P_{c,t}} \right)^{-\epsilon_{\pi_c}} Y_{c,t}.$$

The adjustment-cost term penalizes deviations of individual price changes, $\frac{P_{c,t}(j)}{P_{c,t-1}(j)}$, from lagged indexed inflation, $\pi_{c,t-1}^{\iota_{\pi_c}}$. As equation (A12) shows, retailer profits are discounted by the stochastic discount factor, $\beta^t \frac{u_{c_c,t}}{u_{c_c,0}}$, which depends on the marginal utility of consumption at time t.

The first-order condition of the retailer's problem yields the Phillips curve for core-sector prices:

$$\begin{aligned} &1 - \pi_{c,t}\eta_{\pi_c} \left(\pi_{c,t} - \pi_{c,t-1}^{\iota_{\pi_c}} \right) + \beta \eta_{\pi_c} \mathbb{E}_t \left[\pi_{c,t+1} \frac{u_{c_c,t+1}}{u_{c_c,t}} \left(\pi_{c,t+1} - \pi_{c,t}^{\iota_{\pi_c}} \right) \frac{Y_{c,t+1}}{Y_{c,t}} \right] \\ &= \left(1 - \frac{1}{X_{c,t}} \right) \epsilon_{\pi_c}. \end{aligned} \tag{A13}$$

Profits of retail firms in the core sector are given by

$$\Pi_{r_c,t} = \left(1 - \frac{1}{X_{c,t}} \right) Y_{c,t} - \frac{\eta_{\pi_c}}{2} \left(\pi_{c,t} - \pi_{c,t-1}^{\iota_{\pi_c}} \right)^2 Y_{c,t}.$$

Unions

We report the optimization problem of labor unions in the two sectors. Unions face sector-specific labor demand schedules, $n_{i,t}(j) = \left(\frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\epsilon_w} n_{i,t},\ i \in \{c, e\}$, and incur quadratic adjustment costs *à la* Rotemberg for wage changes. These adjustment costs depend on lagged inflation, with indexation governed by ι_w:

$$\begin{aligned} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & u_{c_c,t} \left[\frac{W_{i,t}(j)}{P_{i,t}} n_{i,t}(j) - \frac{\eta_w}{2} \left(\frac{W_{i,t}(j)}{W_{i,t-1}(j)} - \pi_{i,t-1}^{\iota_w} \right)^2 \frac{W_{i,t}}{P_{i,t}} \right] \\ & - \frac{a_{\zeta,t} \varphi^i a_{\varphi,t} n_{i,t}(j)^{1+\nu_i}}{1+\nu_i} \Bigg\}. \end{aligned}$$

The first-order conditions imply the following wage Phillips curves:

- *Wage Phillips curve for $\mathcal{S}_c$:*

$$\eta_w \omega_{c,t}\left(\omega_{c,t}-\pi_{c,t-1}^{\iota_w}\right)=\beta\eta_w\mathbb{E}_t\frac{u_{c_c,t+1}}{u_{c_c,t}}\left(\omega_{c,t+1}-\pi_{c,t}^{\iota_w}\right)\frac{\omega_{c,t+1}^2}{\pi_{c,t+1}} + (1-\epsilon_w)\,n_{c,t}+\epsilon_w\left(\frac{\varphi^c a_{\varphi,t} n_{c,t}^{1+\nu_c}}{w_{c,t}u_{c_c,t}}\right). \tag{A14}$$

- *Wage Phillips curve for $\mathcal{S}_e$:*

$$\eta_w \omega_{e,t}\left(\omega_{e,t}-\pi_{e,t-1}^{\iota_w}\right)=\beta\eta_w\mathbb{E}_t\frac{u_{c_c,t+1}}{u_{c_c,t}}\left(\omega_{e,t+1}-\pi_{e,t}^{\iota_w}\right)\frac{\omega_{e,t+1}^2}{\pi_{e,t+1}} + (1-\epsilon_w)\,n_{e,t}+\epsilon_w\left(\frac{\varphi^e a_{\varphi,t} n_{e,t}^{1+\nu_e}}{w_{e,t}u_{c_c,t}}\right). \tag{A15}$$

Here, $\omega_{c,t}$ and $\omega_{e,t}$ are the nominal wage inflation rates, namely $\omega_{i,t}=\frac{W_{i,t}}{W_{i,t-1}}=\frac{P_{i,t}w_{i,t}}{P_{i,t-1}w_{i,t-1}}=\pi_{i,t}\frac{w_{i,t}}{w_{i,t-1}}$. Profits of unions are given by net margins minus adjustment costs:

$$\Pi_{u_c,t}=\left(1-\frac{1}{X_{w_c,t}}\right)w_{c,t}\,n_{c,t}-\frac{\eta_w}{2}\left(\omega_{c,t}-\pi_{c,t-1}^{\iota_w}\right)^2 w_{c,t}n_{c,t},$$

$$\Pi_{u_e,t}=\left(1-\frac{1}{X_{w_e,t}}\right)w_{e,t}\,n_{e,t}-\frac{\eta_w}{2}\left(\omega_{e,t}-\pi_{e,t-1}^{\iota_w}\right)^2 w_{e,t}\,n_{e,t}.$$

The profit term Π_t appearing in the household budget constraint in equation (14) is therefore:

$$\Pi_t=\Pi_{u_c,t}+p_{e,t}\Pi_{u_e,t}+\Pi_{r_c,t}+p_{e,t}\Pi_{r_e,t}.$$

Finally, total nominal adjustment costs in the two sectors, which enter the market-clearing conditions in equations (18) and (19), are given by:

$$\begin{aligned}\Xi_{c,t}&=\Xi_{\pi_c,t}+\Xi_{w_c,t}\\&=\left[\frac{\eta_{\pi_c}}{2}\left(\pi_{c,t}-\pi_{c,t-1}^{\iota_{\pi_c}}\right)^2\right]Y_{c,t}+\frac{\eta_w}{2}\left(\omega_{c,t}-\pi_{c,t-1}^{\iota_w}\right)^2 w_{c,t}n_{c,t},\end{aligned} \tag{A16}$$

and

$$\begin{aligned}\Xi_{e,t}&=p_{e,t}\Xi_{\pi_e,t}+p_{e,t}\Xi_{w_e,t}\\&=\left[\frac{\eta_{\pi_e}}{2}\left(\pi_{e,t}-\pi_{e,t-1}^{\iota_{\pi_e}}\right)^2\right]p_{e,t}Y_{e,t}+\frac{\eta_w}{2}\left(\omega_{e,t}-\pi_{e,t-1}^{\iota_w}\right)^2 p_{e,t}w_{e,t}n_{e,t}.\end{aligned} \tag{A17}$$

Households

The functional forms of the investment and capacity-utilization costs appearing in the household budget constraint in equation (14) are:

$$\Psi_{k_c,t} = \frac{\eta_k}{2}\left(\frac{k_{c,t}}{k_{c,t-1}} - 1\right)^2 k_{c,t-1}, \qquad \Psi_{k_e,t} = \frac{\eta_k}{2}\left(\frac{k_{e,t}}{k_{e,t-1}} - 1\right)^2 k_{e,t-1},$$

$$\Psi_{u_c,t} = \left[\frac{1}{\beta} - (1-\delta_{k_c})\right]\left[\frac{\left(\frac{\eta_u}{1-\eta_u}\right)}{2} + \frac{\left(\frac{\eta_u}{1-\eta_u}\right)}{2}u^2_{k_c,t} + u_{k_c,t}\left(1 - \frac{\eta_u}{1-\eta_u}\right) - 1\right],$$

$$\Psi_{u_e,t} = \left[\frac{1}{\beta} - (1-\delta_{k_e})\right]\left[\frac{\left(\frac{\eta_u}{1-\eta_u}\right)}{2} + \frac{\left(\frac{\eta_u}{1-\eta_u}\right)}{2}u^2_{k_e,t} + \left(1 - \frac{\eta_u}{1-\eta_u}\right)u_{k_e,t} - 1\right].$$

The total adjustment-cost term in the household budget constraint is therefore:

$$\Psi_t = \Psi_{k_c,t} + p_{e,t}\Psi_{k_e,t} + \Psi_{u_c,t}k_{c,t-1} + p_{e,t}\Psi_{u_e,t}k_{e,t-1}.$$

The household optimization problem implies the following first-order conditions.

- *Euler equation:*

$$u_{c_c,t} = \beta R_t \mathbb{E}_t\left(\frac{u_{c_c,t+1}}{\pi_{c,t+1}}\right).$$

- *Intratemporal consumption condition*:

$$\frac{u_{c_e,t}}{p_{e,t}} = u_{c_c,t}.$$

The marginal utilities of non-energy ($u_{c_c,t}$) and energy ($u_{c_e,t}$) consumption are given by:

  - Marginal utility of non-energy goods:

$$u_{c_c,t} = \frac{1-h}{1-\beta h}\left[\frac{a_{\zeta,t}}{\bar{c}_t - h\bar{c}_{t-1}} - \mathbb{E}_t\frac{h\beta a_{\zeta,t+1}}{\bar{c}_{t+1} - h\bar{c}_t}\right]\left[\frac{\bar{c}_t}{c_{c,t}}\right]^{\frac{1}{\sigma_c}}\omega_{c_c}^{\frac{1}{\sigma_c}}.$$

  - Marginal utility of energy goods:

$$u_{c_e,t} = \frac{1-h}{1-\beta h}\left[\frac{a_{\zeta,t}}{\bar{c}_t - h\bar{c}_{t-1}} - \mathbb{E}_t\frac{h\beta a_{\zeta,t+1}}{\bar{c}_{t+1} - h\bar{c}_t}\right]\left[\frac{\bar{c}_t}{c_{e,t}}\right]^{\frac{1}{\sigma_c}}(1-\omega_{c_c})^{\frac{1}{\sigma_c}}a_{j,t}^{\frac{1}{\sigma_c}}.$$

- Labor supply to $\mathcal{S}_c$:

$$a_{\zeta,t}a_{\varphi,t}\varphi^c n_{c,t}^{\nu_c} = \frac{w_{c,t}u_{c_c,t}}{X_{w_c,t}}.$$

- *Labor supply to* $\mathcal{S}_e$:

$$a_{\zeta,t}a_{\varphi,t}\varphi^e n_{e,t}^{\nu_e} = \frac{p_{e,t}w_{e,t}u_{c_c,t}}{X_{w_e,t}}.$$

- *Capital supply to* $\mathcal{S}_c$:

$$\begin{aligned} &u_{c_c,t}\left[1+\eta_k\left(\frac{k_{c,t}}{k_{c,t-1}}-1\right)\right]\frac{1}{a_{k,t}} \\ &=\beta\mathbb{E}_t u_{c_c,t+1}\left[r_{k_c,t+1}u_{k_c,t+1}+(1-\delta_{k_c})\frac{1}{a_{k,t+1}}-\Psi_{u_c,t+1}\frac{1}{a_{k,t+1}}+\frac{\eta_k}{2}\left(\frac{k_{c,t+1}^2}{k_{c,t}^2}-1\right)\frac{1}{a_{k,t+1}}\right]. \end{aligned}$$

- *Capital supply to* $\mathcal{S}_e$:

$$\begin{aligned} &p_{e,t}u_{c_e,t}\left[1+\eta_k\left(\frac{k_{e,t}}{k_{e,t-1}}-1\right)\right]\frac{1}{a_{k,t}} \\ &=\beta\mathbb{E}_t p_{e,t+1}u_{c_e,t+1}\left[r_{k_e,t+1}u_{k_e,t+1}+(1-\delta_{k_e})\frac{1}{a_{k,t+1}}-\Psi_{u_e,t+1}\frac{1}{a_{k,t+1}}+\frac{\eta_k}{2}\left(\frac{k_{e,t+1}^2}{k_{e,t}^2}-1\right)\frac{1}{a_{k,t+1}}\right]. \end{aligned}$$

- *Capacity utilization in* $\mathcal{S}_c$ *condition*:

$$\frac{r_{k_c,t}}{\frac{1}{\beta}-(1-\delta_{k_c})} = 1-\frac{\eta_u}{1-\eta_u}+\frac{\eta_u}{1-\eta_u}u_{k_c,t}. \tag{A18}$$

- *Capacity utilization in* $\mathcal{S}_e$ *condition*:

$$\frac{r_{k_e,t}}{\frac{1}{\beta}-(1-\delta_{k_e})} = 1-\frac{\eta_u}{1-\eta_u}+\frac{\eta_u}{1-\eta_u}u_{k_e,t}. \tag{A19}$$

B. Steady State

This section derives the steady state of the model. Variables without time subscripts denote steady-state values. For convenience, define the following ratios:

$$\zeta_1 \equiv \frac{\bar{k}_c}{Y_c}, \qquad \zeta_2 \equiv \frac{k_e}{Y_e}, \qquad \zeta_3 \equiv \frac{V_e}{Y_e}, \qquad \zeta_4 \equiv \frac{m_e}{k_c},$$
$$\zeta_5 \equiv \frac{V_o}{V_e}, \qquad \zeta_6 \equiv \frac{V_g}{V_e}, \qquad \zeta_7 \equiv \frac{V_c}{V_e}, \qquad \zeta_* \equiv \frac{V_{gc}}{V_e}.$$

We set steady-state hours in the two sectors equal to their calibration targets:

$$n_c = n_c^{ss}, \qquad n_e = n_e^{ss}.$$

The Phillips curves in equations (A13), (A8), (A14), and (A15) imply the steady-state markups

$$X_c = \frac{\epsilon_{\pi_c}}{\epsilon_{\pi_c} - 1}, \qquad X_e = \frac{\epsilon_{\pi_e}}{\epsilon_{\pi_e} - 1}, \qquad X_{w_c} = \frac{\epsilon_w}{\epsilon_w - 1}, \qquad X_{w_e} = \frac{\epsilon_w}{\epsilon_w - 1}.$$

The model is solved around a zero-inflation equilibrium, where the gross interest rate equals the inverse of the discount rate. Moreover, utilization rates are normalized to one in steady state and all adjustment costs are zero, namely:

$$\pi_c = 1, \qquad \pi_e = 1, \qquad \omega_c = 1, \qquad \omega_e = 1, \qquad R = \frac{1}{\beta},$$
$$u_{k_c} = 1, \quad u_{k_e} = 1, \quad \Psi_{k_c} = 0, \quad \Psi_{k_e} = 0, \quad \Psi_{u_c} = 0, \quad \Psi_{u_e} = 0, \quad \Xi_c = 0, \quad \Xi_e = 0.$$

We normalize the relative price of refined energy to one:

$$p_e = 1.$$

The capacity-utilization conditions in equations (A18) and (A19) imply the steady-state rental rates of capital:

$$r_{k_c} = \frac{1}{\beta} - 1 + \delta_{k_c}, \qquad r_{k_e} = \frac{1}{\beta} - 1 + \delta_{k_e}.$$

We set the relative prices of oil, gas, and coal equal to their calibration targets:

$$p_o = p_o^{ss}, \qquad p_g = p_g^{ss}, \qquad p_c = p_c^{ss}.$$

The exogenous processes are fixed at their unconditional means in steady state:

$$a_j = 1, \quad a_\zeta = 1, \quad a_\varphi = 1, \quad a_{z_c} = 1, \quad a_{z_e} = 1, \quad a_k = 1, \quad a_r = 1,$$

$$a_{V_o} = 1, \quad a_{V_g} = 1, \quad a_{V_c} = 1, \quad a_{p_c} = 0, \quad a_{p_g} = 0, \quad a_{p_o} = 0.$$

B1. Core sector

From the demand for raw capital in equation (A10) and the demand for refined energy in equation (A11), the share parameter ω_{k_c} can be written as a function of the calibration target $s_{m_e,k_c} \equiv m_e/k_c$:

$$\omega_{k_c} = \left(1 + s_{m_e,k_c} r_{k_c}^{-\sigma_{k_c}}\right)^{-1}.$$

Define the price index of the CES composite $\bar{k}_c$:

$$\bar{r}_{k_c} = \left[\omega_{k_c} r_{k_c}^{1-\sigma_{k_c}} + (1-\omega_{k_c}) p_e^{1-\sigma_{k_c}}\right]^{\frac{1}{1-\sigma_{k_c}}}.$$

In steady state, the demand for $\bar{k}_c$ by the core sector implies

$$\zeta_1 = \frac{\bar{k}_c}{Y_c} = \alpha_c \frac{1}{X_c} \frac{1}{\bar{r}_{k_c}}.$$

Using the production function in equation (10), we obtain

$$Y_c = n_c \zeta_1^{\frac{\alpha_c}{1-\alpha_c}}, \qquad \bar{k}_c = \zeta_1 Y_c.$$

Using the demand conditions for raw capital and refined energy, we obtain

$$\zeta_4 = \frac{m_e}{k_c} = \left[r_{k_c}(1-\omega_{k_c})^{\frac{1}{\sigma_{k_c}}} \omega_{k_c}^{-\frac{1}{\sigma_{k_c}}}\right]^{\sigma_{k_c}}.$$

Substituting $m_e = \zeta_4 k_c$ into equation (11) yields

$$k_c = \bar{k}_c \left[\omega_{k_c}^{\frac{1}{\sigma_{k_c}}} + (1-\omega_{k_c})^{\frac{1}{\sigma_{k_c}}} \zeta_4^{\frac{\sigma_{k_c}-1}{\sigma_{k_c}}}\right]^{\frac{\sigma_{k_c}}{1-\sigma_{k_c}}}.$$

B2. Energy sector

Using the definition of the crude-energy composite V_e and its price index p_v, the demand for crude energy by the energy sector implies

$$\alpha_e(1-\omega_{k_e}) \frac{1}{X_e} \frac{Y_e}{V_e} = p_v. \tag{B1}$$

Matching the calibration target $s_{p_v V_e, Y_e} \equiv p_v V_e / Y_e$ implies

$$\omega_{k_e} = 1 - s_{p_v V_e, Y_e} \frac{1}{\alpha_e} X_e.$$

From the demand for raw capital in equation (A2), we obtain

$$\zeta_2 \equiv \frac{k_e}{Y_e} = \frac{1}{X_e}\alpha_e \omega_{k_e} \frac{1}{r_{k_e}}.$$

Equation (B1) also implies

$$\zeta_3 \equiv \frac{V_e}{Y_e} = \frac{1}{X_e}\alpha_e (1-\omega_{k_e}) \frac{1}{p_v}.$$

Define the scaled level of production in the energy sector as

$$\hat{Y}_e \equiv Y_e \left(a_{z_e}^{ss}\right)^{\frac{1}{\alpha_e - 1}} = n_e \left[\zeta_2^{\omega_{k_e}} \zeta_3^{1-\omega_{k_e}}\right]^{\frac{\alpha_e}{1-\alpha_e}}.$$

B3. Crude-energy nest

From the demand conditions for gas and coal in the inner CES aggregator,

$$\frac{V_g}{V_c} = \frac{\omega_g}{1-\omega_g}\left(\frac{p_c}{p_g}\right)^{\sigma_{gc}}.$$

Imposing the calibration target $s_{V_g,V_c} \equiv V_g/V_c$, we obtain

$$\frac{\omega_g}{1-\omega_g} = s_{V_g,V_c}\left(\frac{p_g}{p_c}\right)^{\sigma_{gc}},$$

so that

$$\omega_g = \frac{s_{V_g,V_c}\left(\frac{p_g}{p_c}\right)^{\sigma_{gc}}}{1 + s_{V_g,V_c}\left(\frac{p_g}{p_c}\right)^{\sigma_{gc}}}.$$

The corresponding price index of the gas-coal composite is:

$$p_{gc} = \left[\omega_g \, p_g^{1-\sigma_{gc}} + (1-\omega_g)\, p_c^{1-\sigma_{gc}}\right]^{\frac{1}{1-\sigma_{gc}}}.$$

Let $s_{V_o,V_c} \equiv V_o/V_c$. The share parameter ω_o in the outer nest satisfies

$$\frac{\omega_o}{1-\omega_o} = s_{V_o,V_c}(1-\omega_g)\left(\frac{p_{gc}}{p_c}\right)^{\sigma_{gc}}\left(\frac{p_o}{p_{gc}}\right)^{\sigma_{o,gc}}.$$

Solving for ω_o:

$$\omega_o = \frac{s_{V_o,V_c}(1-\omega_g)\left(\frac{p_{gc}}{p_c}\right)^{\sigma_{gc}}\left(\frac{p_o}{p_{gc}}\right)^{\sigma_{o,gc}}}{1+s_{V_o,V_c}(1-\omega_g)\left(\frac{p_{gc}}{p_c}\right)^{\sigma_{gc}}\left(\frac{p_o}{p_{gc}}\right)^{\sigma_{o,gc}}}.$$

The implied steady-state ratios are

$$\zeta_* \equiv \frac{V_{gc}}{V_e} = (1-\omega_o)\left(\frac{p_v}{p_{gc}}\right)^{\sigma_{o,gc}},$$

$$\zeta_{**} \equiv \frac{V_g}{V_{gc}} = \omega_g\left(\frac{p_{gc}}{p_g}\right)^{\sigma_{gc}}, \qquad \zeta_{***} \equiv \frac{V_c}{V_{gc}} = (1-\omega_g)\left(\frac{p_{gc}}{p_c}\right)^{\sigma_{gc}}.$$

Hence,

$$\zeta_5 = \frac{V_o}{V_e} = \omega_o\left(\frac{p_v}{p_o}\right)^{\sigma_{o,gc}}, \qquad \zeta_6 = \frac{V_g}{V_e} = \zeta_{**}\zeta_*, \qquad \zeta_7 = \frac{V_c}{V_e} = \zeta_{***}\zeta_*,$$

and therefore

$$V_o = \zeta_5 V_e, \quad V_g = \zeta_6 V_e, \quad V_c = \zeta_7 V_e.$$

B4. Consumption, wages, and profits

The steady-state resource constraints are

$$c_c + \delta_{k_c}k_c + \delta_{k_e}k_e = Y_c,$$

$$c_e + m_e + s_o^* p_o V_o + s_g^* p_g V_g + s_c^* p_c V_c = Y_e.$$

Using the resource constraints, we obtain the normalization constant $a_{z_e}^{ss}$:

$$a_{z_e}^{ss} = \left[\frac{Y_c - \delta_{k_c}k_c + \frac{\omega_{c_c}}{1-\omega_{c_c}}m_e}{\frac{\omega_{c_c}}{1-\omega_{c_c}}\hat{Y}_e - \frac{\omega_{c_c}}{1-\omega_{c_c}}\left(s_o^* p_o\zeta_5 + s_g^* p_g\zeta_6 + s_c^* p_c\zeta_7\right)\hat{Y}_e + \delta_{k_e}\zeta_2\hat{Y}_e}\right]^{1-\alpha_e}.$$

This implies:

$$Y_e = a_{z_e}^{ss\,\frac{1}{1-\alpha_e}}\hat{Y}_e,$$

$$k_e = \zeta_2 Y_e, \quad V_e = \zeta_3 Y_e.$$

Consumption of energy and core goods is:

$$c_e = Y_e - s_o^* p_o V_o - s_g^* p_g V_g - s_c^* p_c V_c - m_e, \qquad c_c = \frac{\omega_{c_c}}{1-\omega_{c_c}}c_e.$$

The aggregate consumption composite is:

$$\bar{c} = \left[\omega_{c_c}^{\frac{1}{\sigma_c}} c_c^{\frac{\sigma_c-1}{\sigma_c}} + (1-\omega_{c_c})^{\frac{1}{\sigma_c}} c_e^{\frac{\sigma_c-1}{\sigma_c}}\right]^{\frac{\sigma_c}{\sigma_c-1}}.$$

The corresponding marginal utilities are:

$$u_{c_c} = \omega_{c_c}^{\frac{1}{\sigma_c}} \bar{c}^{\frac{1}{\sigma_c}-1} c_c^{-\frac{1}{\sigma_c}}, \qquad u_{c_e} = (1-\omega_{c_c})^{\frac{1}{\sigma_c}} \bar{c}^{\frac{1}{\sigma_c}-1} c_e^{-\frac{1}{\sigma_c}}.$$

The labor-demand conditions in equations (A1) and (A9) imply:

$$w_e = (1-\alpha_e)\frac{Y_e}{X_e n_e}, \qquad w_c = (1-\alpha_c)\frac{Y_c}{X_c n_c}.$$

Hence the labor-disutility parameters satisfy:

$$\varphi^c = \frac{w_c u_{c_c}}{X_{w_c} n_c^{\nu_c}}, \qquad \varphi^e = \frac{p_e w_e u_{c_c}}{X_{w_e} n_e^{\nu_e}}.$$

Steady-state profits of retailers and unions are:

$$\Pi_{r_c} = \left(1 - \frac{1}{X_c}\right) Y_c, \quad \Pi_{r_e} = \left(1 - \frac{1}{X_e}\right) Y_e,$$
$$\Pi_{u_c} = \left(1 - \frac{1}{X_{w_c}}\right) w_c n_c, \quad \Pi_{u_e} = \left(1 - \frac{1}{X_{w_e}}\right) w_e n_e.$$

The aggregate profit term entering the household budget constraint is therefore

$$\Pi = \Pi_{u_c} + p_e \Pi_{u_e} + \Pi_{r_c} + p_e \Pi_{r_e}.$$

B5. Aggregate quantities

Aggregate GDP is

$$GDP = Y_c - p_e m_e + p_e \left(Y_e - p_v V_e\right).$$

Aggregate inflation is defined using sectoral value-added shares:

$$\pi = \pi_c^{\frac{Y_c - p_e m_e}{GDP}} \pi_e^{\frac{p_e(Y_e - p_v V_e)}{GDP}}.$$

Aggregate investment is

$$i = \delta_{k_c} k_c + p_e \delta_{k_e} k_e.$$

The capital-energy composite in the energy sector is:

$$\bar{k}_e = k_e^{\omega_{k_e}} V_e^{1-\omega_{k_e}}.$$

The expenditure share of energy goods in total consumption and the share of energy value added in GDP are

$$s_{c_e} = \frac{p_e c_e}{p_e c_e + c_c}, \qquad s_{eVA,tVA} = \frac{p_e(Y_e - p_v V_e)}{GDP}.$$

Total employment and total hours are

$$E = n_c + n_e, \qquad N = E.$$

B6. SVAR Block

Regarding the crude energy block, we impose $\log p_f = \log p_f^{ss}$. The SVAR block then pins down the steady-state value of world output growth:

$$\Delta \log \left(GDP^{\mathrm{W}}\right) = \frac{c_{ocse} + (a_{1,12} + a_{2,12}) \log p_f}{1 - a_{1,11} - a_{2,11}}.$$

International commodity prices coincide with domestic ones:

$$p_g^* = p_g, \quad p_o^* = p_o, \quad p_c^* = p_c.$$

B7. Measurement Equations

The measurement equations in (20) imply the following steady-state relationships, where growth rates are zero, inflation rates are constant, and level observables are proportional to scaling parameters:

$$\begin{aligned}
&\Delta GDP^{data} = 0, && \Delta c^{data} = 0, \\
&\Delta i^{data} = 0, && \Delta l^{data} = 0, \\
&\pi^{data} = \gamma_{\pi}, && \pi_e^{data} = \gamma_{\pi_e}, \\
&\pi_o^{data} = \gamma_{\pi_o}, && \pi_c^{data} = \gamma_{\pi_c}, \\
&\pi_g^{data} = \gamma_{\pi_g}, && \omega^{data} = \gamma_{\omega}, \\
&V_o^{data} = \gamma_{V_o} V_o, && V_c^{data} = \gamma_{V_c} V_c, \\
&V_g^{data} = \gamma_{V_g} V_g, && R^{data} = 400(R-1), \\
&\Delta \log GDP^{W,data} = \Delta \log \left(GDP^{\mathrm{W}}\right).
\end{aligned}$$

The loading parameters must satisfy

$$\lambda_o = \frac{p_o^{ss}}{p_f^{ss}}, \qquad \lambda_g = \frac{p_g^{ss}}{p_f^{ss}}, \qquad \lambda_c = \frac{p_c^{ss}}{p_f^{ss}}.$$

C. Data Construction

Gross Domestic Product

Real gross domestic product (GDP) in the EA from 1990:Q1 to 1995:Q1 is retrieved from the Gross Domestic Product series (YER code) in the Area-Wide Model (AWM) database constructed by Fagan et al. (2005), and from 1995:Q1 onward from Eurostat, retrieved via FRED (CLVMEURSCAB1GQEA19). The series is seasonally adjusted and measured in millions of chained 2010 euros. We divide GDP by total EA population, obtained from the World Bank and retrieved from FRED (SPPOPTOTLEMU), to express the series in per capita terms. The resulting series is transformed into log differences and demeaned.

Consumption

Consumption in the EA from 1990:Q1 to 1995:Q1 is retrieved from the Individual Consumption Expenditure (PCR series) in the AWM database. From 1995:Q1 onward, we use Eurostat quarterly national accounts data on final consumption expenditure of households and non-profit institutions serving households (NPISH) for the EA (19 countries). The series is seasonally and calendar adjusted and measured in millions of chain-linked 2010 euros. We divide it by total EA population (SPPOPTOTLEMU) to obtain per capita consumption. The resulting series is expressed in log differences and demeaned.

Investment

Investment in the EA from 1990:Q1 to 1995:Q1 is measured as Gross Fixed Capital Formation (ITR series) in the AWM database. From 1995:Q1 onward, we use Eurostat quarterly national accounts data on gross fixed capital formation for the EA (19 countries). The series is seasonally and calendar adjusted and measured in millions of chain-linked 2010 euros. We divide it by total EA population (SPPOPTOTLEMU) to obtain per capita investment. The resulting series is expressed in log differences and demeaned.

Employment

Employment in the EA from 1990:Q1 to 1995:Q1 is retrieved from the Total Employment series (LNN) in the AWM database. This series is seasonally and calendar adjusted and measured in thousands of persons. From 1995:Q1 onward, we use Eurostat quarterly national accounts data on total employment (domestic concept) for the EA (19 countries). The series is seasonally and calendar adjusted and measured in thousands of persons. We divide employment by the total population of the EA (SPPOPTOTLEMU) to obtain per capita employment. The series is taken in log differences.

Inflation

Inflation in the EA from 1990:Q1 to 1995:Q1 is measured using the GDP Deflator (YED series) in the AWM database. From 1995:Q1 onward, we use Eurostat data (table `namq_10_gdp_1`). We take log differences of the deflator to obtain inflation.

Energy inflation

Energy inflation in the EA from 1990:Q1 to 1995:Q1 is measured using the HICP for energy (HEGSYA) from the AWM database. From 1995:Q1 onward, we use OECD data retrieved from FRED (ENRGY0EZ19M086NEST). The series is a seasonally and calendar-adjusted index. We take log differences to obtain energy inflation.

Oil price inflation

The oil price is measured using the Brent crude oil spot price series (DCOILBRENTEU). Because the series is denominated in U.S. dollars, we convert it into euros using the euro-dollar exchange rate: the EXR series from the AWM database before 1995:Q1 and, from 1995:Q1 onward, the DEXUSEU series provided by the Board of Governors of the Federal Reserve System. We then take log differences to obtain oil price inflation.

Natural gas price inflation

The price of natural gas in the European Union is obtained from the International Monetary Fund (PNGASEUUSDM), retrieved from FRED. We convert the series into euros using the same procedure as for oil prices and then take log differences to obtain natural gas price inflation.

Coal price inflation

The coal price is measured using the global price of coal, Australia, provided by the International Monetary Fund (PCOALAUUSDQ), retrieved from FRED. This is the representative world market price, determined by the largest exporter of the commodity; see `https://www.imf.org/en/Research/commodity-prices`. As above, we convert the series into euros and take log differences to obtain coal price inflation.

Nominal wage inflation

Nominal wage inflation in the EA is constructed as total compensation divided by total employment. From 1990:Q1 to 1995:Q1, both series are taken from the AWM database: employment from LNN and total compensation from WIN. Both series are seasonally and calendar adjusted; employment is measured in thousands of persons and compensation in millions of euros at current prices. From 1995:Q1 onward, total employment is taken from Eurostat (table NAMQ_10_PE, total employment, domestic concept) and total compensation from Eurostat (table NAMQ_10_GDP, compensation of employees). These series are seasonally and calendar adjusted, with employment measured in thousands of persons and compensation in millions of euros at current prices. We take log differences of the resulting nominal wage series to obtain wage inflation.

Oil, natural gas and coal quantities

Quantities of oil, coal, and natural gas are measured using Eurostat's *gross available energy* series from the Simplified energy balances (dataset nrg_bal_s). Eurostat defines gross available energy as an energy-balance aggregate representing, over a given period, the quantity of energy required to satisfy total energy demand in the reference area (Eurostat, 2026a,c). The series are measured in tonnes of oil equivalent (toe). We divide them by EA population (Eurostat series SPPOPTOTLEMU) to obtain per capita quantities.

Interest rate

The nominal short-term interest rate is measured by the three-month Euribor. Before 1995:Q1, we use the STN series from the AWM database; from 1995:Q1 onward, we use Eurostat data (table irt_st_q).

Global GDP

Global GDP is measured using quarterly GDP for OECD countries, retrieved from the OECD (VPVOBARSA series). The series is seasonally adjusted and expressed in purchasing-power-parity terms. We take log differences to obtain global GDP growth.

D. SVAR Estimation

As discussed in the main text, the growth rate of global GDP, $\Delta \log\left(GDP_t^{\mathrm{W}}\right)$, and the log of the common factor in oil, coal and gas prices, $\log\left(p_{f,t}\right)$, are determined by the following bivariate SVAR:

$$\text{(D1)} \qquad \boldsymbol{A}_0 \begin{bmatrix} \Delta \log\left(GDP_t^{\mathrm{W}}\right) \\ \log\left(p_{f,t}\right) \end{bmatrix} = \boldsymbol{c} + \sum_{j=1}^{2} \boldsymbol{A}_j \begin{bmatrix} \Delta \log\left(GDP_{t-j}^{\mathrm{W}}\right) \\ \log\left(p_{f,t-j}\right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{W,t} \\ \varepsilon_{p_f,t} \end{bmatrix}.$$

The coefficient matrices in equation (D1) are:
(D2)

$$\boldsymbol{A}_0 \equiv \begin{bmatrix} 1 & 0 \\ a_{0,21} & 1 \end{bmatrix}, \boldsymbol{c} \equiv \begin{bmatrix} c_W \\ c_{p_f} \end{bmatrix}, \boldsymbol{A}_1 \equiv \begin{bmatrix} a_{1,11} & a_{1,12} \\ a_{1,21} & a_{1,22} \end{bmatrix} \text{ and } \boldsymbol{A}_2 \equiv \begin{bmatrix} a_{2,11} & a_{2,12} \\ a_{2,21} & a_{2,22} \end{bmatrix}.$$

The SVAR parameters in equation (D2) are estimated jointly with the remaining model parameters using the RWMH algorithm. Their priors and posteriors are reported in Table D1.

*Table D1—: **Estimation Results for the SVAR Parameters**. The table reports each parameter symbol (Symbol), the prior distribution (Prior), the prior mean and standard deviation (Mean, St. Dev.), and the posterior mean and standard deviation (Post. Mean, Post. St. Dev.). $\mathcal{N}$ denotes the Normal distribution.*

SVAR parameters				
Symbol	Prior	Mean, St. Dev.	Post. Mean	Post. St. Dev.
$100 \times c_W$	$\mathcal{N}$	(0.14, 0.03)	0.17	0.02
$a_{0,21}$	$\mathcal{N}$	(-0.18, 0.04)	-0.18	0.03
$a_{1,11}$	$\mathcal{N}$	(0.76, 0.15)	0.66	0.06
$a_{1,12}$	$\mathcal{N}$	(0.02, 0.00)	0.01	0.00
$a_{1,21}$	$\mathcal{N}$	(-0.41, 0.08)	-0.38	0.08
$a_{1,22}$	$\mathcal{N}$	(0.98, 0.20)	0.87	0.05
$a_{2,11}$	$\mathcal{N}$	(-0.16, 0.03)	-0.15	0.03
$a_{2,12}$	$\mathcal{N}$	(-0.02, 0.00)	-0.01	0.00
$a_{2,21}$	$\mathcal{N}$	(-0.10, 0.02)	-0.10	0.02
$a_{2,22}$	$\mathcal{N}$	(-0.14, 0.03)	-0.23	0.03

Online Appendix - Not for Publication - for "Energy Shocks, Pandemics and the Macroeconomy"

By Luisa Corrado, Stefano Grassi, Aldo Paolillo and Francesco Ravazzolo.

E. Additional Results

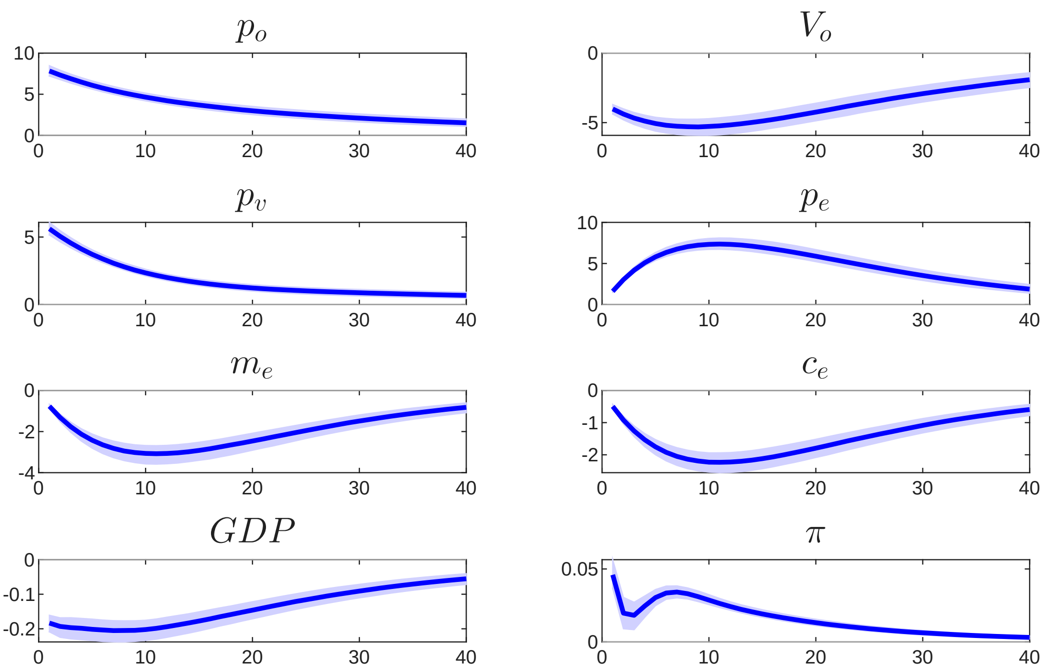

Figure E1. : ***Impulse Responses Functions (IRFs) to a Oil Price Shock.*** *The figure reports the responses of the oil price (p_o), oil use in energy production (V_o), the crude-energy price index (p_v), the refined-energy price (p_e), intermediate energy demand by core-sector firms (m_e), household energy consumption (c_e), real GDP (GDP), and headline inflation (π) to a positive one-standard-deviation shock to the idiosyncratic component of the oil price. Shaded areas denote the 20th and 80th percentile credible bands associated with parameter uncertainty. The IRFs represent percentage deviations from the steady state.*

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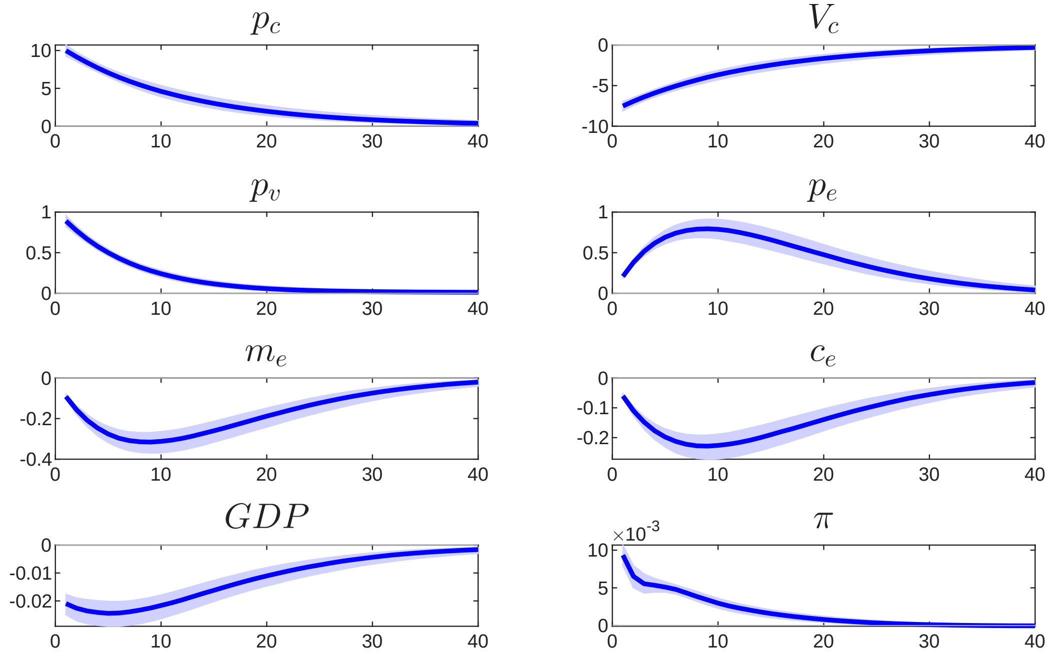

Figure E2. : ***Impulse Responses Functions (IRFs) to a Coal Price Shock.*** *The figure reports the responses of the coal price (p_c), coal use in energy production (V_c), the crude-energy price index (p_v), the refined-energy price (p_e), intermediate energy demand by core-sector firms (m_e), household energy consumption (c_e), real GDP (GDP), and headline inflation (π) to a positive one-standard-deviation shock to the idiosyncratic component of the oil price. Shaded areas denote the 20th and 80th percentile credible bands associated with parameter uncertainty. The IRFs represent percentage deviations from the steady state.*

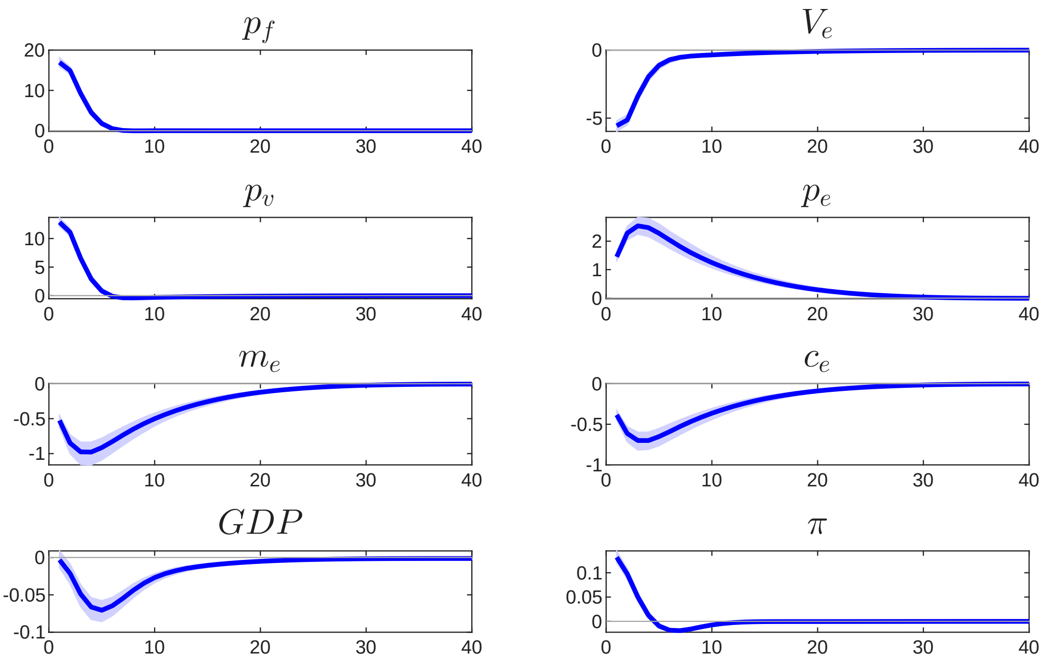

Figure E3. : ***Impulse Responses Functions (IRFs) to a Common Factor Shock to the Price of Crude Energy.*** *The figure reports the responses of the common component of crude energy price (p_f), crude energy use in energy production (V_e), the crude-energy price index (p_v), the refined-energy price (p_e), intermediate energy demand by core-sector firms (m_e), household energy consumption (c_e), real GDP (GDP), and headline inflation (π) to a positive one-standard-deviation shock to the idiosyncratic component of the oil price. Shaded areas denote the 20th and 80th percentile credible bands associated with parameter uncertainty. The IRFs represent percentage deviations from the steady state. .*

Pandemic counterfactual

Figure E4 reports a counterfactual exercise in which all structural shocks are set to zero from 2020:Q2 onward. The figure compares the observed series (solid blue lines) with the counterfactual paths (dashed red lines), where all shocks from 2020:Q2 onward are set to zero. The shaded area denotes the pandemic period. The upper panels report normalized levels of energy prices (HICP Energy), headline inflation (HICP), and Real GDP. Energy prices exhibit the strongest response: they fall sharply below the counterfactual in 2020:Q2–Q3, reflecting the collapse in demand, and then increase persistently during the recovery, rising well above the counterfactual by 2022. The aggregate price index initially moves little, but from 2021 onward it increases more strongly in the smoothed series than in the counterfactual, reflecting the contribution of recovery and energy shocks. Real GDP shows a similar pattern: in the absence of shocks, the sharp contraction in 2020:Q2 would have been avoided, as well as the subsequent rebound, resulting in a much smoother path.

The lower panels report percentage deviations from the counterfactual. Energy prices decline by more than 10% in 2020:Q2–Q3 and then increase by up to 30% by 2022:Q3, reflecting the combined effect of demand recovery and energy shocks. The aggregate price

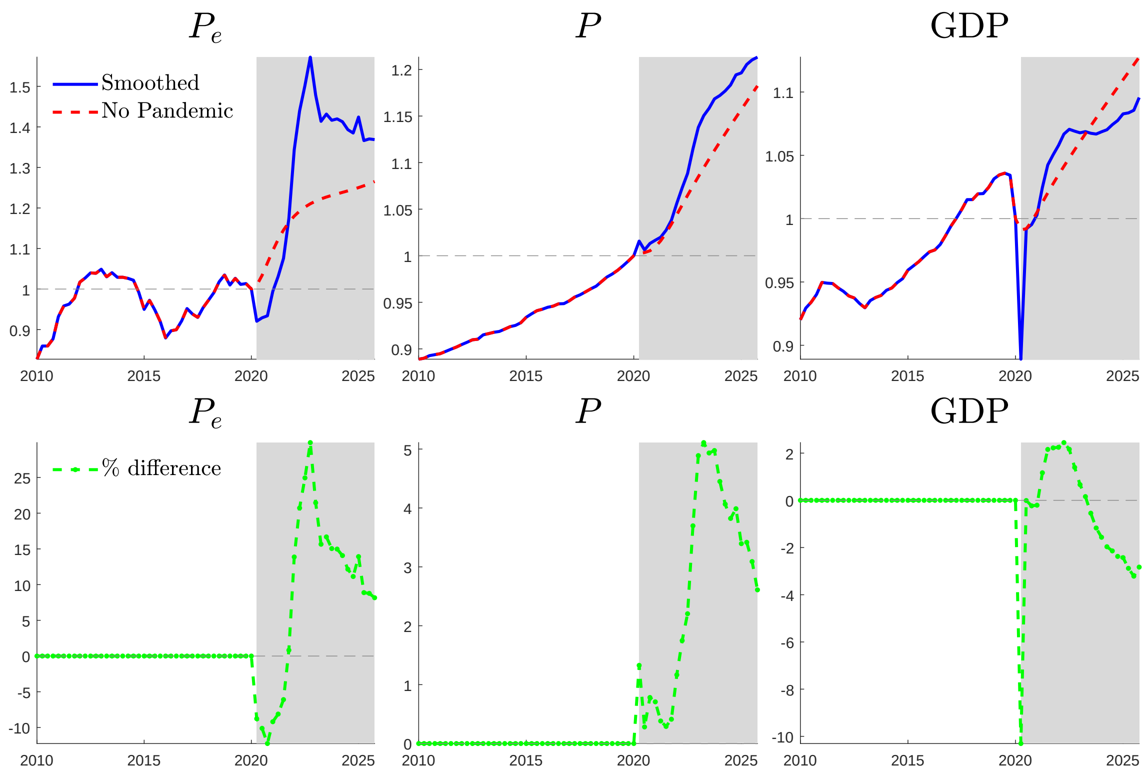

Figure E4. : ***Pandemic Counterfactual****. The top row reports the smoothed (realized) series (solid blue lines) together with their counterfactual counterparts (dashed red lines), obtained by shutting down all structural shocks from 2020:Q2 onward. The three columns report, respectively, the HICP of energy products (P_e), the aggregate HICP (P), and real gross domestic product (GDP). The bottom row reports the percentage difference between the smoothed and counterfactual series (green dash-dotted lines), which measures the joint contribution of pandemic-related and energy shocks over time. All level variables are normalized to one in 2020:Q1, the quarter immediately preceding the start of the counterfactual, so that deviations can be interpreted as percentage changes relative to the pre-counterfactual level. The shaded area denotes the period following the onset of the COVID-19 pandemic.*

index rises by about 5% above the counterfactual by 2022:Q3. Real GDP also displays large deviations, confirming that both the contraction in 2020 and the subsequent recovery were primarily driven by the shocks realized during 2020–2022.

The role of complementarities

The parameters σ_{k_c} and σ_c govern the elasticity of substitution between energy and non-energy goods in production and consumption, respectively, and are estimated to be below one, see Table 2. We assess their quantitative role by computing IRFs to a gas price shock (ε_{p_g}) for alternative values of these parameters.

Figure E5 reports the responses for different values of σ_{k_c}. Lower values of σ_{k_c} amplify the decline in GDP, reflecting more limited substitution away from energy inputs. The lower panel shows that the share of energy costs in production, $s_{m_e} \equiv \frac{p_e m_e}{r_{k_e} u_{k_e} k_e + p_e m_e}$, rises when $\sigma_{k_c} < 1$, remains constant in the Cobb-Douglas case, and falls when inputs are substitutes.

Figure E9 presents the analogous exercise for consumption. A gas price shock raises the price of refined energy and changes the composition of household expenditure. Lower values of σ_c imply a smaller reallocation across goods and a weaker response of aggregate consumption and GDP. At the same time, the share of household expenditure on energy, $s_{c_e} \equiv \frac{p_e c_e}{c_c + p_e c_e}$, rises after the shock when $\sigma_c < 1$.

The corresponding exercises for production are reported in Figures E6, E7, and E8 which consider oil shocks (ε_{p_o}), coal shocks (ε_{p_c}), and common crude-energy shocks (ε_{p_f}), respectively. The analogous exercises for consumption are reported in Figures E10, E11, and E12. The results are qualitatively similar across cases: lower elasticities of substitution amplify the contractionary effects of energy price shocks and increase the share of energy expenditures in both production and consumption.

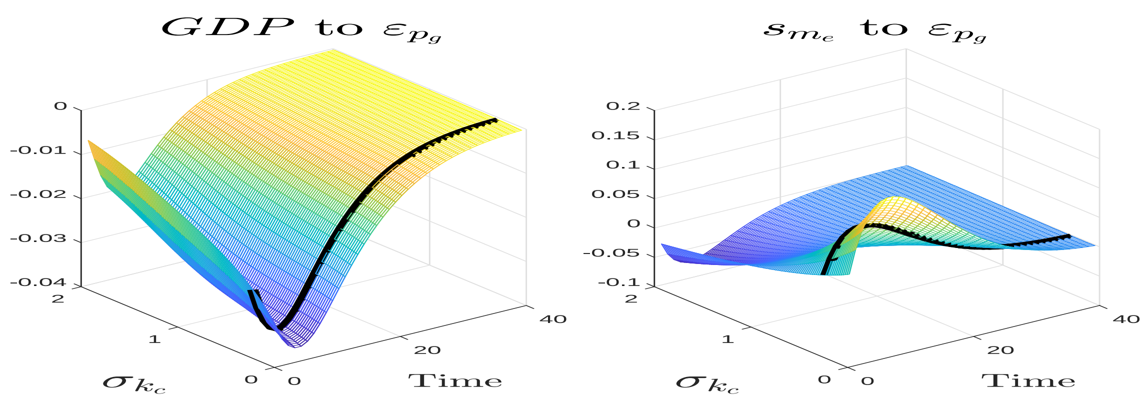

Figure E5. : ***Elasticity of Substitution in Production and the Transmission of Gas Price Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_g}) to the price of natural gas (p_g), computed for alternative values of the elasticity of substitution parameter σ_{k_c}. The black line corresponds to the response at the estimated value of σ_{k_c}. IRFs are expressed as percentage deviations from steady state.*

The role of wage indexation

The wage-indexation parameter ι_w shapes the transmission of energy price shocks. Our estimate, $\iota_w = 0.96$, implies substantial wage adjustment to price movements. Figure E13 in the upper left panel reports the response of GDP to a positive natural gas price shock, ε_{p_g}, for alternative values of ι_w. Higher wage indexation amplifies the contraction in GDP. Relative to a medium-indexation case, $\iota_w = 0.50$, the cumulative effect on output after 20 quarters is about three times larger at the estimated value. Under stronger indexation, higher prices feed more quickly into wages, amplifying production costs and further depressing output.[1] The remaining three panels in Figure E13 report the responses to coal price shocks (ε_{p_c}), oil price shocks (ε_{p_o}), and shocks to the common crude energy

[1]On the empirical side, Donoval et al. (2010) provide reduced-form evidence that oil price shocks have larger effects in European countries with stronger wage indexation. Amiti et al. (2024) provides a broader analysis of the pass-through from wage inflation to price inflation after the COVID-19 pandemic.

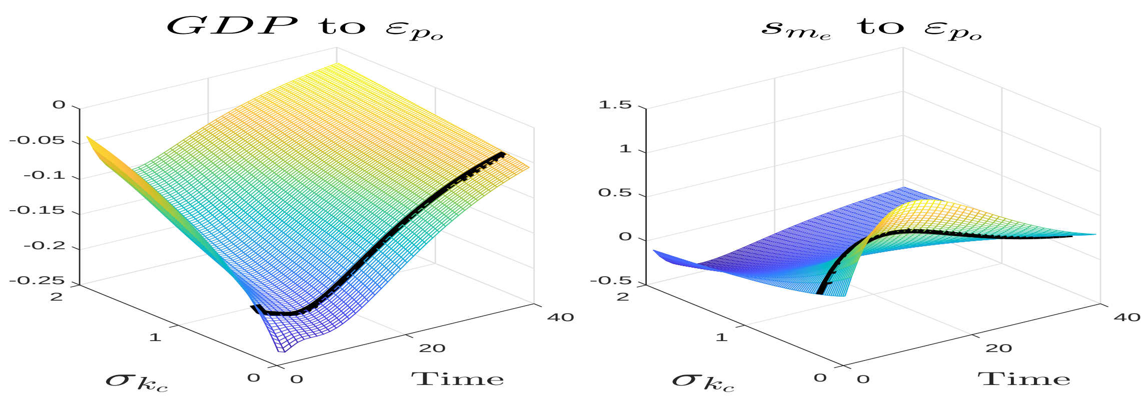

Figure E6. : ***The Role of the Elasticity of Substitution in Production and the Transmission of Oil Price Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_o}) to the price of oil (p_o), computed for alternative values of σ_{k_c}. The black line corresponds to the response at the estimated value of σ_{k_c}. Responses are expressed as percentage deviations from steady state for GDP and as deviations from steady state for s_{m_e}.*

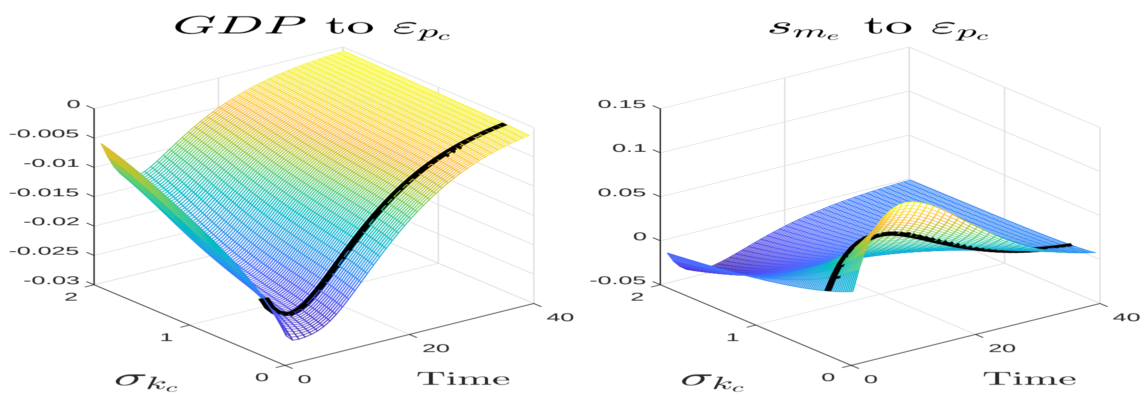

Figure E7. : ***The Role of the Elasticity of Substitution in Production and the Transmission of Coal Price Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_c}) to the price of coal (p_c), computed for alternative values of σ_{k_c}. The black line corresponds to the response at the estimated value of σ_{k_c}. IRFs are expressed as percentage deviations from steady state for GDP and as deviations from steady state for s_{m_e}.*

price factor (ε_{p_f}). The results are qualitatively similar: higher wage indexation amplifies the contractionary effects of energy price shocks on GDP.

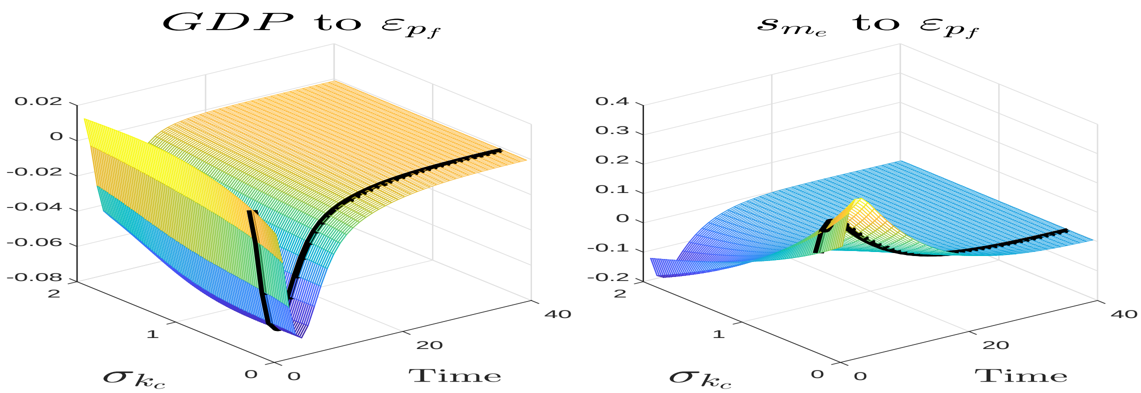

Figure E8. : ***The Role of the Elasticity of Substitution in Production and the Transmission of Common Crude-Energy Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_f}) to the common factor in crude-energy prices (p_f), computed for alternative values of σ_{k_c}. The black line corresponds to the response at the estimated value of σ_{k_c}. IRFs are expressed as percentage deviations from steady state for GDP and as deviations from steady state for s_{m_e}.*

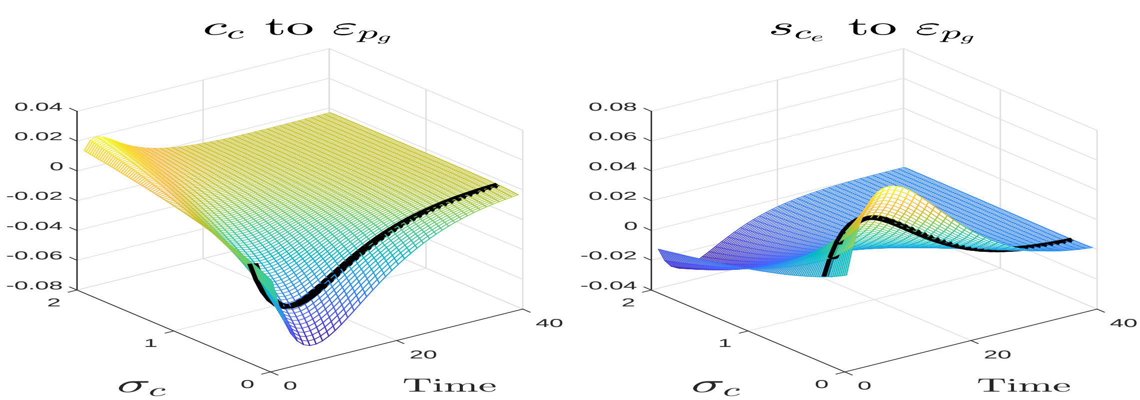

Figure E9. : ***The Role of the Elasticity of Substitution in Consumption in the Transmission of Gas Price Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_g}) to the price of natural gas (p_g), computed for alternative values of the elasticity of substitution parameter σ_c. The black line corresponds to the response at the estimated value of σ_c. The IRFs represent percentage deviations from the steady state.*

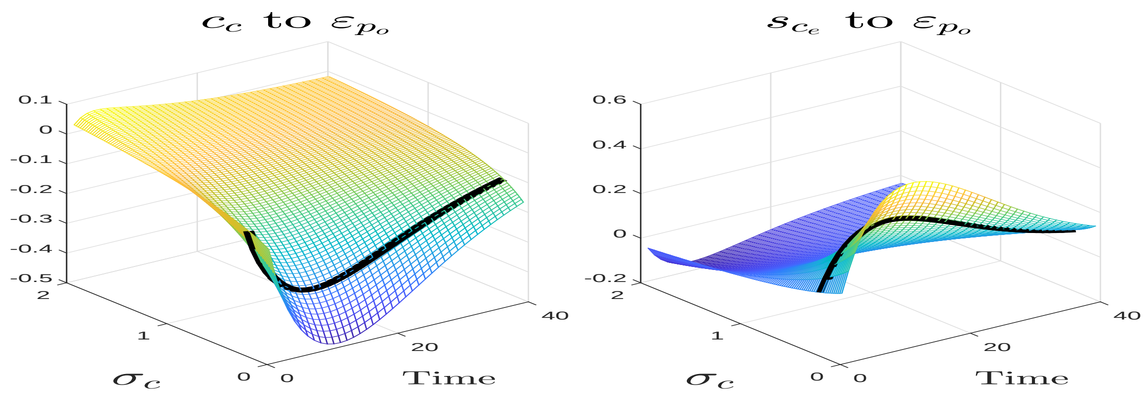

Figure E10. : ***The Role of the Elasticity of Substitution in Consumption in the Transmission of Oil Price Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_o}) to the price of oil (p_o), computed for alternative values of σ_c. The black line corresponds to the response at the estimated value of σ_c. The IRFs represent percentage deviations from the steady state for c_c and deviations from the steady state for s_{c_e}.*

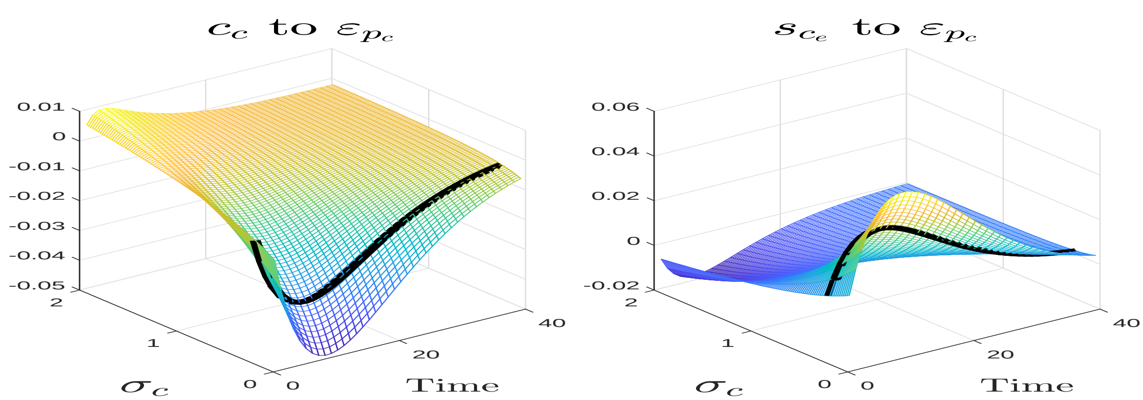

Figure E11. : ***The Role of the Elasticity of Substitution in Consumption in the Transmission of Coal Price Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_c}) to the price of coal (p_c), computed for alternative values of σ_c. The black line corresponds to the response at the estimated value of σ_c. The IRFs represent percentage deviations from the steady state for c_c and deviations from the steady state for s_{c_e}.*

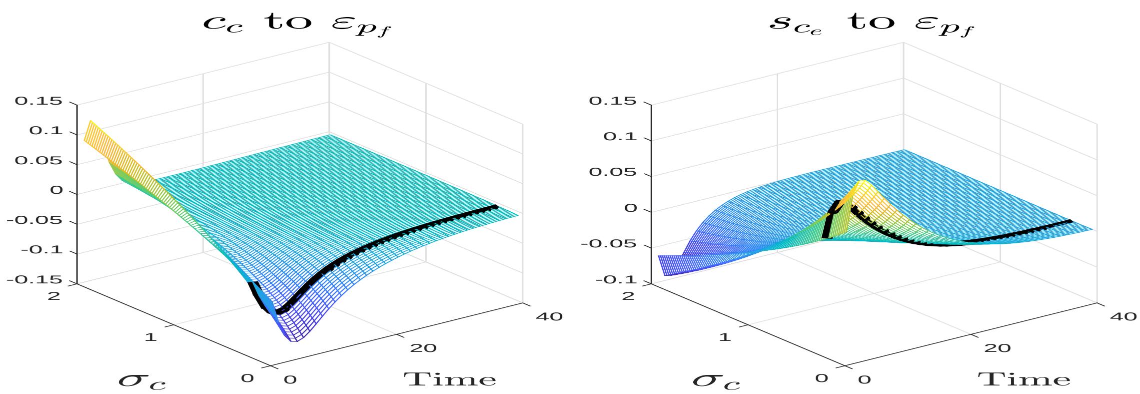

Figure E12. : ***The Role of the Elasticity of Substitution in Consumption in the Transmission of Common Crude-Energy Shocks.*** *The figure reports impulse response functions (IRFs) to a positive one-standard-deviation shock (ε_{p_f}) to the common crude-energy factor (p_f), computed for alternative values of σ_c. The black line corresponds to the response at the estimated value of σ_c. The IRFs represent percentage deviations from the steady state for c_c and deviations from the steady state for s_{c_e}.*

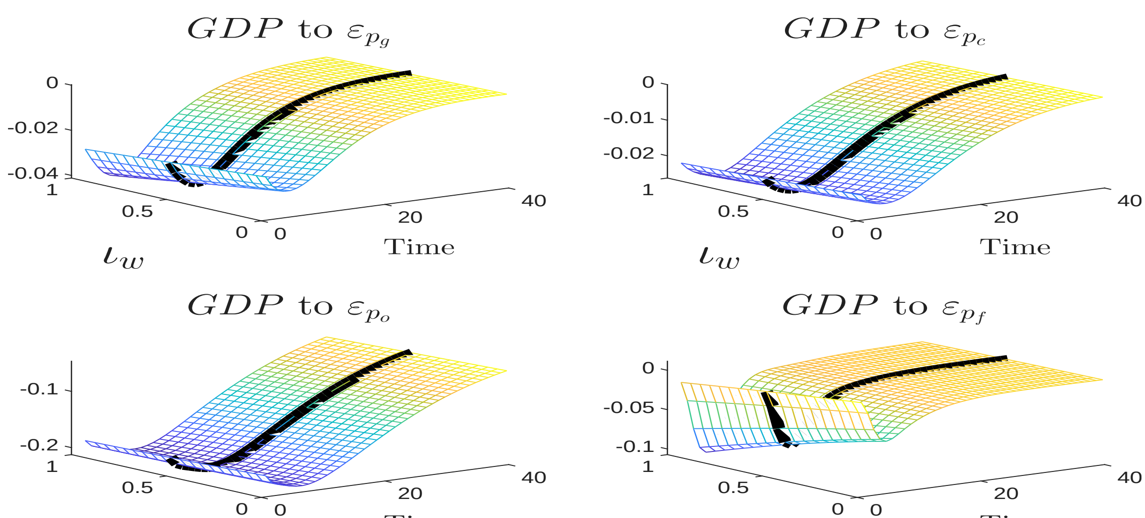

Figure E13. : ***The Role of Wage Indexation in the Transmission of Energy Shocks.*** *The figure reports impulse response functions (IRFs) of GDP to a positive one-standard-deviation natural gas price shock (ε_{p_g}), coal price shock (ε_{p_c}), oil price shock (ε_{p_o}), and shock to the common component of crude energy (ε_{p_f}), for alternative values of the wage-indexation parameter ι_w. The black line corresponds to the response at the estimated value of ι_w. The IRFs represent percentage deviations from the steady state.*

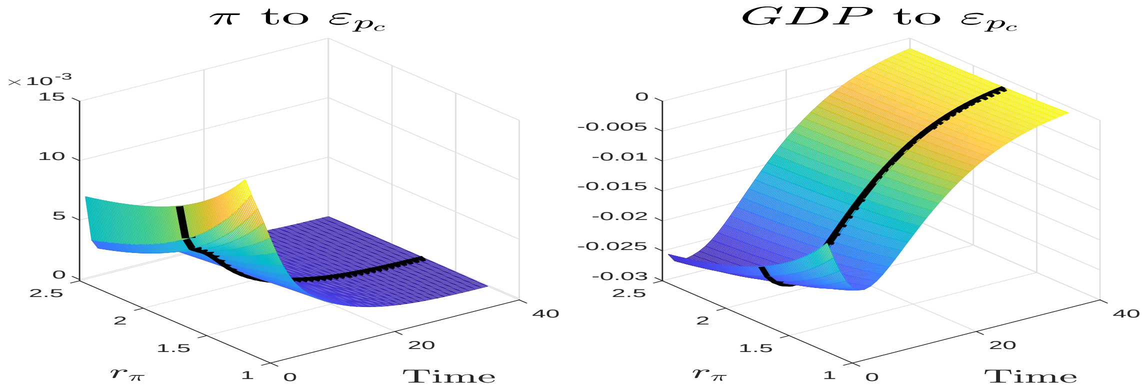

Figure E14. : ***The Role of Monetary Policy in the Transmission of Coal Price Shocks.*** *The figure reports the impulse response functions (IRFs) of inflation and GDP to a positive one-standard-deviation shock to the price of coal (ε_{p_c}), computed for alternative values of the central bank's inflation-response coefficient, r_π. The IRFs represent percentage deviations from the steady state.*

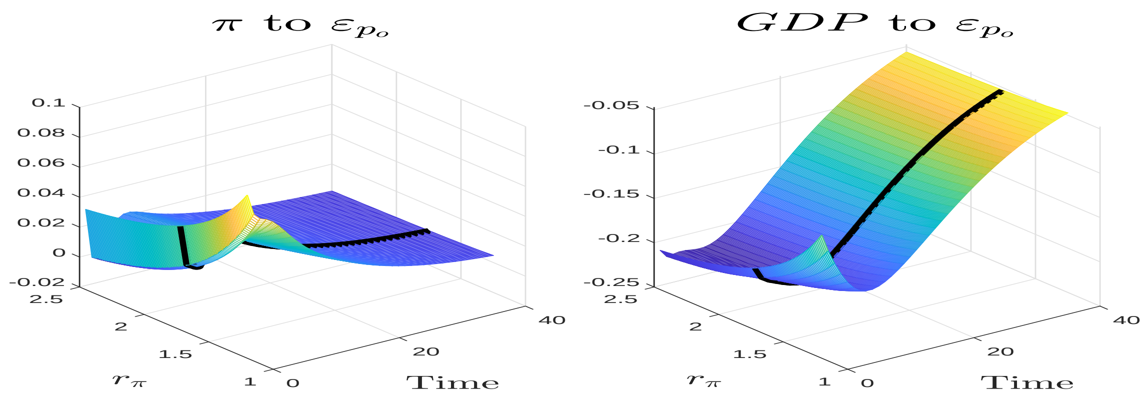

Figure E15. : ***The Role of Monetary Policy in the Transmission of Oil Price Shocks.*** *The figure reports the impulse response functions (IRFs) of inflation and GDP to a positive one-standard-deviation shock to the price of oil (ε_{p_o}), computed for alternative values of the central bank's inflation-response coefficient, r_π. The IRFs represent percentage deviations from the steady state.*

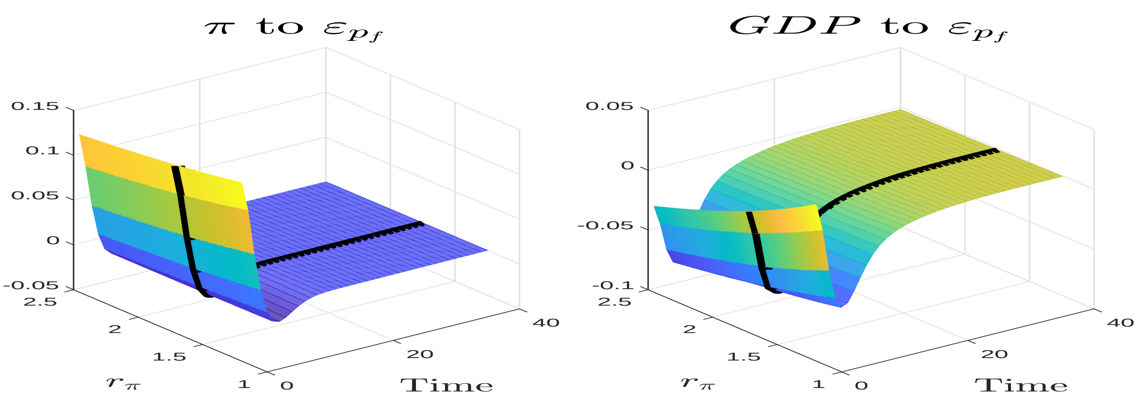

Figure E16. : ***The Role of Monetary Policy in the Transmission of Common Crude-Energy Shocks.*** *The figure reports the impulse response functions (IRFs) of inflation and GDP to a positive one-standard-deviation shock to the common factor in crude-energy prices (ε_{p_f}), computed for alternative values of the central bank's inflation-response coefficient, r_π. The IRFs represent percentage deviations from the steady state.*